$4\hbar\omega$ isoscalar monopole giant resonance in ^{208}Pb and resonance trapping

S. E. Muraviev,[1] I. Rotter,[2,3] S. Shlomo,[4] and M. H. Urin[1]
[1]*Moscow Engineering Physics Institute, 115409 Moscow, Russia*
[2]*Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Strasse 38, D-01187 Dresden, Germany*
[3]*Technische Universität Dresden, Institut für Theoretische Physik, D-01062 Dresden, Germany*
[4]*Cyclotron Institute, Texas A&M University, College Station, Texas 77840*
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In the framework of the random phase approximation in the continuum we calculate the strength function of the $4\hbar\omega$ isoscalar monopole giant resonance in ^{208}Pb. The one-particle continuum plays an important role in the formation of the structure of the strength function. Most interesting is the appearance of some narrow resonances at large excitation energy. We discuss the results obtained from the point of view of resonance trapping which is known to appear due to the strong coupling of the resonance states via the continuum. [S0556-2813(99)02704-1]

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I. INTRODUCTION

Nuclear states at high excitation energy are embedded in the continuum and have a finite lifetime. This important fact has to be taken into account in the theoretical description of giant resonances (GR's) in order to obtain an adequate description of the reaction and to study the internal properties of the system which are influenced by the continuum.

The strong coupling of open quantum systems to the continuum has been investigated in different papers, e.g., [1–3]. The results show nontrivial modifications in the excitation spectra under the influence of the coupling to the continuum when the resonances start to overlap. The most pronounced modification is the appearance of narrow resonances in the spectra at strong coupling, if the number N of states embedded in the continuum is larger than the number K of open decay channels. This effect, called resonance trapping, is caused by the alignment of some resonance states with the open channels. As a result, we see a separation of time scales: the widths of K states become large under the influence of the continuum, while the widths of the remaining $N-K$ states become small. We have K states aligned with the K decay channels and $N-K$ trapped resonance states.

The structure of a GR under the influence of its coupling to the one-particle continuum is investigated in [3] on the basis of a schematic model. The results depend strongly on the ratio λ between external (via the continuum) and internal (configurational) mixing of the basis states. If this ratio is large, we see shifts of the gross structure components in energy as well as changes in the transition strengths and escape widths of the individual components. In particular, some narrow resonances may appear, at large external coupling, near the maximum of the giant resonance which carry a considerable amount of transition strength [3].

Calculations of this type have been performed, up to now, only in the framework of a schematic model. The parameters used in these calculations are not adjusted to the well-known parameters of mean field and residual interactions in nuclei. Moreover, the assumed energy independence of the coupling strength to the continuum is not justified in realistic cases. It is therefore an interesting question whether the coupling of the discrete states to the continuum will influence the structure of GR's in realistic cases in a measurable manner. In this work we will study the structure of GR's using the well-known method for describing the coupling of nuclear excited states to the one-particle continuum within the random phase approximation (continuum RPA) [4–6]. It is the aim of the present paper to consider the $4\hbar\omega$ isoscalar monopole giant resonance (ISMGR) in ^{208}Pb. This resonance lies at a high excitation energy (about 30 MeV) and resonance trapping is expected to appear in its structure.

The higher harmonics of GR's ($n\hbar\omega$ giant resonances with $n>2$) have been investigated experimentally as well as theoretically [7–11]. The best known example is the $3\hbar\omega$ isoscalar dipole GR, which is observed in the cross section of (α,α') scattering [7,8]. There are some indications that a resonance found in the (^{13}C,^{13}N) reaction [12,13] is the $3\hbar\omega$ isovector dipole GR [14]. The $4\hbar\omega$ states have not been observed up to now. It is expected [15], however, that experimental studies of the $n\hbar\omega$ GR's will be continued and the results of our investigation will be useful in analyzing future experimental data.

In our investigation we concentrate on two questions. First, does the $4\hbar\omega$ ISMGR exist as an isolated resonance in spite of the high excitation energy? This is an interesting question because the relaxation is expected to be large at high excitation energy especially for those states which are coupled strongly to the continuum. Second, do the peculiarities described in the schematic model of Ref. [3] appear in the structure of the $4\hbar\omega$ ISMGR and do narrow (trapped) states exist in its structure?

In the next section, the basic relations of the schematic model and the continuum-RPA model are sketched, both of which allow a solution to the problem with one particle in the continuum. Furthermore, the model parameters used in the continuum-RPA calculations are given. The results of numerical calculations are presented and discussed in Sec. III. Our attention is focused mainly on the nature and the properties of narrow (trapped) resonances in the strength function. We also comment briefly on the signature of a possible observation of the $4\hbar\omega$ ISMGR. The results are summarized in Sec. IV.

