Three-body Faddeev calculation for ^{11}Li with separable potentials

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The halo nucleus ^{11}Li is treated as a three-body system consisting of an inert core of ^{9}Li plus two valence neutrons. The Faddeev equations are solved using separable potentials to describe the two-body interactions, corresponding in the n-^{9}Li subsystem to a $p_{1/2}$ resonance plus a virtual s-wave state. The experimental ^{11}Li energy is taken as input and the ^{9}Li transverse momentum distribution in ^{11}Li is studied. [S0556-2813(99)01703-3]

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Recent experiments with radioactive beams have permitted the study of properties of nuclei close to the neutron drip line, that is, close to the stability line for decay through neutron emission. These nuclei have been shown to possess neutron halos characterized by exceptionally large radii and narrow momentum distributions of the decay fragments in break-up experiments [1–2]. Among these nuclei special attention has been paid to ^{11}Li, a nucleus with a radius of about 3 fm, and a two-neutron separation energy of only 0.3 MeV. In the case of ^{11}Li two lines of theoretical calculations have been followed; in one, the conventional shell model or Hartree-Fock approach is used [3], in the other a cluster model assuming a core of ^{9}Li plus two neutrons is taken [4–8]. Because of the halo, shell model calculations require a very large single particle basis for the diagonalization of the Hamiltonian. The cluster model seems particularly suited to the case of ^{11}Li, considering the small $2n$ separation energy, the fact that ^{9}Li is a normal nucleus with a neutron separation energy of 4 MeV and that ^{11}Li is a Borromean nucleus, that is, no two-body subsystem of the three-body system does form a bound state. In the three-body model for ^{11}Li, calculations are hampered by a lack of information with regard to the n-^{9}Li interaction. Several calculations [4–7,24] were performed using local potentials for this interaction and there is also one calculation using separable potentials [8]. The parameters of the potentials were adjusted to produce a n-^{9}Li resonance, which has most frequently been assumed in the $p_{1/2}$ channel, although the experimental data are not conclusive. According to Wilcox *et al.* [9] a resonance occurs at 0.80 ± 0.25 MeV. Following more recent work [10] there is a 2^{+} resonance situated at 0.42 MeV and a 1^{+} state at 0.80 MeV. In addition one now has strong evidence for an enhancement of the production of ^{10}Li near threshold in reactions involving ^{11}Be, ^{11}Li, and ^{11}B [11,12]. This is interpreted as due to an intruder virtual s-wave state of the n-^{9}Li system near zero energy corresponding to a scattering length of -20 fm or less [12]. A three-body study [13] indicated that this barely unbound state in ^{10}Li is able to explain the extra narrowness of the momentum distribution of ^{9}Li in the fragmentation of ^{11}Li. With a s-wave scattering length of -44 fm and a $p_{1/2}$ resonance energy of 0.35 MeV, the $2n$ separation energy in ^{11}Li and the momentum distribution of the ^{9}Li fragment are fitted in Ref. [13] assuming the sudden approximation [14] and neglecting the final state interaction in the break-up reaction. It is our purpose here, to make a similar fit applying the three-body model with separable potentials developed earlier [15] and which proved to be effective in the description of the structure of ^{18}O and ^{18}F as well as ^{16}O(d,p) stripping.

