Hyperfine anomaly of Be isotopes and anomalous large anomaly in ^{11}Be

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A new result of investigations of the hyperfine structure (hfs) anomaly in Be isotopes is presented. The hfs constant for ^{11}Be is obtained by using the core plus neutron type wave function $|2s_{1/2}\rangle + |1d_{5/2}\times 2^+;\frac{1}{2}^+\rangle$. A large hfs anomaly of ^{11}Be is found, which is mainly due to the large radius of the halo single-particle state. [S0556-2813(99)04701-9]

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I. INTRODUCTION

Recently, much interest has been paid to the magnetic hyperfine structure (hfs) for various nuclear isotopes [1–6]. On the experimental side, there has been some progress in observing the transition of the hyperfine levels for atomic ground states. In fact, the accuracy of the measurement of the hfs splitting has improved a great deal, and even for lighter nuclei, there is a good chance of observing the hfs anomaly. This becomes possible due to the ion trap method, which can isolate the atoms. This ion trap method [7,8] can measure the hfs anomaly with an accuracy of the order of 10^{-6}.

The hyperfine structure has a sensitivity to the magnetization distribution in a nucleus. It can, therefore, present a unique way to measure the neutron distribution in nucleus. In particular, there is a strong evidence that ^{11}Li has a neutron halo [9] which may extend quite far over the typical nuclear radius of neighboring light nuclei. Moreover, ^{11}Be has an anomalous spin-parity state for the ground state [10], and there may be some chance that it also has a large neutron radius.

In this paper, we present a model calculation of the hfs anomaly for Be isotopes. For ^{7}Be and ^{9}Be nuclei, we can use the Cohen-Kurath wave functions [11]. On the other hand, ^{11}Be has an anomalous spin-parity state, which is $\frac{1}{2}^+$ instead of $\frac{1}{2}^-$. A simpleminded shell model wave function does not give a proper structure of the ground state.

Here we calculate the matrix element of the hfs operator for the ground state of ^{11}Be using the core plus neutron type wave function $\alpha|2s_{1/2}\rangle + \beta|1d_{5/2}\times 2^+:\frac{1}{2}^+\rangle$. It turns out that the hfs anomaly ϵ for ^{11}Be is quite large in magnitude compared to those of ^{7}Be and ^{9}Be. This is mainly related to the fact that the ground state of ^{11}Be has a large radius compared to those of ground states of other Be isotopes.

The paper is organized as follows. In the next section, we briefly explain the theory of the magnetic hyperfine structure in electronic atoms. Section III treats the core plus neutron type wave function. We evaluate the matrix elements of the hfs operator using this wave function. In Sec. IV, numerical results of the isotope shifts of the hfs anomaly for Be isotopes are presented. A summary is given in Sec. V.

II. MAGNETIC HYPERFINE STRUCTURE

An atomic electron which is bound by a nucleus feels the magnetic interaction in addition to the static Coulomb force. The magnetic interaction between the electron and the nucleus can be described as

$$H' = -\int \mathbf{j}_N(\mathbf{r})\cdot\mathbf{A}(\mathbf{r})\,d^3r, \tag{2.1}$$

where the nuclear current $\mathbf{j}_N(\mathbf{r})$ can be written as

$$\mathbf{j}_N(\mathbf{r}) = \frac{e\hbar}{2Mc}\sum_i g_s^{(i)}\boldsymbol{\nabla}\times s_i\,\delta(\mathbf{r}-\mathbf{R}_i) + \sum_i \frac{eg_l^{(i)}}{2M}[\mathbf{P}_i\delta(\mathbf{r}-\mathbf{R}_i)+\delta(\mathbf{r}-\mathbf{R}_i)\mathbf{P}_i]. \tag{2.2}$$

$\mathbf{A}(\mathbf{r})$ denotes the vector potential which is created by the atomic electron, and it can be written as

$$\mathbf{A}(\mathbf{r}) = \int \frac{\mathbf{j}_e(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,d^3r', \tag{2.3}$$

where $\mathbf{j}_e(\mathbf{r})$ denotes the current density of an electron and can be written as

$$\mathbf{j}_e(\mathbf{r}) = -e\boldsymbol{\alpha}\delta(\mathbf{r}-\mathbf{r}_e), \tag{2.4}$$

where $\boldsymbol{\alpha}$ denotes the Dirac matrix.

