New type of pion interferometry formula

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A new type of pion interferometry formula is derived, in which not only the correlator but also the pion multiplicity is taken into consideration. The method to obtain information about the initial emission probability of unsymmetrized bosons is given. [S0556-2813(99)03403-2]

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Two-particle Bose-Einstein (BE) interferometry as a method to obtain information on the space-time geometry and dynamics of high-energy collisions has recently received intensive theoretical and experimental attention [1]. However, the old, widely used, two-pion interferometry formula has a shortcoming in that it does not take into account multipion BE symmetrization effects, which have aroused great interest among physicists in the fields of high-energy [2–19] and statistical [20] physics. In this paper, we derive a new multipion interferometry formula which is determined not only by the correlator, but also explicitly by the pion multiplicity distribution. The main results are summarized as follows: (1) If the bosons are emitted independently [19] and only BE symmetrization effects are present, the standard two-particle and higher-order interferometry formulas are reproduced. (2) If the bosons are not emitted independently (i.e., they are correlated even without BE symmetrization), then these initial correlations will lead to a new pion interferometry formula which depends on the initial emission probability of the unsymmetrized boson explicitly. (3) One can obtain information about the initial emission probability of the unsymmetrized boson through the study of two-pion interferometry.

The multipion state can be written as

$$|\phi\rangle=\sum_{n=0}^{\infty} a_n|n\rangle, \tag{1}$$

where a_n is a parameter connected with the pion multiplicity distribution. $|n\rangle$ is the n-pion state which can be expressed as

$$|n\rangle=\frac{\left[\int d\mathbf{p}\int j(x)a^{\dagger}(\mathbf{p})\exp(ip\cdot x)\right]^n}{n!}|0\rangle. \tag{2}$$

Here $a^{\dagger}(\mathbf{p})$ is the pion-creation operator and $j(x)$ is the pion current which can be expressed as

$$j(x)=\int d^4y d^4p j(y,p)\gamma(y)\exp[-ip(x-y)], \tag{3}$$

where $j(y,p)$ is the probability amplitude of finding a pion with momentum p, emitted by the emitter at y. Here $\gamma(y)=\exp[i\phi(y)]$ is a random phase factor which has been extracted from $j(y,p)$. All emitters are uncorrelated in coordinate space when assuming [11]

$$\{\gamma^*(x)\gamma(y)\}=\delta^4(x-y). \tag{4}$$

Here $\{\cdots\}$ means phase average. Equation (4) corresponds to the assumption that the source is totally chaotic. If the source is totally coherent, we have

$$\phi(x)=0, \quad \gamma(x)=1. \tag{5}$$

According to Eq. (1), the normalized pion multiplicity distribution can be expressed as

$$P(n)=\frac{|a_n|^2\{\langle n|n\rangle\}}{\sum_{n=0}^{\infty}|a_n|^2\{\langle n|n\rangle\}} \tag{6}$$

and the n pion inclusive distribution reads

$$N_i(\mathbf{p}_1,\ldots,\mathbf{p}_i)=\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1 d\mathbf{p}_2\cdots d\mathbf{p}_i} =\frac{\{\langle\phi|a^{\dagger}(\mathbf{p}_1)\cdots a^{\dagger}(\mathbf{p}_i)a(\mathbf{p}_i)\cdots a(\mathbf{p}_1)|\phi\rangle\}}{\{\langle\phi|\phi\rangle\}}, \tag{7}$$

with

$$\int\prod_{k=1}^{i} d\mathbf{p}_k N_i(\mathbf{p}_1,\ldots,\mathbf{p}_i)=\langle n(n-1)\cdots(n-i+1)\rangle. \tag{8}$$

$N_i(\mathbf{p}_1,\ldots,\mathbf{p}_i)$ can be interpreted as the probability of finding i pions with momentum $\mathbf{p}_1,\ldots,\mathbf{p}_i$ in a *given event*. Then the i pion correlation function reads [21,22]

$$C_i(\mathbf{p}_1,\ldots,\mathbf{p}_i)=\frac{N_i(\mathbf{p}_1,\ldots,\mathbf{p}_i)}{\prod_{j=1}^{i}N_1(\mathbf{p}_j)}. \tag{9}$$

In the following we study first multipion BE correlation effects on the pion multiplicity distribution. Then we will derive a new type n-pion correlation formula which depends not only on the correlator but also explicitly on the pion multiplicity distribution. Finally we will study multipion BE correlation effects on two-pion interferometry.

