

J/ψ suppression in heavy ion collisions at the CERN Super Proton Synchrotron

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We reexamine the production of J/ψ and other charmonium states for a variety of target-projectile choices at the CERN Super Proton Synchrotron, in particular for the interesting comparison between S+U at 200 GeV/c and Pb+Pb at 158 GeV/c as observed in the experiments NA38 and NA50, respectively. For this study we use a newly constructed cascade code LUCIFER II, which yields acceptable descriptions of both hard and soft processes, specifically Drell-Yan and meson production. This code divides the ion-ion collision into an initial phase involving hard interactions of the original nucleons and no soft energy loss, followed after the meson formation time by a “normal” low energy cascade among the secondary particles. The modeling of the charmonium states differs from that of earlier workers in its unified treatment of the hidden charm meson spectrum, which is introduced from the outset as a set of coupled states $\{\psi, \chi^i, \psi'\}$. The result is a description of the NA38 and NA50 data in terms of a conventional, hadronic picture. The apparently anomalous suppression found in the most massive Pb+Pb system arises in the present simulation from three sources: destruction in the initial nucleon-nucleon cascade phase, use of coupled channels to exploit the larger breakup in the less bound χ^i and ψ' states, and comover interaction in the final low energy phase. [S0556-2813(99)01803-8]

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I. INTRODUCTION

The possible use of J/ψ suppression as a signal of unusual behavior in relativistic ion collisions, first suggested by Matsui and Satz [1], has attracted considerable experimental and theoretical study. Great interest has attached to the results obtained by the NA50 Collaboration for charmonium production in Pb+Pb collisions at 158 GeV/c: to the early findings presented at the Quark Matter 1996 meeting [2] as well as to the startling data later released at RHIC'97 [3]. The success of Glauber-like calculations of J/ψ production and breakup in the $p+A$ and S+U [4–6] systems, coupled with a failure of Glauber to provide an equally good description of the apparently accelerated absorption in Pb+Pb has been widely interpreted [2,3,5] as a signal of QCD plasma creation in these collisions. The very sharp behavior of the J/ψ yield as a function of transverse energy E_t , seen in the later experiment [3] has especially attracted attention.

We attempt to retrace this ground theoretically, employing a new, two phase cascade approach, described in detail elsewhere [7,8], combined with a variation of the Satz-Kharzeev model for production and annihilation of charmonium in the initial baryonic collisions. This modeling described below, allows the coupled-channel aspect of the hidden charm spectroscopy, $\{\psi, \chi^i, \psi'\}$ to play a more central role. The comparison with Glauber theory based models is done with two purposes in mind: first to understand the differences with the cascade if any, and second to help place the cascade on a firmer foundation, paradoxically, by indicating how similar the first high energy phase of the cascade is to the Glauber model. In this first application, we include partons in a minimal fashion, to describe for example Drell-Yan production. Hence we are testing a “purely” hadronic description of the anomalous Pb+Pb measurements. From the evidence presented in Figs. 9, 10, 14, and 15, it would appear such a test is justified.

It has been pointed out that a hadronic picture might succeed [6] without invoking quark-gluon plasma (QGP) creation, if at least part of the seemingly anomalous suppression in Pb+Pb could be produced by comover annihilation, i.e., by interactions of the J/ψ with secondary mesons generated in the ion-ion collision. The second phase in LUCIFER II, which is a low energy cascade, perforce includes the effect of J/ψ destruction through such comover rescattering. Also included are later comover interactions between the charmonium states and baryons, a significant component.

We begin with a description of the motivation behind LUCIFER II and a brief outline of the two step cascade. We attempt to separate hard and soft processes by time scale, so as to permit partonic and hadronic cascading to be joined naturally, in a modular fashion. The separation is effected through the use of a short time scale, automatically present at high energies: the time T_{AB} taken for the two interacting nuclei A and B to traverse each other in the global collision frame. The uncertainty principle allows hard interactions involving sufficiently high energy-momentum transfer, i.e., for $Q^{-1} \leq T_{AB}$, to take place in the first and very rapid cascade. Soft processes involving low transverse momentum are not completed until later. Thus in the initial fast cascading the nucleons *lose no energy* but are still aware of the number and nature of the two-particle collisions they have undergone.

Specifically, the method [8] consists of running the cascade in two stages. The first is a high energy fast-time mode in which collision histories are recorded and fast processes (here only Drell-Yan and charmonium production) are allowed to occur. Using the entire space-time and energy-momentum history of this stage, a reinitialization of the cascade is performed using elementary hadron-hadron data as a strict guide. The final positions and momenta of baryons in the first phase, and the number of collisions they suffer are recorded and used to generate produced mesons together with their initial momentum and space-time coordinates. In

the initial ion-ion collision the interacting nucleon paths are almost along light cones. The second cascade begins at T_{AB} , the time of the last nucleon-nucleon collision, with initial conditions specified by the reinitialization, but no secondary interactions are allowed until a formation time for produced mesons has passed. The participants in the second phase are generic mesons, thought of as of $q\bar{q}$ -like in character with masses centered near $M_{q\bar{q}}=700$ MeV and in the range $M_{q\bar{q}}\sim 0.3-1.1$ GeV. Generic baryons consisting of qqq are also included and are excited to rather light masses, $M_{qqq}\sim 0.94-2.0$ GeV [8]. All the generic hadrons decay *via* sequential pion emission. Normal stable mesons and baryons are also present, and terminate the decay chains.

Many cascades [8–15] have been constructed to consider relativistic heavy ion collisions. Since the eventual aim of experiments designed to study such collisions is the creation of a regime in which the quark-gluon structure of hadronic matter becomes evident, it will ultimately be necessary to include the partonic degrees of freedom in such cascades. However, since at SPS and even at RHIC energies it is by no means clear that all initial or subsequent hadron-hadron collisions occur with sufficient transverse momentum to free all partons [16], at least a part of the eventual simulation must deal with collisions both of the initial baryons, in fact nucleons, and of all the produced mesons.

Kharzeev and Satz [5] employ a hadronic model based on Glauber theory describing production and breakup of the J/ψ in ion-ion collisions, to demonstrate that such a picture cannot account for the degree of suppression seen in Pb+Pb collisions at the SPS. Reasoning similarly, we can make a close comparison of our treatment with their work. The required initial production of a $c\bar{c}$ pair is handled within an effective hadronic formulation both in our work and in that of Kharzeev *et al.* There are, naturally, specific and important differences between Glauber theory and a cascade model, and it is partly these differences which permit the so-called anomalous suppression in Pb+Pb to be explained within a purely hadronic framework. Additionally, the cascade provides a real calculation of E_t as a function of centrality, i.e., impact parameter, with no adjustable parameters available for producing agreement with the experimental measurements. Equally important, the interaction of $c\bar{c}$ states with comoving secondary mesons is treated dynamically in the soft phase of our two step cascade.

