Doorway concept at high excitation energy

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The spreading of transition strength is studied in the doorway picture by taking the coupling to the continuum explicitly into account. The internal (configurational) coupling mixes the doorway state with the background states and is responsible for all the states actually having a decay width. Therefore, external mixing of the states via the continuum is induced. If it is large, resonance trapping appears and the states will be demixed. Thus the residual interaction is effectively reduced. As a consequence, the spreading of transition strength can be small also at high excitation energy. The relation to the golden rule is discussed. [S0556-2813(99)02001-4]

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I. INTRODUCTION

Recently, the giant dipole resonance observables have been investigated experimentally in hot rotating nuclei [1]. The results showed, among others, that collisional damping at the temperature $T \approx 2$ MeV is practically the same as at zero temperature. This result is a challenge for the theory. It is necessary to prove the doorway concept under the conditions of extremely high excitation energy. In doing this, one has to consider the quantum system as being open, i.e., one should take into account that the nuclear states are embedded in the continuum of decay channels.

The properties of open quantum systems are discussed in recent theoretical papers; see e.g., [2–5] and further references therein. At high level density, short-lived states exist whose collectivity originates from their coupling to the continuum. These states appear together with long-lived states as a result of resonance trapping. This so-called external collectivity occurs additionally to the well-known collectivity of intrinsic nature known from the properties of giant resonances at low excitation energy [5].

At high level density, the interference of external and internal collectivity influences significantly the distribution of the dipole strength over the states as well as their widths and positions in energy [5]. The interference pattern shows sudden changes in the distribution of the collective strength at a critical value of the level density. It coincides qualitatively with experimental results obtained at high excitation energy in nuclei (see e.g., [6]) under the assumption that the spreading is so small that the interference picture is not washed out.

The spreading of the strength of an originally simple state over states with a more complicated structure is described usually under the following assumptions [7]. The background states R are chosen to be equidistant with level spacing D and to have vanishing decay width. The squares of the coupling matrix elements v_R of these states to the simple state are substituted by the energy independent average value $\langle V^2 \rangle$. It follows then for $\langle V^2 \rangle > D^2$, that the strength distribution has a Breit-Wigner shape with the spreading width $\Gamma^{\downarrow}$ given by the golden rule [7],

$$\Gamma^{\downarrow} = 2\pi \frac{\langle V^2 \rangle}{D} \quad . \tag{1}$$

The same result is found by considering the average cross section [8]. If $\langle V^2 \rangle$ scales with D, as assumed in e.g., [9], Eq. (1) means that $\Gamma^{\downarrow}$ saturates as a function of the level density.

The spreading widths $\Gamma^{\downarrow}$ of isobar analogue resonances obtained from experimental data are, indeed, almost independent of the excitation energy [10]. The same is true for the imaginary part of the optical potential used in analyzing scattering data [11]. General theoretical arguments in favor of a saturation of $\Gamma^{\downarrow}$ as a function of the excitation energy are given in [12]. They are based on the idea of chaotization of the intrinsic dynamics [13]. A doorway state interacting through an internal interaction with a large set of background states, whose escape widths are nonzero, is considered in [13].

The interplay between giant resonances and background states is investigated in the framework of the continuum shell model in [14]. In this model, the direct internal mixing of the states as well as their external mixing via the continuum are taken into account. The results showed the following two results. First, the mixing of the doorway state with the background states is effectively reduced at high level density due to the external mixing leading to resonance trapping. Secondly, missing spectroscopic strength in the standard method of analyzing the data appears when the unitarity of the S-matrix is not taken properly into account. The unitarity is important especially at the top of resonances. Here the contributions from other resonances to the cross section should be added by accounting for the phases in order not to violate unitarity. Indeed, a selective transparency at the top of resonances is observed experimentally [15]. Thus, the interplay between giant resonances and background states via the continuum leads to results which are unexpected from the point of view of bound state calculations.

The properties of the long-lived states are studied in some other papers. For example in [16], experimental data on neutron resonances in ^{53}Cr are analyzed on the basis of the

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doorway concept and shown to resemble trapped states. In [17], the $4\hbar\omega$ isoscalar monopole giant resonance in ^{208}Pb is investigated in the random phase approximation with continuum. Some narrow resonances appear in the cross section which are shown to arise from resonance trapping.

In [18], the restructuring of an open quantum system taking place at high level density under critical conditions is studied in detail in the one channel case. Under certain conditions, the restructuring is a second-order phase transition. While the short-lived state is a collective state with contributions of all the N resonance states, the wave functions of the long-lived trapped states are mixed at most with those of their neighbors.

We will present, in the following, results obtained for the spreading of transition strength in the doorway picture. According to this picture only the doorway state is, due to its spectroscopic properties, coupled directly to the decay channel. We take this coupling explicitly into account. Our results are as follows: The internal mixing of the doorway state with the background states creates an ensemble of resonance states with nonvanishing decay widths. As a consequence, the interplay between internal and external interaction is important and determines the value of the spreading of transition strength. If the states are overlapping, the external mixing causes resonance trapping, that is narrow resonance states are created together with a broad one. This reduces effectively the mixing of the states and leads to a saturation (or even to a decrease) of the spreading at high level density. The original situation of one broad state overlapping many long-lived states is restored.

In Sec. II the model based on the doorway concept is presented and in Sec. III the model used is studied analytically. The interplay between internal and external mixing is illustrated numerically and discussed in Sec. IV. The results of our investigations are discussed in the last section.

