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Collective flow in central Au-Au collisions at 150, 250, and 400A MeV

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Radial collective flow and thermalization are studied in gold on gold collisions at 150, 250, and 400A MeV bombarding energies with a relativistically covariant formulation of a QMD code. We find that radial flow and ''thermal'' energies calculated for all the charged fragments agree reasonably with the experimental values. The experimental hardware filter at small angles used in the FOPI experiments at higher energies selects mainly the thermalized particles. [S0556-2813(99)00603-2]

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In recent years central-collisions studies became a focus of attention in the intermediate energy domain (100−−500A MeV) [1,2]. One of the measurables concentrated on by the experiments is connected to the flow. It is a well-known fact [3] that at larger impact parameters there is a sideward flow and a squeeze-out flow; these quantities were measured [4] and calculated [5]. The sideward and squeeze-out flows mostly disappear in very central collisions. However, calculations [5] predicted a large collective energy (radial flow) in central collisions and this was confirmed later by experiments [6]. This collective energy can be visualized with a blast model [1,7], where the system expands spherically around the center of mass.

Recently we deduced a momentum dependent, relativistically invariant two-body force [8], which can be applied to QMD calculations [9]. To check the validity of the force, we made detailed calculations for central gold on gold collisions in the energy domain 150–400 A MeV [8] and compared the results with experimental data [1]. The agreement turned out to be highly satisfactory; even the number of the intermediate mass fragments (IMF) is very close to the experimentally measured value. It is worth examining what can we learn about the radial flow using this model.

First we study to what extent one may speak about thermalization of the nucleons in central and semicentral collisions. The system is usually assumed to be thermalized if the nucleons collided a few (~ 3) times. From Fig. 1 one can see that for higher energies (250–400 A MeV) only 20% of the nucleons did not collide more than twice (and may be considered as unthermalized). At low energy (150A MeV) this fraction amounts to 60%, due to the large Pauli blocking (for 150A MeV 65% of the possible collisions were blocked, while for 400A MeV the blocking was only 25%). We followed the path of some nucleons in collisions with 400A MeV energy. Those nucleons which did not collide at all (less than 3%) are generally positioned in the outer layer of the colliding nuclei.

Assuming that nucleons having collided at least three times are thermalized, one may conclude from Fig. 1, that at least for high energies the thermalization rate is $\sim 80\%$. For a spatially homogeneous system this would mean an isotropic momentum distribution. For this reason it is interesting to examine the momentum distribution by making a contour plot of the invariant cross section $d^2\sigma/p_T dp_T dy$ of the outgoing single protons in the two dimensional space of the transverse momentum scaled with the projectile momentum, p_T^p, and the rapidity y scaled with the projectile rapidity y^p in the center of mass system. We note that in the energy domain of interest (up to 600 MeV) the scaled rapidity and scaled momentum are equal to each other within 5%. As a consequence, a thermalized distribution should appear as circles in the y-p plane. First we examined the contour plots without experimental filters at $b=0.5$ fm and 400A MeV collision energy after 150 fm/c evolution. One can see from Fig. 2, that in contrast to a thermalized system, the raw data do not show an isotropic distribution (see the most inner-highest multiplicity contour, having a distortion factor ~ 1.5). However, applying *only* the seemingly negligible small laboratory forward angle filter (excluding particles at $\theta_{\text{lab}} < 1.2°$), which means the exclusion of a small number of protons only, a nearly isotropic distribution is recovered. The interpretation of this result is the following: the unthermalized (not yet collided) protons leave the collision region with high energy in the forward direction; thus, the remaining proton distribution is already thermalized. Applying further angular filters does not change this behavior.

Figures 3 and 4 show contour plots for protons at 150 and 400A MeV, respectively, for central and peripheral events. In order to compare our result with the experimental one, where the effect of the filters in the range $1.2° < \theta_{\text{lab}} < 30°$ was averaged out, we took only the relevant $1.2° < \theta_{\text{lab}}$ and $\theta_{\text{lab}} < 30°$ filters. One can see that for central events the filtered distributions are almost isotropic (except for the $\theta_{\text{lab}} < 30°$ filters), however, the raw, unfiltered values are not. This fact is very pronounced in the case of 150A MeV. In the case of large impact parameters the distributions are always distorted. The same observation was made in connection with the experimental measurements [1].

