

## Statistical properties of quasiparticle spectra in deformed nuclei

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The statistical properties of quasiparticle spectra in deformed heavy nuclei are studied. It has been found that at the ground state and at minima of the potential, Poisson-like spacing distributions are observed both in the quasiparticle spectra and in the single-particle ones. However, at the saddle point configuration in an even system, the original Wigner-like single-particle spacing distribution changes to a mixture of Wigner- and Poisson-like quasiparticle distributions, whereas for an odd system the pairing hardly changes the Wigner-like distribution. Therefore, from this study we have observed the tendency of the pairing effect to act to make the system more stable. [S0556-2813(99)05701-5]

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### I. INTRODUCTION

The statistical property of quantum spectra has been one of the topics of current interest in nuclear structure studies [1–4]. Many analyses have been performed on experimental nuclear level spectra. A typical one is an analysis of the ‘‘complete’’ spectroscopic information near the threshold of neutron and proton emission. It provides us with evidence that the nuclear system at low spin and high excitation energy is ‘‘chaotic’’ [1,3,5,6]. When one extends this kind of analysis to lower excitation energy, a pure Poisson-type distribution is obtained. It shows that at low energy the nuclear systems show an ordered motion [7–10]. Recently, the theoretical investigation of statistical feature of single-particle spectra has been extended to a highly deformed configuration [11–13]. Many new features of the single-particle motion have been revealed. For example, at the minima of the potential ordered motion occurs and at the saddle points chaotic motion of the single-particles occurs [12]. We should notice, however, that the experimental spectra contain the collective states (rotation and vibration states) in addition to the single-particle states. Thus, in order to make a comparison between the analysis of experimental spectra and theoretical calculations, we need to develop a model which contains the correlation among single-particle motion. As a first step to this end, we examine the effect of the pairing correlation by studying the statistical properties of the quasiparticle spectra.

The nearest neighbor level spacing distribution is commonly used to explore the fluctuation properties of energy levels. One knows that if the spacing distribution is close to the Wigner type, the system is mainly in the chaotic state, and if it obeys the Poisson-like distribution, ordered behavior of the system is expected. To calculate the quasiparticle levels, the BCS approach is employed [14]. We start with a Hamiltonian that contains a pure single-particle part plus pairing interaction

$$\hat{H} = \sum_k \epsilon_k \hat{a}_k^\dagger \hat{a}_k + \sum_{k,k' > 0} \langle k, -k | V | k', -k' \rangle \hat{a}_k^\dagger \hat{a}_{-k}^\dagger \hat{a}_{-k'} \hat{a}_{k'} . \quad (1)$$

In the simplest case one may assume a constant matrix element for the pairing interaction  $-G$ . Then the Hamiltonian is given by

$$\hat{H} = \sum_k \epsilon_k \hat{a}_k^\dagger \hat{a}_k - G \sum_{k,k' > 0} \hat{a}_k^\dagger \hat{a}_{-k}^\dagger \hat{a}_{-k'} \hat{a}_{k'} . \quad (2)$$

The value of the pairing strength  $G$  is taken to be (16.6–2.8) /A MeV for nucleons in heavy nuclei. This range of pairing strength covers the realistic values used for the calculation of heavy nuclear systems. An analytic solution is not available for the above Hamiltonian but there is a solution based on the BCS approximation

$$|\text{BCS}\rangle = \prod_{k>0}^{\infty} (u_k + v_k \hat{a}_k^\dagger \hat{a}_{-k}^\dagger) |0\rangle . \quad (3)$$

In this state each pair of single-particle levels  $(k, -k)$  is occupied with a probability  $|v_k|^2$  and is unoccupied with probability  $|u_k|^2$ . The parameters  $u_k$  and  $v_k$  are determined through the variational principle as

$$v_k^2 = \frac{1}{2} [1 - (\epsilon_k - \epsilon_f) / \sqrt{(\epsilon_k - \epsilon_f)^2 + \Delta^2}] , \quad (4)$$

$$u_k^2 = \frac{1}{2} [1 + (\epsilon_k - \epsilon_f) / \sqrt{(\epsilon_k - \epsilon_f)^2 + \Delta^2}] . \quad (5)$$