The overall degree of suppression in Pb+Pb, insofar as it differs from earlier work [5,6], results from a combination of effects; these are baryonic, coupled channel and comover in kind, with substantial contributions arising from both phases of the cascade. There are potential unknown variables: the production and dynamic time evolution of each charmonium state, the breakup probabilities against both baryons and mesons, the density of secondary mesons. This last is to a large extent predicted by the cascade, which must agree with actual inclusive final state meson and baryon distributions. There also exist constraints on the basic charmonium variables. The production is in principle determined in elementary nucleon-nucleon collisions, the baryonic breakup in $p+A$ collisions. The $\psi\pi$ breakup cross-sections are not known directly from any measurements. If the relative ψ production in NN and πN systems may be taken as a guide

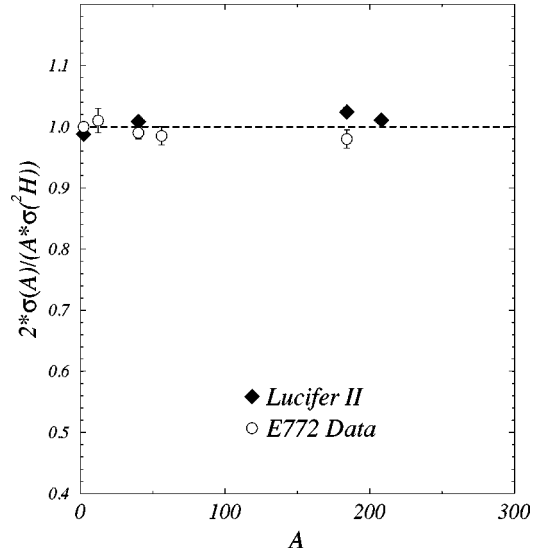


FIG. 1. A dependence of Drell-Yan in $p+A$ at 800 GeV/c: E772 (FNAL) vs LUCIFER. Minimum bias dimuon production as shown is calculated microscopically using NA3 structure functions [17] but could as well have been obtained directly from considerations of the total collision number. The same argument applies to the high mass dimuon cross section as a function of E_t .

here, the $\psi\pi$ and ψN breakup cross sections could be expected to be directly comparable. We have used a factor 2/3 to relate the charmonium-meson to charmonium-baryon breakup cross sections, but also employed equal charmonium-meson as a test.

The success obtained in describing the meson spectra has already been presented in [8], and of course is relevant to the degree of charmonium destruction by comovers. In particular, the selection of a meson formation time is tied to this latter issue. The differences between S+U and Pb+Pb, which exhibit a considerable increase in the product $A \times B$, arise both from baryonic processes and meson production.

The source code to LUCIFER II is made available on the world wide web at <http://bnlnth.phy.bnl.gov>, and may be downloaded either directly from the web site or by anonymous ftp to bnlnth.phy.bnl.gov. It is in C and should be relatively easy to port to any UNIX system. Linux, AIX, SunOS, IRIX and HP/UX ports have at one time or another been made.

II. THE TWO PHASE CASCADE

We present in this work a mere outline of the cascade architecture, details having been provided in earlier work [8]. We already noted the global time scales which divide the cascade into two steps, the first loosely designated as “hard,” the second as “soft.” Energy loss and meson production associated with low transverse momentum p_t are slow, processes. In contrast stand fast or “hard” processes, involving large p_t , of which production of high mass Drell-Yan pairs [17–19] is a good example. The A dependence of minimum bias Drell-Yan data [18] in $p+A$ collisions, see Fig. 1, suggests that high mass lepton pair production occurs only at the highest collision energy. For a theory of charmonium suppression to be taken seriously, it must simulta-

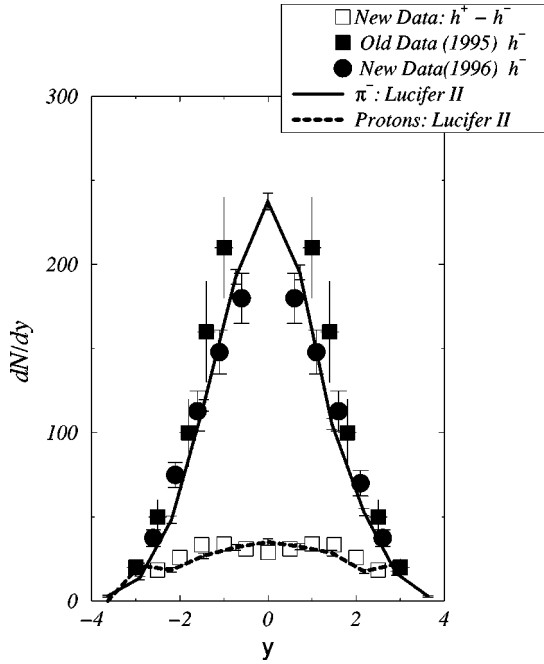


FIG. 2. The calculated Pb+Pb rapidity spectra [7,8] at 158 GeV/c for π^- and protons compared to measurements by NA49. The latter are for total h^- and $h^+ - h^-$, respectively.

neously explain this striking feature of Drell-Yan data, and the substantial soft energy loss experienced by the projectile nucleon. If not, a spurious A -dependent suppression in $p + A$ might be built into the model, attributable to production in successive nucleon-nucleon collisions occurring at lower and lower energies, where the chance of J/ψ production is considerably less. The present model does well in this regard as may be seen in Fig. 1.

Nevertheless, calculations with a purely hadronic cascade [7] describe very well the nucleon energy loss and inclusive pion spectrum seen in massive Pb+Pb collisions at SPS energies (see Fig. 2). These apparently contradictory features of the data, considerable soft energy loss occurring together with A -independent Drell-Yan production, can in fact be united in a resonance based multiscattering picture, which takes account of the different basic time scales involved. Following the high energy cascade stage in which collision histories are recorded and hard processes engage, the cascade is reinitialized and a second hadronic cascade is carried out at greatly reduced energy.

A. First phase: high energy collisions

As indicated, the procedure used here is relatively straightforward and in outline resembles the eikonal or Glauber calculations made by previous researchers [4–6], but retains the random, fluctuating, collisional nature of a cascade. This stage serves to establish the space-time geometry of the interactions between the target and projectile nucleons. Any actual hard processes which occur in this stage result in real energy loss, but soft processes do not occur yet, they are delayed until the reinitialization. So, if a Drell-Yan pair is produced, its energy is immediately subtracted, similarly for a $c\bar{c}$ state or an open charm pair (we do not produce open charm at the moment, except by breakup of

hidden charm mesons). Clearly jets or partons could be produced in the same way and their evolution followed up to hadronization, when the products would simply be included during the reinitialization before the second phase.

In practice, since Drell-Yan and J/ψ production are very rare processes, it is not necessary actually to conserve energy globally in the simulation; this would in any case lead to considerable difficulty since the basic production rates must be considerably increased artificially to make calculations possible in a finite time. Therefore, energy is conserved in the first stage in the sense that realistic rapidity, p_t and mass (for Drell-Yan pairs) distributions are employed for the appropriate elementary NN collision energy.

Needless to say, the first stage interactions of preresonant charmonium states produced in the first stage cannot be neglected. This is because the $c\bar{c}$ pairs are produced relatively quickly, and may undergo high energy collisions with target or projectile nucleons still in the way. Therefore these interactions are also counted for each charmonium, though actual breakup or feeding to another channel is left until the reinitialization.

B. Reinitialization

The fast cascade history is used to set up initial conditions for the second low energy cascade. To begin, one needs positions and momenta for the baryons and for the mesons *expected* to be produced from the initial baryon-baryon collisions. In the reinitialization, the nucleon-nucleon interaction history, together with the entire trajectory of each participant, is used to set up groups of nucleons which have mutually interacted in the first stage. The structure of the groups is virtually dictated by consideration of the $p + A$ system, where the projectile proton collides successively with those target nucleons which are directly in its path. This simple grouping can be easily generalized to $A + B$ collisions by using a procedure which kinematically symmetrizes each group with respect to target and projectile [8].

