Deuteron stripping and pick-up on halo nuclei

N. K. Timofeyuk* and R. C. Johnson
Physics Department, University of Surrey, Guildford, Surrey GU2 5XH, United Kingdom
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We present an approach to halo transfer in (p,d) and (d,p) reactions in which excitation and breakup effects are treated in a simple way. The method assumes that halo excitation energies are small compared with kinetic energies in the channel containing halo nucleus. It is shown that in the zero-range approximation to this approach the transfer amplitude has a formal resemblance to the conventional distorted wave Born approximation amplitude in which the distorted wave in the channel containing the halo nucleus is replaced by an effective distorted wave. Finite-range corrections are included using a generalization of the local-energy approximation. As a first application of the method, numerical calculations have been performed for the $^{16}O(d,p)^{17}O$, $^{10}Be(d,p)^{11}Be$, and $^{11}Be(p,d)^{10}Be$ reactions. Deuteron breakup was included within the adiabatic approach of Johnson and Soper. It was found that including recoil excitation and breakup of the ^{17}O and ^{11}Be nuclei increases the calculated cross sections and thus decreases the deduced spectroscopic factors. This effect is expected to increase with incident energy and decrease when the mass of the core of the halo nucleus increases relative to the mass of the halo. [S0556-2813(99)03203-3]

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I. INTRODUCTION

Recently, nuclear reactions involving transfer of a halo particle have attracted attention. Because of the simplicity of the theoretical interpretation of these reactions, they are thought to provide an important source of the information about the structure of halo nuclei. Some experiments with radioactive beams have already been performed. For example, the cross sections for the neutron transfer reaction $^{10}Be(d,p)^{11}Be$ were measured using the radioactive target ^{10}Be almost twenty years ago [1]. Measurements of the inverse reaction $^{11}Be(p,d)^{10}Be$ have recently been performed with the aim of studying the role of the core excitation in the structure of ^{11}Be [2].

An important motivation for studying transfer reactions is to extract spectroscopic factors. In the standard procedure, spectroscopic factors are determined as ratios of the experimental transfer cross sections to those calculated within the distorted wave Born approximation (DWBA). The latter assumes that the initial and final channel wave functions are entirely dominated by their elastic components. However, one would expect that in the case of weakly bound halo nuclei, channel wave function components involving excitation of the halo degree of freedom by the target nucleus would be important and that standard DWBA might not be reliable. The situation is further complicated by the fact that for halo nuclei these excitations are likely to include unbound continuum states.

These effects are best illustrated by reference to the case of the simplest ''halo'' nucleus, the deuteron. The DWBA assumes that the only way breakup influences the initial channel wave function in a (d,p) reaction is by contributing extra absorption in the elastic component. However, transfer can occur directly out of the breakup components themselves. In the transfer matrix element these components can interfere constructively or destructively with the elastic component. The effect in many cases is that when deuteron breakup is included, the spectroscopic factors extracted from the experimental data are about 20–30 % smaller than those obtained with the conventional DWBA. However, at present there are no simple theories which estimate the role of breakup effects in transfer reactions involving halo nuclei in initial or final channels.

One approach to including the effects of halo excitation and breakup in the transfer amplitude, is to solve continuum-discretized coupled channels (CDCC) equations for the channel containing the halo. However, this requires considerable computational effort and physical information about all possible bound state-continuum and continuum-continuum form factors which is never completely available. Convergence problems may also become more serious when very weakly bound nuclei are involved.

In this paper we introduce a simple approach to the treatment of break-up and excitation in halo transfer (p,d) and (d,p) reactions within the framework of a three-body model. This approach is based on an alternative exact representation of the transfer amplitude in which the transition operator is the p-n potential and any effects due to remnant terms are included in the three-body wave function in the initial or final channel.

We consider cases where the energy of halo excitations mixed into the relevant channel wave functions is small compared with the channel kinetic energies. It is shown below that in these circumstances we can incorporate these excitations into the transfer amplitude by using the adiabatic wave function of Johnson, Al-Khalili, and Tostevin [3] in one channel and Johnson and Soper [4] in the other. The latter form of the adiabatic wave function is valid only for the deuteron within the range of the n-p interaction.

The application of the adiabatic three-body wave function of Johnson, Al-Khalili, and Tostevin [3] to the elastic scattering of halo nuclei has given a useful insight into the main

*Permanent address: Institute of Nuclear Physics, Ulugbek, Tashkent 702132, Uzbekistan.

features of the elastic scattering of these nuclei. We show here that in the zero-range limit the use of the same wave function in the transfer amplitude results in a formula which has some similarity with the conventional DWBA amplitude, but one in which the distorted wave in the halo channel is replaced by an effective distorted wave constructed according to a definite prescription. Unlike DWBA this new amplitude does not require knowledge of optical potentials for the halo nuclei involved. It is determined by nucleon optical potentials which describe elastic scattering from the core. Finite range corrections are included using a generalization of the local-energy approximation as developed by Buttle and Goldfarb [5].

Although the present paper has been motivated by experimental studies of deuteron stripping and pick-up reactions involving halo nuclei, the formalism developed here is more generally applicable. We have in mind, for example, (d,p) and other one nucleon transfer reactions involving both halo and nonhalo nuclei, two neutron transfer reactions and other reactions with heavy ions at energies high enough for our adiabatic approximations to be valid.

In Sec. II we choose an appropriate form for the exact transition amplitude and derive zero-range and local-energy approximation in the adiabatic approach. In Sec. III we apply our approach to the ^{16}O$(d,p)^{17}$O, ^{10}Be$(d,p)^{11}$Be, and ^{11}Be$(p,d)^{10}$Be reactions. The results of our calculations are discussed in Sec. IV.

II. TRANSFER AMPLITUDE

We consider a halo transfer reaction

$$P+(h+C)\rightarrow(P+h)+C,$$

where P is a projectile (proton), h is a halo particle, and C is a core to which the halo particle is weakly bound in the entrance channel. The proton P carries away a halo particle h leaving the core C. The amplitude of such a reaction can be written in either prior or post form as [6]

$$T_{\rm prior}=\langle\Psi^{(-)}|V_{Ph}(\vec{r}_{Ph})+V_{PC}(\vec{r}_{PC}) - U_{\alpha}(\vec{r}_{\alpha})|\chi^{(+)}_{\alpha}(\vec{k}_{\alpha},\vec{r}_{\alpha})\psi_{Ch}(\vec{r}_{Ch})\rangle, \quad (1)$$

$$T_{\rm post}=\langle\chi^{(-)}_{\beta}(\vec{k}_{\beta},\vec{r}_{\beta})\psi_{Ph}(\vec{r}_{Ph})|V_{Ch}(\vec{r}_{Ch}) + V_{PC}(\vec{r}_{PC})-U_{\beta}(\vec{r}_{\beta})|\Psi^{(+)}\rangle, \quad (2)$$

where $\vec{r}_{ij}$ is the relative coordinate between nuclei i and j, V_{ij} is the interaction potential between nuclei i and j, $\Psi^{(\pm)}$ are exact solutions of the many-body Schrödinger equation. The distorted waves χ_{α} and χ_{β} are the solutions of the two-body Schrödinger equation with optical potentials U_{α} and U_{β} in the entrance and exit channels, respectively.