Experimentally the collective radial flow is determined from the kinetic energy distribution of the large fragments. However, since at 400 MeV the total mass of all large fragments for central collisions is less than 3% of all the particles, such a method produces very poor statistics in our case. In order to determine the radial flow we considered two possibilities: calculating the flow in the (1) *gas scenario*, when all the nucleons are considered to be individual objects

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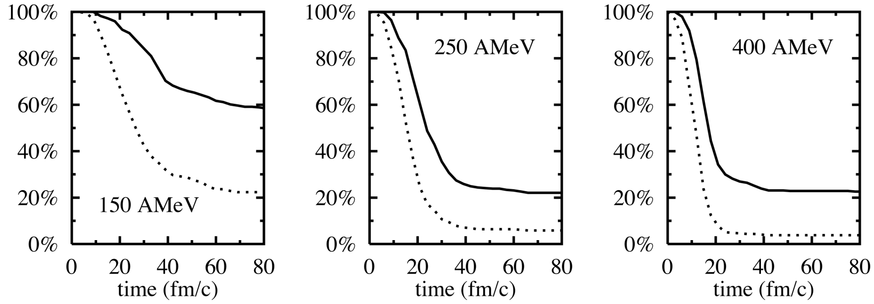

FIG. 1. Number of particles that collided less than three times (solid line), and without collisions (dotted line) for three incident energies, at $b=0.5$ fm impact parameter, as the function of time. Initially $(t=0)$ the nuclei are at a distance of 2 fm from each other and the total overlap for free evolution would occur at 29, 23, and 19 fm/c for the three energies, respectively.

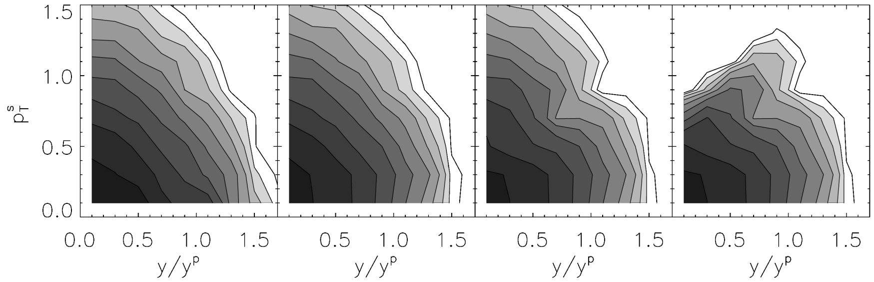

FIG. 2. Contour plots averaged over 360 events for 400A MeV collisions with $b=0.5$ fm impact parameter. Shown are the raw proton distribution (far left), the distribution with a $\theta_{\mathrm{lab}}>1.2°$ experimental hardware filter (center left), with an additional $\theta_{\mathrm{lab}}\sim 21°$ filter (center right), and with an extra $\theta_{\mathrm{lab}}<30°$ filter (far right) in the scaled momentum-rapidity plane. The hardware filter around $\theta_{\mathrm{lab}}\sim 5°$ is not visible. The contours are separated by factors 1.5.

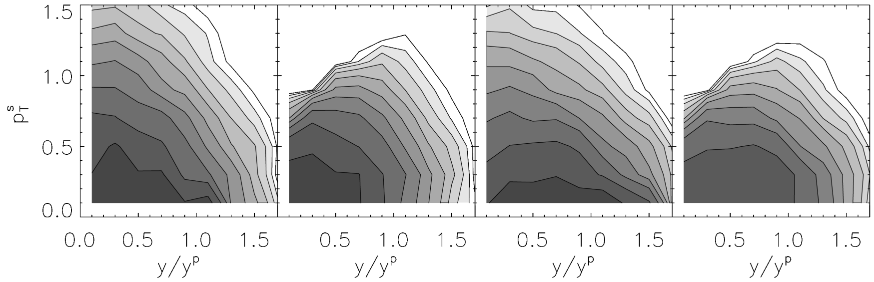

FIG. 3. Contour plots for proton distributions at 150A MeV, $b=1.5$ fm (left block of 2 panels), and $b=7.5$ fm (right block of 2 panels) in the scaled momentum-rapidity plain. The left figures of each block show the raw data, while the right-hand ones show the data with the most relevant $\theta_{\mathrm{lab}}>1.2°$ and $\theta_{\mathrm{lab}}<30°$ hardware filters.

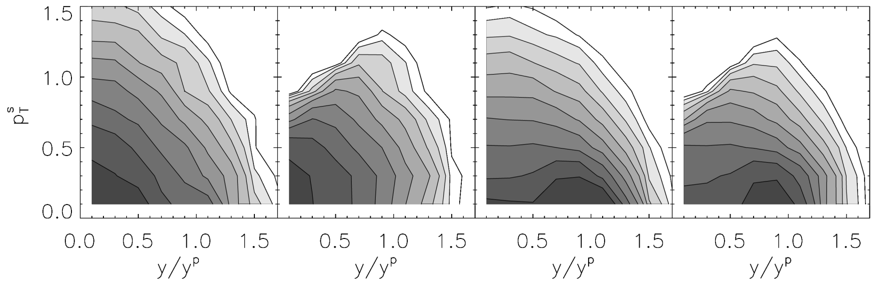

FIG. 4. Same as Fig. 3 at 400A MeV.

TABLE I. Average flow energies (MeV), ‘‘disoriented/thermal’’ energy, and flow velocity at 400 (upper), 250 (middle), and 150A MeV (lower part) at $b=1$ fm impact parameter for the gas (g) and cluster (c) assumption.