Here,  $\epsilon_f$  is the Fermi energy. The pairing gap  $\Delta$  can be obtained iteratively by the so-called gap equation

$$\Delta = G/2 \sum_{k>0} [\Delta / \sqrt{(\epsilon_k - \epsilon_f)^2 + \Delta^2}] , \quad (6)$$

where the value of the Fermi energy  $\epsilon_f$  is determined from the conservation of the nucleon number. Finally, the quasiparticle energy is given by

$$E_k = \sqrt{(\epsilon_k - \epsilon_f)^2 + \Delta^2} . \quad (7)$$

In this way, the pairing correlation has been simplified considerably: The ground state now contains the correlation between the nucleons via the fractional occupation number and the excited states can be approximated as consisting of noninteracting quasiparticles with their energies related to the single-particle energies.

## II. MODEL

The single-particle levels are generated by using the two-center shell model (TCSM) [15,16], which is very suitable

for a description of highly deformed nuclei, especially for a description of the fission and heavy-ion collisions. The single-particle Hamiltonian of the TCSM in cylinder coordinates  $z$ ,  $\rho$ , and  $\phi$  is as follows:

$$H = -\frac{\hbar^2 \nabla^2}{2m_0} + V(\rho, z) + V_{LS}(p, s) + V_{L^2}(l). \quad (8)$$

The momentum-independent part of the TCSM potential is axially symmetric with respect to the  $z$  axis and is taken to be

$$V(\rho, z) = \begin{cases} \frac{1}{2} m_0 \omega_{z_1}^2 z'^2 + \frac{1}{2} m_0 \omega_{\rho_1}^2 \rho^2, & z < z_1, \\ \frac{f_0}{2} m_0 \omega_{z_1}^2 z'^2 (1 + c_1 z' + d_1 z'^2) + \frac{1}{2} m_0 \omega_{\rho_1}^2 (1 + g_1 z'^2) \rho^2, & z_1 < z < 0, \\ \frac{f_0}{2} m_0 \omega_{z_2}^2 z'^2 (1 + c_2 z' + d_2 z'^2) + \frac{1}{2} m_0 \omega_{\rho_2}^2 (1 + g_2 z'^2) \rho^2, & 0 < z < z_2, \\ \frac{1}{2} m_0 \omega_{z_2}^2 z'^2 + \frac{1}{2} m_0 \omega_{\rho_2}^2 \rho^2, & z > z_2, \end{cases} \quad (9)$$

with the abbreviation

$$z' = \begin{cases} z - z_1, & z < 0, \\ z - z_2, & z > 0. \end{cases}$$

We denote the positions of the centers of the two fragments by  $z_1$  and  $z_2$ , with  $z_1 \leq 0 \leq z_2$ .

The momentum-dependent part of the potential consists of a spin-orbit coupling term

$$V_{LS}(p, s) = \begin{cases} \left\{ -\frac{\hbar k_1}{m_0 \omega_{0_1}}, & (\nabla V \times p) \cdot s \right\}, & z < 0, \\ \left\{ -\frac{\hbar k_2}{m_0 \omega_{0_2}}, & (\nabla V \times p) \cdot s \right\}, & z > 0, \end{cases} \quad (10)$$

and a  $l^2$  term,

$$V_{L^2}(l) = \begin{cases} \left\{ -\frac{1}{2} \left( \frac{\hbar k_1 \mu_1}{m_0^2 \omega_{0_1}^3}, & l^2 \right) + \frac{1}{2} \hbar k_1 \mu_1 \omega_{0_1} N_1 (N_1 + 3) \delta_{if}, & z < 0, \\ \left\{ -\frac{1}{2} \left( \frac{\hbar k_2 \mu_2}{m_0^2 \omega_{0_2}^3}, & l^2 \right) + \frac{1}{2} \hbar k_2 \mu_2 \omega_{0_2} N_2 (N_2 + 3) \delta_{if}, & z > 0. \end{cases} \quad (11)$$