II. MODEL DESCRIPTION

Following the doorway concept, the spreading of the transition strength of the doorway state is described by the $(N+1)\times(N+1)$ Hamiltonian [10]

$$\mathcal{H}=H-iAA^{+} \quad . \tag{2}$$

Here, H consists of three parts: (i) the $N\times N$ part h with eigenvalues e_R describing the N discrete states $|R\rangle$, $R=1,\ldots,N$, (ii) the 1×1 part corresponding to the unperturbed doorway state $|0\rangle$ and (iii) the real coupling vector V between doorway and discrete states with elements v_R. The second part $-iAA^{+}$ of $\mathcal{H}$ couples the system to the decay channel and makes the Hamilton operator non-Hermitian. According to the doorway concept, only $[AA^{+}]_{00}\equiv\Gamma^{\uparrow}/2\neq 0$ where $\Gamma^{\uparrow}$ is the escape width of the doorway state. Thus $\mathcal{H}$ reads

$$\mathcal{H}=\begin{pmatrix} 0 & V^T \\ V & h \end{pmatrix}-\frac{i}{2}\begin{pmatrix} \Gamma^{\uparrow} & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{3}$$

In this paper we mostly use the following assumptions for h and V: h is diagonal with diagonal elements e_R distributed according to a picket-fence, i.e., $D=$const. We number the $N+1$ states from $-N/2$ to $N/2$ and write $e_R=DR$. The state $R=0$ is the doorway state. The real coupling matrix elements $v_R=v$ are chosen constant and are normalized according to [9,19],

$$\sum_{R\neq 0} v_R^2/L=v^2/D\equiv W. \tag{4}$$

Here, the sum runs over the N background states lying in the interval $[-L/2,L/2]$. [In contrast to the doorway concept involved in the Hamiltonian (3), the Hamiltonian used in [13] couples all states directly to the continuum from the very beginning.]

The effective Hamilton operator Eq. (3) is complex, i.e., non-Hermitian. The eigenvalues of $\mathcal{H}$ give the energies E_R and half-widths $\Gamma_R/2$ of all $N+1$ states. The eigenfunctions

$$\Phi_R=\sum_{R'} c_{RR'}\varphi_{R'} \tag{5}$$

form a biorthogonal set with $\langle\Phi_R|\Phi_R\rangle\geqslant 1$ (for details, see [3]). The φ_R are defined by $\{\varphi\}=\{\chi\}\oplus\eta$ where the $\{\chi\}$ are the N eigenfunctions of h and η is the wave function of the doorway state. To study the wave functions of the resonance states, we normalize the coefficients according to

$$|b_{RR'}|^2=\frac{|c_{RR'}|^2}{\langle\Phi_R|\Phi_R\rangle}=\frac{|c_{RR'}|^2}{\Sigma_{R''}|c_{RR''}|^2} \quad . \tag{6}$$

Using this normalization we have the following relation:

$$\Gamma_R=\Gamma^{\uparrow}|b_{R0}|^2 \quad . \tag{7}$$

In the following we are interested in the sum of the widths of the narrow states,

$$\Gamma_v=\sum_{R\neq 0}\Gamma_R=\Gamma^{\uparrow}-\Gamma_0 \quad . \tag{8}$$

Γ_v describes how much of the escape width of the doorway state is spread to the background states $R\neq 0$.

III. ANALYTICAL STUDIES

A. The two resonance approximation

Let us study the interaction between the doorway state and one of the background states. This interaction is described by

$$\mathcal{H}_2=\begin{pmatrix} 0 & v \\ v & e_R \end{pmatrix}-\frac{i}{2}\begin{pmatrix} \Gamma^{\uparrow} & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{9}$$

The eigenvalues of Eq. (9) are

$$\lambda^{\pm}=\frac{e_R-\frac{i}{2}\Gamma^{\uparrow}}{2}\pm\frac{e_R+\frac{i}{2}\Gamma^{\uparrow}}{2}\sqrt{1+\frac{4v^2}{\left(e_R+\frac{i}{2}\Gamma^{\uparrow}\right)^2}}$$

$$\approx\frac{e_R-\frac{i}{2}\Gamma^{\uparrow}}{2}\pm\frac{e_R+\frac{i}{2}\Gamma^{\uparrow}}{2}\left(1+\frac{2v^2}{\left(e_R+\frac{i}{2}\Gamma^{\uparrow}\right)^2}\right) . \tag{10}$$

In the last step, it was assumed that v is small. Using Eq. (10) one can show

$$\lambda^{+}\equiv E_R-\frac{i}{2}\Gamma_R$$
$$\approx e_R\left(1+\frac{v^2}{e_R^2+\frac{(\Gamma^{\uparrow})^2}{4}}\right)-\frac{i}{2}\frac{\Gamma^{\uparrow}v^2}{e_R^2+\frac{(\Gamma^{\uparrow})^2}{4}} . \tag{11}$$

Here $E_R\approx e_R$ (for sufficiently small v) and

$$\Gamma_R\approx\frac{\Gamma^{\uparrow}v^2}{e_R^2+\frac{(\Gamma^{\uparrow})^2}{4}}=\Gamma(e_R) \tag{12}$$

are the energy and width of the background state R. Further, the width Γ_0 of the broader state can be found from Eq. (10),

$$\Gamma_0=-2\,\mathrm{Im}\,\lambda^{-}=\Gamma^{\uparrow}-\Gamma_R . \tag{13}$$

According to these equations, Γ_R increases with v and Γ_0 decreases with v for fixed $\Gamma^{\uparrow}$. This corresponds to the well-known result that the internal mixing v spreads the strength away from the original doorway state.

Equations (12) and (13) show, however, also the effect of resonance trapping as can be seen by fixing v and varying $\Gamma^{\uparrow}$. For isolated resonance states, i.e., $e_R\gg\Gamma^{\uparrow}/2$, it follows $\Gamma_R\approx v^2\Gamma^{\uparrow}/e_R^2$ which increases with $\Gamma^{\uparrow}$ (for fixed v). Further, $\Gamma_0/\Gamma^{\uparrow}\approx 1-v^2/e_R^2=$const, i.e., the ratio between the strength which remains at the broad state and the total strength does not depend on $\Gamma^{\uparrow}$. In the opposite case (overlapping states), $e_R\ll\Gamma^{\uparrow}/2$, it holds $\Gamma_R\approx 4v^2/\Gamma^{\uparrow}$ which is independent of energy and inversely proportional to $\Gamma^{\uparrow}$. The width Γ_R of the state R *decreases* with increasing $\Gamma^{\uparrow}$, i.e., it is a trapped state. In contrast to the width Γ_R, the width Γ_0 of the broad state increases with increasing $\Gamma^{\uparrow}$ also in this case, $\Gamma_0/\Gamma^{\uparrow}=1-4v^2/(\Gamma^{\uparrow})^2\rightarrow 1$ for large $\Gamma^{\uparrow}$. Thus, as a result of resonance trapping, the broad state accumulates almost all of the available width $\Gamma^{\uparrow}$ whereas the width of the other state becomes very small. For detailed investigations on resonance trapping, see e.g., [2–4].