The experimentally known averages and fluctuations inherent in NN scattering, in energy loss, multiplicity, and character (flavor, etc.) of produced mesons are all used to produce additional generic mesons associated with each nucleon group. Momentum, charge, baryon number and flavor conservation are imposed on each group of baryons and associated mesons. These generic mesons are the principal cascaders in the second stage, along with the baryons.

The present work adds the possibility of producing and destroying hidden charm $c\bar{c}$ states in each of the two stages of the cascade. The production is accomplished almost totally in the high energy phase, and uses elementary production cross sections normalized to pp measurements. Breakup is done using the collision history for the charmonium collected in the first stage, through an interaction matrix described in the section on coupled channels. These processes are constrained by pp and $p + A$ measurements [2,18,20].

The final step in the reinitialization places mesons and baryons in position and time to restart the cascade. The four-momenta, in the global frame, of all particles are known, as are the space-time coordinates of the initial nucleons. It was thought best to distribute mesons produced within a group randomly along the paths of the baryons in that group. This

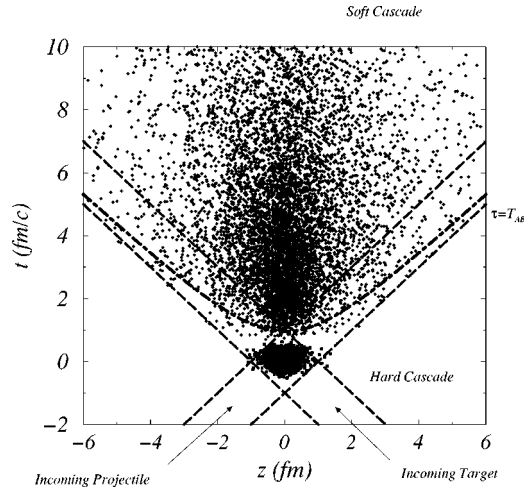


FIG. 3. Time evolution of the ion-ion collision. The distribution in space-time of collisions and decays in hard and soft cascades is shown for the minimum bias Pb+Pb system. The initial paths of the incoming nuclei are close to lightlike and result in the dense initial blob of hard collisions. The soft cascade occurs after the formation time of mesons has passed, as indicated roughly by the constant proper time surface.

choice would seem reasonably consistent with locality principles. In any case, sins committed in this way are remediated by the formation time that must be attributed to each meson before it can begin to interact in the last phase. See Fig. 3 for a spacetime picture of the collisions in the entire cascade, both soft and hard phases.

C. Soft phase

The second phase involves, for the most part, generic resonances, baryonic and mesonic, having the quantum numbers of the N^* , Δ , π , and ρ . As stated, these have masses between $M_N=0.939$ GeV and about 2 GeV for baryons and from 0.3 GeV to 1.1 GeV for nonstrange mesons, and appropriately higher for strange mesons. The mesonic resonances are not allowed to interact until a sufficient formation time has transpired. This time, τ_f , is a *real* adjustable parameter in the model, to be fixed perhaps from $p+A$, or as in earlier work from light nucleus collisions (S+S) [8]. We imagine that these resonances are broad s -wave excitations of the underlying representatives, not series of special and very narrow states such as are tabulated in the particle data book. These narrow states cannot be excited very much in actual ion-ion collisions, and do not carry much of the pp cross section. Finally, these generic resonances decay by sequential pion emission into lower mass excitations, losing mass with each decay until physical π 's, ρ 's, and K 's, or stable baryons are the only open channels.

The decay time for the generic excitations is a second real parameter, perhaps to be taken inversely proportional to the excitation mass, but here for simplicity fixed at $\tau_d \sim 1/125$ MeV $^{-1}$. It is often assumed that one's lack of knowledge of resonance-resonance scattering opens up to cascade models a deep well of adjustable parameters. This is not the case here. We employ, as did Gottfried [21], a "universality" principle for soft interactions. Surely, for soft baryon-baryon interactions all fine details, excepting perhaps

String-Like Model for Hadron-Hadron Scattering

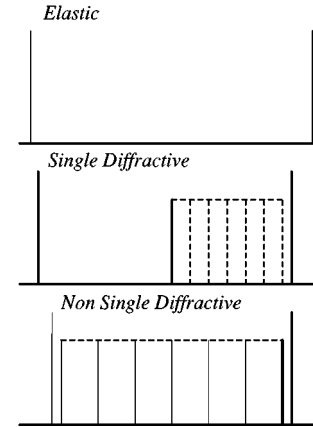


FIG. 4. Shown are graphic representations of the elements of the model for the elementary hadron-hadron collision: elastic, single diffractive (SD), and nonsingle diffractive (NSD). The meson groups introduced in both SD (with a rapidity gap) and NSD have a stringlike character but divided into our generic resonances. It is customary to associate SD with a three Pomeron coupling and NSD with one, two, or more Pomeron exchange.

size and mass thresholds, are irrelevant in determining what must be, basically, interactions driven by many gluon exchange. We ignore size differences for the moment, with the exception of such very small objects as charmonium mesons. This limits the number of free parameters in the model to a minimum, in fact only a few, so far the two times τ_f and τ_d . The important pp , $p\bar{p}$, and πp data, determined from experimental measurements over a wide range energies at which cascading takes place, are the primary inputs, and serve to fix all of the baryon-meson and baryon-baryon interactions. Meson-meson interactions are fixed by appealing to constituent quark model counting.

III. MODEL FOR HADRON-HADRON INTERACTION

The objective of the cascade approach to ion-ion collision is to proceed from a knowledge of elementary hadron-hadron collisions to a prediction of the far more complex many body event. Many approaches have been put forward [11,13,14] including strings [9,10], but we prefer to retain a particle nature for the cascade.

The required input is a model for the elementary hadron-hadron system, beginning with nucleon-nucleon but easily extended to meson-nucleon and ultimately applied to any two body hadron-hadron collision. The basic processes are elastic scattering and inelastic production of mesons. The latter we divide into the well known categories [23]: diffractive scattering, referred to as single diffractive (SD), and nonsingle diffractive (NSD) [24]. A graphic description of these processes is given in Fig. 4. These diagrams are the basis for our hadron-hadron model but must be supplemented by an intermediate picture which allows us to apply them, not only to hadron-hadron interactions in free space but also inside a nuclear environment.

The generic mesons depicted in Fig. 4 and generic baryons, having rather light masses selected in the ranges suggested above, constitute the basic elements for rescattering in

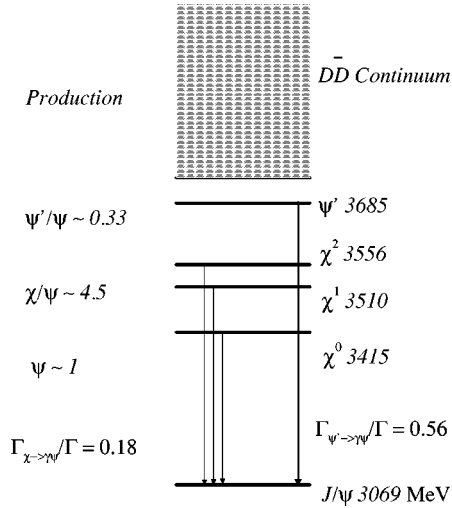


FIG. 5. Charmonium spectroscopy including higher mass states which are significantly produced in pp and which feed strongly to the J/ψ . Electromagnetic and hadronic decays of χ^i (a weighted average) and ψ' are both included in the indicated branching ratios. The production ratios are suggested by direct measurement.

the second-phase cascade. Our principal phenomenological sources are tied to the rapidity [23] and multiplicity [24–26,22] information obtainable from the elementary collisions. For example we invoke KNO scaling [27] though there are slight, measured deviations at higher SPS energies. Details are given in Ref. [8]. The crucial point is that our soft meson and baryon spectra in $A+B$ collisions are not arbitrary but to a large extent determined by free space hadron-hadron dynamics.