We consider the halo nucleus ^{11}Li as a three-body system consisting of a ^{9}Li core (particle 3), which stays inert, plus two valence neutrons (particles 1 and 2). The valence neutrons couple with the orbital angular momentum of the core to total angular momentum and parity $J^{\pi}=0^{+}$, the spin and parity of ^{11}Li being then due to the value $3/2^{-}$ of the ^{9}Li core. The neutron-^{9}Li system is assumed to have a $p_{1/2}$ resonance of width $\Gamma = 0.15 \pm 0.07$ MeV at an energy $E_r = 0.42 \pm 0.05$ MeV [10] and also a $s_{1/2}$ virtual state close to zero energy [12,16]. We must also take into account the Pauli principle which does not allow a valence neutron to occupy the $1s_{1/2}$ and the $1p_{3/2}$ single particle states which are already filled in the core. To fix the energies $\epsilon_{1s_{1/2}}$ and $\epsilon_{1p_{3/2}}$ of these states, we proceed as follows. First by doing an interpolation between the experimental value -4.053 MeV of $\epsilon_{1p_{3/2}}$ for $A=9$ [17] and the values in Figs. 2–30 of Bohr-Mottelson's book [18] we obtain $\epsilon_{1p_{3/2}} \simeq -7$ MeV for the $A=10$ system. To obtain $\epsilon_{1s_{1/2}}$ we suppose that the separation between the $1s_{1/2}$ level and the centroid of the levels $1p_{3/2}$ and $1p_{1/2}$ is $\hbar\omega$ and use the prescription $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ MeV, which is appropriate for light nuclei [19]. For $A=10$, one has $\hbar\omega = 15.501$ MeV and $\epsilon_{1s_{1/2}}$ results equal to -20.028 MeV. To account for the Pauli blocking of the states $1s_{1/2}$ and $1p_{3/2}$, we use the projection method of Kukulin [20]. From now on, we consider $\hbar = 1$.

To describe the neutron-^{9}Li interaction we use a separable potential which acts on the $s_{1/2}$, $p_{3/2}$, and $p_{1/2}$ waves:

$$\langle \mathbf{P}_i | V_i | \mathbf{P}_i' \rangle = \sum_{lj\alpha} -\frac{\Lambda_{lj}^{(\alpha)}}{2m} v_{lj}^{(\alpha)}(P_i) v_{lj}^{(\alpha)}(P_i') \times \sum_{\mu} \langle \hat{\mathbf{P}}_i | y_{lj\mu} \rangle \langle y_{lj\mu} | \hat{\mathbf{P}}_i' \rangle \quad (i=1,2), \tag{1}$$

where $lj=0(1/2)$, $1(3/2)$, and $1(1/2)$, m is the reduced mass of the n-^{9}Li system ($m=9/10M$, M being the nucleon mass), and $\mathbf{P}_i$ is the momentum of neutron i with respect to ^{9}Li. The $s_{1/2}$ potential is a three-term ($\alpha=1,2,3$) potential, with the form factors chosen as

$$v^{(1)}_{0(1/2)}(q)=(q^2+\alpha^2_{0(1/2)})\exp(-\beta_0^2q^2/2), \tag{2}$$

$$v^{(2)}_{0(1/2)}(q)=\left(\frac{3}{2}-\beta_0^2q^2\right)\exp(-\beta_0^2q^2/2), \tag{3}$$

$$v^{(3)}_{0(1/2)}(q)=\exp(-\beta_0^2q^2/2). \tag{4}$$

The first term is chosen in such a way that, alone, it reproduces the $1s_{1/2}$ state. Taking for $\alpha_{0(1/2)}$ and $\Lambda^{(1)}_{0(1/2)}$ the special values

$$\alpha_{0(1/2)}=\sqrt{2m|\epsilon_{1s_{1/2}}|} \tag{5}$$

and

$$\Lambda^{(1)}_{0(1/2)}=\left[\int_0^\infty dqq^2\frac{[v^{(1)}_{0(1/2)}(q)]^2}{q^2-2m\epsilon_{1s_{1/2}}}\right]^{-1}, \tag{6}$$

we get a bound state of energy $\epsilon_{1s_{1/2}}$ and wave function

$$\Phi_{0(1/2)\mu}(\mathbf{q})=N_{0(1/2)}\exp(-\beta_0^2q^2/2)y_{0(1/2)\mu}(\hat{\mathbf{q}}), \tag{7}$$

which is precisely the $1s_{1/2}$ oscillator function in momentum space if we set $\beta_0=1/\sqrt{m\omega}$. From the values chosen for $\epsilon_{1s_{1/2}}$ and ω, we obtain $\alpha_{0(1/2)}=0.932\,\mathrm{fm}^{-1}$, $\Lambda^{(1)}_{0(1/2)}=8.417\,\mathrm{fm}^5$ and $\beta_0=1.724\,\mathrm{fm}$.