Now we calculate the accumulated width of the background states, Γ_v, for $N,L\rightarrow\infty$ at a fixed $D=L/N$. We assume that the width of each background state follows from the two resonance approximation, Eq. (12). The energy shifts of the states are small because each E_R must lie between e_R and $e_{R\pm1}$. For a dense spectrum (which is in the regime of resonance trapping, $\Gamma^{\uparrow}>D$) it follows

$$\Gamma_v\approx\frac{1}{D}\int_{-\infty}^{+\infty}de\,\Gamma(e)=\frac{v^2}{D}\int_{-\infty}^{+\infty}de\frac{\Gamma^{\uparrow}}{e^2+\frac{(\Gamma^{\uparrow})^2}{4}}=2\pi W , \tag{14}$$

where D and v_R are assumed to be independent of energy. Equation (8) means $\Gamma^{\uparrow}>\Gamma_v$ since $\Gamma_0\geqslant 0$. Thus, Eq. (14) makes sense only under the condition

$$\Gamma^{\uparrow}>2\pi W. \tag{15}$$

Note that Eq. (15) does not imply $v>D$ but relates the value $v^2=WD$ to $\Gamma^{\uparrow}$.

A more general derivation of Eq. (14) will be given in Sec. III B while a numerical proof can be found in Sec. IV.

Equation (14) means that, as long as $\Gamma^{\uparrow}>2\pi W$, the sum Γ_v of widths of the narrow states does *not* depend on $\Gamma^{\uparrow}$. The only dependence is on $W=v^2/D$.

B. Solution of the secular equation

Now, let us derive the value Γ_v directly by considering the eigenvalues of $\mathcal{H}$, Eq. (3). They can be found from the solutions of the secular equation,

$$E_R-\frac{i}{2}\Gamma_R+\frac{i}{2}\Gamma^{\uparrow}=v^2\sum_{m=-N/2}^{m=+N/2}\frac{1}{E_R-\frac{i}{2}\Gamma_R-mD} . \tag{16}$$

By splitting Eq. (16) into its real and imaginary part, one can show that $E_R=0$ is a solution for the real part of the equation. The right hand side of Eq. (16) can be exactly calculated for $N\rightarrow\infty$, see e.g., [13]. In this limit and for $E_R\approx 0$, the imaginary part of Eq. (16) reduces to

$$\Gamma^{\uparrow}-\Gamma_R=2\pi W\cdot 1/\tanh\left(\frac{\pi\Gamma_R}{2D}\right) . \tag{17}$$

The broad state $R=0$ lies in the center of the spectrum. Assuming $\Gamma_0\gg D$ it holds $\tanh(\pi\Gamma_0/(2D))\approx 1$ and thus

$$\Gamma_0=\Gamma^{\uparrow}-2\pi W, \tag{18}$$

i.e.,

$$\Gamma_v=\Gamma^{\uparrow}-\Gamma_0=2\pi W . \tag{19}$$

This is the same result as Eq. (14) obtained by using the two resonance approximation. Equation (19) holds for $\Gamma_0>0$ meaning $2\pi W<\Gamma^{\uparrow}$ according to Eq. (18). This condition is the same as Eq. (15).

We can also find the width of background states with energies very close to 0 by assuming that $\Gamma_R\ll D$. In this case $\tanh(\pi\Gamma_R/(2D))\approx\pi\Gamma_R/(2D)$ and thus

$$\Gamma_{R\neq 0}\approx 4v^2/\Gamma^{\uparrow} \tag{20}$$

which is the same result as Eq. (12) taken at $e_R\approx 0$. Equation (18) means that the state $R=0$ can be identified with the broad state formed by trapping the states in its neighborhood while the states $R \neq 0$ near to $E=0$ are trapped states according to Eq. (20). This result corresponds to that obtained in the two resonance approximation, Sec. III A.

C. Spreading width

We consider the Hamiltonian $\mathcal{H}$, Eq. (3), in a base in which the real part H is diagonal. Diagonalizing H with the orthonormal matrix O we get in the new base

$$\mathcal{H}' = H' - i(A')(A')^{+}, \tag{21}$$

where $H' = O^T H O$ and $A' = O^T A$. Because only $A_0 \neq 0$ it holds $A'_R \propto O_{R0}$ where O_{R0} is the component of the doorway state in the wave function of the background state $R \neq 0$. In the picket fence model O_{R0} is given by the strength function. For $v > D$, it follows [7]

$$(A'_R)^2 = \frac{D\Gamma^{\uparrow}}{4\pi} \frac{\Gamma^{\downarrow}}{E_R^2 + \left(\frac{\Gamma^{\downarrow}}{2}\right)^2} \tag{22}$$

with $\Gamma^{\downarrow}$ determined by the golden rule Eq. (1). If the nondiagonal elements of the second part of $\mathcal{H}'$ are small, the resonances are isolated and the eigenvalues are approximately equal to the diagonal elements. This gives $\Gamma_R/2 \approx (A'_R)^2$.

According to Eq. (22), the coupling vectors $(A'_R)^2$ of the states R increase with increasing $\Gamma^{\uparrow}$. When the resonances start to overlap, resonance trapping comes into play. This must be taken into account in the parametrization of the interaction $\langle V^2 \rangle$ in the golden rule (1).

IV. NUMERICAL STUDIES

A. Interplay between internal and external mixing

Due to the internal interaction, not only the doorway state but also all the background states have a decay width. Thus an external mixing of the states is induced, which becomes important if the states are overlapping. In the following we study numerically the influence of this mixing onto Γ_v.