IV. COUPLED CHANNEL MODEL FOR CHARMONIUM

The treatment of the hidden charm $c\bar{c}$ mesons within a “purely” hadronic code presents some problems, perhaps not fully solvable within the effective hadronic treatment of such states. We do not deviate much in spirit from the work of previous researchers [4–6], but the devil lies sufficiently in the details to produce some quantitative effects. The production of charmonium mesons is almost completely limited to that coming from nucleon-nucleon collisions at the highest energies, i.e., in the initial high energy cascade, not by fiat but by the greatly reduced collision energies in the second phase. Destruction of the charm meson precursors, in contrast, can occur in the first baryonic phase and also later in collisions with generic mesons and baryons in the second, low energy phase, i.e., on comovers. It is in the destruction of the charmonium states that we differ most, ascribing a more direct role to the presence of the higher mass χ and ψ' mesons, for which in fact breakup is far easier. We include in Fig. 5 a level diagram showing the relevant charmonium states to make the picture as clear as possible.

In the actual calculations of the above cited references both production and breakup are treated as instantaneous. There is no J/ψ formation time in the high energy phase or its near cousin, the Glauber or eikonal modeling. Kharzeev *et al.* [5] in fact justify such a choice by referring to microscopic production of charmed quark pairs, the subsequent formation of a preresonant state from which all charmonium

mesons emanate, and the breakup in relatively hard scattering by gluons radiating from nearby nucleons. All of this can be incorporated in the initial cascade by ascribing to NN collisions a production cross section normalized to the measured elementary pp and pn data and similarly a baryonic breakup deduced from say $p+A$ measurements. Nevertheless, the eventual result is an effective hadronic modeling for the charmonium states, ascribing to a produced $c\bar{c}$ pair a hadronic state which can be destroyed.

We imagine that the primordial $c\bar{c}$ pairs are originally produced essentially in plane wave states. Clearly, both singlet and octet color states are involved. This view would seem to be a reasonable one given the predominance of open charm production over hidden charm production in free space NN collisions. We further suppose that in elementary NN collisions the $c\bar{c}$ pair eventually coalesces, with a state dependent probability, into a $J/\psi, \psi'$ or χ . The time which elapses will be determined by the size of the bound state and the probability that a transition occurs. The probability of formation will depend critically on the relative momentum of the coalescing pair as well as on their spatial separation. In any picture of charmonium generation there must needs be some formation time τ_b for the bound state, which may be longer than the total duration of the fast, baryonic cascade T_{AB} . Then, in an ion-ion collision, additional transitions may be induced: into the continuum, i.e., breakup, or perhaps also between bound states.

Therefore, whether one sees the early evolution of the eventual charmonium as a preresonant state or as plane waves may be immaterial. Given the small size of the J/ψ as opposed to the much larger ψ' and χ , the separation of the c and \bar{c} in the plane wave picture could equally well serve as a distinguishing feature. What additionally differentiates our calculation from earlier models is the possibility of transitions occurring between charmonium states dynamically in a nuclear environment. Such transitions may be easily implemented by allowing them to occur, with some probability at each collision of a charmonium. Certainly, the χ^1, χ^2 , and χ^3 states are produced considerably more copiously in basic pp collisions, with perhaps as high a ratio as $\chi/\psi = 4-5$ [28], and they decay appreciably into J/ψ , with branching ratios in the range $\Gamma_b/\Gamma \sim 12-25\%$ [29]. The ψ' also feed some 57% into J/ψ . It follows that one cannot ignore their presence.

This point becomes even more significant when one considers what the breakup probabilities for the higher mass charmonium states are likely to be, either in the fast or slow cascades. These heavier objects are considerably larger spatially and might well have total cross sections on baryons or mesons proportional to the square of their color dipole radius [30]. In any case, in the precursor states of the initial rapid cascade the spatially larger charmonia will have more time for collisions before forming, and should then possess larger effective breakup strengths. One of our conclusions will be that a considerable portion of the anomalous suppression seen in Pb+Pb, even for quite large impact parameters, is a result of breakup in the higher charmonium states and an extinction of their free space feeding down.

What extra parameters has our model introduced relative to other treatments? We introduce breakup cross sections for

each of the charmonium states and in principle a matrix R_{ij} between these states, permitting transitions to occur between the states in the nuclear environment, i.e., only after their initial production. The breakup cross sections for each of the charmonium states are specified in the sections on specific calculations. The transition matrix, which should be constrained by detailed balance, is for the present purposes just taken as $R_{ij} = \delta_{ij}$. That is, in this work, *no feeding between the states is allowed*.

The only significant off-diagonal component that we might consider necessary is that for the ψ to ψ' transition, expected to be small but potentially influential. The dynamics consists then of producing the charmonium states in hadron-hadron collisions, with probability weighted by their production cross sections inferred from experiment, allowing them to propagate through the cascade, and either elastically scatter or breakup, again through weighting by assigned cross sections, as specified elsewhere in this work. The breakup matrix elements in the second phase cascade, i.e., for breakup on comoving mesons, are scaled from the first phase by counting constituent quarks.

The formation time for secondary mesons, τ_f , controls the initiation of the second cascade and thus the onset of comover destruction of charmonium and the density of comovers. A reasonable choice for this parameter is $\tau_f \sim 0.5 - 1$ fm/c, and this is in fact consistent with the production of π mesons in ion-ion collisions at the SPS [8,31,32]. The high density of comovers which obtains at these times implies they play a considerable role. The effective formation time is actually somewhat longer, since it is increased by the duration of the fast cascade, i.e., $\tau_{\text{eff}} = \tau_f + T_{AB}/2$.

The energy dependence of the elementary J/ψ production cross sections is shown in Fig. 6. The sharp dependence of $\sigma_{J/\psi}$ on energy near the SPS values $\sqrt{s} = 17 - 20$ GeV implies that virtually all production occurs in the high energy phase. Drell-Yan exhibits a similar behavior.

V. DRELL-YAN

The high energy phase, designed to record the initial interactions of the nucleons in the two colliding nuclei also provides the basis for our estimate of massive dilepton production, i.e., Drell-Yan, an important side of the quandary we faced at the start. We limit ourselves to the canonical FNAL [18] $p+A$ measurement at 800 GeV/c, but in fact the method of calculation guarantees agreement with the lower energy $p+A$ and $A+B$ collected by NA50 [2]. Drell-Yan is generally considered to be calculable perturbatively for dilepton pairs with masses in excess of $M_{\mu\mu} = 4$ GeV. Production in the short time defined by such masses proceeds without energy loss and leads to the A dependence shown in Fig. 1, implying very close to linear A dependence. To perform the Drell-Yan microscopically we have introduced parton structure functions [8]. But the curves in Fig. 1 could have been obtained purely geometrically from the elementary production rates and the high energy phase only; very little production comes from the second, low energy phase.