II. FORMULATION OF THE MODEL

A. Basic relations

In the schematic model for GR's the effective Hamiltonian

$$\mathcal{H}=H_0+DD^T-i\mathcal{V}\mathcal{V}^\dagger=\mathcal{H}_0-i\mathcal{V}\mathcal{V}^\dagger \tag{1}$$

is studied in Ref. [3] and consists of three parts. The part H_0 is the Hamiltonian of the closed nucleus with N discrete states. The part DD^T describes the internal interaction between these states. The additional part $i\mathcal{V}\mathcal{V}^\dagger$ describes the coupling of the states via the continuum. The $\mathcal{V}$ are the corresponding coupling vectors between the N states and the K open decay channels. The effective Hamiltonian $\mathcal{H}$ is non-Hermitian. Its eigenvalues provide the energies as well as the widths of the resonance states. The strength function is calculated by means of the eigenfunctions of $\mathcal{H}$. The influence of the continuum can be studied by comparing the eigenvalues and eigenfunctions of $\mathcal{H}$ with those of $\mathcal{H}_0$ as well as the corresponding strength functions.

In the continuum RPA [4,5] used in our quantitative analysis, the strength function is calculated, while the wave functions, positions, and widths of the resonance states are not directly determined. The strength function is proportional to the excitation probability of a nucleus under the action of an external field. Its energy dependence contains information about the distribution of strength over all resonance states and, therefore, about their positions and widths. The basic states for this approach are one-particle–one-hole (1p-1h) states including all states with one particle in the continuum.

Let us first sketch the basic relations of this approach according to Refs. [4,5,16] as applied to the analysis of isoscalar excitations in nuclei. The strength function corresponding to an external field $\hat{V}=\Sigma_a V(\mathbf{r}_a)\exp(-i\omega t)$, where the index a runs over all nucleons, is determined by [5]

$$S_V(\omega)=-\frac{1}{\pi}\mathrm{Im}\int V(\mathbf{r})A(\mathbf{r}\mathbf{r}',\omega)\tilde{V}(\mathbf{r}')d\mathbf{r}d\mathbf{r}'. \tag{2}$$

Here ω is the excitation energy, $A(\mathbf{r}\mathbf{r}',\omega)$ is the particle-hole propagator, and $\tilde{V}(\mathbf{r}')$ is the effective field which differs from the external field $V(\mathbf{r})$ because of the polarization of the nucleus. The effective field satisfies the equation [5]

$$\tilde{V}(\mathbf{r})=V(\mathbf{r})+\int F(\mathbf{r},\mathbf{r}')A(\mathbf{r}'\mathbf{r}'',\omega)\tilde{V}(\mathbf{r}'')d\mathbf{r}'d\mathbf{r}'', \tag{3}$$

where $F(\mathbf{r},\mathbf{r}')$ is the effective (residual) particle-hole interaction. The particle-hole propagator can be expressed in terms of single-particle wave and Green functions and single-particle energies in the shell-model potential [4,5] as

$$A(\mathbf{r}\mathbf{r}',\omega)=\sum_{\lambda,m_\lambda} n_\lambda \phi^*_{\lambda m_\lambda}(\mathbf{r})\phi_{\lambda m_\lambda}(\mathbf{r}')[g(\mathbf{r}\mathbf{r}',\varepsilon_\lambda+\omega)+g(\mathbf{r}\mathbf{r}',\varepsilon_\lambda-\omega)], \tag{4}$$

where λ,m_λ compose a set of single-particle quantum numbers (λ includes the radial quantum number, orbital, and total momenta; m_λ is the magnetic quantum number), n_λ, $\phi_{\lambda m_\lambda}(\mathbf{r})$, and ε_λ are the occupation numbers, wave functions, and energies of the single-particle states, respectively, and $g(\mathbf{r}\mathbf{r}',\varepsilon)$ is the single-particle Green function. For simplicity we omit in Eq. (4) the isospin index. In Eqs. (2)–(4) the one-particle continuum is taken into account in the following manner. The Green functions in Eq. (4) are calculated by means of two linearly independent solutions of the single-particle Schrödinger equation with the appropriate boundary conditions [4,17]. Within this method the energy dependence of the strength function, calculated using Eqs. (2)–(4), exhibits resonances with widths determined by the coupling of the internal states to the continuum.

To calculate the strength function $S_V(\omega)$, Eq. (2), we have to choose an external field $V(\mathbf{r})$. Since we are interested in the excitation of the isoscalar monopole states, the external field $V(\mathbf{r})$ has to act, therefore, on protons and neutrons in the same manner and its angular dependence has to contain the spherical function $Y_{00}(\mathbf{n})$: $V(\mathbf{r})=V(r)Y_{00}(\mathbf{n})$. The radial dependence of the field can, in principal, be chosen arbitrarily, because the energies and widths of all considered states are independent of this field. Only the values of the strength function depend on this choice. Usually the field is chosen so that (i) the strength function has maximum values and (ii) the integral of the energy-weighted strength function (sum rule) can be calculated in a manner that is almost model independent. In the case of the $4\hbar\omega$ ISMGR it is convenient to choose this field in the form

$$V(r)=(r/R)^2[1-(r/R)^2], \tag{5}$$

where R is the radius of the nucleus. This choice is dictated by the fact that $V(r)$ has large matrix elements for quasiparticle transitions over four shells (which can be seen, for example, from the harmonic oscillator model) and therefore the external field (5) excites the $4\hbar\omega$ states with a high probability. We note that the field $V(r)$, Eq. (5), coincides with the function $j_0(qr)-1$, where j_0 is the spherical Bessel function, at the momentum transfer $q\sim 2/R$. Therefore, the cross section of a reaction with excitation of monopole states is connected with the strength function corresponding to the field (5).