If $a_n = 1$, $n \in (0,\infty)$ in Eq. (1) and the source is totally coherent, then we have

$$\{\langle n|n\rangle\} = \frac{n_0^n}{n!}, \tag{10}$$

with

$$n_0 = \int |\tilde{j}(\mathbf{p})|^2 d\mathbf{p}, \quad \tilde{j}(\mathbf{p}) = \int j(x)\exp(ipx)d^4x. \tag{11}$$

The normalized unsymmetrized pion multiplicity distribution for the above source can be expressed as [using Eq. (6)]

$$P(n) = \frac{n_0^n}{n!}\exp(-n_0). \tag{12}$$

If $a_n = n!^{1/4}$, $n \in (0,\infty)$, and the source is coherent, then we have

$$P(n) = \left(\sum_{n=0}^{\infty} \frac{n_0^n}{\sqrt{n!}}\right)^{-1} \frac{n_0^n}{\sqrt{n!}}. \tag{13}$$

If $a_n = n!^{-1/8}$, $n \in (0,\infty)$, and the source is coherent, then we have

$$P(n) = \left(\sum_{n=0}^{\infty} \frac{n_0^n}{n!^{1.25}}\right)^{-1} \frac{n_0^n}{n!^{1.25}}. \tag{14}$$

In the following, the above three states (corresponding to different choices of a_n) will be denoted as $D1$, $D2$, and $D3$, respectively. Here $D1$ corresponds to the case where the pions are emitted independently while $D2$ and $D3$ correspond to the case in which the pions are not emitted independently, even without BE symmetrization. In the following we will study multipion BE correlation effects on the pion multiplicity distribution. Using the notation $\omega(n) = \{\langle n|n\rangle\}$ and assuming the source is totally chaotic, we find [5]

$$\omega(n) = \frac{1}{n}\sum_{i=1}^{n} \omega(n-i)\int d\mathbf{p}G_i(\mathbf{p},\mathbf{p}), \tag{15}$$

with

$$G_i(\mathbf{p},\mathbf{q}) = \int \rho(\mathbf{p},\mathbf{p}_1)d\mathbf{p}_1\rho(\mathbf{p}_1,\mathbf{p}_2)\cdots d\mathbf{p}_{i-1}\rho(\mathbf{p}_{i-1},\mathbf{q}), \tag{16}$$

where $\rho(\mathbf{p},\mathbf{q})$ is the Fourier transformation of the source distribution $g(x,K)$,

$$\rho(\mathbf{p},\mathbf{q}) = \int g\left(x, \frac{p+q}{2}\right)\exp[i(p-q)x]d^4x. \tag{17}$$

We assume the source distribution as

$$g(\mathbf{r},t,\mathbf{p}) = n_0\left(\frac{1}{\pi R^2}\right)^{3/2}\exp\left(-\frac{\mathbf{r}^2}{R^2}\right)\delta(t)\left(\frac{1}{2\pi\Delta^2}\right)^{3/2} \times\exp\left(-\frac{\mathbf{p}^2}{2\Delta^2}\right). \tag{18}$$

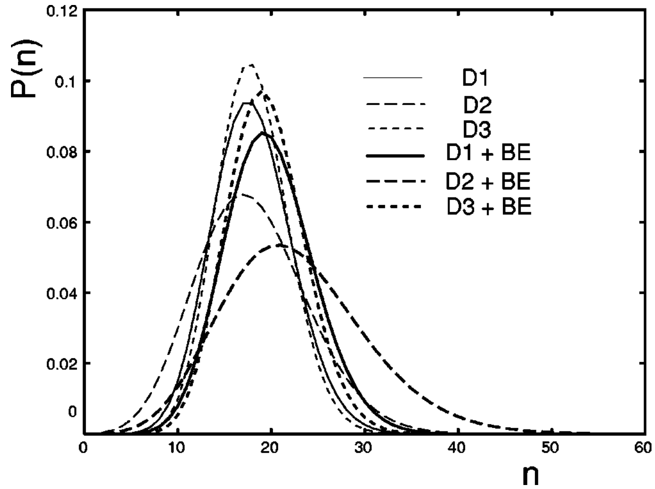

FIG. 1. Multipion BE correlation effects on the pion multiplicity distribution. The thin solid line, dashed line, and dotted line correspond to the unsymmetrized pion multiplicity distribution of states $D1$, $D2$, and $D3$, respectively. The input values of n_0 for $D1$, $D2$, and $D3$, are 18, 4.183, and 37.38, which ensure the mean unsymmetrized pion multiplicity $\langle n\rangle_{D1} = \langle n\rangle_{D2} = \langle n\rangle_{D3} = 18$. The wider solid line, dashed line, and dotted line correspond to the BE symmetrization effects on $D1$, $D2$, and $D3$, respectively. The input values of R and Δ are 5.3 fm and 0.18 GeV, respectively.