The addition of the second term to the $s_{1/2}$ potential does not affect the bound state generated by the first term. This is a consequence of the orthogonality relation

$$\int_0^\infty dqq^2[\exp(-\beta_0^2q^2/2)]\left[\left(\frac{3}{2}-\beta_0^2q^2\right)\exp(-\beta_0^2q^2/2)\right]=0. \tag{8}$$

However, the scattering states are affected and the two terms together can give rise to a virtual $s_{1/2}$ state. For $\Lambda^{(2)}_{0(1/2)}=2.696\,\mathrm{fm}$, we obtain a virtual state placed at an energy $\epsilon_v=-40\,\mathrm{keV}$ on the second Riemann sheet. The corresponding scattering length is $a_{s_{1/2}}=-20\,\mathrm{fm}$. We remark here that the virtual state has a $2s_{1/2}$ character, since by increasing slightly the strength of the second term ($\alpha=2$) in the $s_{1/2}$ wave potential, it becomes a bound state with two nodes.

The third term of the $s_{1/2}$ potential is the projection operator for the forbidden $1s_{1/2}$ state, constructed according to the prescription of Kukulin [20]. The corresponding form factor [Eq. (4)], being proportional to the wave function of the bound state produced by the first two terms, is orthogonal to the scattering states generated by these terms. Therefore the third term does not affect the scattering and the virtual state remains unchanged. It is not so for the $1s_{1/2}$ bound state. Although the wave function given by expression (7) remains an eigenfunction for the three-term potential, the corresponding energy is affected. By considering the third term repulsive ($\Lambda^{(3)}_{0(1/2)}<0$), we remove the bound state to the continuum part of the spectrum (therefore, it becomes a continuum bound state) and, by making $\Lambda^{(3)}_{0(1/2)}\to-\infty$, the forbidden $1s_{1/2}$ state is projected out.

For the $p_{3/2}$ potential, we consider a two-term potential with form factors

$$v^{(1)}_{1(3/2)}(q)=q(q^2+\alpha^2_{1(3/2)})\exp(-\beta_1^2q^2/2), \tag{9}$$

$$v^{(2)}_{1(3/2)}(q)=q\exp(-\beta_1^2q^2/2). \tag{10}$$

The first term of the potential is chosen so as to reproduce the $1p_{3/2}$ bound state. With the choice $\alpha_{1(3/2)}=\sqrt{2m|\epsilon_{1p_{3/2}}|}$ and

$$\Lambda^{(1)}_{1(3/2)}=\left[\int_0^\infty dqq^2\frac{[v^{(1)}_{1(3/2)}(q)]^2}{q^2-2m\epsilon_{1p_{3/2}}}\right]^{-1}, \tag{11}$$

the potential produces a bound state of energy $\epsilon_{1p_{3/2}}$ and wave function identical to the $1p_{3/2}$ harmonic oscillator wave function

$$\Phi_{1(3/2)\mu}(\mathbf{q})=N_{1(3/2)}q\exp(-\beta_1^2q^2/2)y_{1(3/2)\mu}(\hat{\mathbf{q}}). \tag{12}$$

For the choice $\beta_1=\beta_0=1.724\,\mathrm{fm}$ and $\epsilon_{1p_{3/2}}=-7\,\mathrm{MeV}$, we obtain $\alpha_{1(3/2)}=0.551\,\mathrm{fm}^{-1}$ and $\Lambda^{(1)}_{1(3/2)}=20.018\,\mathrm{fm}^7$.

The second term in the $p_{3/2}$ potential, with $\Lambda^{(2)}_{1(3/2)}\to-\infty$, is nothing but the projection operator which projects out the forbidden $1p_{3/2}$ state, the scattering states remaining the same as those produced by the first term alone. Thus, the $p_{3/2}$ phase shift is dominated by the occupied $1p_{3/2}$ bound state, there being no resonances as it might occur in the case of a local potential.