Under the assumption that the nuclear density is constant inside the nucleus, the energy weighted sum rule is [18]

$$\int_0^\infty \omega S_V(\omega)d\omega=\frac{23}{420\pi}\frac{\hbar^2 A}{mR^2} \tag{6}$$

for the external field given by Eq. (5). Here A is the number of nucleons and m is the nucleon mass.

B. Qualitative analysis of the basic relations

For a qualitative analysis we consider the basic equations (2)–(4) within the Tamm-Dancoff approximation (TDA). For this purpose we neglect the second term in the brackets in Eq. (4) and take into account only those poles of the Green function $g(\mathbf{r}\mathbf{r}',\varepsilon_\lambda+\omega)$ in Eq. (4) which correspond to unoccupied single-particle states, to avoid Pauli violating excitations. Among these unoccupied states, there are both stationary and quasistationary states in the shell-model potential. In the vicinity of such a state with quantum numbers μ, the Green function inside the nucleus can be represented as [17]

$$g(\mathbf{r}\mathbf{r}',\varepsilon_\lambda+\omega)=\sum_{m_\mu}\frac{\tilde{\phi}_{\mu m_\mu}(\mathbf{r})\tilde{\phi}^*_{\mu m_\mu}(\mathbf{r}')}{\varepsilon_\lambda+\omega-\varepsilon_\mu+i\Gamma_\mu/2}, \qquad (7)$$

where ε_μ and Γ_μ are the energy and the width of the quasistationary state μ, respectively, $\tilde{\phi}_{\mu m_\mu}(\mathbf{r})$ is the solution of the Schrödinger equation at the energy ε_μ, which is normalized to unity within the volume of the nucleus [in the case of a stationary state $\Gamma_\mu\to 0$ and $\tilde{\phi}_{\mu m_\mu}(\mathbf{r})\to\phi_{\mu m_\mu}(\mathbf{r})$].

As a result of the shell structure of the nucleus, the energy distances $\varepsilon_\mu-\varepsilon_\lambda$ between the energies of two different single-particle states are grouped near values which are close to a multiple of the energy interval between shells and determine the $1\hbar\omega$, $2\hbar\omega$, $3\hbar\omega$, etc., GR's. In realistic shell-model potentials for nuclei, the widths Γ_μ of the quasistationary states are smaller than the energy interval between single-particle states with the same values of angular momentum and parity. Thus for a qualitative analysis of the $4\hbar\omega$ excitations (near an energy of 30 MeV in ^{208}Pb) we should take into account only one pole of the Green function in each term of the expression (4). Further, we substitute the modified (TDA) expression (4) with the Green function (7) into Eqs. (2) and (3), multiply Eq. (3) by $\tilde{\phi}^*_{\mu m_\mu}(\mathbf{r})\phi_{\lambda m_\lambda}(\mathbf{r})$, and integrate over $\mathbf{r}$. After summation over the magnetic quantum numbers, we obtain

$$S_V(\omega)=-\frac{1}{\pi}\mathrm{Im}\sum_{\lambda\mu}\frac{2j_\lambda+1}{4\pi}\frac{V_{\lambda\mu}\tilde{V}_{\mu\lambda}}{\varepsilon_\lambda+\omega-\varepsilon_\mu+i\Gamma_\mu/2} \qquad (8)$$

for the strength function (2) and the system of equations

$$\tilde{V}_{\mu\lambda}=V_{\mu\lambda}+\sum_{\nu\kappa}\frac{2j_\lambda+1}{4\pi}\frac{F_{\mu\lambda\nu\kappa}\tilde{V}_{\kappa\nu}}{\varepsilon_\nu+\omega-\varepsilon_\kappa+i\Gamma_\kappa/2} \qquad (9)$$

for the matrix elements of the effective field. Here $F_{\mu\lambda\nu\kappa}$ is the matrix element of the residual interaction associated with four single-particle wave functions. In these expressions, the quantum numbers of a hole λ and of a particle μ (or ν and κ) are connected by the monopole transition rules ($j_\lambda=j_\mu$ and $l_\lambda=l_\mu$, where j and l are the total and orbital angular momenta of single-particle states, respectively) and the corresponding radial quantum numbers differ by 2 (i.e., the hole states and the particle states are separated by four shells).

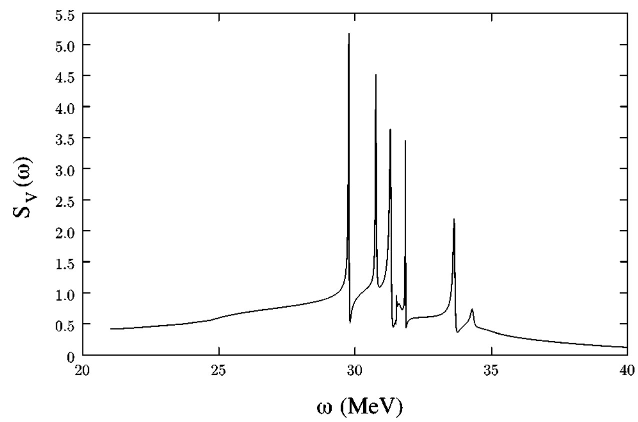

FIG. 1. Energy dependence of the strength function (in arbitrary units) of the $4\hbar\omega$ ISMGR in ^{208}Pb calculated according to Eqs. (2)–(5). The parameters (12)–(15) are used.