A. hfs anomaly

The magnetic hyperfine splitting energy W can be written as

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$$W = \langle IJ{:}FF|H'|IJ{:}FF\rangle = \frac{1}{2}[F(F+1)-I(I+1)-J(J+1)]a_I, \tag{2.5}$$

where I, J, and F denote the spin of the nucleus, the spin of the atomic electron, and the total spin of the atomic system, respectively. a_I is called the hyperfine structure constant. Following Ref. [12], we can write the expression for the a_I as

$$a_I = a_I^{(0)}(1+\epsilon), \tag{2.6}$$

where $a_I^{(0)}$ is the hfs constant for the point charge and can be written as

$$a_I^{(0)} = -\frac{2ek\mu_N}{IJ(J+1)}\,\mu\int_0^\infty F^{(kJ)}G^{(kJ)}dr, \tag{2.7}$$

where μ is the magnetic moment of the nucleus in units of the nuclear magneton μ_N, and $F^{(kJ)}$ and $G^{(kJ)}$ are the large and small components of the relativistic electron wave function for the kJ state. ϵ is called the hfs anomaly and can be written as

$$\epsilon = -\frac{0.62b^{(kJ)}}{\mu}\left\langle II\left|\sum_{i=1}^{A}\left(\frac{R_i}{R_0}\right)^2\mu_i\right|II\right\rangle - \frac{0.38b^{(kJ)}}{\mu}\left\langle II\left|\sum_{i=1}^{A}\left(\frac{R_i}{R_0}\right)^2 g_s^{(i)}\Sigma_i^{(1)}\right|II\right\rangle, \tag{2.8}$$

where μ_i is the single-particle operator of the magnetic moment, that is, $\mu_i = g_s^{(i)}s_i + g_l^{(i)}l_i$, and $\Sigma_i^{(1)}$ is defined as

$$\Sigma_i^{(1)} = s_i + \sqrt{2\pi}[sY^{(2)}]_i^{(1)}. \tag{2.9}$$

R_0 is a nuclear radius and can be given as $R_0 = r_0A^{1/3}$ with $r_0 = 1.2$ fm. On the other hand, $b^{(kJ)}$ is a constant, which is calculated in terms of relativistic electron wave functions [13] and is given as

$$b^{(kJ)} = 0.23k_0^2R_0\gamma(1-0.2\gamma^2) \Big/ \int_0^\infty F^{(kJ)}G^{(kJ)}dr. \tag{2.10}$$

m_e denotes the electron mass, k_0^2 is a normalization constant, and $\gamma = Z\alpha$.

The isotope shift of the hfs anomalies of the two isotopes Δ_{12} is defined as

$$\Delta_{12} = \frac{a_{I_1}g_2}{a_{I_2}g_1} - 1, \tag{2.11}$$

where g_1 and g_2 are the total nuclear g factors for isotopes 1 and 2, respectively. Since the hfs anomaly ϵ is quite small, Δ_{12} becomes

$$\Delta_{12} \approx \epsilon_1 - \epsilon_2. \tag{2.12}$$

The hfs anomaly ϵ can be calculated once we know the nuclear wave function. Here we employ simple-minded shell model wave functions with core polarization taken into account. We take the following approach which considers only the $\Delta l = 0$ core polarization for the $\Sigma_i^{(1)}$ operator. In this case, we can obtain the matrix element of the $\Sigma_i^{(1)}$ without introducing any free parameters as discussed in Ref. [12].

B. $\Delta l = 0$ core polarization

In this case, we can express the effect of the core polarization on the $\Sigma_i^{(1)}$ operator in terms of the core polarization of the magnetic moment. Following Ref. [12], we can write the expectation value of the $\Sigma_i^{(1)}$ as

$$\left\langle II\left|\sum_{i=1}^{A} g_s^{(i)}\Sigma_i^{(1)}\right|II\right\rangle = \pm g_s^{(VN)}\frac{3(I+\frac{1}{2})}{4(I+1)} + \frac{3g_s^{(VN)}}{4(g_s-g_l)^{(VN)}}(\mu-\mu_{\mathrm{sp}}-\delta\mu^{\mathrm{mes}}), \tag{2.13}$$

for $I = l \pm \frac{1}{2}$. Here $g_s^{(VN)}$ denotes the g factor of the valence nucleon for the single-particle state we are considering. μ_{sp} is the single-particle value of the magnetic moment. $\delta\mu^{\mathrm{mes}}$ comes from the meson exchange current and can be given approximately as

$$\delta\mu^{\mathrm{mes}} \approx 0.1\tau_e l.$$

Therefore, we do not have any free parameters in the evaluation of the expectation value of the $\Sigma_i^{(1)}$. As the Be isotopes have orbits with small l, the exchange current effects are not important and we can safely neglect the term $\delta\mu^{\mathrm{mes}}$.