DWBA calculations are often based on several assumptions. These include the following.

(i) The cancellation hypothesis. This assumes that $[V_{PC}(\vec{r}_{PC})-U_{\alpha}(\vec{r}_{\alpha})]\chi^{(+)}_{\alpha}(\vec{k}_{\alpha},\vec{r}_{\alpha})\psi_{Ch}(\vec{r}_{Ch})\approx 0$ in the prior form or $[V_{PC}(\vec{r}_{PC})-U_{\beta}(\vec{r}_{\beta})]\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r}_{\beta})\psi^{*}_{Ph}(\vec{r}_{Ph})\approx 0$ in the post form.

(ii) The approximation of the many-body wave function Ψ either by $\chi_{\beta}(\vec{k}_{\beta},\vec{r}_{\beta})\psi_{Ph}(\vec{r}_{Ph})$ in the prior DWBA or by $\chi_{\alpha}(\vec{k}_{\alpha},\vec{r}_{\alpha})\psi_{Ch}(\vec{r}_{Ch})$ in the post DWBA.

The first assumption is reasonably well satisfied for a single nucleon transfer between heavy tightly bound nuclei. However, in the case of halo nuclei the optical potential between the projectile and a halo target may differ strongly from the projectile-core interaction potential V_{PC} because in the former case there will be important channels involving excitation of the halo degrees of freedom which are obviously not present when internal excitations of the core alone are involved. Similar considerations suggest that the approximation of the exact many-body wave function Ψ by an elastic distorted wave times halo bound state wave function, i.e., its projection onto an elastic channel, may not be valid.

We start with the transfer amplitude in the the prior form (1). If Ψ is the exact solution of the many-body problem, then the optical potential U_{α} in the transition operator amplitude can be chosen arbitrarily. In this case the corresponding entrance channel wave function must be the solution of the Schrödinger equation with potential $V_{Ch}(\vec{r}_{Ch})+U_{\alpha}(\vec{r}_{\alpha})$. If we replace the optical potential $U_{\alpha}(\vec{r}_{\alpha})$ by the potential $V_{PC}(\vec{r}_{PC})$ then the entrance channel wave function (which we now denote $\tilde{\phi}$) would be a solution of the Schrödinger equation in which the interaction between nucleon h and nucleons of projectile P is switched off

$$[\hat{T}+V_{Ch}(\vec{r}_{Ch})+V_{PC}(\vec{r}_{PC})-E]\tilde{\phi}^{(+)}_{\alpha}=0, \quad (3)$$

where $\hat{T}$ is the kinetic energy operator. Therefore, the exact transfer amplitude is given by the expression

$$T=\langle\Psi^{(-)}|V_{Ph}|\tilde{\phi}^{(+)}_{\alpha}\rangle. \quad (4)$$

A rigorous derivation of this equation for the case of a (d,p) reaction is given in the book of Goldberger and Watson [7] on pp. 838–839.

It was shown in Refs. [3] and [8] that when the projectile energy is sufficiently large the ''adiabatic'' (or ''sudden'') approximation can be used to solve Eq. (3). The adiabatic solution of this equation can be written without further approximation as[1]

$$\tilde{\phi}^{ad(+)}_{\alpha}=\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}_{CP})\psi_{Ch}(\vec{r}_{Ch})e^{-i\alpha\vec{k}_{\alpha}\vec{r}_{Ch}}, \quad (5)$$

where $\alpha=m_h/(m_h+m_C)$ and m_i is the mass of nucleus i. The wave function $\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}_{CP})$ is a ''distorted-wave'' solution of the two-body Schrödinger equation for a particle of mass $\mu_{\alpha}=m_P(m_h+m_C)/(m_P+m_h+m_C)$ in the field of potential V_{PC}.

Substituting Eq. (5) to Eq. (4) gives

$$T_{ad}=\langle\Psi^{(-)}|V_{Ph}(\vec{r}_{Ph})|\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}_{CP})\psi_{Ch}(\vec{r}_{Ch})e^{-i\alpha\vec{k}_{\alpha}\vec{r}_{Ch}}\rangle. \quad (6)$$

[1]Here we use the minus sign in the exponent because in this paper the direction of the vector $\vec{r}_{CP}=\vec{r}_{C}-\vec{r}_{P}$ has different definition than that used in Ref. [3].

The advantage of Eq. (6) is that the transfer amplitude contains only the short-range potential V_{Ph} as a transition operator and there is no need for any cancellation hypothesis.

The meaning of the result Eq. (6) is best understood by first considering the exact Eq. (4). The function $\tilde{\phi}_\alpha^{(+)}$ contains components in which the relative motion of C and h in the nucleus $(C+h)$ is in its ground state or excited to one of its bound or continuum breakup states. The latter components are mixed in by the interaction between P and C alone, i.e., by the recoil of C being transmitted to h through V_{Ch}. The interaction between P and the halo particle h plays no role at this stage. When the range of $(C+h)$ excitation energies is small compared with the incident kinetic energy in the center of mass system then these admixtures may be adequately described by an adiabatic approximation in which the internal Hamiltonian of the $(h+C)$ system is replaced by a constant (its ground state value). The resulting two-body Schrödinger equation is referred to as the adiabatic approximation to the three-body equation (3). The correct three-body wave function corresponding to an incident wave in a channel with $(C+h)$ in its ground state is the wave function $\tilde{\phi}_\alpha^{ad(+)}$ of Eq. (5). Because of its complicated dependence on $\vec{r}_{Ch}$ for a fixed $\vec{r}_{P(Ch)}$ this function has a nonvanishing overlap with excited states of the halo which may be bound or in the continuum. Exact quantum mechanical scattering theory tells us that all effects of the projectile-halo particle V_{Ph} are fully taken into account through the factors V_{Ph} and $\Psi^{(-)}$ in Eq. (4).

To distinguish the amplitude (6) from the usual DWBA, we will refer to recoil excitation and breakup (REB) effects. We will discuss the results of calculations with and without REB effects.

A. Zero-range approximation

Since V_{Ph} in Eq. (4) is short range, the many-body wave function $\Psi^{(-)}$ is needed only when the halo particle h is close to the projectile P. This piece of $\Psi^{(-)}$ is easy to construct if the relative motion of P and h can be treated adiabatically as was the C-h relative motion in the last section. This was first shown by Johnson and Soper [4,9,10] in the (d,p) and (p,d) case. They showed that within the range of V_{Ph} the wave function $\Psi^{(-)}$ can be approximated by

$$\Psi^{(-)} \approx \chi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta)\psi_{Ph}(\vec{r}_{Ph}), \tag{7}$$

where $\chi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta)$ is a solution of a two-body Schrödinger equation with the adiabatic Johnson-Soper potential. The DWBA uses an approximation for $\Psi^{(-)}$ of the same general form but with a very different choice for the potential generating $\chi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta)$, namely a potential which fits elastic scattering of $(P+h)$ and C. The Johnson-Soper potential is designed to generate the three-body wave function at $P-h$ coincidence and this will be a superposition of components in which the $(P+h)$ system may be excited or broken up. This potential may not describe elastic scattering because it is not designed to. For the analysis that follows we assume only that $\Psi^{(-)}$ has the form of Eq. (7) and make it clear which theory is being used in the context of specific calculations in the following sections.