The number of equations in Eq. (9) coincides with the number of occupied states λ which have a corresponding partner μ among the unoccupied states. The index μ relates to either a stationary state or a quasistationary state in the shell-model potential, and the widths of the quasistationary states are included in the equations. In the case $\Gamma_\mu\neq 0$, the basic state (λ,μ) is coupled to the continuum directly and determines one open decay channel. The other basic states are coupled to the continuum by the effective interaction. The latter are associated with hole states λ in deep shells, because in this case the energy of the corresponding particle state μ can be smaller than the threshold energy in the shell-model potential, and hence, this state is a stationary one. In the realistic shell-model potential for ^{208}Pb [Eqs. (12) and (13) below], there are 34 basic states of $4\hbar\omega$ type; 15 of them are in the proton subsystem and 19 in the neutron one. Out of these states, 29 basic states are coupled directly to open decay channels $(\Gamma_\mu>0)$; for two of them the widths Γ_μ are very small. It means that within the 1p-1h approximation, the $4\hbar\omega$ ISMGR in ^{208}Pb has 29 open decay channels, two of which are almost closed.

The solution of the inhomogeneous system of equations (9) is inversely proportional to the determinant

$$\begin{vmatrix} \omega-\omega_{\mu\lambda}-F_{\mu\lambda\lambda\mu}+\frac{i}{2}\Gamma_\mu & -F_{\mu\lambda\lambda_1\mu_1} & -F_{\mu\lambda\lambda_2\mu_2} & \dots \\ -F_{\mu_1\lambda_1\lambda\mu} & \omega-\omega_{\mu_1\lambda_1}-F_{\mu_1\lambda_1\lambda_1\mu_1}+\frac{i}{2}\Gamma_{\mu_1} & -F_{\mu_1\lambda_1\lambda_2\mu_2} & \dots \\ -F_{\mu_2\lambda_2\lambda\mu} & -F_{\mu_2\lambda_2\lambda_1\mu_1} & \dots & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}, \qquad (10)$$

where $\omega_{\mu\lambda}=\varepsilon_\mu-\varepsilon_\lambda$ and the factor $(2j+1)/4\pi$ is included in F. The poles of the strength function as a function of ω are, therefore, determined by the zeros of this determinant and hence coincide with the eigenvalues of the non-Hermitian matrix

$$\begin{pmatrix} \omega_{\mu\lambda}+F_{\mu\lambda\lambda\mu} & F_{\mu\lambda\lambda_1\mu_1} & \dots \\ F_{\mu_1\lambda_1\lambda\mu} & \omega_{\mu_1\lambda_1}+F_{\mu_1\lambda_1\lambda_1\mu_1} & \dots \\ \dots & \dots & \dots \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_\mu & 0 & 0 \\ 0 & \Gamma_{\mu_1} & 0 \\ 0 & 0 & \dots \end{pmatrix}. \tag{11}$$

The matrix (11) coincides with the matrix of the Hamiltonian (1) for a specific choice of the coupling vector $\mathcal{V}$ (each basic state couples to only one decay channel). As shown in Refs. [1–3], there are narrow (trapped) states among eigenstates of the Hamiltonian (1) at large coupling to the continuum, if the number N of basic states is larger than the number K of open decay channels. We therefore expect the appearance of trapped resonance states also in the continuum-RPA results at large excitation energies. We emphasize that our numerical results given in the present paper (Sec. III) are obtained on the basis of the exact continuum-RPA equations (2)–(5), and *not* on the basis of Eqs. (7)–(11) which are given here only for a qualitative analysis.

C. Choice of the model parameters

To calculate the strength function $S_V(\omega)$ according to Eqs. (2)–(5) it is necessary to choose the nuclear mean field, which determines the energies ε_λ, wave functions $\phi_{\lambda m_\lambda}(\mathbf{r})$, and Green functions $g(\mathbf{r}\mathbf{r}',\varepsilon)$ in expression (4) for the propagator, and the effective interaction $F(\mathbf{r},\mathbf{r}')$ used in Eq. (3). In our calculations we use a shell-model potential which includes the central isoscalar potential, spin-orbital interaction, symmetry potential, and Coulomb potential (for protons). The shell-model potential has the form

$$U(\mathbf{r})=U_0 f_{\rm WS}(r)+U_{\rm so}(\boldsymbol{\sigma}\cdot\mathbf{l})\frac{1}{r}\frac{df_{\rm WS}(r)}{dr}\pm U_{\rm sym}f_{\rm WS}(r) + U_C(r), \tag{12}$$

where $f_{\rm WS}(r)=1/\{1+\exp[(r-R)/a]\}$ is the Woods-Saxon function (a is the nucleus diffuseness), $(1/2)\boldsymbol{\sigma}$ and $\mathbf{l}$ are the operators of spin and orbital angular momenta of a nucleon, the ‘‘plus’’ and ‘‘minus’’ signs relate to protons and neutrons, respectively. The Coulomb potential $U_C(r)$ is that associated with a uniformly charged sphere with radius R. Phenomenological parameters of the potential (12) are chosen according to Ref. [19],

$$R=7.34\ \text{fm},\quad a=0.65\ \text{fm},\quad U_0=-53.3\ \text{MeV},$$
$$U_{\rm so}=19.8\ \text{MeV fm}^2,\quad U_{\rm sym}=-7.05\ \text{MeV}. \tag{13}$$

This set of parameters allows us to reproduce, within the shell-model approach with the continuum, both the nucleon single-particle (binding) energies and the low-lying levels in odd-even nuclei in the vicinity of ^{208}Pb.