III. CORE PLUS NEUTRON MODEL

Recently, Suzuki *et al.* [14] investigated the magnetic moment of the ground state of ^{11}Be. They describe the ground state of ^{11}Be as

$$\left|{}^{11}\mathrm{Be}\left(\frac{1^+}{2}\right)\right\rangle = \alpha\left|{}^{10}\mathrm{Be}(0^+)\times\nu 2s_{1/2}:\frac{1^+}{2}\right\rangle + \beta\left|{}^{10}\mathrm{Be}(2^+)\times\nu 2d_{5/2}:\frac{1^+}{2}\right\rangle. \tag{3.1}$$

In this case, the magnetic moment of the ^{11}Be ground state can be expressed as

$$\mu = \alpha^2\mu_{\nu(2s_{1/2})} + \tfrac{7}{15}\,\beta^2\mu_{\nu(1d_{5/2})} - \tfrac{1}{3}\,\beta^2\mu_{(2^+)}, \tag{3.2}$$

where $\mu_{\nu(2s_{1/2})}$, $\mu_{\nu(1d_{5/2})}$, and $\mu_{(2^+)}$ denote the magnetic moment of the neutron $2s_{1/2}$, $1d_{5/2}$ states and the 2^+ (3.37 MeV) state of ^{10}Be, respectively. Here we evaluate the hfs anomaly ϵ for the two-component wave function, Eq. (3.1). The hfs anomaly of Eq. (2.8) can be written as

$$\epsilon=-\frac{0.62b^{(1s)}}{\mu}\left[\alpha^2\mu_{\nu(2s_{1/2})}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{2s}+\frac{7}{15}\beta^2\mu_{\nu(1d_{5/2})}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{1d}-\frac{1}{3}\beta^2\mu_{(2^+)}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{1p}\right]$$
$$-\frac{0.38b^{(1s)}}{\mu}\left[\alpha^2\Sigma_{\nu(2s_{1/2})}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{2s}+\frac{7}{15}\beta^2\Sigma_{\nu(1d_{5/2})}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{1d}-\frac{1}{3}\beta^2\Sigma_{(2^+)}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{1p}\right], \tag{3.3}$$

where

$$\Sigma_j=\left\langle jj\left|\sum_i g_s^{(i)}\Sigma_i^{(1)}\right|jj\right\rangle \tag{3.4}$$

and i runs over the nucleons in the state that has the total angular momentum j. Here $\mu_{\nu(2s_{1/2})}=\mu_{\nu(1d_{5/2})}=\frac{1}{2}g_s^{(n)}$, where $g_s^{(n)}=-3.826$ is the spin g factor for the neutron, and $\mu_{(2^+)}=1.787$ [14]. The values of $\Sigma_{\nu(2s_{1/2})}$, $\Sigma_{\nu(1d_{5/2})}$, and $\Sigma_{(2^+)}$ are obtained as $\Sigma_{\nu(2s_{1/2})}=\frac{1}{2}g_s^{(n)}$, $\Sigma_{\nu(1d_{5/2})}=\frac{9}{14}g_s^{(n)}$, and $\Sigma_{(2^+)}=-1.034$, respectively. The $\mu_{(2^+)}$ and the $\Sigma_{(2^+)}$ are obtained by using the Cohen-Kurath wave function, TBE (8)–(16) [11,15]. Now, the recent measurement of the magnetic moment of ^{11}Be gives [16]

$$\mu(^{11}\mathrm{Be})=-1.682(3)\mu_N. \tag{3.5}$$

This indicates that the value of α^2 is close to

$$\alpha^2=0.5 \tag{3.6}$$

from the comparison of the observation with Fig. 2a of Ref. [14]. In this way, we obtain the hfs anomaly ϵ for ^{11}Be. For the ^{7}Be and ^{9}Be isotopes, we can calculate the hfs anomaly using the Cohen-Kurath wave function [11]. The hfs anomaly ϵ for this case is written similarly as

$$\epsilon=-\frac{0.62b^{(1s)}}{\mu}\mu_{[(3/2)^-]}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{1p}-\frac{0.38b^{(1s)}}{\mu}\Sigma_{[(3/2)^-]}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{1p}. \tag{3.7}$$