In these formulas  $\{A, B\} = AB + BA$  denotes the anticommutator of two quantities and  $\delta_{if}$  is a pure diagonal operator.  $N_i = n_{z_i} + 2n_\rho + |m|$  ( $i=1$ , for  $z < 0$ ;  $i=2$ , for  $z > 0$ ), in which  $n_{z_i}$ ,  $n_\rho$ , and  $m$  are the good quantum numbers in a two-center-oscillator basis [15]. The parameters of  $k_i$  and  $\mu_i$  in Eqs. (10) and (11) are mass dependent, which are taken to be  $k_i = ek + 1.2 dk A_i^{1/3}/R_0$  fm,  $\mu_i = em + 1.2 dmy A_i^{1/3}/R_0$  fm ( $i=1$ , for  $z < 0$ ;  $i=2$ , for  $z > 0$ ) with  $R_0 = 1.2249 A^{1/3}$  fm ( $A$  is the nuclear mass number). For protons  $ek = 0.0768$ ,  $em = 0.0638$ ,  $dk = -0.003$ , and  $dmy = 0.003$ ; for neutrons  $ek = 0.0509$ ,  $em = 0.0919$ ,  $dk = 0.002$ , and  $dmy = -0.095$  [17]. In addition,  $\omega_{0_i} = 41 \text{ MeV}/A_i^{1/3}$  and the  $m_0$  appearing in Eqs. (8)–(11) is the nucleon mass.

All the parameters appearing in above formula are related to five shape parameters, by which the nuclear shape can be described very well. They are the separation of the two centers  $\Delta z = z_2 - z_1$ , the neck parameter  $\epsilon$  ( $\epsilon = 0$  corresponds to no-neck shape and  $\epsilon = 1$  to well-necked-in shape), the mass asymmetry  $X_i = (A_1 - A_2)/(A_1 + A_2)$  with  $A_1$  and  $A_2$  the mass numbers of the fragments ( $X_i$  ranges from 0 to 1), and finally the ellipsoidal deformations of the fragments,  $\beta_1$  and  $\beta_2$ .

Based on this model, we calculate the energy levels for the heavy nuclei of Cf, in which there is the partially filled shell and the pairing force is expected to be important.

According to BCS theory the quasiparticle has the same angular momentum as the original single particle. In our

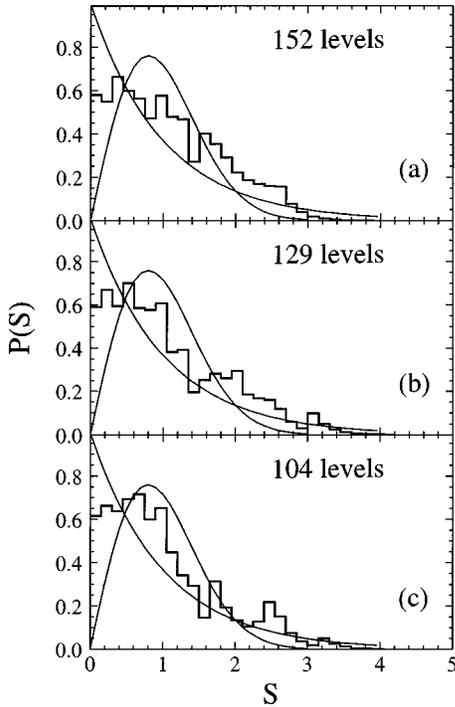


FIG. 1. The nearest-neighbor level spacing distributions of quasiparticle spectra for the element Cf as a function of unfolded energies, which are calculated at different neutron numbers involved and at the saddle point. The solid lines denote a Wigner distribution and a Poisson distribution. The histograms are our numerical results.

present case, the mean field is deformed but has rotational symmetry along the  $z$  axis. The conserved quantity is  $J_z$ , the projection of the total angular momentum onto this symmetry axis. We therefore analyze the statistical properties of levels in the subspace of fixed  $J_z$ .

Since the level density of quasiparticle spectra greatly changes with the energy range of levels, we introduce an unfolding procedure [18] in order to study its local statistical properties. This means that for a given stretch of levels, one has to divide them into several sets (form an ensemble). In each set, the local mean level spacing  $\bar{s}$  is defined as

$$\bar{s} = \frac{\Delta E}{\Delta N},$$

where  $\Delta E$  and  $\Delta N$  denote the energy interval and the level number in a set, respectively. The value of  $\Delta N$  has to be chosen carefully. If  $\Delta N$  is too small, the quantities will lose their statistical meaning. If  $\Delta N$  is too large, the local mean level spacing  $\bar{s}$  is meaningless. Through numerical tests,  $\Delta N$  is finally taken to be 30–40 in our calculations.