The effective interaction $F(\mathbf{r},\mathbf{r}')$ is chosen in the Landau-Migdal form

$$F(\mathbf{r},\mathbf{r}')=\frac{1}{2}F(r)\,\delta(\mathbf{r}-\mathbf{r}'),$$
$$F(r)=350[f_{\rm ex}-(f_{\rm in}-f_{\rm ex})f_{\rm WS}(r)]\ \text{MeV fm}^3, \tag{14}$$

where $f_{\rm ex}$ and $f_{\rm in}$ are phenomenological parameters determining the strength of the residual interaction inside ($f_{\rm in}$) and outside ($f_{\rm ex}$) the nucleus. We use [5]

$$f_{\rm ex}=-3.9,\quad f_{\rm in}=-0.15. \tag{15}$$

These parameters allow us to obtain a satisfactory description of the $2\hbar\omega$ isoscalar monopole and quadrupole giant resonances [16] and the $3\hbar\omega$ isoscalar dipole giant resonance [20] in nuclei within the continuum RPA.

In connection with the choice of model parameters, Eqs. (12)–(15), we note the following: In a self-consistent approach (see, for example, [9,21]) the strength function $S_V(\omega)$ for the isoscalar dipole field $V(\mathbf{r})=rY_{10}(\mathbf{n})$ has one maximum at zero energy with 100% strength which corresponds to a spurious state. In our approach, with parameters (12)–(15), the strength of the spurious state lying at zero energy exhausts 91% of the sum rule as shown in [20]. This small difference from 100% characterizes the degree of nonconsistency of our model. We will show in the next section that the structure of the $4\hbar\omega$ ISMGR is mainly determined by its strong coupling to the continuum and is only slightly dependent on the residual interaction. It means that the small nonconsistency in our model has a negligible influence on our results. In the following section we use Eqs. (2)–(5) with the parameters (12)–(15) in calculating the strength function of the $4\hbar\omega$ ISMGR.

III. NUMERICAL RESULTS AND DISCUSSION

The strength function of the $4\hbar\omega$ ISMGR in ^{208}Pb obtained in the continuum-RPA approach is given in Fig. 1. The calculated strength function exhibits the following structure: There is one broad resonance with an energy of 29 MeV and total escape width of 9 MeV. It exhausts 40% of the sum rule (6) within the energy interval of 21–40 MeV. Seven narrow asymmetrical resonances are near the maximum of the broad resonance. These narrow resonances exhaust altogether about 4% of the sum rule (6). According to our calculations the remainder of the strength is exhausted mainly by the $2\hbar\omega$ ISMGR and the high-energy tail of the $4\hbar\omega$ ISMGR. The energies ω_n, widths Γ_n, and strengths s_n of the narrow resonances are given in Table I. Below we will investigate this structure in detail. Here, we note only that this structure is expected for the $4\hbar\omega$ ISMGR in ^{208}Pb from the theoretical point of resonance trapping [1,2]. As discussed in Sec. II B, there are 34 neighboring basic states and 29 open decay channels, two of which are almost closed. We expect therefore 27 resonance states appearing as one broad resonance in the strength function due to overlapping and seven narrow resonances with energies near the maximum of the broad resonance.

TABLE I. Parameters of the narrow resonances in the energy dependence of the strength function.

n	ω_n (MeV)	Γ_n (keV)	s_n (%)	Basic state	ω_n^{ph} (MeV)	s_n^{ph} (%)
1	29.8	26	0.64	$2s^n_{1/2}-4s^n_{1/2}$	29.9	0.49
2	30.8	39	0.71	$1p^p_{3/2}-3p^p_{3/2}$	30.9	0.55
3	31.3	66	0.96	$1d^n_{5/2}-3d^n_{5/2}$	31.4	0.91
4	31.5	12	0.04	$1p^p_{1/2}-3p^p_{1/2}$	31.6	0.16
5	31.9	11	0.40	$1d^n_{3/2}-3d^n_{3/2}$	31.9	0.49
6	33.6	73	0.77	$1d^p_{5/2}-3d^p_{5/2}$	33.8	0.95
7	34.3	146	0.39	$1d^p_{3/2}-3d^p_{3/2}$	34.4	0.45

The results shown in Fig. 1 differ substantially from those obtained, using the same approach, for resonances with smaller energies (see, e.g., [9,16]). In the last case, one or just a few resonances exhausting the main part of the sum rule are shifted in energy with respect to the energies of the other states. As an example we show in Fig. 2(a) the strength function of the $2\hbar\omega$ ISMGR in ^{208}Pb, calculated according to Eqs. (2)–(4) for the external field $V(\mathbf{r})=r^2Y_{00}(\mathbf{n})$ with the parameters (12)–(15). We see from this figure that one resonance with a large strength is shifted in energy in relation to several weak components (satellites). The width of the main resonance is approximately equal to the sum of the satellite widths. The difference in the resonance structures at large and small energies is caused by the strong influence of the continuum.