Any cascade which does not correctly describe this feature of Drell-Yan is in danger of producing spurious charmonium suppression by means of premature energy loss. Drell-Yan and J/ψ production both can occur in second, third and

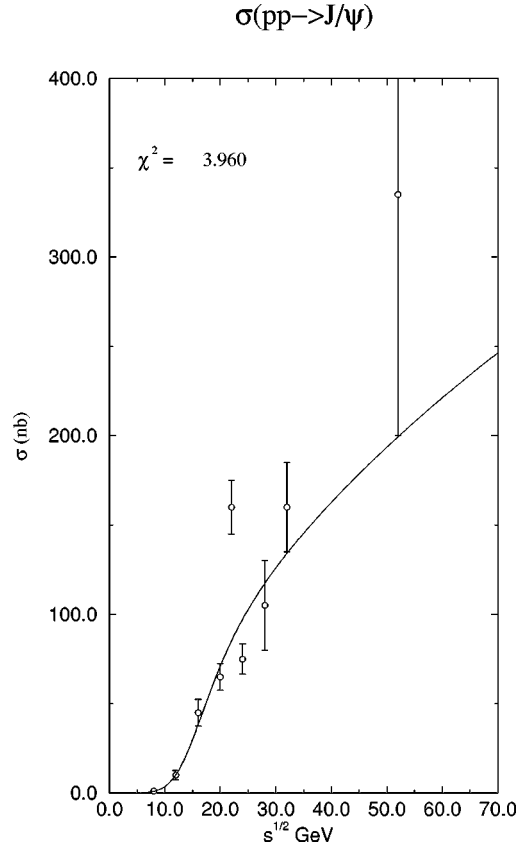


FIG. 6. Production of J/ψ from pp as a function of energy. The πp cross section is also known, and in fact is very similar to that for pp , but rarely plays a role with production generally significant only at the highest energies.

higher order collisions of initial nucleons as well as in the first collision, and if energy is lost by the nucleons immediately following each collision, the result will be A -dependent “suppression,” since the production of all charmonium states drops sharply with decreasing energy.

Later we use a survival probability for J/ψ , which has as its denominator, aside from a nucleon-nucleon normalization, the Drell-Yan yield, corresponding to the charmonium yield in the numerator. Thus, apart from overall normalization the ratio $\sigma(\psi)/\sigma(DY)$ and the J/ψ survival probability $P_s = N_{\psi}(\text{survived})/N_{\psi}(\text{produced})$ are one and the same thing. This is because elementary Drell-Yan production and J/ψ production are treated in exactly the same way in our cascade, and the number of high mass lepton pairs tracks the initial number of charmonia.

VI. CHARMONIUM SUPPRESSION IN NUCLEAR COLLISIONS

A. Minimum bias: $p+A$ and comparison to Glauber

We begin with the suppression in $p+A$ for which meson comovers play little role. Even here, however, the first stage high energy cascade does not suffice for an accurate description, since some of the suppression on baryons occurs only in the second stage, as slow J/ψ 's emerging from the interaction region are caught by nucleons, or interact at low energy in the target. The nucleon-nucleus data provides a necessary constraint on the basic parameters to be used in baryon-

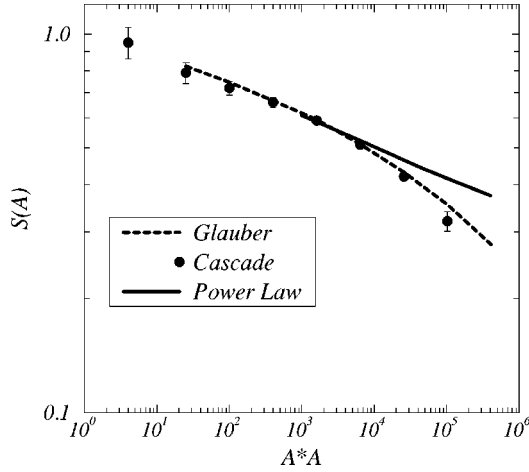


FIG. 7. Comparison for $A+A$ between Glauber theory and cascade, the latter in a purely J/ψ mode and both calculations employ $\sigma_{br}=7.3$ mb. The deviation from a power law is apparent for large $A \times A$. A hard sphere form is used for the nuclear density.

baryon production and breakup. This simplified system also provides a fruitful ground for comparison between the eikonal approach and LUCIFER.

To facilitate a comparison with the cascade we have made our own calculations with the Glauber formalism, relying on the formula:

$$\begin{aligned} \frac{dS_{Gl}}{d^2b} &= \frac{1}{AB \sigma(NN \rightarrow \psi)} \left[\frac{d\sigma(AB)}{d^2b} \right] \\ &= \int d^2s dz dz' \rho_A(s, z) \\ &\quad \times \rho_B(b-s, z') I_A(s, z) I_B(b-s, z'), \end{aligned} \quad (1)$$

$$I_A(s, z) = \left[-(A-1) \int_z^\infty dz_A \rho_A(s, z_A) \sigma_{abs} \right], \quad (2)$$

for the differential survival probability of J/ψ produced in $p+A$ collisions, with all integrals, including the one over b , carried out numerically to obtain the total survival probability. Here σ_{abs} is the J/ψ breakup cross section and is to be determined from $p+A$ data. We follow Ref. [5] in this development but employ a simpler, hard sphere, version of the nuclear density $\rho_A(b, z)$ for the purpose of comparing Glauber theory and LUCIFER II. It was instructive to extend this comparison to $A+A$ collisions to demonstrate that even Glauber theory does not reproduce the canonical power law, implied in the experimental descriptions [2,20,31] which always are compared with a straight line fit on a log-log plot, supposedly arising from purely baryonic breakup. These results are displayed in Fig. 7 for the J/ψ without coupled channels to make an easier comparison between cascade and Glauber theory. The J/ψ absorption cross section is taken so as to reproduce the $p+A$ observations at 800 GeV/c [18], $\sigma_{abs} \sim 7.0$ mb with the hard sphere configuration. But this value is equally successful for the lower SPS energies.

A second comparison, again for minimum bias production of J/ψ , appears for $p+A$ in Fig. 8. In this we use the coupled channel modeling, whose details we now elaborate

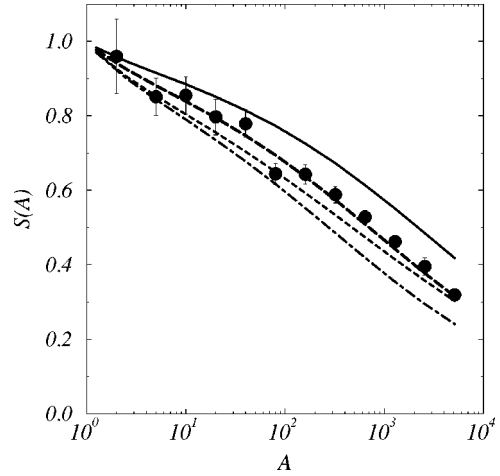


FIG. 8. Comparison for $p+A$ of survival probabilities for Glauber theory and cascade, the latter appearing both in pure J/ψ and coupled channel modes. A variety of absorptive strengths are illustrated; the solid, dashed, and dot-dashed lines are pure J/ψ Glauber theory with 5.0, 7.3, and 10.0 mb breakup, respectively, while the solid circles and dotted lines are the cascade (LUCIFER) with no coupling and a breakup of 7.3 mb and a coupled cascade with breakup cross sections of 5.0 mb and 15.0 mb for the J/ψ and (χ, ψ') pair, respectively.

further. The relative production of the different charmonium states is taken so as to reproduce the pp data from the ISR [28] for the χ to J/ψ ratio, i.e., $\chi/\psi \sim 4.5$, and for appropriate ψ' production [33]. A general ballpark for the measured χ to J/ψ production ratio in pp is a factor of 4–5 [28,33] and our final results are rather insensitive to a choice in this range, since a decrease could easily be compensated by adding a small transition matrix element between J/ψ and χ .