As is well known, the pairing effect mainly plays a role around the Fermi energy. Thus in order to obtain a correct local statistical property for quasiparticles we have to choose an adequate number of levels. It should not be too many or too few. In Fig. 1 we show the influence of the quasiparticle numbers on the nearest-neighbor spacing distributions. It is calculated at a large deformed heavy nucleus of  $^{252}_{98}\text{Cf}$ . The histograms from the upper, middle and lower portions of Fig. 1 represent the statistical results for 152-, 129-, and 104-

quasiparticle (neutron) levels, respectively. Two solid curves indicate Poisson and Wigner distributions. By comparing the behaviors shown in the three histograms, we can see that the change of level numbers in this range causes no big difference in the statistical results. Therefore, we will use about 120 levels in the neighborhood of the Fermi surface in the following calculations.

### III. RESULTS AND DISCUSSION

In order to explore the statistical properties of the quasiparticle spectra and compare them with that of the single-particle spectra we consider two extreme cases: one is at the ground state configuration in which the strong shell structure exists and the single-particle spectra show a typical Poisson distribution. It means that the single-particles undergo an ordered motion. The other one is at a large deformed configuration in which the single particles undergo a chaotic motion. For understanding how the pairing effect plays a role in the statistical property of quasiparticle spectra, we make a comparison between even and odd systems. The numerical calculations are carried out mainly for heavy nucleus of  $^{98}\text{Cf}$  with neutron numbers of 152, 154, 156, and 158 for the even case and 151, 153, 155, and 157 for the odd case. We notice that the statistical properties are rather common for even systems with 152, 154, 156, and 158 neutrons. Similarly, we have also observed common statistical properties for odd systems with neutron numbers of 151, 153, 155, and 157, while properties for even and odd systems are quite different. So in the following we show the results of the system with a 154 neutron number representing the even system and results of the system with a 153 neutron number representing the odd system without loss of generality.

In Fig. 2 we show the spacing distributions for the quasiparticle spectra at the ground state configuration. The histograms at the upper, middle, and lower portions are for quasiparticle spectra of an even system with 154 neutrons and an odd system with 153 neutrons as well as for the single-particle spectra of a system with 154 neutrons, respectively. For the sake of comparison the Poisson and Wigner distributions have also been shown in each subfigure. From Fig. 2 we can see that all numerical results show a perfect Poisson-like distribution. This implies that the pairing force cannot change the statistical properties of levels when the system is stabilized by the strong shell effect.

We now turn to a large deformed configuration of Cf, especially to an asymmetric saddle point configuration, which is of crucial importance for the study of the fission and hyperdeformation. At these deformations the pairing correction has been known to be of minor importance compared to the shell correction from the point of view of the contribution to the potential energy. However, from the point of view of the interplay between the order and chaos, pairing effects may play an important role. A remarkable property of the level statistics was found in Ref. [12], that at the saddle point the nearest-neighbor level-spacing single-particle distribution is more likely a Wigner type. It means that the single-particle motion at this configuration is unstable against chaos. This feature gives us insight into properties of fission statics. Now we will take the pairing effect into account in addition to the shell effect. In Fig. 3 we show the nearest-

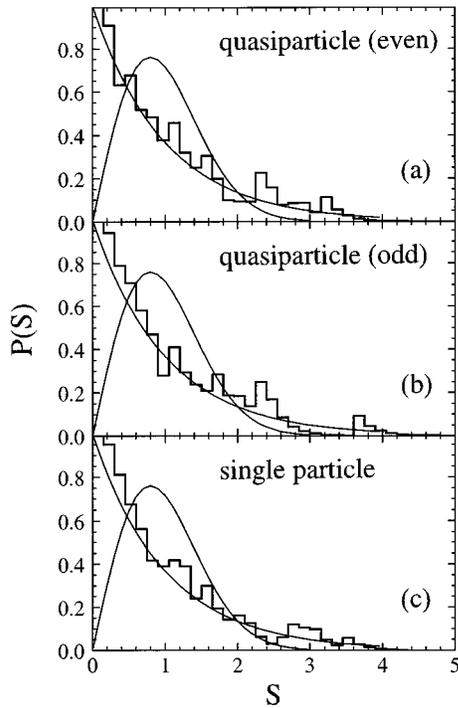


FIG. 2. The level spacing distributions of quasiparticle spectra at the ground state configuration of the element Cf. The histograms from the upper, middle, and lower portions are for the systems with even and odd neutron numbers as well as for the single-particle spectra, respectively. The solid lines denote a Wigner distribution and a Poisson distribution.