Let us consider values of W which are not restricted by the condition (15). In Fig. 1(a) we show the eigenvalue picture $\Gamma_R/2$ versus E_R (in units of $L/2$) for some $\Gamma^{\uparrow}$ (marked near the corresponding curves), $N=298$, $D=0.007$, and $W=0.01$, i.e., $v/D=1.2>1$. In all cases, $L=ND=2>\Gamma^{\uparrow}$. Due to the internal mixing, *all* states actually have a decay width. At small values of $\Gamma^{\uparrow}$ the internal interaction mixes the states so strongly that the original doorway state is not much broader than its neighbors, i.e., it loses its identity. At large $\Gamma^{\uparrow}$, however, the widths of all states (which are shown in the figure) but one are equal and much smaller than the width of the state in the center of the spectrum. This corresponds to the original assumption of equal and very small (zero) widths of the background states.

In Fig. 1(a) also $\Gamma(E)/2$ calculated according to the two resonance approximation Eq. (12) for $E_R<0$ (full lines on the left) and $(A'_{R\neq 0})^2$ for $E_R>0$ from Eqs. (22) and (1) with $\langle V^2 \rangle = v^2$ (full lines on the right) are shown. (Both curves are, of course, symmetric around $E_R=0$.) The agreement between $\Gamma_{R\neq 0}/2$ and $\Gamma(E)/2$ is good for $\Gamma^{\uparrow}=0.1$ and 1

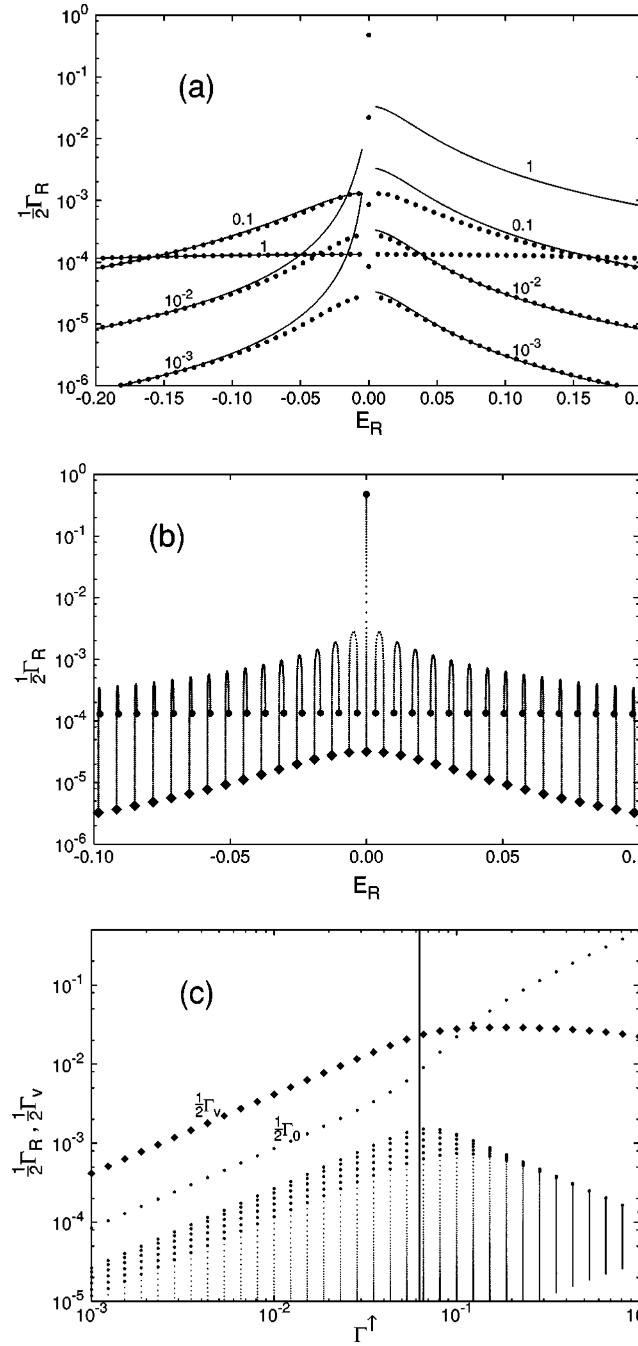

FIG. 1. (a) $\Gamma_R/2$ and E_R (in units of $L/2$) for different $\Gamma^{\uparrow}$ (marked in the plot). The full lines on the left side are calculated from Eq. (12) for $E_R<0$ and the full lines on the right from Eq. (22) for $E_R>0$. The unperturbed energies e_R of the background states are $\pm 1/N, \pm 2/N, \ldots$ (in units of $L/2$). (b) $\Gamma_R/2$ and E_R (in units of $L/2$) by varying $\Gamma^{\uparrow}$ from $\Gamma^{\uparrow}=10^{-3}$ (diamonds) to $\Gamma^{\uparrow}=1$ (full circles). The e_R are $\pm 1/(2N), \pm 3/(2N), \ldots$ (in units of $L/2$). (c) $\Gamma_R/2$ (points) and $\Gamma_v/2$ (diamonds) versus $\Gamma^{\uparrow}$ (all in units of $L/2$). The e_R are as in (a). The full line marks $\Gamma^{\uparrow}=2\pi W=0.063$. In (a), (b), and (c), $W=10^{-2}$, $L=2$, and $N=298$.

where Eq. (15) is fulfilled. By way of contrast, the bound state calculation describes well the results only for small $\Gamma^{\uparrow}$, i.e., for $\Gamma^{\uparrow}=10^{-3}$ and 10^{-2}. For larger $\Gamma^{\uparrow}$, the state at zero energy has a width Γ_0 much larger than the widths $\Gamma_{R\neq 0}$ of all the other states due to resonance trapping. The $\Gamma_{R\neq 0}$ are much smaller than the values obtained from Eqs. (22) and (1) with $\langle V^2\rangle = v^2$. For $\Gamma^\uparrow = 1$, the widths of all states but one are more than three orders of magnitude smaller than the width of the state in the center. There are N long-lived states overlapped by a broad state what can be described by Eq. (1) only when the residual interaction $\langle V^2\rangle$ is parametrized in a proper manner.

