Rotational alignment near $N=Z$ and proton-neutron correlations

S. Frauendorf[1] and J. A. Sheikh[1,2,3]
[1]*IKHP, Research Center Rossendorf, PF 510119, D-01314 Dresden, Germany*
[2]*Department of Physics, University of Surrey, Guildford, Surrey GU2 5XH, United Kingdom*
[3]*Tata Institute of Fundamental Research, Bombay 400 005, India*
(Received 3 June 1998)

The effects of the residual proton-neutron interactions on bandcrossing features are studied by means of shell model calculations for nucleons in a high-j intruder orbital. The presence of an odd-nucleon shifts the frequency of the alignment of two nucleons of the other kind along the axis of rotation. It is shown that the anomalous delayed crossing observed in nuclei with aligning neutrons and protons occupying the same intruder subshell can be partly attributed to these residual interactions. [S0556-2813(99)00503-8]

PACS number(s): 21.60.Cs, 21.10.Hw, 21.10.Ky, 27.50.+e

I. INTRODUCTION

The cranked shell model (CSM) approach [1,2] accounts well for the overall systematics of band-crossing phenomena in rapidly rotating medium and heavy mass nuclei. In this model the protons and neutrons move independently in a fixed rotating potential. As a consequence of this assumption, the alignment of a pair of neutrons with the rotational axis occurs at the same rotational frequency for different proton configurations. However, recently substantial deviations from this simple picture have been reported using the CSM approach in the high-spin study of nuclei near $N=Z$ [3–10] and for nuclei with aligning neutrons and protons occupying the same intruder subshell [11–16]. These deviations, concerning the crossing between the g-band and the s-band in the even-neutron systems referred to as the (AB) crossing, include the following. The crossing in the odd-proton nuclei is considerably delayed as compared to the neighboring even-even nuclei. In the case of $N=Z$ even-even nuclei, the (AB) crossing is substantially delayed as compared to the neighboring $N\neq Z$. Figure 1 summarizes the experimental evidence.

The observed delay of the neutron alignment has been attributed to an increase of the deformation induced by the odd high-j proton. This mechanism has first been pointed out in Ref. [2] as the possible origin of the delayed $i_{13/2}$ neutron backbend if an $h_{9/2}$ proton is present. It has been substantiated by systematic calculations of the equilibrium deformations for bands with an odd-proton in the $h_{11/2}$, $h_{9/2}$, and $i_{13/2}$ orbitals (cf., e.g., [15,12,14,16]). Though being in the right order of magnitude, the calculations tend to underestimate the experimental shifts. The $p-n$ residual interaction has been invoked to be responsible for the remaining discrepancy [15,12]. In a study of a QQ-type $p-n$ interaction acting in the particle-hole channel Ref. [16] finds only very small shifts. In the present paper we further investigate the role of the residual interaction. At variance with Ref. [16], we use a short range force and calculate the exact solutions for the model system of interacting protons and neutrons in a j shell, exposed to a deformed potential.

The strong short-range proton-neutron ($p-n$) attraction probes the relative orientation of the high-j orbitals, because of their strongly anisotropic, torus like density distributions. It favors the antiparallel ($J=0,1$) and the parallel ($J=2j$) coupling of a $p-n$ pair, the energies being comparable. Since there are many low-J pairs available, they tend to form a correlated state ($p-n$ pairing). Processes in which the high-j particles change their relative orientation should be good means to study these aspects of the $p-n$ interaction. In this manuscript, we investigate a special kind of such reorientation, the alignment of a pair of neutrons with the axis of rotation. By means of a shell model calculation it will be demonstrated that the rotational frequency at which the neutron alignment occurs changes when additional protons are present and that the frequency shifts are sensitive to the $p-n$ interaction and the correlations it generates. It will also be shown that the $p-n$ interaction causes a delay of the first double band crossing in $N=Z$ model systems studied as compared to cases with $N\neq Z$ which may be related to the delayed band crossing observed in $N=Z$ nuclei [6].

II. THE SHELL MODEL

In our model the protons and neutrons in a j-shell interact via a delta force and move in the deformed rotating potential generated by the nucleons outside the j shell. The model Hamiltonian is the same as used in Refs. [17,18], where one may look for the details, and is given by

$$H'=-\sum_i\left[4\left(\frac{4\pi}{5}\right)^{1/2}\kappa Y_{20}(\hat r_i)+\omega j_i^{(x)}\right]-G\sum_{i<j}\delta(|\hat r_i-\hat r_j|). \quad (1)$$

The first term is the deformed quadrupole field where κ is related to the deformation parameter β by $\kappa=51.5A^{1/3}\beta$ in units of the coupling strength G of the delta force. The second, cranking term describes the uniform rotation about the x-axis with the frequency ω and the third term is the interaction. The eigenfunctions of the Hamiltonian are found by numerical diagonalization. They can be classified by the isospin and the signature [symmetry with respect to $R_x(\pi)$] which are conserved by the model Hamiltonian.