We assume that the $(P+h)$ system is in a $0s$-wave bound state and introduce a zero-range approximation

$$V_{Ph}(\vec{r}_{Ph})\psi_{Ph}(\vec{r}_{Ph}) = D_0\delta(\vec{r}_P - \vec{r}_h), \tag{8}$$

where

$$D_0 = \int d\vec{r}\, \psi_{Ph}(\vec{r})V_{Ph}(\vec{r}). \tag{9}$$

Then, after a substitution of Eqs. (7) and (8) into Eq. (6), the transfer amplitude becomes

$$T_{ad}^{ZR} = D_0 \int d\vec{r}\, \chi_\beta^{(-)*}(\vec{k}_\beta, \vec{r})\psi_{Ch}(\vec{r})\tilde{\chi}_\alpha(\vec{k}_\alpha, \vec{r}). \tag{10}$$

Here a new distorted wave function

$$\tilde{\chi}_\alpha(\vec{k}_\alpha, \vec{r}) = e^{-i\alpha\vec{k}_\alpha\vec{r}}\chi_{PC}^{(+)}(\vec{k}_\alpha, \vec{r}) \tag{11}$$

has been introduced. This function plays the role of an effective distorted wave in the entrance channel.

It is possible to perform a partial wave expansion of $\tilde{\chi}_\alpha(\vec{k}_\alpha, \vec{r})$. In the absence of the spin-orbit interactions this expansion is

$$\tilde{\chi}_\alpha(\vec{k}_\alpha, \vec{r}) = \frac{4\pi}{k_\alpha r}\sum_{LM} i^L \tilde{\chi}_L(k_\alpha r)Y_{LM}^*(\hat{k}_\alpha)Y_{LM}(\hat{r}), \tag{12}$$

where

$$\tilde{\chi}_L(k_\alpha r) = \sum_{L'=0}^{\infty} \chi_{L'}(k_\alpha r)\sum_\lambda (-)^{(L'-\lambda-L)/2}(2\lambda+1) \times (L0\lambda 0|L'0)^2 j_\lambda(\alpha k_\alpha r) \tag{13}$$

and $\chi_{L'}(k_\alpha r)$ is the partial wave obtained from the expansion of $\chi_{PC}(\vec{k}_\alpha, \vec{r})$.

Let us compare the adiabatic zero-range amplitude (10), which contains explicit $(C+h)$ excitation and breakup effects of the type described in the last section, with the standard DWBA zero-range amplitude

$$T_{\text{standard}}^{ZR} = D_0 \int d\vec{r}\, \chi_\beta^{(-)*}(\vec{k}_\beta, \vec{r})\psi_{Ch}(\vec{r})\chi_\alpha(\vec{k}_\alpha, \mu\vec{r}), \tag{14}$$

where $\mu = m_C/(m_C+m_h)$.

We see that these two amplitudes have exactly the same structure. However, the interpretation of the entrance channel distorted wave is different. The standard DWBA uses a distorted wave χ_α which is a solution of the Schrödinger equation with optical potential U_α while the adiabatic zero-range approximation uses the effective distorted wave (11) which is constructed on the basis of the projectile-core distorted wave $\chi_{PC}^{(+)}$. In addition, the argument of the effective distorted wave (11) is not scaled with a factor μ. Therefore, to treat the recoil excitation and break-up effects of the $(C+h)$ in the zero-range approximation one can use available DWBA zero-range computer codes after replacing the entrance channel distorted wave χ_α by the effective distorted wave (11)–(13).

B. Beyond the zero-range approximation

To estimate how important finite-range effects are, let us rewrite the amplitude (6) as

$$T_{ad}=\int d\vec{r}_{Ch}d\vec{r}_{Ph}\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r}_{\beta})\psi^{*}_{Ph}(\vec{r}_{Ph})V_{Ph}(\vec{r}_{Ph}) \times\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}_{CP})\psi_{Ch}(\vec{r}_{Ch})e^{-i\alpha\vec{k}_{\alpha}\vec{r}_{Ch}}, \quad (15)$$

where $\vec{r}_{\beta}=\vec{r}_{Ch}+\gamma\vec{r}_{Ph}$, $\gamma=-m_P/(m_h+m_P)$ and $\vec{r}_{CP}=\vec{r}_{Ch}-\vec{r}_{Ph}$. Then, by use of the translation operator, the distorted waves $\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r}_{\beta})$ and $\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}_{CP})$ may be written as

$$\chi^{(-)}_{\beta}(\vec{k}_{\beta},\vec{r}_{\beta})=e^{\gamma\vec{r}_{Ph}\vec{\nabla}_{Ch}}\chi^{(-)}_{\beta}(\vec{k}_{\beta},\vec{r}_{Ch}), \quad (16)$$

$$\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r})=e^{-\vec{r}_{Ph}\vec{\nabla}_{CP}}\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}_{Ch}). \quad (17)$$

Using the well-known Taylor expansion of the exponential operator

$$e^{\vec{r}\vec{O}}=1+\vec{r}\vec{O}+\frac{1}{6}r^2O^2\left(1+\frac{8\pi}{5}Y_2(\hat{r})\cdot Y_2(\hat{O})\right)+\cdots \quad (18)$$

one can present the amplitude T_{ad} as

$$T_{ad}=T_0+T_1+T_2+\cdots, \quad (19)$$

where $T_0=T^{ZR}_{ad}$,

$$T_1=\vec{D}_1\int d\vec{r}\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}[\gamma\vec{\nabla}\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r}) -\vec{\nabla}\chi_{PC}(\vec{k}_{\alpha},\vec{r})],$$

$$\vec{D}_1=\int d\vec{r}\vec{r}\psi^{*}_{Ph}(\vec{r})V_{Ph}(\vec{r}). \quad (20)$$

In our special case, when the $P+h$ system is in an s state, T_1 does not contribute to the reaction amplitude because $\vec{D}_1=0$. However, in the general case, the dipole corection to the adiabatic zero-range amplitude has an order of magnitude of $\gamma R_{Ph}k_{\beta}+k_{\alpha}$ where R_{Ph} is the radius of the $P-h$ interaction potential. It is clear that when the adiabatic approximation is valid, $k_{\alpha}>1$ fm, the contribution of this term can be large and exact finite-range calculations should be performed.