For illustration, we show in Fig. 1(b) the motion of the eigenvalues $E_R - (i/2)\Gamma_R$ in the complex plane as a function of increasing $\Gamma^\uparrow$ for the resonance states lying in the center of the spectrum [W and N are the same as in Fig. 1(a)]. In order to avoid the "hole" in the density of the background states at $E=0$ (where the doorway state lies), the spectrum of the discrete background states is shifted, in this calculation, by $\pm 1/(2N)$ (in units of $L/2$) as compared to Fig. 1(a). Internal mixing creates a spectrum which is almost equidistant in the center and has a distribution of the widths symmetrical around the center [diamonds in Fig. 1(b)]. By resonance trapping, the widths of all states but one equilibrate and the N trapped states return back to the original positions of the N discrete (unmixed) background states at $e_R = \pm 1/(2N), \pm 3/(2N) \ldots$ (in units of $L/2$) [full circles in Fig. 1(b)]. This picture shows very clearly that at high level density, there is again one short-lived state which overlaps N long-lived states.

In Fig. 1(c) we show $\Gamma_R/2$ and $\Gamma_v/2$ versus $\Gamma^\uparrow$ for the same W and N as in Fig. 1(a). For small $\Gamma^\uparrow$, the width Γ_0 of the broadest state is in the same order of magnitude as the width of its neighbors and consequently $\Gamma_v > \Gamma_0$. This is true as long as $\Gamma^\uparrow \leqslant 2\pi W = 0.063$. In the region $\Gamma^\uparrow \approx 2\pi W$ (marked with a line in the plot), the external mixing starts to play a role. The widths of background states being overlapped by the doorway state start to decrease with increasing $\Gamma^\uparrow$ (resonance trapping) and Γ_0 approaches $\Gamma^\uparrow$. As a consequence, $\Gamma_{R\neq 0} < 2D$ for all $\Gamma^\uparrow$ and $\Gamma_v < \Gamma_0$ for large $\Gamma^\uparrow$. The states at the borders of the spectrum, not being overlapped by the broad state, continue to increase in width and thus Γ_v saturates (Γ_v is the sum over *all* $\Gamma_{R\neq 0}$). The numerical saturation value, $\Gamma_v = 5.8W$, is in good agreement with the analytical result $\Gamma_v = 2\pi W$, Eq. (14).

Figure 1 shows that at $\Gamma^\uparrow < 2\pi W$ the spreading is so large that the doorway state can hardly be identified. This situation is described by the bound state calculation with the internal interaction v. At $\Gamma^\uparrow \gg 2\pi W$, however, the original situation of one state being much broader than all the other ones is restored by resonance trapping. This situation is well described by the two resonance approximation. Here, the residual interaction v is effectively reduced.

We study now the wave functions of the states. In Fig. 2(a) we show $|b_{00}|^2$ [compare Eqs. (5) and (6)] of the doorway state versus $\Gamma^\uparrow$ (in units of $L/2$) for $N=298$, $L=2$, and some different $W=$const. When $|b_{00}|^2 = 1$, the doorway state is aligned with the decay channel, i.e., the state is pure in the channel base. Figure 2(a) shows the following results. (i) The value $|b_{00}|^2$ decreases with increasing W. (ii) At small $\Gamma^\uparrow$, $|b_{00}|^2 \ll 1$, i.e., the internal interaction v mixes the doorway state strongly with its neighbors. (iii) At $\Gamma^\uparrow = 2\pi W$ [marked by crosses in Fig. 2(a)] resonance trapping starts to take place due to the external mixing, as can be seen for all curves corresponding to different constant values of W. (iv) At $\Gamma^\uparrow > 2\pi W$, the state $R=0$ aligns with the decay channel and $|b_{00}|^2 \to 1$. This means that the mixing of the states is effectively reduced due to the external mixing.

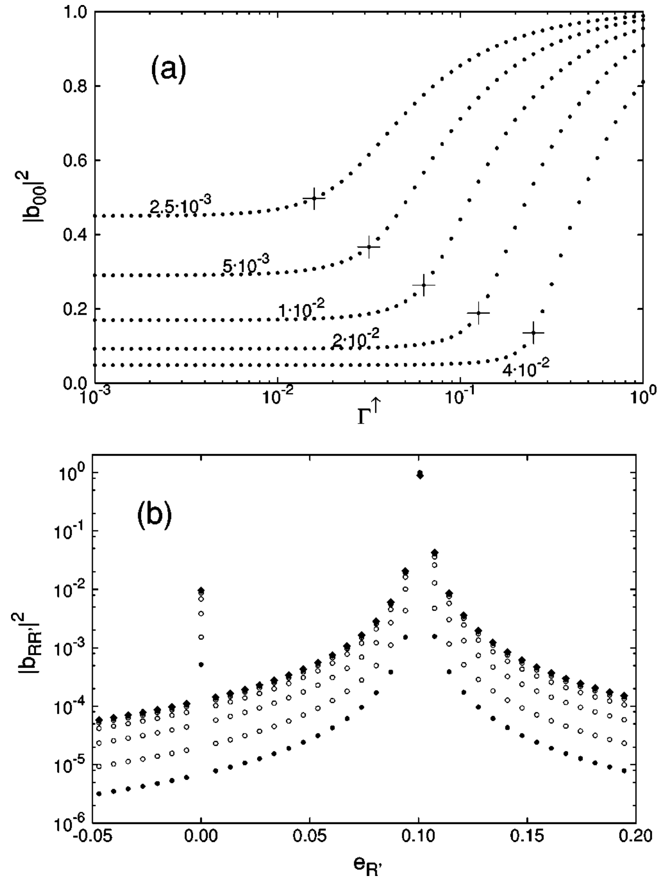

FIG. 2. (a) $|b_{00}|^2$ versus $\Gamma^\uparrow$ (in units of $L/2$) for $N=298$ and some W (marked in the plot). The crosses correspond to $\Gamma^\uparrow = 2\pi W$. (b) $|b_{RR'}|^2$ for the background state with $e_R = 0.1$ for some $\Gamma^\uparrow$ (in units of $L/2$); $N=298$ and $W=0.02$. $\Gamma^\uparrow = 0$ is marked with diamonds and $\Gamma^\uparrow = 1$ with full circles. The four curves marked by open circles are for $\Gamma^\uparrow = 0.1$, 0.18, 0.32, and 0.56.