To prove this statement we performed some calculations for the strength function similar to those in Ref. [3]. In these calculations we varied both the residual interaction and the intensity of the coupling to the continuum in the following manner. The residual interaction was varied by varying the parameters f_{in} and f_{ex} of Eq. (14) and the coupling to the continuum was varied by adding an additional potential barrier outside the nucleus into the shell-model potential (12). This potential barrier has the form

$$U_b(r)=\alpha U_0 \exp\left\{-\left(\frac{r-\beta R}{\gamma R}\right)^2\right\}, \tag{16}$$

where $\beta=1.25$ and $\gamma=0.15$ are chosen. The parameter α characterizing the height of the barrier is varied within the limits $0\leqslant\alpha\leqslant0.9$. In this way we change strongly the coupling of the basic states to the continuum, i.e., the widths Γ_μ in Eqs. (7)–(11). The energies of the states are only slightly changed.

As an example, the strength function of the $4\hbar\omega$ ISMGR calculated by using the interactions (14) and (15) and the shell-model potential (12) and (13) with $\alpha=0.7$ in the barrier (16) is given in Fig. 2(b). The strength function shown is qualitatively similar to that given in Fig. 2(a). Two collective resonance states exhaust the main part of the total strength and suffer a shift in energy. The widths are distributed over all resonances more or less uniformly.

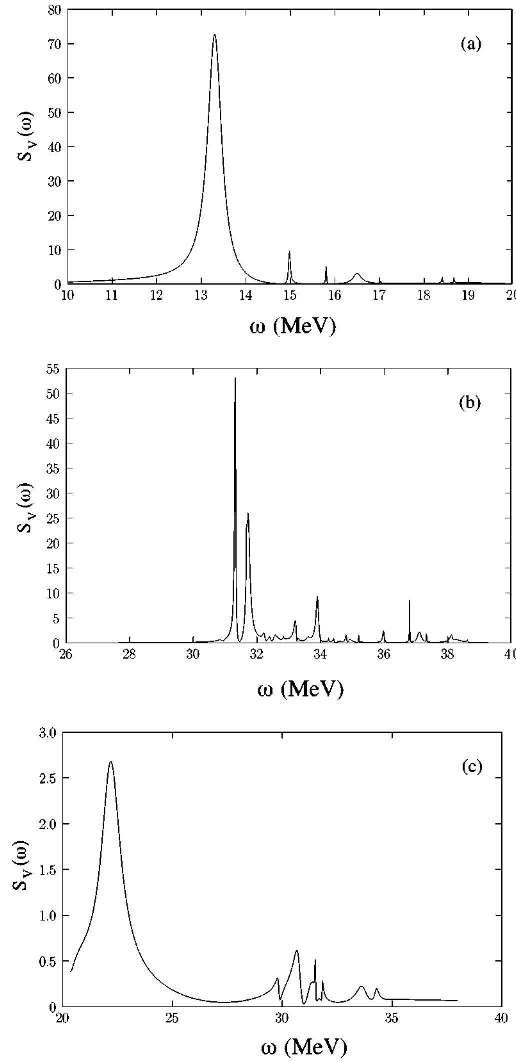

FIG. 2. Energy dependence of the strength functions (in arbitrary units) in ^{208}Pb. (a) The $2\hbar\omega$ ISMGR, calculated according to Eqs. (2)–(4) for the external field $V(r)=(r/R)^2$ with parameters (12)–(15). (b) The $4\hbar\omega$ ISMGR, calculated according to Eqs. (2)–(5), using the mean fields (12) and (13) with the barrier (16) with $\alpha=0.7$ and the residual interactions (14)–(15). (c) The $4\hbar\omega$ ISMGR, calculated according to Eqs. (2)–(5) using the mean fields (12) and (13). The residual interaction is increased by a factor of 4 in comparison with Eq. (15).

The shift of the collective states as well as the ratio between the widths of the satellites and the collective states decreases with increasing coupling to the continuum. For illustration we show in Fig. 3(a) the dependence of the quantity $\delta_G=(\overline{\omega}_x-\omega_G)/\overline{\omega}_x$ on the parameter α ($\overline{\omega}_x$ is the average energy of the basic states, and ω_G is the average energy of the collective states). This quantity characterizes the relative energy shift of the GR. In Fig. 3(b) the dependence of the ratio $\gamma_G=\Gamma_G/(\Gamma_G+\Gamma_s)$ on the parameter α is shown, where Γ_G (Γ_s) is the sum of the widths of the collective states (satellites). The results are as follows. Introducing the barrier with the height $\alpha=0.9$ leads to a 40% increase in the relative shift δ_G of the GR, while the ratio γ_G between the widths of the GR and satellites decreases from 0.96 at α