The ψ' to J/ψ production in pp is taken near 0.33 so as to reproduce the final J/ψ contribution from the eventual decay of the ψ' seen in a variety of experiments at a range of energies [2,33], i.e., implying a final ψ' to J/ψ ratio ~ 0.15 . The free decay of the ψ' into J/ψ is of course given the standard [29] value 0.57.

The χ^1 and χ^2 (and all other charmonium states) are assigned their correct masses, to properly include threshold effects in breakup. But the χ branching into J/ψ is taken the same, 0.18, a weighted average over the measured electromagnetic values together with a very small hadronic component, $\leq 0.5\%$ [29]. Again, small variations $\sim 1-2\%$ in this branching ratio have little effect on suppression in any of the charmonium states and can be compensated for by commensurate changes in, for example, overall cross section normalization.

A first inference to be drawn from Fig. 7 for $A+A$ and Fig. 8 for $p+A$ is that the first high energy cascade produces pure J/ψ dynamics very much like that in Glauber theory. For the above choices a $\sigma_{abs} \sim 7.0$ mb leads to very nearly the same yield with the cascade as with Glauber theory. A second lesson, key to our development, is that the coupled channel model reproduces the Glauber result for J/ψ , using a smaller direct breakup cross section, here taken as $\sigma_{abs}(J/\psi) = 5-6.0$ mb, and including indirect destruction via the considerably larger $\sigma_{abs}(\chi) = 3 \sigma_{abs}(J/\psi)$ for χ and perhaps higher for ψ' . The increased spatial sizes of the

higher states strongly support the use of larger absorption cross sections. For collisions extending to $A \times B$ values encompassing S+U and Pb+Pb, pure Glauber theory and the first stage cascade, both produce lines curving appreciably downwards on log-log plots, deviating from any power law. One gathers there is a little bit of “anomalous” suppression even in a bare bones, no comover, theory.

One should keep in mind that the true charmonium states in $p+A$ collisions are produced mostly outside the nucleus and we and other workers are, for this first cascade, considering charmonium progenitors, perhaps preresonances or perhaps just comoving $c\bar{c}$ pairs at a certain separation. The effect is however the same as using effective charmonium states, instantaneously produced, as we and previous researchers [4–6] are doing. This situation is altered when one turns to the second stage cascade which begins later, when all mesons, charmonium and others, may have had time to precipitate.

B. Suppression in $A+B$ collisions

To complete the picture one must allow the soft cascade to go forward, especially for ion-ion collisions where the production of mesons becomes very significant. A key parameter in the second cascade is the delay, τ_f or more properly τ_{eff} , afforded by the time scale involved in the “soft” formation of mesons. We reemphasize that we use a more or less standard value $\tau_f \sim 0.5 - 1.0$ fm/c, but are constrained by the production of mesons in the most massive system considered here, Pb+Pb. This production was considered extensively in Ref. [8]. Perhaps a 10–20% uncertainty might attach to τ_f .

There are two sets of data to be considered: first, minimum bias J/ψ cross sections as a function of the product $A \times B$ of nuclear atomic numbers, and second the ratio of J/ψ yield to Drell-Yan yield as a function of centrality, or more specifically transverse energy E_t .

Our results for minimum bias are displayed for the combined effect of both cascade phases in Fig. 9. The anomalous suppression in Pb+Pb is well reproduced by the totality of our two step, but otherwise conventional, hadronic dynamics. Part of the additional suppression in Pb+Pb relative to S+U already arises from the high energy cascade, coming from the increased χ and ψ' breakup in the more massive nuclear collision. But a considerable differential suppression arises from comovers, some 40% of the difference between S+U and Pb+Pb, with a similar amount arising from the hard cascade. Thus our conventional, hadronic explanation of the interesting NA50 measurements is multifaceted, being rooted in both of the two cascade stages, in the coupling of charmonium channels, and in the presence of comovers. Part of the anomaly suggested in Fig. 9, however, is perhaps illusory in view of the “curving down” seen for large $A \times B$ in Fig. 7 and Fig. 8.

The calculated minimum bias ψ' suppression is compared to data in Fig. 10. The strong drop occasioned by the large increase from $p+W$ to S+U or Pb+Pb is clearly present in the theory. As is evident in this figure the ψ' breakup strength inferred from $p+A$ proves sufficient for both S+U and Pb+Pb. Our calculations predict some drop in the ψ' to J/ψ ratio for $p+A$, but we point out that the introduction of

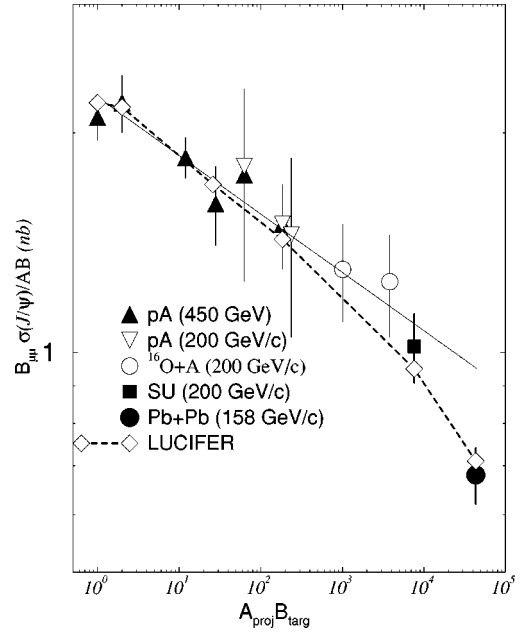


FIG. 9. Minimum bias J/ψ suppression: for a whole range of target-projectile sets from pp and $p+D$ to Pb+Pb, calculated in the cascade and compared to SPS measurements at various energies. The absolute theoretical values are obtained by normalization to nucleon-nucleon. The solid line is a supposed power law, which is of course not realized in Glauber theory or cascade, even without comovers. The experimental data have been rescaled to 200 GeV/c. The theoretical points (open diamonds) are connected by a dotted line.

a small off-diagonal element in the transition matrix R_{ij} and small changes in the ψ' breakup cross sections could essentially independently vary the ψ' production, without appreciably altering our J/ψ results. This difficult to measure observable would perhaps benefit from further attention experimentally, even for nucleon-nucleon.

The breakup cross sections in these simulations are 6.6, 20.0, and 25.2 mb for the ψ , χ^i , and ψ' , respectively. These

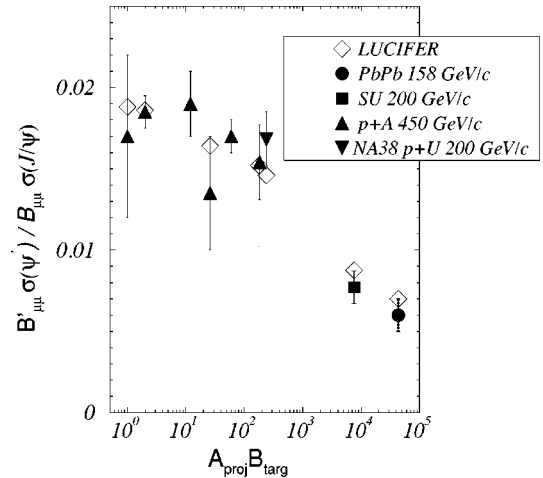


FIG. 10. Comparison of experiment vs simulation for minimum bias ψ' . The Pb+Pb data from NA50 was rescaled to 200 GeV/c by the collaboration. The S+U data is taken from NA38. The cascade calculations, again normalized to nucleon-nucleon, reproduce the sharp drop in the ψ' to J/ψ branching ratios for the massive nuclear collisions, but indicate some variation with A for $p+A$.