In Fig. 2(b) we show $|b_{RR'}|^2$ for the background state with $e_R = 0.1$ calculated for some different $\Gamma^\uparrow$ and $W=0.02$. As long as $\Gamma^\uparrow < 2\pi W$, the $|b_{RR'}|^2$ are almost independent of $\Gamma^\uparrow$ and the corresponding values are covered by the values for $\Gamma^\uparrow = 0$ (diamonds). For $\Gamma^\uparrow > 2\pi W$, the wave function gets purer with increasing $\Gamma^\uparrow$ (in the base of h). It is $|b_{R0}|^2 \gg |b_{RR'\neq 0}|^2$ for $|R'-R| \gg 1$ because $|b_{R0}|^2 \propto \Gamma_R$ [see Eq. (7)].

Figures 1 and 2 illustrate the interplay between internal and external interaction. A strong internal interaction destroys the doorway picture but induces an external mixing of all the resonance states. If the external interaction gets strong, the original picture of one short-lived state together with N long-lived states will be restored: one state becomes much broader than all the other ones and the wave functions of all states become more or less pure. Effectively, the residual interaction, which is responsible for the mixing of the wave functions, starts to decrease with increasing $\Gamma^\uparrow$ at $\Gamma^\uparrow \approx 2\pi W$.

B. The role of *W*

In this subsection we keep $\Gamma^\uparrow$ constant. First, we study the behavior of the resonances as a function of the mean level

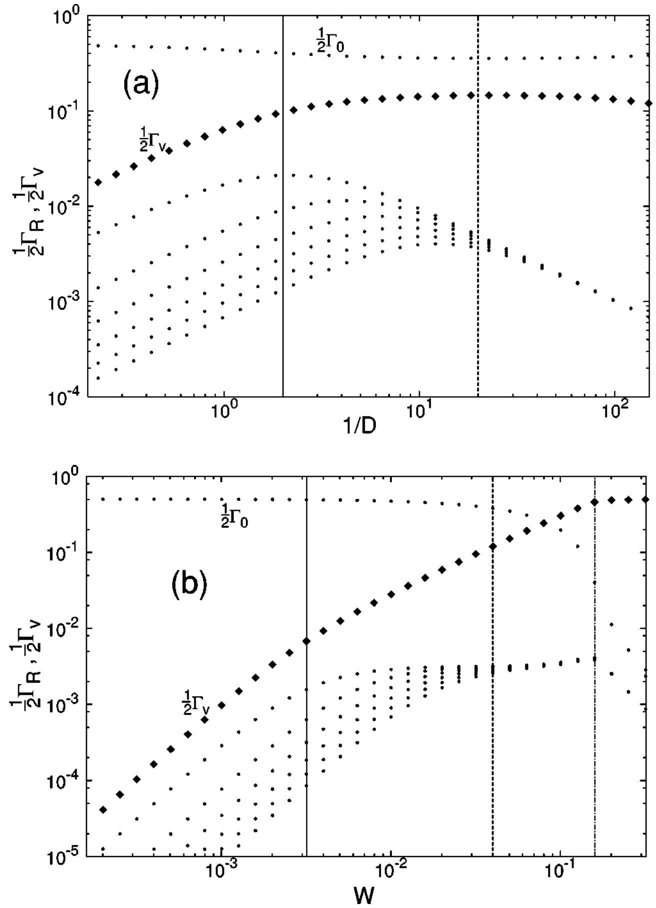

FIG. 3. (a) $\Gamma_v/2$ (diamonds) and some of the largest $\Gamma_R/2$ (points) versus $1/D$ (all in units of $\Gamma^{\uparrow}$); $N=298$, $W=\text{const}=0.05$. (b) $\Gamma_v/2$ and some $\Gamma_R/2$ versus W (all in units of $\Gamma^{\uparrow}$). $N=998$ and $v=\text{const}=0.04$ (instead of $W=\text{const}$). In (a) and (b) $\Gamma^{\uparrow}=1$ and D is varied by pushing the spectrum together. The full lines in (a) and (b) mark $\Gamma^{\uparrow}=2D$, the dashed lines $v=D$ and the dash-dotted line [in (b)] $\Gamma^{\uparrow}=2\pi W$.

distance $D=L/N$. One way to decrease D is to push the spectrum together, i.e., to change L. To keep $W=\text{const}$, Eq. (4), the v_R must be multiplied by a factor $\sqrt{D}$.

Figure 3(a) shows $\Gamma_v/2$ and some of the broadest $\Gamma_R/2$ versus $1/D=0.05/v^2$ (in units of $\Gamma^{\uparrow}$) calculated by pushing the spectrum together for $W=0.05$, $\Gamma^{\uparrow}=1$, and $N=298$. As long as resonance trapping is not important, all $\Gamma_{R\neq 0}$ as well as Γ_v increase with $1/D$. At $1/D\approx 2$, corresponding to $\Gamma^{\uparrow}\approx 2D$ (marked with a full line in the plot) trapping of the resonance states overlapped by the doorway state starts to take place. Both Γ_v and Γ_0 saturate, $\Gamma_v\rightarrow 2\pi W=0.3$. Note that in these calculations the length L of the spectrum is varied from 2000 to 2. Trapping starts to play a rôle at $1/D\approx 2$ corresponding to $L\approx 150$, which is much larger than the fixed value $\Gamma^{\uparrow}=1$.

It is $v=D$ at $1/D=20$ in Fig. 3(a), dashed line. Around $1/D\approx 20$, the behavior of the system does not change. This is in contrast to the changes in the system taking place at the onset of resonance trapping at $1/D\approx 2$.

A possibly more natural way to decrease D is to increase the number N of resonance states in a fixed energy interval L. Numerical calculations confirm the saturation of Γ_v as a function of N as soon as the doorway state overlaps its neighbors.

TABLE I. Γ_v for fixed $\Gamma^{\uparrow}$ and v.