Let us estimate the second order correction T_2 to the zero-range amplitude:

$$T_2=\frac{1}{6}D_2\int d\vec{r}\,\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}(\gamma\vec{\nabla}_{\beta}-\vec{\nabla}_{CP})^2 \times\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r})$$

$$=\frac{1}{6}D_2\int d\vec{r}\{\gamma^2[\nabla^2\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})]\psi_{Ch}(\vec{r})\tilde{\chi}_{\alpha}(\vec{k}_{\alpha},\vec{r}) +\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}[\nabla^2\chi_{PC}(\vec{k}_{\alpha},\vec{r})] -2\gamma\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}[\vec{\nabla}\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})]\cdot[\vec{\nabla}\chi_{PC}(\vec{k}_{\alpha},\vec{r})]\},$$

$$D_2=\int d\vec{r}\,r^2\psi^{*}_{Ph}(\vec{r})V_{Ph}(\vec{r}). \quad (21)$$

The term containing the product of the first derivatives may be eliminated with the help of the equation

$$2\int d\vec{r}\,\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}[\vec{\nabla}\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})]\cdot[\vec{\nabla}\chi_{PC}(\vec{k}_{\alpha},\vec{r})]$$

$$=-\int d\vec{r}\,\psi_{Ch}(\vec{r})[\nabla^2\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})]\tilde{\chi}_{\alpha}(\vec{k}_{\alpha},\vec{r}) -\int d\vec{r}\,\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}[\nabla^2\chi_{PC}(\vec{k}_{\alpha},\vec{r})] +\int d\vec{r}\,\chi_{PC}(\vec{k}_{\alpha},\vec{r})\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r})\{\nabla^2[\psi_{Ch}(\vec{r})e^{-i\alpha\vec{k}_{\alpha}\vec{r}}]\}.$$

The terms containing second derivatives may be eliminated using the Schrödinger equations

$$\nabla^2\chi^{(-)}_{\beta}(\vec{k}_{\beta},\vec{r})=\frac{2\mu_{\beta}}{\hbar^2}[U_{\beta}(r)-E_{\beta}]\chi^{(-)}_{\beta}(\vec{k}_{\beta},\vec{r}),$$

$$\nabla^2\psi_{Ch}(\vec{r})=\frac{2\mu_{Ch}}{\hbar^2}[V_{Ch}(r)+B_{Ch}]\psi_{Ch}(\vec{r}),$$

$$\nabla^2\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r})=\frac{2\mu_{\alpha}}{\hbar^2}[V_{PC}(r)-E_{\alpha}]\chi^{(+)}_{PC}(\vec{k}_{\alpha},\vec{r}). \quad (22)$$

Then the amplitude T_{ad} can be written in a form similar to the local-energy approximation (LEA) of the standard DWBA:

$$T_{ad}\approx T^{\mathrm{LEA}}_{ad}=D_0\int d\vec{r}\,\chi^{(-)*}_{\beta}(\vec{k}_{\beta},\vec{r}) \times\left[1+\frac{1}{6}\rho^2\Lambda_{ad}(r)+\Delta(\vec{r})\right]\psi_{Ch}(\vec{r})\tilde{\chi}_{\alpha}(\vec{k}_{\alpha},\vec{r})$$

$$\Lambda_{ad}(r)=\gamma(1+\gamma)\frac{2\mu_{\beta}}{\hbar^2}[U_{\beta}(r)-E_{\beta}]+(1+\gamma)\frac{2\mu_{\alpha}}{\hbar^2} \times[V_{PC}(r)-E_{\alpha}]-\gamma\frac{2\mu_{Ch}}{\hbar^2}[V_{Ch}(r)+B_{Ch}] +\gamma\alpha^2\frac{2\mu_{\alpha}E_{\alpha}}{\hbar^2}$$

$$\Delta(\vec{r})=\frac{1}{3}i\rho^2\alpha\gamma[\vec{k}_{\alpha}\cdot\vec{\nabla}\psi_{Ch}(\vec{r})]/\psi_{Ch}(\vec{r}), \quad (23)$$

where $\rho^2=D_2/D_0$ is the root-mean square radius of the function $\psi_{Ph}V_{Ph}$. We see that $\Lambda_{ad}(r)$ has more complicated structure than in the standard LEA [5,6,11]

$$\Lambda_{\text{standard}}(r)=\frac{2\mu_{Ph}}{\hbar^2}\left[V_{Ch}(r)+U_\alpha\left(\frac{m_C}{m_c+m_h}r\right)-U_\beta(r)+B_{Ph}\right], \quad (24)$$

where $\mu_{Ph}=m_P m_h/(m_P+m_h)$.

Another difference with the standard LEA is that the term $\Delta(\vec{r})$ containing a first derivative is still present in Eq. (23). However, this term is imaginary (at least asymptotically). If the contribution from the nuclear interior is neglected, then

$$T^{\text{LEA}}=T^{ZR}\left(1+\frac{1}{6}\rho^2\Lambda_{ad}(\infty)+i|\Delta(\infty)|\right) \quad (25)$$

and

$$|T^{\text{LEA}}|^2=|T^{ZR}|^2\left[\left(1+\frac{1}{6}\rho^2\Lambda_{ad}(\infty)\right)^2+|\Delta(\infty)|^2\right]. \quad (26)$$

It is clear that when $|\Delta(\infty)|^2\ll 1$, its contribution to the cross section is negligible even if $|\Delta(\infty)|\approx\frac{1}{6}\rho^2\Lambda_{ad}(\infty)$.

In the case of the (p,d) reactions on $1p$-shell halo nuclei $|\Delta(\infty)|$ has an order of magnitude of $\alpha\gamma k_\alpha k_{Ch}R_{Ph}^2/3$ where $k_{Ch}=\sqrt{2\mu_{Ch}B_{Ch}/\hbar^2}$, B_{Ch} is the binding energy of the halo particle and $1/2\leqslant\gamma\leqslant 1$. For typical values $k_{Ch}=0.1\sim 0.2$ fm^{-1}, $k_\alpha=1$ fm^{-1}, and $R_{Ph}=2$ fm and $\alpha=1/10$ this contribution to the (p,d) amplitude is about 1%. The contribution of this term will increase with projectile mass m_P when both k_α and interaction radius of the $P-h$ system become larger.

III. NUMERICAL CALCULATIONS

To test our approach, we have calculated cross sections of the (d,p) reactions on ^{16}O and ^{10}Be nuclei and of the (p,d) reaction on ^{11}Be. For the stripping case we also use the results obtained using Sec. II B because stripping and pick-up amplitudes are related by a time reversal transformation. These results can also be obtained for stripping reactions if we start with the exact post-form amplitude and use similar ideas to those given in Sec. II.

We have performed calculations according to the LEA derived in Sec. II with a finite-range radius ρ of the $p-n$ interaction chosen to be 1.5 fm. When describing the proton channel, we have neglected spin-orbit term in the optical potential thus facilitating calculation of the effective distorted wave (12). Our own experience is that the contribution of the spin-orbit optical potential in the proton channel to unpolarized observables is negligible in the standard DWBA.