Here n_0 is a parameter. Using this source distribution, we can study multipion BE correlations effects on the pion multiplicity distribution, pion spectrum distribution, and two-pion interferometry since $G_i(\mathbf{p}\cdot\mathbf{q})$ can be calculated by recurrence relations [4,5] or analytically [9,10]. Multipion BE correlation effects on the pion multiplicity distribution are shown in Fig. 1. One clearly sees that BE symmetrization shifts the pion multiplicity distributions of $D1$, $D2$, and $D3$ to the right side. The unsymmetrized pion multiplicity distribution for $D1$, $D2$, and $D3$ are also shown in Fig. 1.

According to Eq. (7), the one-pion inclusive distribution reads

$$N_1(\mathbf{p}_1) = \frac{\{\langle\phi|a^\dagger(\mathbf{p}_1)a(\mathbf{p}_1)|\phi\rangle\}}{\{\langle\phi|\phi\rangle\}} = \frac{\sum_{n=0}^{\infty}|a_n|^2\{\langle n|a^\dagger(\mathbf{p}_1)a(\mathbf{p}_1)|n\rangle\}}{\sum_{n=0}^{\infty}|a_n|^2\omega_n}. \tag{19}$$

Similar to Ref. [5], one can obtain the relation

$$\{\langle n|a^\dagger(\mathbf{p}_1)a(\mathbf{p}_1)|n\rangle\} = \sum_{i=1}^{n}\omega(n-i)G_i(\mathbf{p}_1,\mathbf{p}_1), \tag{20}$$

and the one-pion inclusive distribution can be expressed as

$$N_1(\mathbf{p}) = \frac{\sum_{n=1}^{\infty}|a_n|^2\sum_{i=1}^{n}\omega(n-i)G_i(\mathbf{p},\mathbf{p})}{\sum_{n=0}^{\infty}|a_n|^2\omega(n)}. \tag{21}$$

The two-pion inclusive distribution can be expressed as

$$N_2(\mathbf{p}_1,\mathbf{p}_2)=\frac{\{\langle\phi|a^\dagger(\mathbf{p}_1)a^\dagger(\mathbf{p}_2)a(\mathbf{p}_2)a(\mathbf{p}_1)|\phi\rangle\}}{\{\langle\phi|\phi\rangle\}}=\frac{\sum_{n=0}^{\infty}|a_n|^2\{\langle n|a^\dagger(\mathbf{p}_1)a^\dagger(\mathbf{p}_2)a(\mathbf{p}_2)a(\mathbf{p}_1)|n\rangle\}}{\sum_{n=0}^{\infty}|a_n|^2\omega_n}. \quad (22)$$

Using the relationship [5]

$$\{\langle n|a^\dagger(\mathbf{p}_1)a^\dagger(\mathbf{p}_2)a(\mathbf{p}_2)a(\mathbf{p}_1)|n\rangle\}=\sum_{i=2}^{n}\sum_{m=1}^{i-1}[G_m(\mathbf{p}_1,\mathbf{p}_1)G_{i-m}(\mathbf{p}_2,\mathbf{p}_2)+G_m(\mathbf{p}_1,\mathbf{p}_2)G_{i-m}(\mathbf{p}_2,\mathbf{p}_1)]\omega(n-i), \quad (23)$$

the two-pion inclusive distribution reads

$$N_2(\mathbf{p}_1,\mathbf{p}_2)=\frac{1}{\sum_{n=0}^{\infty}|a_n|^2\omega(n)}\sum_{n=2}^{\infty}|a_n|^2\sum_{i=2}^{n}\sum_{m=1}^{i-1}[G_m(\mathbf{p}_1,\mathbf{p}_1)G_{i-m}(\mathbf{p}_2,\mathbf{p}_2)+G_m(\mathbf{p}_1,\mathbf{p}_2)G_{i-m}(\mathbf{p}_2,\mathbf{p}_1)]\omega(n-i). \quad (24)$$