	Γ_v
$N+1$ non-overlapping states	
$2D>\Gamma^{\uparrow}$; $D>v$	$\propto W^2$
Doorway picture (with trapping)	
$2\pi W<\Gamma^{\uparrow}$; $D>v$	$2\pi W$
Doorway picture (with trapping)	
$2\pi W<\Gamma^{\uparrow}$; $D<v$	$2\pi W$
$N+1$ states with comparable widths	
$2\pi W>\Gamma^{\uparrow}$; $D<v$	$\leq\Gamma^{\uparrow}$

Now let us consider the properties of the system at a fixed value $v=\sqrt{WD}$ instead of W, compare Eq. (4). In Fig. 3(b), $\Gamma_v/2$ and some $\Gamma_R/2$ versus $W=0.0016/D$ (in units of $\Gamma^{\uparrow}$) are shown for a calculation with $\Gamma^{\uparrow}=1$ and increasing W by changing L. We have chosen $N=998$ which is larger than that in Fig. 2(a) and a fixed $v_R=0.04$. For small W, Γ_v increases quadratically with increasing W whereas for $W>0.003$, Γ_v increases linearly with W. The value $W=0.003$ (full line in the plot) corresponds to $\Gamma^{\uparrow}=2D$.

For $W<0.003$, the resonance states do not overlap, $e_R^2\gg(\Gamma^{\uparrow})^2/4$, and Eq. (12) gives $\Gamma_R\propto D^{-2}\propto W^2$ for all $R\neq 0$. Therefore $\Gamma_v\propto W^2$. For $W>0.0032$, resonance trapping is important, and $\Gamma_v\propto D^{-1}\propto W$ according to Eq. (14). Thus resonance trapping makes the increase of Γ_v with W smaller as compared to regions of W where resonance trapping does not play a role. This result is independent of any assumptions on how v scales with D. Note that in this calculation resonance trapping starts at $L\approx 500$ ($N=998$) which is much larger than $\Gamma^{\uparrow}=1$.

Further, in Fig. 3(b), it is $v=D$ at $W\approx 0.04$ (dashed line). The transition from $v<D$ to $v>D$ does not change the behavior of the system. The transition from $2\pi W<\Gamma^{\uparrow}$ to $2\pi W>\Gamma^{\uparrow}$ takes place at $W\approx 0.16$ (dash-dotted line). Here, the properties of the system change drastically from the doorway picture to a situation in which the width of the broadest state is comparable with those of its neighbors. The state $R=0$ and some of its neighbors start to decrease their widths. Γ_v approaches its maximum value determined by $\Gamma^{\uparrow}$ which is fixed in the present calculation.

All these results show the same tendency as those shown in Figs. 1 and 2. Trapping plays a role in the region from $W\gtrsim 2v^2/\Gamma^{\uparrow}$ (corresponding to $\Gamma^{\uparrow}\gtrsim 2D$) up to $W\lesssim\Gamma^{\uparrow}/(2\pi)$.

Note that the border where the range of the inner mixing exceeds the length of the spectrum is $2\pi W=L$ which gives $W=0.5$. Thus the change of the system behavior at $2\pi W\approx\Gamma^{\uparrow}$ is *not* a border effect.

Summarizing the results, we state the following (compare Table I): The behavior of the system depends strongly on resonance trapping, i.e., on the ratio between the two values $\Gamma^{\uparrow}$ and $2\pi W$. It is *not* determined by the ratio between v and D. The whole region from $D\lesssim\Gamma^{\uparrow}/2$ [full line in Fig. 3(b)] up

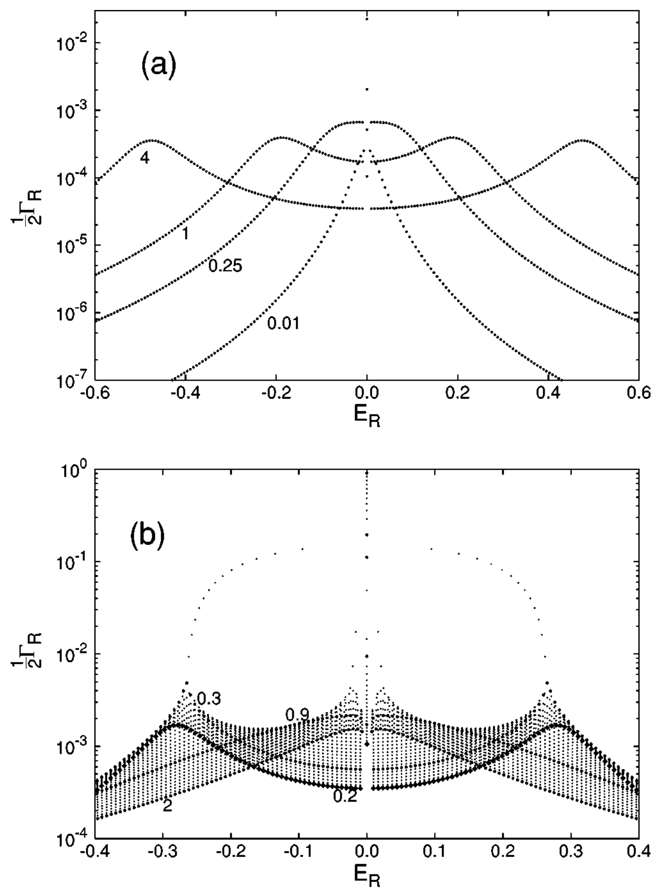

FIG. 4. $\Gamma_R/2$ and E_R (in units of $L/2$) for a Breit-Wigner shaped distribution of the v with width $\alpha=0.3$. $L=2$ and $N=298$. The unperturbed energies e_R are as in Fig. 1(a). (a) $\Gamma^{\uparrow}=0.05$ and some different W (marked in the plot). (b) $W=2$ and $0.2\leqslant\Gamma^{\uparrow}\leqslant 2$ in steps of $lg_{10}\Gamma^{\uparrow}=1/30$. Some $\Gamma^{\uparrow}$ are marked in the plot.

to $\Gamma^{\uparrow}\approx 2\pi W$ [dash-dotted line in Fig. 3(b)] is determined by resonance trapping. Here, Γ_v increases linearly with W: $\Gamma_v\approx 2\pi W$. For $2\pi W>\Gamma^{\uparrow}$, the internal mixing of the states dominates, the doorway picture breaks down and Γ_v reaches its maximum value determined by $\Gamma^{\uparrow}$.