neighbor spacing distribution of quasiparticle spectra for systems with 154 (upper portion) and 153 (middle portion) neutrons. The corresponding single-particle spacing distribution (for the system with 154 neutrons) is also shown in the lower portion of Fig. 3. From the histogram in the upper portion we find that for the even system (fully paired off) the strong pairing effect makes the spacing distribution of quasiparticle spectra deviate from the original Wigner-like distribution of single-particle spectra and approach the Poisson-like distribution. It means that the pairing effect enhances the stability of the system against chaos, whereas for the odd system the pairing effect is relatively weak and therefore the corresponding level spacing distribution remains a Wigner type, as shown by the histogram of the middle portion of Fig. 3.

In order to understand the remarkable behaviors as shown above, we try to make a further analysis. We denote the single-particle levels and the corresponding Fermi energy by  $\epsilon_k$  and  $\epsilon'_f$ , the quasiparticle levels and the corresponding Fermi energy by  $E_k$  and  $\epsilon_f$ . Here we will show the statistical results for spectra of  $[(\epsilon_k - \epsilon_f)^2 + \Delta^2]^{1/2}$ ,  $|\epsilon_k - \epsilon_f|$ ,  $|\epsilon_k - \epsilon'_f|$ , and  $\epsilon_k$  in Figs. 4 and 5 for even (154 neutrons) and odd (153 neutrons) cases, respectively. Since the spacing distribution for the spectrum of  $|\epsilon_k - \epsilon'_f|$  [Figs. 4(c) and 5(c)] is just obtained by taking the absolute values of  $\epsilon_k - \epsilon'_f$  and then reordering the resultant levels according to the energy, the original correlation (repulsion) between the neighborhood single-particle levels is partly lost. Therefore, the spacing distributions for them (in both even and odd cases) show an evident deviation from the original Wigner type (comparing with Figs. 4(d) and 5(d)]. In Figs. 4(b) and 5(b) the

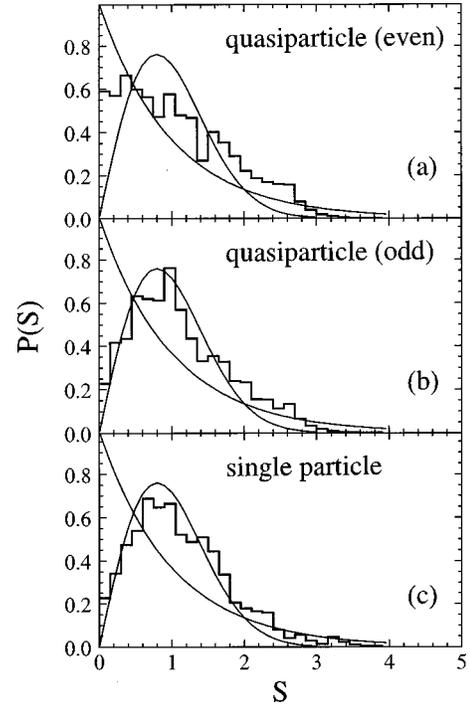


FIG. 3. The level spacing distribution of quasiparticle spectra as a function of unfolded energies for the saddle point of the system of Cf. The histograms from the upper, middle, and lower portions are for the systems with even (154) and odd (153) neutron numbers as well as for the single-particle spectra (with 154 neutrons), respectively. The solid lines denote a Wigner distribution and a Poisson distribution.