	$E_{\rm flow}$	$E^{\rm exp}_{\rm flow}$	$E_{\rm do}$	$E^{\rm expt}_{\rm th}$	β	$\beta^{\rm expt}$
g (400)	63.2±2.5		37.2±3		0.345±0.006	
c (400)	59.7±2.5	56.8±6.3	32.7±2	32.8±6.3	0.338±0.006	0.334±0.017
g (250)	41.2±1.2		18.2±0.5		0.285±0.002	
c (250)	34.8±1.0	34.0±3.9	16.8±0.6	21.5±3.9	0.264±0.002	0.263±0.014
g (150)	21.6±0.5		12.9±0.8		0.21±0.002	
c (150)	18.3±0.3	19.9±2.3	10.2±0.5	12.6±2.3	0.194±0.002	0.204±0.011

contributing to the radial flow; or in the (2) *cluster scenario*, when all the charged fragments (single protons and clusters) are taken into account. For comparison we present results for both.

In order to determine the radial flow of the nucleons we divided the solid angle into 32 equal pieces and calculated the flow velocity $\beta_{(k)}$ within each sector k ($k=1,32$) as

$$u_{(k)}=\sum_{i\in\Omega_k}\frac{\vec{r}_i\vec{p}_i}{|\vec{r}_i|}\Bigg/\sum_{i\in\Omega_k}m_i\,,\quad \beta_{(k)}=\frac{u_{(k)}}{\sqrt{1+u^2_{(k)}}},\qquad (1)$$

where $u_{(k)}$ is the four velocity associated to sector k and the summation is extended for all the charged particles within the solid angle Ω_k, with m_i being the mass of a fragment, while r_i is the position of the charged particle at the end of the calculation (150 fm/c). We repeated the same for the gas scenario with each m_i being the nucleon mass.

The fluctuation of the flow velocity is less than 2% in all the 32 sections for each energy at $b=0.5$ fm both for the cluster and gas algorithm. Furthermore, the fluctuation of the number of particles within the sectors is at most 3–4 %. The extracted values are also stable against changing the end time of the calculation: from the freeze-out time up to twice the freeze-out time the change of the flow energy is 10% and does not change further for larger end times. These results show that the radial flow can be determined in a very reliable way. Encouraged by the isotropy of the system and by the lack of other collective excitations (side flow or squeeze-out) we define a disoriented (‘‘thermal’’) energy as the rest kinetic energy,[1]

$$E_{\rm do}=\frac{1}{\sum_{i=1}^{M}A_i}\sum_{i=1}^{M}\left(\sqrt{m_i^2+p_i^2}-m_i\right)-E_{\rm flow}\quad\text{with}$$

$$E_{\rm flow}=\frac{1}{\sum_{i=1}^{M}A_i}\sum_{i=1}^{M}\left(\sqrt{m_i^2+\frac{(\vec{r}_i\vec{p}_i)^2}{r_i^2}}-m_i\right),\qquad (2)$$

where A_i is the mass number of a cluster, and M is the total multiplicity of the clusters (and $M=A$, the total number of particles, for the gas algorithm).

In Table I we give the calculated flow velocity, the flow energies, and the ‘‘disoriented/thermal’’ energies evaluated both for the gas and for the fragments (cluster). As a comparison, we give the values extracted from Ref. [1], where they use a blast wave model fit to the experimentally measured kinetic energies of heavy fragments. We note that these quantities are model dependent, and evaluating them we used the natural way for the QMD model, not the experimental procedure. We find the agreement surprising.

Finally we make a remark comparing our result to the one of the EOS [2] group. In this experiment the selection criteria were based on the multiplicity only and the deduced values of the radial flow velocity are considerably lower compared to the flow velocities of the FOPI experiment. In our model we find that the multiplicity trigger is not sufficient enough for selecting the central collisions; we got a considerable amount of events contributing from $b=4.5$–5 fm. As a result, using the multiplicity trigger, our flow velocities, β, are reduced by ~15–20 % in agreement with the EOS result. Consequently the ‘‘thermal’’ energy is increasing by the same amount.

In conclusion, we have investigated collective radial flow in the case of Au+Au collision at 150, 250, and 400A MeV. The results were compared to FOPI experiments [1] and we found a reasonable agreement. Furthermore the experimental setup automatically filters out the unthermalized particles for higher energies. These results suggest that the compressibility and the momentum dependence of the used force is highly satisfactory. The application of our force for higher energies when particle creation is important, is in progress.

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[1] The energy defined here is *not* exactly the thermal energy, since a thermal energy can be defined for completely thermalized systems only in the local rest frame. As the complicated experimental procedure obtaining the thermal energy cannot be reproduced within the given model, we consider the disoriented energy $E_{\rm do}$ to be close to the thermal one.

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