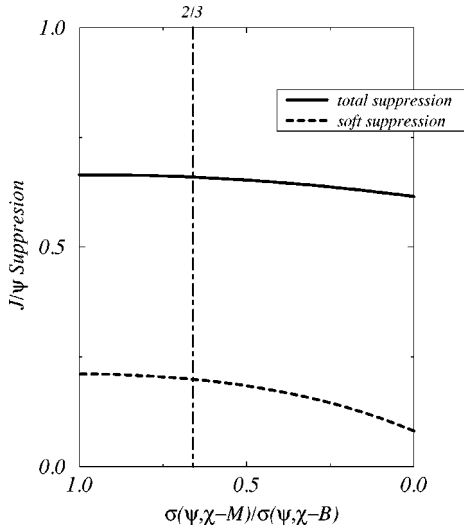


FIG. 11. Variation of J/ψ suppression with the charmonium-meson cross sections. We use $2/3$ as the ratio to charmonium-baryon for the calculations in the paper, but appreciable changes in this factor would not alter the results much. There is evidently a nonlinear saturation in the suppression occasioned by the coupling of the J/ψ to the higher charmonium states.

represent absorption in charmonium-baryon collisions, and are reduced by the constituent quark factor $2/3$ in ψ -meson. Variation of these meson-meson cross sections upwards to full equality with charmonium-baryon leads to a small change in the overall J/ψ suppression for Pb+Pb (see Fig. 11). This change is muted by two circumstances: in the soft cascade the suppression from charmonium-baryon collisions is still $\sim 35\%$ of the total, and there is a saturation effect in the degree of J/ψ suppression with increasing cross section. Hence, one obtains a robustness in the predictions for J/ψ .

In Fig. 12 we display the time history of charmed meson collisions with the mesons and baryons in the cascade. These are shown for a central Pb+Pb collision. Clearly the high

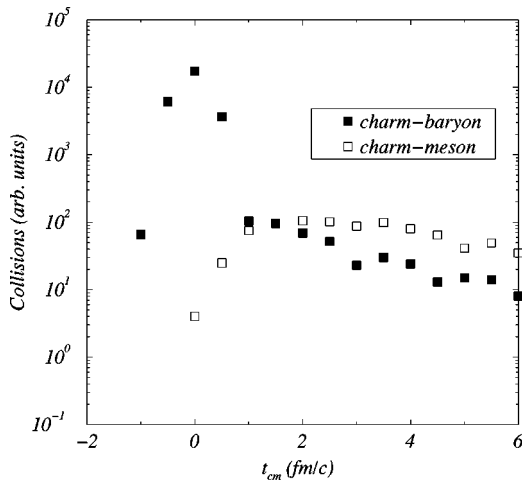


FIG. 12. Time histograms for collisions of charmonium states with baryons and mesons in a central Pb+Pb collision at SPS energies. Though baryon-charm collisions outnumber meson-charm collisions by a significant factor the meson-charm collisions which occur in the second stage of the cascade are important in determining the differential J/ψ suppression in Pb+Pb and S+U.

density in the first phase leads to a preponderance of charm collisions in this stage, but from the previous levels of suppression, detailed for minimum bias, breakup in the second phase is also significant.

The perhaps surprising nonlinearity for the J/ψ interaction arises because in our model this state is intrinsically tied up with the higher states. Introducing off diagonal elements R_{ij} would produce a family of solutions. We have left well enough alone; the present few modeling parameters, now determined independently of S+U or Pb+Pb data, surely having produced already an adequate description of observations.

The survival probabilities for the J/ψ in S+U are 0.50 and 0.87 in the hard and soft cascades, respectively. The same figures for Pb+Pb are 0.42 and 0.775. These results are both obtained using the model parameters described above and an effective formation time $\tau_{\text{eff}} \sim 0.6 + 0.9/2.0 \text{ fm}/c$, i.e., close to $1 \text{ fm}/c$. Comovers play a significant role, increasingly more so in Pb+Pb, but one must restate that baryons as well as mesons are comovers in the soft cascade. The ratio of breakup on mesons vs that on baryons is $\sim 2:1$ for Pb+Pb and less for S+U.

C. Centrality: Dependence on transverse energy

Perhaps the most striking features of the NA50 [3] measurements are contained in their plot of J/ψ suppression vs E_t . Unlike the existing Glauber calculations of transverse energy the cascade provides a built in E_t scale, which does not necessarily agree exactly with the experimental determination. NA50 plots for J/ψ and Drell-Yan use the neutral E_t within the pseudorapidity range $\eta = 1.1 - 2.3$. To establish a calibration from LUCIFER II we first refer to their earlier Pb+Pb results [31] using a more central rapidity range $\eta = 2.1 - 3.4$, and including both electromagnetic (neutral) and hadronic calorimeters to estimate E_t . This comparison is shown in Fig. 13 and indicates that LUCIFER II, with standard parameters [8], provides a reasonable representation of the measurements. The small discrepancy between cascade and experimental endpoints, some 10%, should be kept in mind when examining the NA50 charmonium data.

Charmonium breakup in the second phase cascade occurs both on baryons and on mesons, generic and stable. Since meson numbers can be large, in particular for Pb+Pb, their contribution to suppression may be appreciable. For collisions of J/ψ with generic mesons of mass greater than the charmonium binding, i.e., the energy interval $\delta E \sim 600 \text{ MeV}$ from the charmonium mass to the $D\bar{D}$ continuum, we employ a properly exothermic cross section. We use $\sigma_{\text{abs}} \propto k_f/k_i$ for the enhancement of the reaction cross section [34]. However, the enhancement due to the inverse k_i factor is not large numerically, since in general breakup occurs well above threshold.

Figures 14 and 15 display the results of simulations for the two massive ion-ion collisions. The magnitudes use the calculated survival probabilities, normalized by the pp or $p + D$ experiments and could be altered in normalization by changes in the latter. The rather low E_t value at which the measured J/ψ suppression becomes pronounced obtains equally well in the simulation, and the same low level of J/ψ 's is reproduced. The results reinforce the perception al-

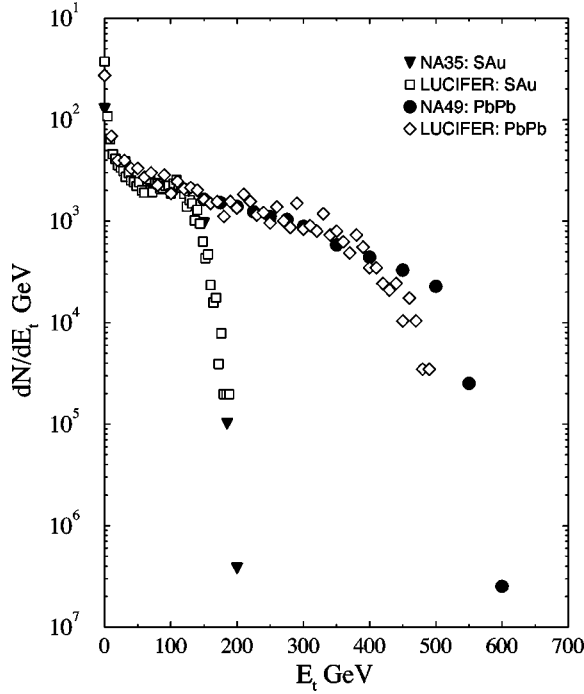


FIG. 13. Transverse energy distributions from LUCIFER compared to experiment (NA49) for all charges of hadrons. The purely neutral energy inferred from this figure should give an upper limit for that seen in the more peripheral cut used below for NA50.

ready created by the comparison with the minimum bias data. The hadronic two-step cascade is capable of describing the charmonium yields: J/ψ and at least broadly ψ' as well. Again, the source of the suppression is multifold, perhaps half coming from comover interactions and half from the hard cascade, both strongly influenced by our treatment of the charmonium states. The beginning of strong suppression in J/ψ for very peripheral collisions is a reflection of the important role the heavier charmonium states play.