We have calculated the effective distorted wave according to Eq. (13) starting from p-^{16}O and p-^{10}Be optical potentials and read the calculated distorted wave into a zero-range version of the DWBA program TWOFNR [12] in which the new corrections for finite range derived in Sec. II B have been introduced. The term $\Delta(\vec{r})$ containing the first derivative in Eq. (23) has been neglected.

To describe the deuteron channel we used either conventional optical model or adiabatic Johnson-Soper model [4] potentials. When the Johnson-Soper model was used, the adiabatic deuteron potential was constructed from proton optical potentials at half the deuteron energy assuming that neutron and proton optical potentials are the same.

All calculations in the following subsections are done without nonlocality corrections. Our calculations have shown that their influence is small for the reactions considered. Nonlocality corrections for halo transfer are expected to be small because they correct the transition amplitude in the nuclear interior, but the long tail of the halo wave function makes the internal contributions less important.

A. ^{16}O$(d,p)^{17}$O reaction

The ^{16}O$(d,p)^{17}$O reaction has been chosen because the ^{17}O nucleus is the best candidate for the core+valence particle structure because of the shell closure and absence of the low-lying excited states of the ^{16}O core. Although the neutron binding energy is too large to consider ^{17}O as a halo nucleus, the conditions of the applicability of the adiabatic model are still satisfied if the incident deuteron energy is high enough.

These calculations can be regarded as a test case of the new theory. The ^{16}O mass is large enough that ^{17}O excitation effects produced by recoil might be expected to be small but not negligible. Spectroscopic factors we obtain must not be unreasonable for these well studied nuclei.

We have chosen $E_d=36$ MeV. At this energy the ^{16}O$(d,p)^{17}$O reaction has been studied earlier in great detail [13]. We use the parameters of the proton and deuteron optical potentials from this paper and select $r_0=1.25$ fm with $a=0.5234$ fm geometric parameters for the neutron bound state wave function.

First of all we compare the results of our calculations with standard LEA DWBA [see Fig. 1(a)] without taking into account the deuteron breakup. One can see that inclusion of the REB effects for ^{17}O slightly changes the shape of angular distribution and increases their absolute values at very forward angles. The difference between ZR and LEA in our approach is quite significant.

The calculated cross sections do not agree with experimental data unless the deuteron breakup is included. We have repeated the calculations with deuteron breakup included according to the Johnson and Soper approach [see Fig. 1(b)]. The difference between no-REB and REB calculations is now small except for the forward angles. The finite-range correction is now very small. The agreement between experimental data and theory is good [Fig. 2(a)]. Although the quality of the description of the angular dependence of the experimental data is the same with and without REB effects, the corresponding spectroscopic factors differ by 7%: $S=0.90$ is obtained for no-REB and $S=0.84$ for REB calculations. The same effect is observed for the ^{16}O$(d,p)^{17}$O$(1/2^+)$ reaction: the recoil excitation and breakup of the ^{17}O increases the calculated cross sections. In Fig. 2(b) the angular distribution of this reaction is shown with spectroscopic factor equal to unity.

The extra exponential factor in Eq. (11) involves the momentum in the ^{17}O channel and therefore the REB effects should increase if the energy of the projectile increases by a fraction of the depth of the proton optical potential. We have performed the calculations of the ^{16}O$(d,p)^{17}$O reaction at

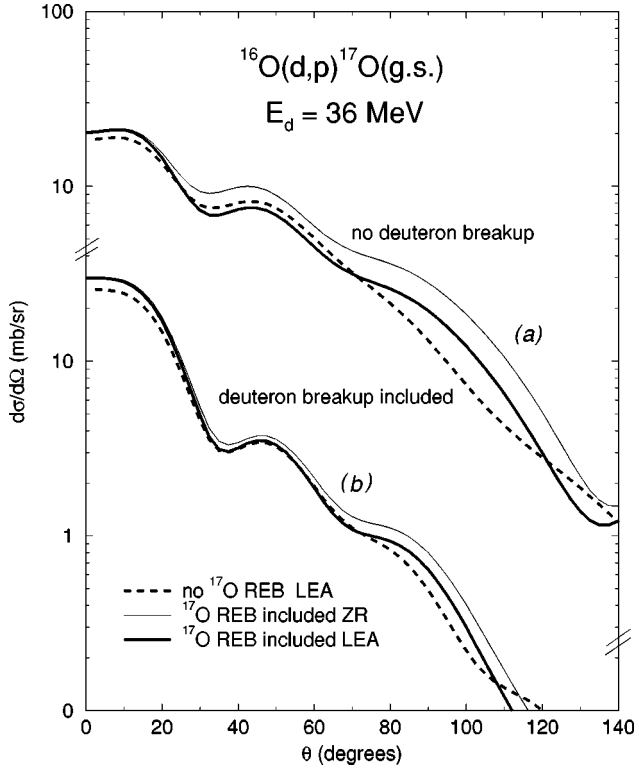

FIG. 1. Cross sections of the $^{16}O(d,p)^{17}O$ reaction at $E_d = 36$ MeV calculated with optical (a) and adiabatic (b) deuteron wave functions.

63.2 MeV to observe this effect. The calculated angular distribution is shown in Fig. 2(c) assuming that the spectroscopic factor is equal to one. The shapes of two curves are very similar but the absolute values differ by 27%.

B. ^{10}Be$(d,p)^{11}$Be reaction

The measurements of the cross sections of the $^{10}Be(d,p)^{11}Be$ reaction at 25 MeV have been reported in Ref. [1] together with the theoretical analysis within the framework of DWBA. The agreement between theoretical cross sections of Ref. [1] and experimental data is excellent. This agreement could be fortuitous, however. In the first place the Becchetti-Greenlees nucleon optical potential [14] which was used in Ref. [1] to describe the $p+^{11}$Be channel, was obtained from systematics for medium mass nuclei and does not describe elastic scattering from $1p$-shell nuclei (see Fig. 3). Secondly, neither ^{11}Be nor deuteron breakup was taken into account in Ref. [1]. When we repeated the standard DWBA calculations for this reaction using the more relevant optical potentials of Watson *et al.* [15], we found that the agreement between the DWBA predictions and experimental data completely disappears. Thus, contrary to the impression given in Ref. [1], the DWBA does not account for these data.

In this section we calculate the cross sections of the $^{10}Be(d,p)^{11}Be$ reaction including both the deuteron breakup and ^{11}Be REB effect. To construct the adiabatic deuteron potential we need the optical potential for $p+^{10}$Be scattering at 12.5 MeV. To construct the effective distorted wave in the exit proton channel according to Eq. (13), we need the $p+^{10}$Be optical potential at $E_p = 20.85$ MeV.