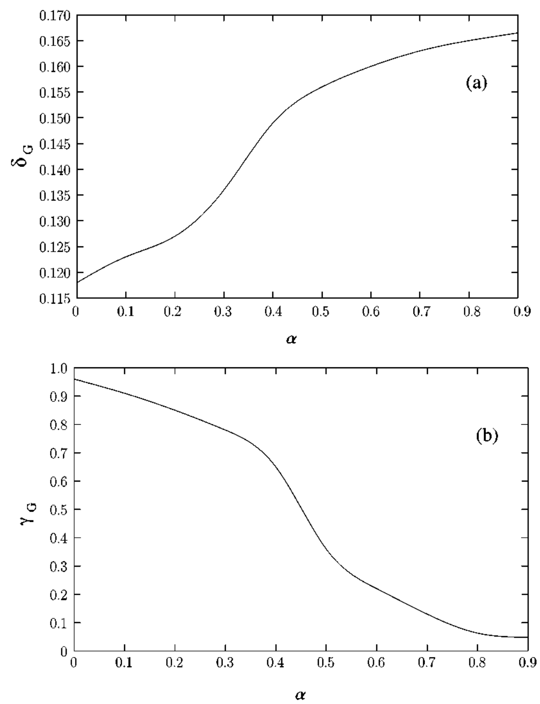

FIG. 3. Dependences of ratios $\delta_G = (\overline{\omega_x} - \omega_G)/\overline{\omega_x}$ (a) and $\gamma_G = \Gamma_G/(\Gamma_G + \Gamma_s)$ (b) on the height of the barrier (16) (see text).

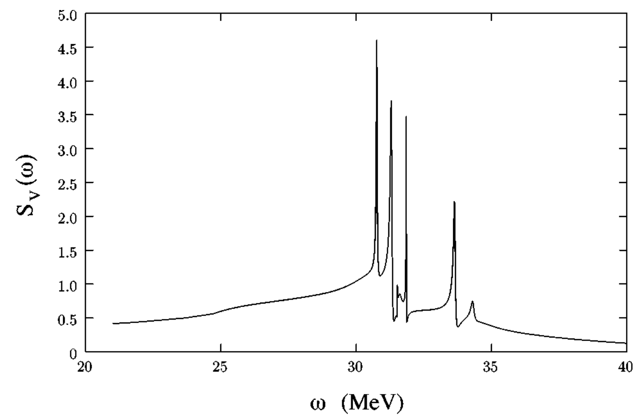

FIG. 4. Energy dependence of the strength function (in arbitrary units) of the $4\hbar\omega$ ISMGR in ^{208}Pb calculated according to Eqs. (2)–(5). The parameters (12)–(15) are used. Different from Fig. 1, the term corresponding to the neutron hole state $2s_{1/2}$ is excluded from the propagator (4).

$=0$ to 0.05 at $\alpha = 0.9$. We also note that all resonance states suffer a small shift to larger energies in calculations with the barrier [Fig. 2(b)] in comparison with those obtained without it (Fig. 1). This is explained by the fact that the barrier (16) leads to a decrease of the ''effective'' radius of the nucleus, and hence to an increase of the energies of all particle-hole states.

In Fig. 2(c) we show the strength function of the $4\hbar\omega$ ISMGR calculated with the shell-model potentials (12) and (13) without a barrier, but with the residual interaction increased by a factor of 4 in comparison with the interaction (15). One can see a large shift of the main resonance to low energies. The widths are distributed more or less uniformly between the GR and satellites. This result is qualitatively similar to that obtained by enlarging the barrier [Fig. 2(b)].

All the results given above show the following: the structure of the strength function depends on both the residual interaction of the basic states (internal interaction) and their decay into the continuum (external interaction). It is governed by the interplay and competition of these two types of interaction. This behavior of the strength function is qualitatively similar to that obtained in Ref. [3] [the different signs between the shift of the $4\hbar\omega$ ISMGR considered here and the GR considered in Ref. [3] are due to the different signs between the isoscalar interaction in Eqs. (14) and (15) and that of the residual interaction used in Ref. [3]]. All these results allow us to identify the narrow resonances in the energy dependence of the strength function as trapped resonances. We note that changes of the mean field and of the residual interaction made above can lead to some nonconsistency of the model. For this reason the results given in Figs. 2 and 3 have an illustrative character and serve the aim to better understand the influence of the continuum on the structure of the strength function.

The peculiarities of the strength function of the $4\hbar\omega$ ISMGR described above can also be qualitatively explained on the basis of the shell model in the continuum. In this model each of the 1p-1h basic states can be coupled to only one decay channel and interacts with the others by means of the residual interaction (see Sec. II B). At large energies (for example, in the case of the $4\hbar\omega$ basic states), the widths of the basic states are very different from one another. The basic states with small (or zero) widths contain a hole in a deep shell (see Sec. II B). The interaction between the basic states with large and small widths is small, because the matrix elements of the realistic residual interactions (14) and (15) [$F_{\mu\lambda\nu\kappa}$ in Eqs. (9)–(11)] are smaller than the energy intervals between these states in the *complex* energy plane. Therefore, the mixing of the basic states having different widths is small, and each narrow resonance in the energy dependence of the strength function relates to a concrete basic state with a small (or zero) width. As a consequence, the energies and strengths of these resonances are close to those of the basic states. The small widths arise from their small interaction with other basic states.