It can be proved that the k-pion inclusive distribution can be expressed as

$$\begin{aligned}N_k(\mathbf{p}_1,\ldots\mathbf{p}_k)&=\frac{1}{\sum_{n=0}^{\infty}|a_n|^2\omega(n)}\sum_{n=k}^{\infty}|a_n|^2\sum_{i=k}^{n}\sum_{m_1=1}^{i-(k-1)}\sum_{m_2=1}^{i-m_1-(k-2)}\cdots\sum_{m_{k-1}=1}^{i-m_1-m_2\cdots-m_{k-2}-1}\sum_{\sigma}G_{m_1}(\mathbf{p}_1,\mathbf{p}_{\sigma(1)})\\&\quad\times G_{m_2}(\mathbf{p}_2,\mathbf{p}_{\sigma(2)})\cdots G_{m_{k-1}}(\mathbf{p}_{k-1},\mathbf{p}_{\sigma(k-1)})G_{i-m_1\cdots-m_{k-1}}(\mathbf{p}_k,\mathbf{p}_{\sigma(k)})\omega(n-i)\\&=\frac{\sum_{n=i=m_1+m_2\cdots+m_k}^{\infty}|a_n|^2\omega(n-i)}{\sum_{n=0}^{\infty}|a_n|^2\omega(n)}\sum_{\sigma}\sum_{m_1=1}^{\infty}G_{m_1}(\mathbf{p}_1,\mathbf{p}_{\sigma(1)})\cdots\sum_{m_k=1}^{\infty}G_{m_k}(\mathbf{p}_k,\mathbf{p}_{\sigma(k)})\\&=\sum_{n=i=m_1+m_2\cdots+m_k}^{\infty}P(n)\frac{\omega(n-i)}{\omega(n)}\sum_{\sigma}\sum_{m_1=1}^{\infty}G_{m_1}(\mathbf{p}_1,\mathbf{p}_{\sigma(1)})\cdots\sum_{m_k=1}^{\infty}G_{m_k}(\mathbf{p}_k,\mathbf{p}_{\sigma(k)}). \quad (25)\end{aligned}$$

Here $\sigma(k)$ denotes the kth element of a permutations of the sequence $\{1,2,\ldots,k\}$, and the sum over σ denotes the sum over all $k!$ permutations of this sequence. It is interesting to notice that if $a_n=1$ $[P(n)=\omega(n)/\Sigma_n\omega(n)]$ as assumed in Ref. [12], this new k pion inclusive distribution is very similar to the old k pion inclusive distribution which does not take account of higher-order BE correlation effects. This similarity warrants the validity of the formula [12] used in earlier studies [24,25]. But in the general situation, the pion interferometry formula depends not only on the correlator $\rho(i,j)$ but also explicitly on the pion multiplicity distribution $P(n)$ as shown in Eq. (25). Multipion BE correlation effects on the single-pion inclusive distribution are shown in Fig. 2. Here $P^{\langle n\rangle}(\mathbf{p})$ is defined as

$$P^{\langle n\rangle}(p)=\frac{N_1(\mathbf{p})}{\langle n\rangle},\quad \langle n\rangle=\int N_1(\mathbf{p})d\mathbf{p}. \quad (26)$$

One clearly sees that multipion BE correlations cause pions to concentrate at the lower momenta region. Similar behavior was observed in Ref. [9]. The above phenomenon is caused by the nature of bosons: pions would like to be in the same state. As the probability of finding a bigger n pion state, $|n\rangle$, in $D2$ is bigger than the probability of finding the same n pion state in $D3$, the multipion BE correlation effects on $D2$ are stronger than the BE correlation effects on $D3$.

According to the definition of the n-pion correlation function [Eq. (9)], one can easily write out the n-pion interferometry formula by using Eq. (25). Multipion BE correlations effects on two-pion interferometry are shown in Fig. 3. It is clear that one can tell the difference among those three different pion states through studies of the two-pion correlation function. For the $D1$ state, the two-pion correlation function's intercept $C_2(q)_{q=0}$ with $q=|\mathbf{p}_1-\mathbf{p}_2|$ is 2 and the value of the two-pion correlation function is 1 if the relative momentum q becomes very large, while for the $D2$ state one sees that $C_2(q)_{q=0}=2.19>2$ and $C_2(q)_{q\to\infty}=1.07>1$. For the $D3$ state, we have $C_2(q)_{q=0}=1.97<2$ and $C_2(q)_{q\to\infty}=0.987<1$. So one can tell the difference among the three types of multipion states by studying the behaviors of two-pion correlation functions at the smaller-q and larger-q regions. Because of the resonance decay and resolution power of the data, it is very difficult to determine the intercept of