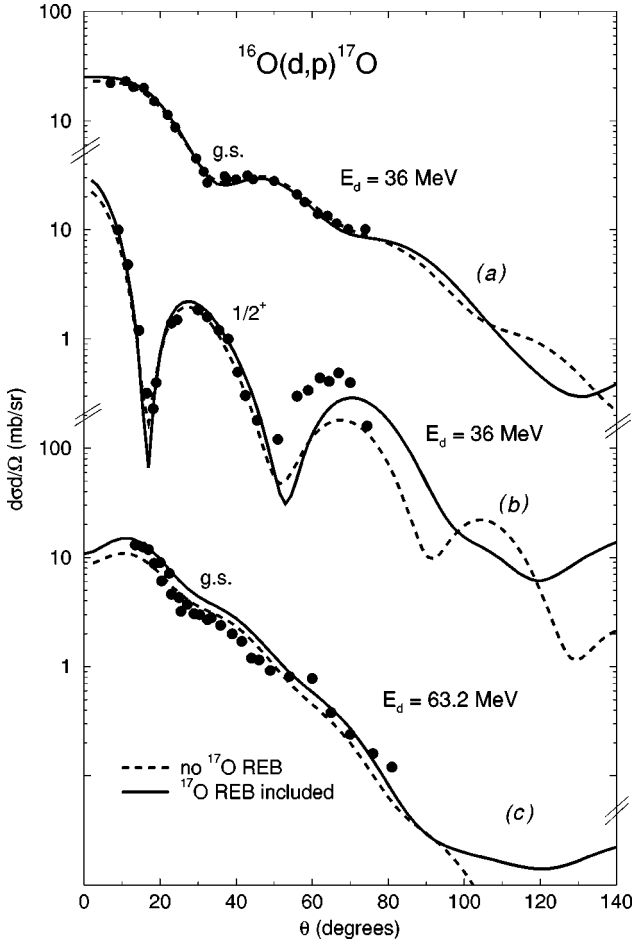

FIG. 2. (a) $^{16}O(d,p)^{17}O$(g.s.) at $E_d = 36$ MeV, (b) $^{16}O(d,p)^{17}O(1/2^+)$ at $E_d = 36$ MeV, and (c) $^{16}O(d,p)^{17}O$(g.s.) at $E_d = 63.2$ MeV calculated with deuteron breakup taken into account. Solid lines denote REB and dashed, no-REB calculations.

The experimental data on the $p+^{10}$Be elastic scattering for proton energy from 12 to 16 MeV are available in Ref. [16]. We have analyzed these data to find a potential which could be extrapolated to 20.85 MeV.

Three sets of potentials have been found: P1 has a linear energy dependence of the real and imaginary depths and was obtained as a fit to all five data sets, P2, an energy independent potential was obtained from fits to the $E = 12$, 13, 14, and 15 MeV data sets, and P3 is a potential without energy dependence found by fitting the 13, 14, 15, and 16 MeV data sets. The parameters of these potentials are listed in Table I. The quality of the description of the angular distributions is illustrated by Fig. 3 in the case of the 14 MeV data.

We use potentials P1 and P2 to construct the adiabatic deuteron potential. The parameters of the deuteron adiabatic potentials are listed in Table II. Potentials P1 and P3 are used to calculate the effective distorted waves in the proton channel.

The neutron bound state wave function has been calculated with $r_0 = 1.25$ fm and $a = 0.65$ fm by varying the well depth to reproduce the neutron separation energy. The calculated cross sections with optical and REB description of the proton channel are compared to each other at Fig. 4(a) assuming that the spectroscopic factors are equal to one in both cases. All calculations include deutron breakup. One can see

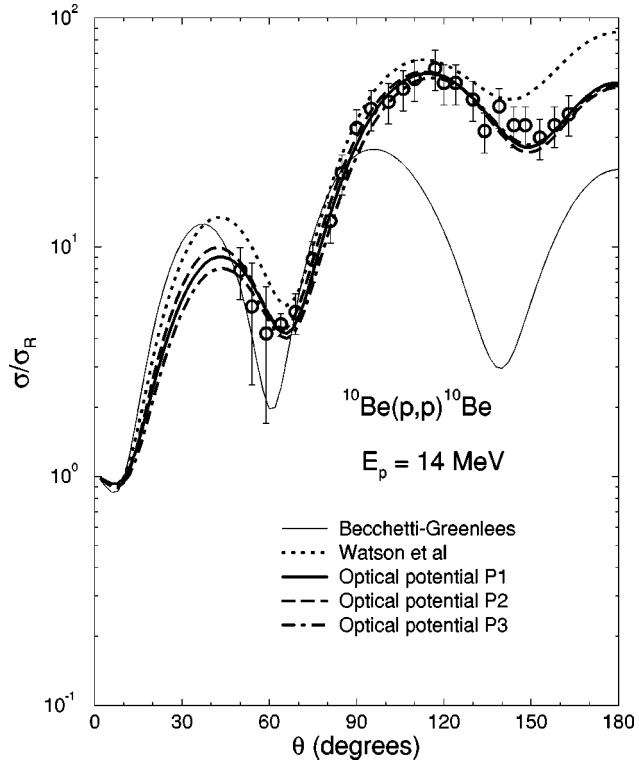

FIG. 3. Elastic scattering of the $p+{}^{10}$Be at $E_p=14$ MeV calculated with different proton optical potentials.

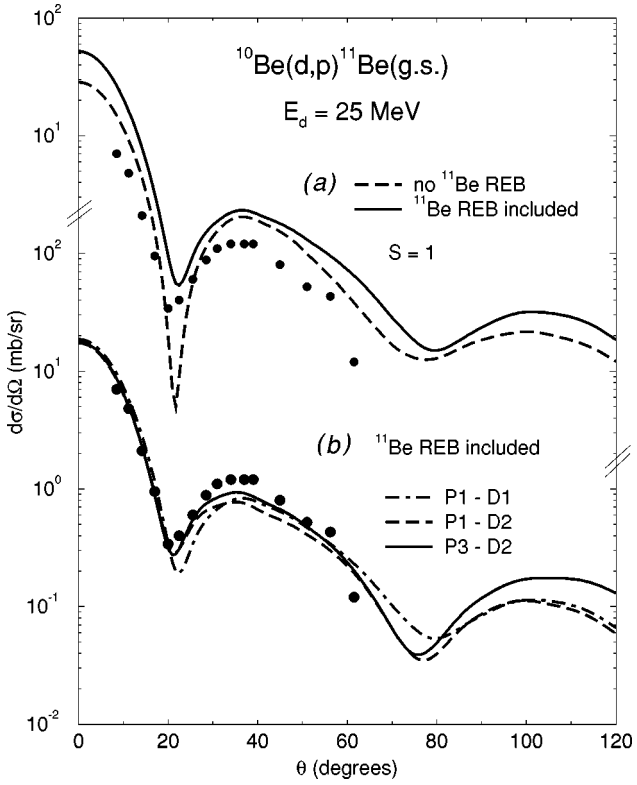

FIG. 4. Cross sections of the ^{10}Be$(d,p)^{11}$Be reaction at $E_d=25$ MeV calculated with adiabatic deuteron wave function. (a) Calculations have been done with proton optical potential P1 and deuteron adiabatic potential D1; spectroscopic factor $S=1$ was used. (b) Different sets of proton and deuteron optical potentials were used and theoretical curves are normalized to the experimental data.

that the inclusion of the REB effects for ^{11}Be strongly increases the cross section and slightly changes its shape. If we renormalize the theoretical curves to the experimental data in the anglular region of 10°–20°, we obtain the spectroscopic factors 0.60 (0.36) without (with) REB effects.