We also consider the case in which the effective g factor for the spin operator, g_s^{eff}, is taken into account. As for the halo orbit, we assume that the second-order effects are rather small and result in little quenching of the spin operator. We therefore take $g_s^{\mathrm{eff}}=1.0$ for the $\nu 2s_{(1/2)^-}$ orbit. In this case, $\mu_{\nu(2s_{1/2})}=\frac{1}{2}g_s^{(n)}=-1.913$, $\mu_{\nu(1d_{5/2})}=\frac{1}{2}g_s^{(n)}g_s^{\mathrm{eff}}=-1.913g_s^{\mathrm{eff}}$, $\mu_{(2^+)}=1.076+0.711g_s^{\mathrm{eff}}$, and $\Sigma_{\nu(2s_{1/2})}=\frac{1}{2}g_s^{(n)}=-1.913$, $\Sigma_{\nu(1d_{5/2})}=\frac{9}{14}g_s^{(n)}g_s^{\mathrm{eff}}=-2.460g_s^{\mathrm{eff}}$, $\Sigma_{(2^+)}=-1.034g_s^{\mathrm{eff}}$, in units of μ_N. The magnetic moment is, then, given by

$$\mu=-1.913\alpha^2-(0.3585+1.1298g_s^{\mathrm{eff}})\beta^2. \tag{3.8}$$

In comparison with the experimental value of the magnetic moment, $-1.682(3)\mu_N$, we obtain $\alpha^2=61\%$. When the contributions from the meson exchange currents are taken into account, α^2 is close to 65%. We give results of the hfs anomaly ϵ for ^{11}Be also for the case with an effective spin g factor. Numerical evaluations are carried out in Sec. IV.

IV. NUMERICAL RESULTS

Now we evaluate numerically the hfs anomaly ϵ for the Be isotopes.

A. Core plus neutron type wave function

In order to evaluate Eq. (3.3), we need to know the magnetic moment μ for the Be isotopes. The magnetic moments of ^{9}Be and ^{11}Be are observed, but the magnetic moment of ^{7}Be is not yet determined experimentally. We evaluate the magnetic moment of ^{7}Be empirically.

For ^{7}Be, we can make use of the magnetic moment of ^{7}Li since they are isospin doublet states. In this case, we predict the magnetic moment for ^{7}Be:

$$\mu^{\mathrm{pred}}(^7\mathrm{Be})=-1.377. \tag{4.1}$$

In Table I, we list the values of the quantities which are necessary to calculate the hfs anomaly ϵ. The rms radii are those for wave functions solved in a Woods-Saxon potential with $R_0=1.2A^{1/3}$ and $a=0.60$ fm. For the $2s_{1/2}$ and $1d_{5/2}$ neutron orbits in ^{11}Be, the wave functions are obtained to reproduce the separation energies, 0.50 and 3.87 MeV, respectively. The anomaly ϵ for ^{11}Be is obtained from Eq. (3.3) to be

$$\epsilon(^{11}\mathrm{Be})=-0.120\,15\alpha^2-0.023\,26\beta^2\ (\%). \tag{4.2}$$

It gives $\epsilon=-0.0717\%$ for $\alpha^2=0.50$ and $\beta^2=0.50$. The magnitude of ϵ gets as large as -0.091% (-0.101%) as α^2

TABLE I. R_0, $b^{(1s)}$, rms radius, and magnetic moment. The values of R_0, $b^{(1s)}$, rms radii for the $1p$ and $2s$ states, and the magnetic moments for Be isotopes are shown. An asterisk indicates that the value of the magnetic moment is empirically extracted from those of other nuclear isotopes.

	^{7}Be	^{9}Be	^{11}Be
R_0 [fm]	2.296	2.496	2.669
$b^{(1s)}$ [%]	0.0170	0.0185	0.0198
$\langle r^2\rangle_{(1p)}^{1/2}$ [fm]	2.553	2.569	2.588
$\langle r^2\rangle_{(2s)}^{1/2}$ [fm]			6.165
$\langle r^2\rangle_{(1d)}^{1/2}$ [fm]			3.551
μ_{exp} [nm]	−1.377*	−1.177	−1.682
μ_{sp} [nm]	−1.913	−1.913	−1.913
$\delta\mu_{CP}$ [nm]	0.536*	0.736	0.231

TABLE II. (a) hfs anomaly ϵ and isotope shift Δ_{12}. The calculated values of the hfs anomaly ϵ and the isotope shift Δ_{12} for the Be isotopes obtained from Eqs. (3.3) and (3.7) are shown. $\alpha^2=\beta^2=0.50$ are used for ^{11}Be. (b) hfs anomaly ϵ and isotope shift Δ_{12}. The calculated values of the hfs anomaly ϵ and the isotope shift Δ_{12} for the Be isotopes obtained from Eq. (4.5) are shown.