C. Beyond the picket fence model

In realistic systems, the v_R can have an energy dependence. In the following we do a small excursion investigating the question whether such an energy dependence influences the conclusions of this paper. We assume that the v_R decrease with an increasing energy difference e between the background and the doorway state,

$$v(e)=\frac{\sqrt{WD}}{2\pi}\;\frac{\alpha}{e^2+\alpha^2/4}\;. \tag{23}$$

Here, α controls the width of the distribution and Eq. (4) still holds.

In Fig. 4(a) we show the eigenvalue picture $\Gamma_R/2$ and E_R (in units of $L/2$) for $\Gamma^{\uparrow}=0.05$, $N=298$, $\alpha=0.3$ and some W ranging from 0.01 to 4. The case $W=0.01$ corresponds to a narrow distribution of the widths Γ_R. As W increases, $\Gamma_{R=0}$ decreases and at $W=0.25$, a broad structure is formed in the distribution of $\Gamma_{R\neq 0}$. For even larger W two hump shapes are formed which move away from the center of the spectrum. As a result, a strong internal coupling repels the strength from the middle of the spectrum to the borders.

In order to study the influence of the coupling to the decay channel we show in Fig. 4(b) the eigenvalue picture $\Gamma_R/2$ and E_R for $N=298$, $\alpha=0.3$ [the same as in Fig. 4(a)], but using a fixed $W=2$ and some $\Gamma^{\uparrow}$ ranging from 0.2 to 2. For small $\Gamma^{\uparrow}$ we see two hump shapes as in Fig. 4(a). At $\Gamma^{\uparrow}\approx 0.3$ the widths of the broadest states in both humps are of the order of the level distance D. Local resonance trapping takes place in each hump which creates two broad states. These states attract each other in energy with further increasing $\Gamma^{\uparrow}$. Two-resonance trapping occurs and creates just one broad state at $\Gamma^{\uparrow}\approx 0.9$. As a result, the two humps disappear and we have, at large $\Gamma^{\uparrow}$, just one broad hump around the energy $E=0$.

Note that Figs. 4(a) and 4(b) have only illustrative character. They illustrate however boldly that the internal interaction spreads the strength whereas the external one gathers it back to the original doorway state.

Additionally, we have also calculated the Γ_v for the case that the levels are distributed in a more realistic manner. We have chosen a level distribution of h according to the GOE and random elements v_R of the internal interaction. We performed the calculations in the same manner as above for the picket-fence level distribution with constant v_R (Figs. 1 and 3). After averaging over a number of random realizations of the Hamiltonian, the results for Γ_v are the same as for the picket fence.

Also in the two cases of an energy dependent internal interaction and a random distribution of e_R and v_R the internal mixing is reduced by the external one and the original picture with coexistence of states of very different lifetime is restored at large $\Gamma^{\uparrow}$.

V. DISCUSSION

In this paper we studied analytically as well as numerically the spreading of transition strength in the doorway concept at high excitation energy. According to this picture, the doorway state is coupled to a large number of background states through the real internal interaction v. Further, it is coupled, due to its spectroscopic properties, to the continuum (decay channel) by $\Gamma^{\uparrow}$ while the background states $\Gamma_{R\neq 0}$ have an access to the continuum only via the doorway state.

For $2\pi W\equiv 2\pi v^2/D>\Gamma^{\uparrow}$ the internal interaction destroys the original doorway picture obtained from the spectroscopic properties of the states. This means, there does not exist a state the width of which is much larger than that of all the other ones and the wave function of which is aligned with the channel wave function. This situation is well described by a calculation without considering the coupling to the continuum.

At $\Gamma^{\uparrow}\approx 2\pi W$ the external interaction starts to reduce the mixing of the states by giving back a large part of the available width $\Gamma^{\uparrow}$ to only one state. This situation is characterized by resonance trapping. For $\Gamma^{\uparrow}>2\pi W$ the original picture with one short-lived and N long-lived states is restored. One of the states has again a much larger width than all the others and the wave function of the doorway state as well as those of the background states are almost pure. Thus, the phenomenological doorway picture finds its justification if the coupling of the states via the continuum is taken into account. Table I shows that it holds always when $\Gamma^{\uparrow} > 2\pi W$ where W contains the internal interaction v.

This picture is proven by us for $\Gamma^{\uparrow} \ll L$ and $2\pi W \ll L$ where L is the length of the spectrum. This means, the restoration of the original picture with coexistence of a short-lived and N long-lived states at large W is *not* a border effect. It appears in realistic systems due to resonance trapping.

In the golden rule $\Gamma^{\downarrow} = 2\pi\langle V^2\rangle/D$, the interaction $\langle V^2\rangle$ appears in parametrized form. The parametrization includes the direct internal coupling of the states as well as the external coupling via the continuum. At high excitation energy (high level density), the external mixing of all the resonance states is large and therefore $\langle V^2\rangle$ is small. The assumption that $\langle V^2\rangle$ scales with D as proposed in [9] describes this situation. That means long-lived resonance states coexist with a few short-lived resonance states also at high level density. The mixing of the wave functions of these two types of resonance states is small.

Thus, resonance trapping correlates high level density (small D) with small $\langle V^2\rangle$ what is unexpected from the point of view of bound state calculations. As a consequence, the spreading widths $\Gamma^{\downarrow}$ do not increase with increasing level density. This outcome explains the experimental result [1] according to which collisional damping at a temperature of about 2 MeV is the same as at zero temperature.

In the doorway picture, only one decay channel is important whereas in nuclei at high excitation energy many decay channels are open. Their coupling strengths to the system differ, however, considerably. In [4] it was shown that this leads effectively to a small number of open decay channels needed for describing the decay of the system in a certain energy region. Therefore, the one channel approximation used in this paper implies no major restriction of the applicability of the results.

As a conclusion, we state that the interplay between internal and external mixing determines the spreading of transition strength in an open quantum system. Pure states may exist even at large excitation energies. An equilibration of the states in relation to their decay widths does not occur.

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