Several other calculations with different sets of optical potential parameters are shown in Fig. 4(b) and the spectroscopic factors are given in Table III. The best quality of the description of the experimental angular distribution is obtained with sets P3 and P2 for proton and deuteron channels, respectively. The corresponding spectroscopic factor for this case is 0.44 which is significantly smaller than the value of 0.82 predicted by the shell model in Ref. [17].

We should mention here, that one should be cautious when extrapolating the optical potential obtained from the 12–16 MeV data to 20.85 MeV. If any anomaly occurs in the behavior of the proton scattering at 12–16 MeV data, the extrapolation to the 20.85 MeV will make the results of the (d,p) analysis of the present paper less reliable. It would be useful to have experimental $p+{}^{10}$Be data at 20 MeV.

C. ^{11}Be$(p,d)^{10}$Be reaction

The measurements of the ^{11}Be$(p,d)^{10}$Be reaction at $E_p=35$ MeV in inverse kinematics have been reported in Ref. [2]. For future comparison to the experimental data, we calculate in this section the cross sections of the ^{11}Be$(p,d)^{10}$Be (g.s.) and ^{11}Be$(p,d)^{10}$Be(2^+) reactions including the ^{11}Be excitation and breakup according to the prescription of Sec. II of the present paper. The deuteron breakup was included adiabatically according to the Johnson and Soper approach. Two different sets of adiabatic potentials in the deuteron channel were used: D3 which was constructed from the $p+{}^{10}$Be potential P3 and D4 which used the proton potential obtained from global systematics by Watson *et al.* The effective distorted wave (13) in the ^{11}Be channel has been calculated with proton optical potentials either from the systematics of Fabrici *et al.* [18] (P4) or from the systematics of Watson *et al.* (P5).

We assumed $s_{1/2}$ transfer to ^{10}Be(0^+), and pure $d_{5/2}$ transfer to the ^{10}Be(2^+) state. The geometric parameters of the bound state potential well for the last neutron were as in previous subsection except that a spin-orbit potential of

TABLE I. Optical model potentials for the $p+{}^{10}$Be elastic scattering. The potential depths are in MeV and radii and diffusenesses are in fm.

Set	V_R	r_R	a_R	W_D	r_D	a_D	V_{so}	r_{so}	a_{so}
P1	65.17–0.233E	1.265	0.410	0.332+1.01E	0.996	0.354	12.60	0.70	0.62
P2	68.817	1.107	0.609	13.037	1.275	0.354	12.86	0.69	0.64
P3	68.461	1.188	0.469	15.623	1.043	0.328	11.79	0.71	0.53

TABLE II. Adiabatic potentials for the $d+{}^{10}$Be elastic scattering constructed for different deuteron energy E_d. The potential depths are in MeV and radii and diffusenesses are in fm.

E_d	Set	V_R	r_R	a_R	W_D	r_D	a_D	V_{so}	r_{so}	a_{so}
25 MeV	D1	118.26	1.265	0.455	22.369	0.996	0.398	12.60	0.70	0.62
	D2	131.81	1.107	0.639	22.806	1.275	0.398	12.86	0.69	0.64
40.6 MeV	D3	130.21	1.188	0.508	26.589	1.043	0.375	11.79	0.71	0.53
40.6 MeV	D4	115.90	1.132	0.602	15.687	1.132	0.531	5.5	1.132	0.57
36.5 MeV	D4	116.96	1.133	0.602	15.890	1.133	0.531	5.5	1.133	0.57

depth 4 MeV was added in the $d_{5/2}$ case. We assumed everywhere that the spectroscopic factors are equal to unity.

In Fig. 5(a) the angular distribution of the ^{11}Be(p,d) reaction calculated without and with REB effects are compared to each other for optical potentials P4 and D3. We see that at small angles the slopes of the two curves are different. Cross sections of the ^{11}Be$(p,d)^{10}$Be(0^+) reaction calculated with REB effects, are larger but decrease faster. The angular distributions of the transfer reaction to the 2^+ state of ^{10}Be at small angles look the same both with and without REB effects but have different absolute values.

The calculations with a different selection of optical potentials are shown in Fig. 5(b). Potential P5 has been used for the proton channel and D4 for the deuteron channel. We see that at small angles the shapes of the theoretical curves corresponding to two different potential sets are almost the same. At larger angles they differ, especially for the (p,d) reaction to the ^{10}Be ground state. The absolute values of the ^{11}Be$(p,d)^{10}$Be(0^+) reaction differ at small angles by 40% but remain the same for the transition to the 2^+ state of ^{10}Be.

IV. DISCUSSION

The (d,p) and (p,d) calculations carried out in previous section, have demonstrated that the inclusion of ^{17}O and ^{11}Be recoil excitation and breakup effects in transfer reactions produces changes in the shape of the angular distributions of these reactions. These effects generally increase the absolute values of the theoretical cross sections and thus lead to the smaller values of the spectroscopic factors extracted from the experimental data. The physical origin of this phenomena lies in the extra paths which become available between the transfer channels when the implications inherent in the halo concept are built into the theory. This effect depends on incident energy, target mass, and quantum numbers of the bound state of the valence nucleon. Because of the exponential factor $e^{-i\alpha\vec{k}_\alpha\vec{r}}$ in the expression for the adiabatic wave function (12), it is expected that the REB effects will be larger when the incident energy increases and the mass of the target decreases. Indeed, at fixed energy the influence of the recoil excitation and breakup in the case of ^{11}Be is stronger than in ^{17}O, and in the case of the ^{16}O$(d,p)^{17}$O reaction they are more important for $E_d=63.2$ MeV than for 36 MeV. As a result the spectroscopic factors for the ^{16}O(g.s.) obtained with and without REB at 36 MeV differ only by 7%, while those obtained at 63.2 MeV differ by 27%.

Our results suggest that the recoil excitation and breakup effects are stronger for $s_{1/2}$ transfer than for $d_{5/2}$ transfer. This may be associated with the node in the bound state wave function in the $s_{1/2}$ case. As a result, the transition

TABLE III. Spectroscopic factors from the ^{10}Be$(d,p)^{11}$Be reaction at $E_d=25$ MeV obtained without and with ^{11}Be recoil excitation and breakup for different sets of optical potentials. Deuteron breakup is included.