behaviors of the spectrum of  $|\epsilon_k - \epsilon_f|$  are demonstrated. Since the pairing interaction is partly taken into account through changing the Fermi energy from the single-particle one ( $\epsilon'_f$ ) to quasiparticle one ( $\epsilon_f$ ), the difference in spacing distributions between spectra of  $|\epsilon_k - \epsilon_f|$  and  $\epsilon_k$  should be observed. By comparing Figs. 4(b) with 4(d), we find that the pairing effect makes the spacing distribution of the spectrum  $|\epsilon_k - \epsilon_f|$  deviate from that of the single-particle spectrum due to being fully paired off in the even case. Whereas for the odd case [comparing Fig. 5(b) with 5(d)] only a small difference on the spacing distributions has been found. Concerning the effect of the energy gap of  $\Delta$  on the spacing distribution, we may compare Fig. 4(a) with 4(b) for the even case and Fig. 5(a) with 5(b) for the odd case. For the former it has been shown that the  $\Delta$  also plays a role for making the spacing distribution of quasiparticle spectra approach to Poisson type, but for the latter case there is only a very small effect on the spacing distribution. From this study we can see that both the energy gap and the change of Fermi energy caused by pairing interaction play a role in making the spacing distribution of quasiparticle spectra deviate from the original Wigner type. However, it seems that the change of the Fermi energy gives a stronger effect on this deviation.

For a further understanding of the odd-even effects in the statistical properties of quasiparticle spectra explicitly, we plot a part of the quasiparticle energy levels as a function of pairing strengths in Fig. 6 at the same deformed configuration of nuclei as in Fig. 3. The left portion of Fig. 6 is for an odd system (153 neutrons), the right portion for an even system (154 neutrons). The arrows on the right side indicate

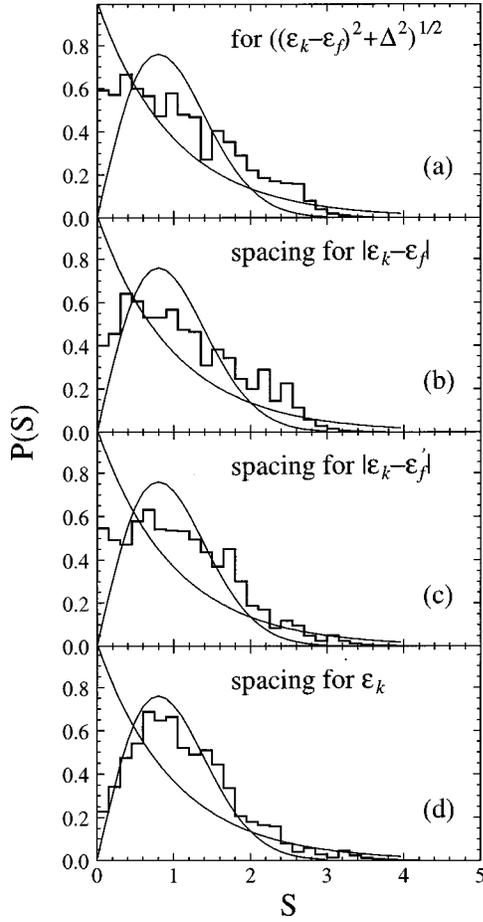


FIG. 4. The level spacing distribution of quasiparticle spectra as a function of unfolded energies for the saddle point of the system of Cf with 154 neutrons. The histograms in the subfigures from the upper to lower portions represent the spacing distributions for the spectra of  $[(\epsilon_k - \epsilon_j)^2 + \Delta^2]^{1/2}$ ,  $|\epsilon_k - \epsilon_j|$ ,  $|\epsilon_k - \epsilon_j|$ , and  $\epsilon_k$ , respectively. The solid lines denote a Wigner distribution and a Poisson distribution.

some levels of the even system in which there exists a quite strong degeneracy compared with the corresponding levels in the odd system. By comparing these two parts of Fig. 6, we see that for the even system there exists a strong pairing effect indeed which results in the stronger degeneracy of levels around the pairing strength of 0.055 MeV (realistic value often used for the calculations of heavy nuclear systems) than that for the odd case. This leads to a change of the spacing distribution from a Wigner type towards a Poisson type.