To illustrate these statements we give in Table I the energies (ω_n^{ph}) and strengths (s_n^{ph}) of the basic states with a hole in a deep shell. The fact that these values are close to those calculated by taking into account the residual interaction supports our interpretation. Further, we performed some additional calculations of the strength function in the following manner. In each calculation, we excluded from the propagator (4) one of the terms which corresponds to a single-particle state λ in a deep shell. This decreases the number of the basic states by 1, but the number of open channels remains unchanged (see Sec. II B). As an example we give in Fig. 4 the strength function calculated according to Eqs. (2)–(5) with the parameters (12)–(15), by excluding the term corresponding to the neutron state $2s_{1/2}$. It is seen from Fig. 4 that the strength function loses the narrow resonance at 29.8 MeV (compare with Fig. 1). These results show that the narrow resonance states are almost pure 1p-1h states. This means that the internal interaction of the 1p-1h states is effectively reduced due to their strong coupling to the continuum.

We emphasize that our results show that the continuum strongly influences not only the widths, but also the energies and strengths of the GR components. In calculations without the continuum, the role of the internal interaction is overestimated. Therefore, the energy shift of the GR as well as its collectivity are overestimated when the influence of the continuum is not taken into account. This conclusion is obtained on the basis of the continuum RPA and is in agreement with the results obtained in the framework of the schematic model for giant resonances of Ref. [3].

The interaction of the 1p-1h resonance states with states of more complicated structure leads to some spreading of the 1p-1h strength and, hence, to some changes in the results obtained. A detailed analysis of the spreading effects (see, for example, [16,21,23]) is beyond the scope of the present work. However, we can understand the main features of the actual strength function of the $4\hbar\omega$ ISMGR on the basis of the experimental widths of other giant resonances. The experimental total widths of the isoscalar GR's in ^{208}Pb are equal to 2.8 MeV for the $2\hbar\omega$ ISMGR lying at the excitation energy of 13.7 MeV [22], about 3 MeV for the octupole resonance at the excitation energy of 19.7 MeV [8], and 3–6 MeV (according to different data) for the dipole giant resonance at the excitation energy of 22 MeV [7,8]. According to these data, the spreading width of the $4\hbar\omega$ ISMGR is expected to be not larger than 5–6 MeV. Because this value is smaller than the escape width of the broad resonance in the RPA strength function, the interaction between the narrow and broad resonances via complicated configurations is not expected to be strong so that only one broad resonance in the actual strength function will appear. Therefore, we expect that the interaction of the $4\hbar\omega$ ISMGR with complicated configurations does not change our conclusion about the existence of two different time scales in its structure. The strength function is expected to exhibit two different resonances. One of them exhausts 40% of the total strength and has a total width of about 14 MeV. The second resonance is a superposition of the considered narrow resonances. It has a total width of about 6–7 MeV and exhausts 4% of the sum rule (6). Therefore, the main part of the strength of the $4\hbar\omega$ ISMGR suffers a strong relaxation mainly due to the coupling to the continuum and can appear in any cross section as a smooth background. The small part of the strength is expected to appear as an isolated resonance over the background.

Taking into account the strengths and widths of the narrow and broad resonances one can obtain an estimation for the ratio of the strength function at the maximum of the narrow resonance to the background (''signal-to-noise'' ratio): $s_n\Gamma_b/s_b\Gamma_n \sim 0.2$, where $s_b = 40\%$, $\Gamma_b \sim 14$ MeV and $s_n = 4\%$, $\Gamma_n \sim 7$ MeV are the strengths and total widths of the broad and narrow resonances, respectively. This value allows us to hope that the $4\hbar\omega$ ISMGR may be, in principle, observed experimentally. One can use, for example, the (α,α') reaction [24] at large momentum transfer, because the excitation probability of the $4\hbar\omega$ ISMGR is small for long-wave external fields. A detailed analysis of the possibility of observing the $4\hbar\omega$ ISMGR experimentally requires the ingredients of transition densities, hadron-nucleon interaction and optical potentials, and a sensitivity of the experimental techniques and thus should be the subject of a separate work. Here we stress only that the observation of the $4\hbar\omega$ ISMGR would provide an important physical insight, because it would show the important role of the continuum at high excitation energy.

IV. SUMMARY

In this work we have investigated the $4\hbar\omega$ ISMGR in ^{208}Pb on the basis of the continuum-RPA approach. By means of some numerical calculations we have shown that the structure of this resonance is mainly determined by its strong coupling to the continuum. This leads to the formation of two different time scales in its structure. On the one hand, there is one broad resonance whose width is approximately twice as big as the distance between the shells in ^{208}Pb. This resonance is therefore relaxed more or less fully. On the other hand, there are several narrow resonances in the energy dependence of the 1p-1h strength function. These narrow resonance states are shown to arise due to resonance trapping. They carry a small part of the strength and can appear in the experimental cross section as an isolated resonance at large excitation energy.

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