Proton OP	Deuteron OP	no ^{11}Be breakup	^{11}Be breakup included
P1	D1	0.60	0.36
P1	D2	0.69	0.44
P3	D2	0.70	0.44

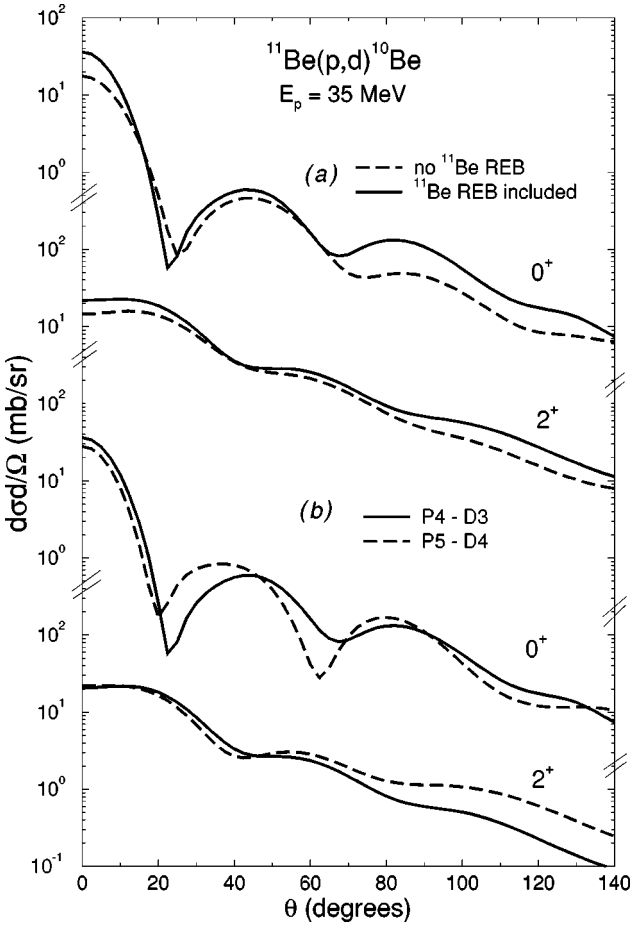

FIG. 5. Cross sections of the ^{11}Be$(p,d)^{10}$Be(g.s.) and ^{11}Be$(p,d)^{10}$Be(2^+) reactions at $E_d=35$ MeV calculated (a) with and without REB effects for optical potentials P4 and D3; (b) with REB and different optical potentials. All cross sections are ploted with $S=1$. Deuteron breakup is included in both cases.

amplitude depends on the cancellations between internal and external parts of the integrand and thus may be more sensitive to the replacement of the conventional DWBA distorted wave by some effective distorted wave. In the case of the $s_{1/2}$ transfer the shape of the differential cross sections changes more strongly at small angles compared to the $d_{5/2}$ transfer. This should influence the ratio of the spectroscopic factors for 0^+ and 2^+ states of ^{10}Be obtained with and without ^{11}Be recoil excitation and breakup.

In all the cases studied, deuteron breakup changed the shapes of angular distribution much more strongly than the breakup of the heavier nucleus. The adiabatic transfer amplitude differs from the DWBA more strongly when the mass of the core decreases. Therefore, in the limit of deuteron the effects of breakup should reach their maximum.

Next, we emphasize that the expressions (1) and (4) are exactly equivalent on shell when $\Psi^{(-)}$ is an exact solution of the many-body Schrödinger equation. In principle, it is therefore possible to include the REB effects we have discussed by evaluating the remnant term $V_{PC}(\vec{r}_{PC})-U_\alpha$ in Eq. (1). This has never been done without drastic approximations of unknown validity. It is, for example, not sensible to use the simplified Johnson-Soper form, Eq. (7), for the adiabatic wave function in the remnant term because $V_{PC}(\vec{r}_{PC})-U_\alpha$ is not short range in V_{Ph}. In order to compare with our calculations which include the crucial deuteron breakup effects it would be necessary to use the full adiabatic three-body deuteron wave function (see Ref. [19], and references therein).

We have made rough comparison of the two approaches in the case of the ^{16}O$(d,p)^{17}$O(g.s.) reaction without the deuteron breakup. The remnant term $V_{PC}(\vec{r}_{PC})-U_\alpha$ was treated in a standard way assuming that the V_{PC} potential is a real potential which binds the $p+^{16}$O system. The results of the calculations are shown in Fig. 6. One can see that including the remnant term almost uniformly increases the cross sections. At small angles this increase agrees qualitively with our adiabatic calculations, however, in our approach the shape of the angular distributions has been changed as well.

Finally we note that our formulation depends heavily on the use of adiabatic approximations to treat excitation effects in the initial and final channels. A recent review of the application of adiabatic approximations to few body models of nuclear reactions can be found in Ref. [20]. Referring to the

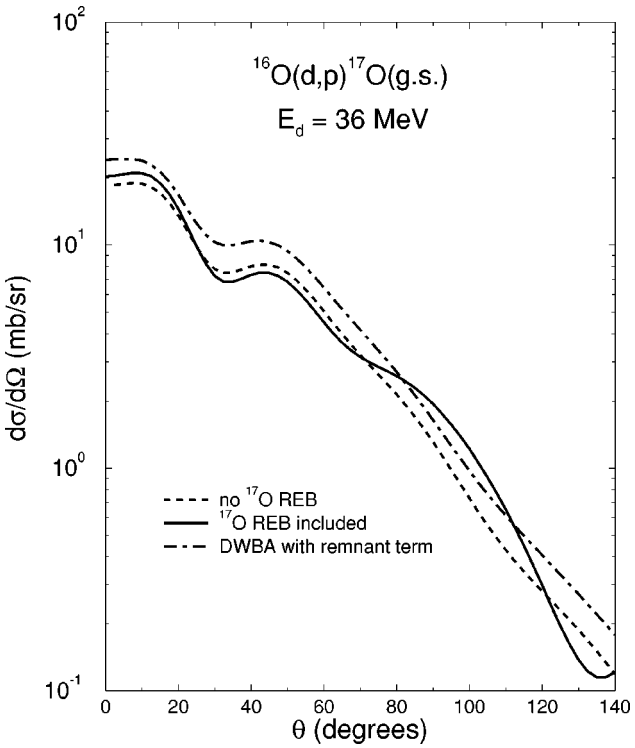

FIG. 6. Cross sections of the ^{16}O$(d,p)^{17}$O reaction at $E_d = 36$ MeV calculated within the standard DWBA without (dashed curve) and with (dot-dashed curve) remnant term and within the adiabatic REB approach of the present paper (solid line).

(d,p) case for definiteness, it is known that corrections to the adiabatic treatment of deuteron breakup effects in transfer reactions can be significant [20]. A practical treatment of these corrections is given in Ref. [21]. Corrections to our adiabatic treatment of the special three-body problem which describes the final channel in a (d,p) reaction are discussed in Refs. [22] and [8]. Future work will examine these corrections in detail.

ACKNOWLEDGMENTS

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