The numerical results in this section seem to indicate that the pairing effect plays an important role in the statistical properties of quasiparticle spectra and the general tendency of the pairing effect on the system is to make it be at the ordered motion. In other words, the pairing effect (including the pairing energy gap and the change of Fermi energy caused by the pairing interaction) has the tendency to stabilize the system against chaos. With respect to this point we may understand this as follows: Because of the transformation from a real single-particle to a quasiparticle in the BCS model, the complexity in the original single-particle Hamiltonian is taken into account and has been partially canceled. Our study shows that this cancellation occurs rather gener-

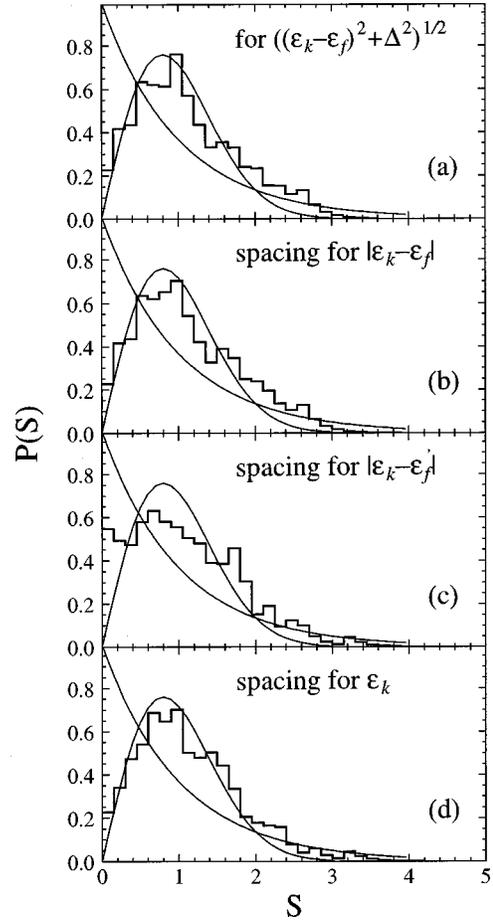


FIG. 5. The level spacing distribution of quasiparticle spectra as a function of unfolded energies for the saddle point of the system of Cf with 153 neutrons. The others are the same as in Fig. 4.

ally for the even case when we choose realistic values of the pairing strength. Thus the correlation between quasiparticles will on average be much smaller than that between the single particles. This leads the quasiparticle spacing distribution to

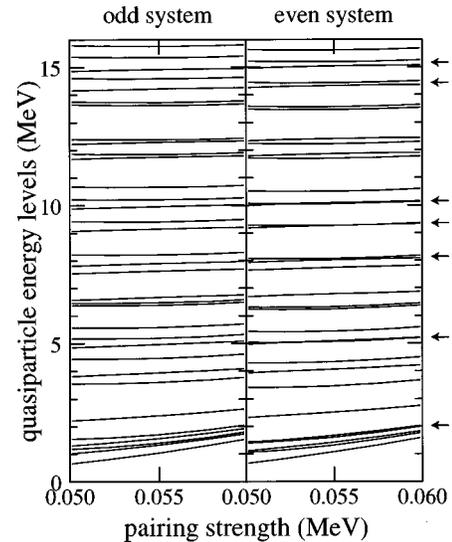


FIG. 6. A part of the quasiparticle energy levels as a function of pairing strengths, which are calculated at the saddle point of Cf. The right part is for the even system with 154 neutrons and left part for the odd system with 153 neutrons.

deviate from a Wigner-like distribution and approach a Poisson-like distribution for the even case.

#### IV. SUMMARY

In summary, we have studied the statistical properties of quasiparticle spectra for heavy nuclei at two typical deformations: ground state deformation (minima) and saddle point (maxima in potential). At the ground state deformation, the spacing distribution of quasiparticle spectra indicates that the pairing effect does not change the statistical characteristics of single particles, that is, quasiparticles and single particles are both in the ordered motion. However, at the saddle point, the pairing effect (including the pairing energy gap and the change of Fermi energy caused by the pairing interaction) in the even system changes the statistical feature from Wigner like characterized by the single-particles spectra to approaching Poisson like and therefore it has the tendency to stabilize the system against chaos, whereas for the odd system the pairing effect is not strong enough to change the statistical

characteristic of the system and the motion of quasiparticles is still unstable against chaos at the saddles. We notice that the above analysis is based on the BCS model, the shortcoming of which is that for levels in the vicinity of the Fermi surface the average particle number is not correct. Since we are mainly interested in the statistical properties of quasiparticle spectra which include more than 100 levels, we expect that the main conclusions given above are believable at least qualitatively.

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