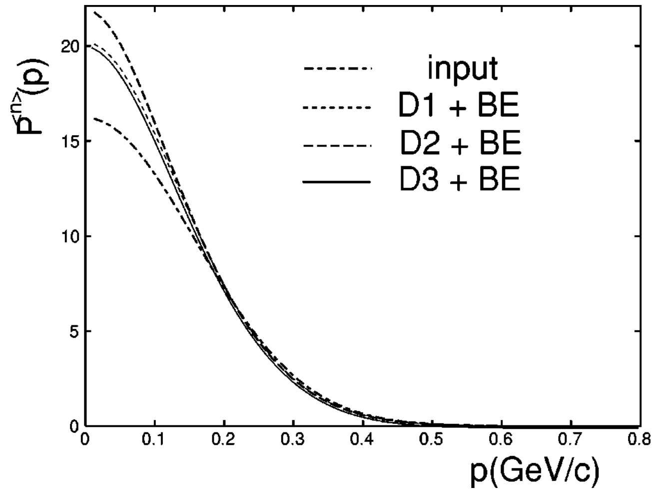

FIG. 2. Multipion correlation effects on the pion momentum distribution. The solid line, dashed line, and dotted line correspond to multipion BE correlation effects on the pion spectrum distribution of pion states $D3$, $D1$, and $D2$, respectively. The mean pion multiplicity $\langle n \rangle = 19$ for the above three cases. The dash-dotted line corresponds to the input momentum distribution $\int g(x,p) d^4x$. The input values of R and Δ are 5 fm and 0.16 GeV, respectively.

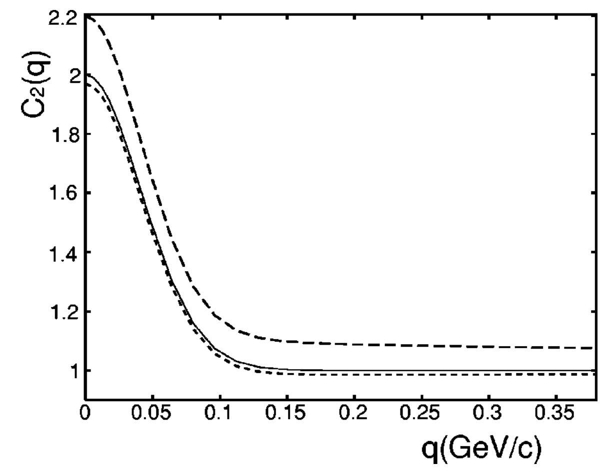

FIG. 3. Multipion correlation effects on two-pion interferometry. The solid line, dashed line, and dotted line correspond to multipion BE correlation effects on two-pion interferometry for pion states $D1$, $D2$, and $D3$, respectively. The mean pion multiplicity for the three cases is 17.7. The input values of R and Δ are 5 fm and 0.16 GeV.

two-pion interferometry without further assumptions. On the other hand, we can determine two-pion interferometry results at the large-q region without any assumptions. As pion pairs at the large-q region are less affected by BE symmetrization, so the tail of two-pion interferometry is determined mainly by the initial emission probability of unsymmetrized pions. According to Eqs. (12)–(14), one can determine $\langle n(n-1) \rangle / \langle n \rangle^2$ as 1.0, 1.07, and 0.986 for $D1$, $D2$, and $D3$, respectively. Those values are consistent with the results obtained from two-pion interferometry. Present heavy-ion experimental results indicate that $C_2(q)_{q\sim 300-400\ \mathrm{MeV}}$ is around 1.01–1.03 [23]. That means that $D1$ is one of the best models (pion states) to be used to analyze experimental results.

In this paper, multipion BE correlation effects on the pion spectrum distribution and pion multiplicity distribution are studied. As a result of multipion BE correlation effects, we derive a new type n-pion correlation function which is determined not only by the correlator but also explicitly by the pion multiplicity distribution. This new pion interferometry formula is sensitive to the multiplicity distribution as well as to the space structure of the source with and without BE symmetrization. We believe that one can obtain information about pion states by the pion interferometry method.

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