The $p_{1/2}$ potential is taken as a one term potential with the form factor given by

$$v^{(1)}_{1(1/2)}(q)=q(q^2+\alpha^2_{1(1/2)})\exp(-\beta_1^2q^2/2). \tag{13}$$

Thus, $v^{(1)}_{1(1/2)}$ is of the same form as $v^{(1)}_{1(3/2)}$. For simplicity we have also assumed the same parameter β_1 in the exponential part of the $p_{1/2}$ and $p_{3/2}$ form factors. We initially make the choice $\alpha_{1(1/2)}=\alpha_{1(3/2)}$, thus getting $\alpha_{1(1/2)}=0.551\,\mathrm{fm}^{-1}$. Using this value and the condition that the $p_{1/2}$ resonance occurs at 0.42 MeV, we obtain $\Lambda^{(1)}_{1(1/2)}=13.535\,\mathrm{fm}^7$. The width of the resonance turns out to be 0.12 MeV and compares with the experimental value mentioned before.

Regarding the interaction between the valence neutrons, we assume a free nucleon-nucleon interaction considering the low nucleon density in the region of the halo. For the 0^+ state which we are considering, if one assumes pure single particle harmonic oscillator states, the $(p_{1/2})^2$ configuration is a superposition of both the $^1S_0$ and $^3P_1$ states (33 and 67%, respectively) of the n-n subsystem, while only $^1S_0$ appears in the $(s_{1/2})^2$ configuration. Calculations by other authors (for instance, Ref. [13]) have shown that the $^3P_1$ potential, which is repulsive, changes the $2n$ separation energy by at least 50%. We therefore use a potential which acts in both $^1S_0$ and $^3P_1$ channels, and take it separable,

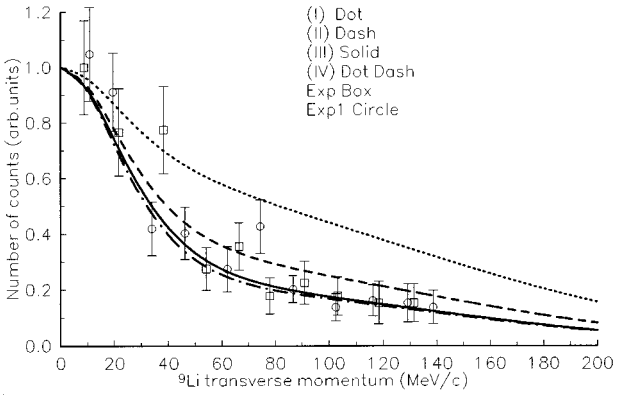

FIG. 1. Transverse momentum distribution of ^{9}Li in the ^{11}Li. The squares and circles are experimental data [26] corresponding to $P_\perp<0$ and $P_\perp>0$, respectively. Curve I corresponds to $\beta_0=\beta_1=1.724$ fm and scattering length $a_{s_{1/2}}=-20$ fm, II to $\beta_0=1.5$ fm, $\beta_1=1.724$ fm, $a_{s_{1/2}}=-20$ fm, III to $\beta_0=1.4$ fm, $\beta_1=1.724$ fm, $a_{s_{1/2}}=-20$ fm, and IV to $\beta_0=1.5$ fm, $\beta_1=1.724$ fm, $a_{s_{1/2}}=-40$ fm.

$$\langle \mathbf{p}|V_{12}|\mathbf{p}'\rangle=\sum_{\lambda SI}-\frac{\Lambda_{\lambda SI}}{M}v_{\lambda SI}(p)v_{\lambda SI}(p')\times\sum_{M_I}\langle\hat{\mathbf{p}}|y_{\lambda S}^{IM_I}\rangle\langle y_{\lambda S}^{IM_I}|\hat{\mathbf{p}}'\rangle, \tag{14}$$

where $\mathbf{p}$ is the relative momentum between the two neutrons, $(\lambda SI)=(000)$ and (111) and the form factors are of Yamaguchi type:

$$v_{000}(p)=[p^2+\alpha_{000}^2]^{-1}, \tag{15}$$

$$v_{111}(p)=p[p^2+\alpha_{111}^2]^{-2}. \tag{16}$$

Using the values $a_{^1S_0}=-17$ fm and $r_{^1S_0}=2.84$ fm for the neutron-neutron scattering length and effective range in the $^1S_0$ channel [21] and $a_{^3P_1}=2.2$ fm^3 and $r_{^3P_1}=-8.0$ fm^{-1} for the $^3P_1$ channel [22], we fix the parameters of the V_{12} potential as $\Lambda_{000}=1.662$ fm^{-3}, $\alpha_{000}=1.130$ fm^{-1}, $\Lambda_{111}=-0.078$ fm^{-5}, and $\alpha_{111}=0.693$ fm^{-1}. The negative value of Λ_{111} means that the $^3P_1$ term is repulsive.

The proposed interactions are used as input in the homogeneous Faddeev equations

$$\Psi^{(1)}=G_0T_1(\Psi^{(2)}+\Psi^{(3)}),$$
$$\Psi^{(2)}=G_0T_2(\Psi^{(3)}+\Psi^{(1)}),$$
$$\Psi^{(3)}=G_0T_{12}(\Psi^{(1)}+\Psi^{(2)}), \tag{17}$$

where $\Psi^{(1)}$, $\Psi^{(2)}$, and $\Psi^{(3)}$ are the Faddeev components of the total wave function

$$\Psi=\Psi^{(1)}+\Psi^{(2)}+\Psi^{(3)}, \tag{18}$$

$G_0=(E-H_0)^{-1}$ is the Green's function, and T_1, T_2, and T_{12} are the T matrices corresponding to the potentials V_1, V_2, and V_{12}, respectively. After performing the angular momentum decomposition, we end up with a homogeneous system of coupled integral equations in one variable. The equations are then transformed into a set of homogeneous algebraic equations using the Gauss quadrature method to approximate each integral by a finite sum. The zero of the determinant of this system of equations gives the separation energy S_{2n} of the two valence neutrons. From the corresponding three-body wave function, we calculate the momentum distributions. We restrict ourselves to the calculation of the transverse momentum distribution of the ^{9}Li core in ^{11}Li. We should mention that, in our model, the transverse and the parallel momentum distributions turn out to be identical. An approximate equality has indeed been verified experimentally for the ^{9}Li fragment in the break-up of ^{11}Li [23]. Corrections due to final state interactions in the case of the momentum distribution of the core fragment should be small according to Ref. [24].

For the chosen values of the parameters, we obtain the value $S_{2n}=0.293$ MeV which agrees with the (average) experimental value 0.294 ± 0.030 MeV, reported in Ref. [25]. However, the width of the transverse momentum distribution of ^{9}Li turns out to be too large, as shown in the figure (dotted line). The experimental data are from Kobayashi *et al.* [26] (actually, the experimental points shown in the figure are taken from Fig. 2 of Ref. [6]). We mention here that, if the $^3P_1$ component of the n-n interaction is suppressed the value of S_{2n} duplicates, becoming equal to 0.597 MeV.

The result may be improved by taking for β_0 a value smaller than the one given by the prescription $\beta_0=1/\sqrt{m\omega}=1.724$ fm. We made a new calculation in which the $s_{1/2}$ and $p_{1/2}$ potentials are modified as follows. Assuming $\beta_0=1.5$ fm and maintaining the previous values $\epsilon_{1s_{1/2}}=-20.028$ MeV and $\alpha_{0(1/2)}=0.932$ fm^{-1}, we determine

TABLE I. Parameters of the two-body $s_{1/2}$ and $p_{1/2}$ potential corresponding to the curves I–IV presented in Fig. 1. These parameters correspond to $\epsilon_{1s_{1/2}}=-20.028$ MeV and $E_r=0.42$ MeV. For the $p_{3/2}$ potential we use $\beta_1=1.742$ fm, $\Lambda^{(1)}_{1(3/2)}=20.018$ fm^{-1}, $\alpha_{1(3/2)}=0.551$ fm^{-1}, corresponding to $\epsilon_{1p_{3/2}}=-7$ MeV. The width Γ of the $p_{1/2}$ resonance and the scattering length $a_{s_{1/2}}$ are also given.