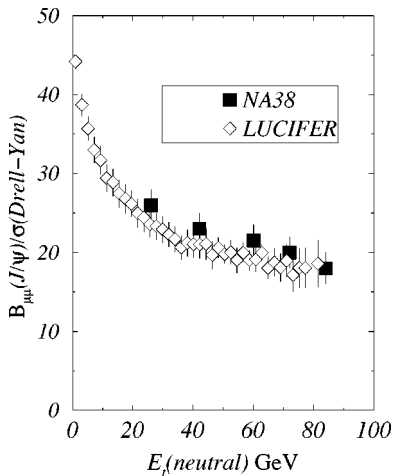


FIG. 14. Ratio of J/ψ to Drell-Yan in S+U. Comparison between the cascade and NA38 neutral transverse energy dependence for J/ψ , for S+U. The pseudorapidity range used here is 1.7–4.3.

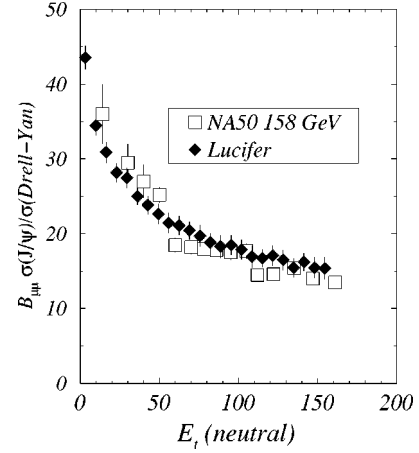


FIG. 15. Ratio of J/ψ to Drell-Yan in Pb+Pb. Comparison between the cascade and NA50 neutral transverse energy dependence for J/ψ . There are no discontinuities, of course, in the LUCIFER yields, but the general shape is reproduced. The pseudorapidity range here is 1.1–2.3, and a factor of 1.2 used to normalize the theoretical energy scale [35]. The experimental data was rescaled to 200 GeV/c by the experimentalists. The theoretical normalization in both this comparison and the previous one are subject to choices made for the elementary pp values.

VII. CONCLUSIONS

It appears that a conventional hadronic explanation of the minimum bias and central J/ψ and ψ' suppression in $A+B$ collisions is possible. This is accomplished here with a cascade, not specially tuned for the charmonium sector alone, but consistent with soft energy loss processes, both meson and proton spectra, and Drell-Yan data. There are parameters in the model, notably the meson formation time, the generic meson decay constant, and certainly the elements of the charmonium reaction matrix. However, we have not made use of all of this freedom in obtaining the present results. We have used a pure diagonal reaction matrix to describe the coupled charmonium channels, no mixing was needed further than that implied in NN . The ratio of charmonium states produced in the elementary nucleon interactions is naturally somewhat uncertain, but strong hints are available from experiment and we followed these indications.

Many of the other parameters are also constrained by $p+A$ data or by reasonable assumptions, for example the relative size of breakup probabilities must be tied to the relative sizes of the charmonium states. The formation time for produced mesons is also strongly tied to inclusive meson production. Some authors [5] suggest theoretically that the J/ψ total cross section on mesons must be drastically smaller than the 3–4 mb we use for breakup. This argument is disputed by others [30], and it seems unlikely, especially at the energies, well above breakup threshold, seen in our calculations. Our direct experimental knowledge of the total and partial cross sections, including any energy dependence, for J/ψ or other charmonium mesons on the lower mass mesons is of course very limited.

Evidently, the initial phase of our model, or that of others, for charmonium creation or breakup is not strictly nor conventionally hadronic. The production and destruction of states in this phase cannot be of standard fully formed charmonium states alone. But whether one imagines these initial

forms to be preresonant or a somewhat excited $c\bar{c}$ may not change the effective dynamics. We have chosen to view the early life of the hidden charm mesons as akin to a c and \bar{c} in relatively narrowly defined continuum wave packets characterized by their relative distance. It is crucial that the $c\bar{c}$ pair be in both color singlet and octet states. Then, despite the lengthier formation time for the more massive states, in fact precisely because of this, these states are harder to form, suffering more collisions. The dependence of an effective cross section on radius then follows.

For an elementary pp collision the eventual coalescence of this pair into a proper charmonium state could be described in a fashion following the formation of deuterons or other clusters [36]. In the case of pp there would be independent formation probabilities for the various spectroscopic states, but the reaction matrix would presumably be diagonal, assuming no significant final state interactions on the charmonium. The coalescence within a nuclear environment would be very different than in free space, with many more interactions preventing bound state formation. The more general matrix we proposed, R_{ij} , would have a simple meaning in an extension of this picture to ion-ion collisions, its elements describing what happens to these early $c\bar{c}$ states as they propagate through the nuclear environment, above and beyond their fate in pure pp .

What then has been learned about excited, dense, nuclear matter from the reduction in J/ψ 's? Our earlier calculations [8] for a broader range of processes, suggested that very high baryonic and mesonic energy densities were achieved in central Pb+Pb interactions, $\rho_B \sim 4-6/(\text{fm})^3$ and $\rho_E \sim 3-4 \text{ GeV}/(\text{fm})^3$ respectively and that these densities persist for quite long times $\tau \sim 3-5 \text{ fm}/c$, in the c.m. frame. Thus appropriate conditions for possible "plasma" creation exist in the most massive collision. This matter density has been sensed in the theoretical comover breakup of charmonium, both J/ψ and ψ' , more so for Pb+Pb than for S+U. But should our model stand the test of time, and it has explained

a good portion of existing data at the SPS, then the case for a nonconventional explanation is hard to establish as yet. It is always of course possible that partons actually are playing a less passive role than portrayed by hadronic modeling, that especially high gluon densities are achieved in the initial phase. If so, the use of purely hadrons seems to mock up the partonic behavior rather closely, and even the early charmonium and Drell-Yan dynamics seem to, effectively at least, fit into this picture. In any case our simulations do not rule out the creation of some form of partonic matter in ion collisions at SPS energies. They only make the necessity thereof less compelling. A microscopic, nonhadronic treatment of the internal structure of the charmonium states might alter the entire picture, and may be necessary at higher energies.

The dynamics of charmonium suppression would seem *ab initio* to be similar at RHIC, but the greatly reduced duration of the initial hard cascade at $s^{1/2} = 200 \text{ GeV}$ might play a spoiler role. The quasihadronic modeling employed in this phase by all practitioners may be inappropriate at such increased energy.

At RHIC energies and beyond, surely even the soft hadronic measurements will be contaminated by contributions originating in harder parton interactions, degraded say, by later hadronization. Nevertheless, it may be hard to find partonic footprints in the soft spectra. The generic mesons and baryons may simply be stand ins for minijets [16], effectively and correctly incorporating divided parton cross sections and hadronization within hadronic collisions, decays and meson production. If appreciable partonic plasma is quickly formed, in a state close to thermalization, the loss of energy to the baryonic background might very well be detectable. We intend to pursue such a path to finding "signals" of collective parton dynamics.

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