	^{7}Be	^{9}Be	^{11}Be
		(a)	
ϵ [%]	-0.0245	-0.0249	-0.0717
$\Delta_{7,A}$ [%]	0	0.0004	0.0472
		(b)	
ϵ [%]	-0.024	-0.023	-0.118
$\Delta_{7,A}$ [%]	0	-0.001	0.094

becomes 0.70 (0.80). The hfs anomalies for ^{7}Be and ^{9}Be are obtained from Eq. (3.7) by using the Cohen-Kurath wave functions [11,15].

In Table II(a), we present the calculated values of hfs anomaly ϵ for the Be isotopes. Now it turns out that the hfs anomaly for ^{11}Be has a very large value compared to other isotopes. This is mainly connected with the fact that the ^{11}Be has a large neutron radius. The rms radius of the halo $2s_{1/2}$ orbit becomes as large as 6.4 fm in deformed Woods-Saxon models [17,18]. When we use a value of 6.37 fm for the rms radius of the halo $2s_{1/2}$ orbit, which is obtained in the deformed Woods-Saxon model [18,14], the hfs anomaly is given by

$$\epsilon=-0.128\,27\alpha^2-0.023\,26\beta^2\ (\%). \tag{4.3}$$

Equation (4.3) leads to $\epsilon=-0.0758\%$ for $\alpha^2=0.5$, which is close to the value obtained from Eq. (4.2). The anomaly ϵ becomes -0.076 to -0.097% for $\alpha^2=0.5$–0.7. We finally give numerical results for the case with the effective spin g factor. We take $g_s^{\mathrm{eff}}=1.0$ for the halo $\nu 2s_{1/2}$ orbit and $g_s^{\mathrm{eff}}=0.85$ [19] for other normal orbits. The hfs anomaly is given by

$$\epsilon=-0.120\,15\alpha^2-0.020\,14\beta^2\ (\%). \tag{4.4}$$

Here the $2s_{1/2}$ orbit obtained in the spherical Woods-Saxon potential is used. The anomaly becomes $\epsilon=-0.0852\%$ for $\alpha^2=0.65$, whose magnitude is larger than the value obtained from Eq. (4.2) about by 0.01%. The anomaly ϵ becomes -0.070 to -0.100% for $\alpha^2=0.5$–0.7.

B. Single-particle model with core polarization

Next, we calculate the hfs anomaly ϵ for Be isotopes using the single-particle model with core polarization. Since the radius of the single-particle state can be different from that of the core polarization state, we modify the expression (2.8) in the same way as Eq. (3.3):

$$\begin{aligned}\epsilon=&-\frac{0.62b^{(1s)}}{\mu}\left[\mu_{\mathrm{sp}}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{\mathrm{sp}}+\delta\mu_{CP}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{CP}\right]\\&-\frac{0.38b^{(1s)}}{\mu}\left[g_s^{(VN)}\Sigma_{\mathrm{sp}}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{\mathrm{sp}}\right.\\&\left.+\frac{3}{4}\,\delta\mu_{CP}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{CP}\right],\end{aligned} \tag{4.5}$$

where Σ_{sp} denotes the expectation value of $\langle II|\Sigma_{i=1}^{A}\Sigma_i^{(1)}|II\rangle$ with the single-particle state and can be written as

$$\Sigma_{\mathrm{sp}}=\frac{3(I+\frac{1}{2})}{4(I+1)}\quad\text{for}\ \ I=l+\frac{1}{2}. \tag{4.6}$$

$\delta\mu_{CP}$ is the magnetic moment which arizes from the core polarization [20]. Here $\langle(R/R_0)^2\rangle_{CP}$ denotes the expectation values with the states involved in the core polarization. In Be isotopes, they are $1p$ states. In Table II(b), we list the calculated values of the hfs anomaly ϵ for the Be isotopes. In the same way as the core plus neutron type calculation, the hfs anomaly for ^{11}Be has a very large value compared to other isotopes. This is due to the fact that the ^{11}Be has a large neutron radius since it has an anomalous spin-parity state.

V. CONCLUSIONS

We have presented the numerical calculations of the magnetic hfs anomaly for the Be isotopes. First, we employ the wave function which has a component coupled to the 2^+ core excitation. This gives a large hfs anomaly for ^{11}Be since the neutron outside the shell is assumed to be $2s_{1/2}$ or $1d_{5/2}$ orbits. On the other hand, we find that the ^{7}Be and ^{9}Be isotopes have a small hfs anomaly. We also evaluate the hfs anomaly using a single-particle shell model with core polarization. We also predict a very large hfs anomaly for ^{11}Be.

It would be extremely interesting to learn whether the very large hfs anomaly of ^{11}Be can be realized in nature or not. Since this is related to the radius of the neutron halo nucleus, it may well help understand the structure of the neutron-rich nuclei. We hope that experimental observations will clarify this point in near future.

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