	β_0 (fm)	$\alpha_{0(1/2)}$ (fm^{-1})	$\Lambda^{(1)}_{0(1/2)}$ (fm^5)	$\Lambda^{(2)}_{0(1/2)}$ (fm)	β_1 (fm)	$\alpha_{1(1/2)}$ (fm^{-1})	$\Lambda^{(1)}_{1(1/2)}$ (fm^7)	Γ (MeV)	$a_{s_{1/2}}$ (fm)
I	1.724	0.932	8.417	2.696	1.724	0.551	13.535	0.12	−20
II	1.5	0.932	4.959	2.219	1.724	0.656	10.628	0.16	−20
III	1.4	0.932	3.788	2.008	1.724	0.746	8.502	0.20	−20
IV	1.5	0.932	4.959	2.304	1.724	0.709	9.328	0.18	−40

TABLE II. Three-body results corresponding to the parameters listed in Table I. S_{2n} is the $2n$ separation energy from ^{11}Li and γ is the HWHM of the calculated momentum distribution of the ^{9}Li core in ^{11}Li. The last four columns give the fractional admixture of n-n states in the ^{11}Li wave function.

	S_{2n} (MeV)	γ (MeV/c)	$^1S_0$	$^3P_1$	$^1D_2$	$^3F_3$
I	0.293	82	0.38	0.59	0.01	0.02
II	0.294	40	0.47	0.48	0.02	0.02
III	0.294	35	0.54	0.41	0.03	0.02
IV	0.294	35	0.55	0.40	0.04	0.02

$\Lambda^{(1)}_{0(1/2)}$ to be 4.959 fm^5. By setting $\Lambda^{(2)}_{0(1/2)}=2.219$ fm the virtual state is positioned at $\epsilon_v=-40$ keV, the corresponding scattering length being $a_{s_{1/2}}=-20$ fm. To determine the $p_{1/2}$ potential, we keep β_1 unchanged and adjust $\Lambda^{(1)}_{1(1/2)}$ and $\alpha_{1(1/2)}$ with the condition that the calculated $p_{1/2}$ resonance energy occurs at 0.42 MeV, the resonance width Γ remains inside the range 0.15 ± 0.07 MeV and, in addition, that the calculated $2n$ separation energy agrees with the experimental value 0.294 MeV. The parameters of the $p_{3/2}$ interaction are the same as in the previous calculation. The corresponding transverse momentum distribution is given by the dashed line of the figure. The half width at half maximum (HWHM) is 40 MeV/c and the three-body state is a superposition mainly of $^1S_0$ (47%) and $^3P_1$ (48%) configurations of the n-n subsystem.

Repeating the previous calculation, with a still smaller value, $\beta_0=1.4$ fm, we get 35 MeV/c for the HWHM. The agreement with experiment is very good (solid line of Fig. 1). A similar result for the transverse momentum distribution is also obtained taking $\beta_0=1.5$ fm, and changing the scattering length to $a_{s_{1/2}}=-40$ fm (dot-dashed line). In Table I we summarize the parameters of the two-body $s_{1/2}$ and $p_{1/2}$ potentials used in curves I–IV and in Table II the corresponding three-body results. The increase of the n-n $^1S_0$ contribution in cases II–IV, points to an increase of the contribution of the $(s_{1/2})^2$ configuration to the three-body wave function.

Our calculation shows that relevant ^{11}Li data may be fitted by a three-body model using simple separable potentials with parameters adjusted to two body data. The fact that one needs for the parameter β_0 a value not in accordance with the prescription $\beta_0=1/\sqrt{m\omega}$ indicates that the effective neutron-^{9}Li potential may differ appreciably from the usual single-particle potential. The use of a local neutron-^{9}Li potential should corroborate this conclusion. In fact, one can verify, by using for instance a simple square-well potential, that in order to obtain an intruder $2s_{1/2}$ state near zero energy a much deeper potential than usually is needed. Such odd potentials may indicate that a more detailed description, which does not consider the core as a structureless object, is required.

The numerical calculations were performed at LCCA-USP.

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