We would like to point out that the present model predictions should only be considered as qualitative indications of the physics expected from more realistic studies. The major restriction of our model is that we consider only a few par-

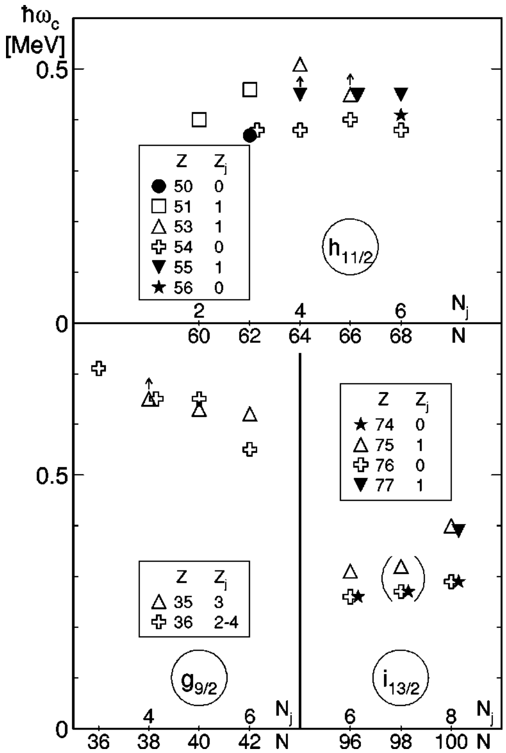

FIG. 1. Experimental crossing frequencies $\hbar\omega_c$ for the first alignment of a pair of neutrons in the $g_{9/2}$, $h_{11/2}$, and $i_{13/2}$ shells. The odd-proton occupies the same j shell as the neutrons. The number N_j is an estimate of the number of neutrons in the j shell, obtained by counting Nilsson levels at the deformation $\beta=0.25$. For the $h_{11/2}$ and $i_{13/2}$ shells the proton occupancy is zero or one, for the $g_{9/2}$ shell it is close to the neutron occupancy. The arrows indicate that only lower limits are known. For $N=98$ the very gradual rise of the function $I(\omega)$ does not permit a reliable determination of $\hbar\omega_c$, however the shift of the two curves gives a reasonable estimate of the difference between the crossing frequencies. In the case of $Z=50$ and 51 the deformed excited bands are shown. The experimental data are from Refs. [3–14]. The even Sr isotopes and the $g_{9/2}$ bands of the Rb isotopes do not show a $g_{9/2}$ alignment that might be a consequence of the $Z=N=38$ gap in the single particle spectrum.

ticles occupying the intruder subshell with the rest of the nucleons giving rise to a potential, the deformation of which is fixed. The assumption that the nucleons outside the intruder shell do not take part in the correlations generated by the short range residual interaction is a serious restriction of the model. The pair correlations are known to involve both types of nucleons. Thus, the model cannot be expected to reproduce the modification of the pairing by the odd particle or by the alignment of a pair of nucleons in a quantitative way. Since we choose a realistic value of the deformation for a given nucleus and estimate the number of particles occupying the high-j orbital by looking at the realistic Nilsson diagrams, the estimates of the frequency where the rotational alignment occurs should be qualitatively right. However, when discussing the shifts of the crossing frequencies the change of the deformation due to the presence of the odd particle or alignments must be taken into account. Due to the lack of the feed back from the j-shell particles to the remaining nucleons one cannot expect the model reproduce the shifts of the crossing frequencies in a quantitative way.

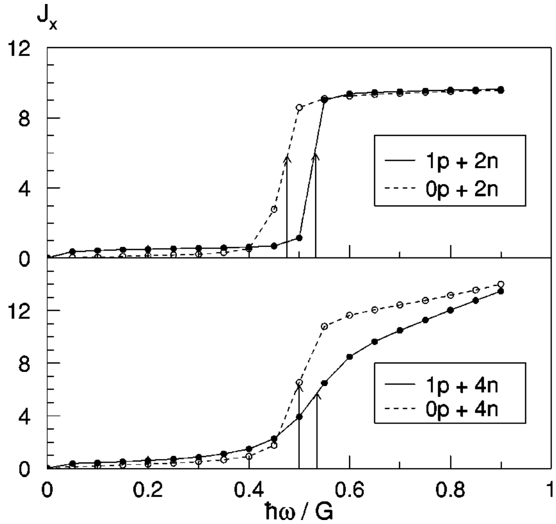

FIG. 2. Angular momentum J_x as a function of the rotational frequency for particles in a $h_{11/2}$ shell with deformation $\beta=0.25$. The arrows indicate the crossing frequencies. For odd-Z a constant of 11/2 is subtracted from J_x.

As another consequence of the restriction to a single j-shell, the total angular momentum is underestimated. This is of little importance, because we study the *frequency* of the alignment processes, which will not be changed by the missing angular momentum from the nucleons outside the j-shell.

III. EXAMPLES FOR DELAYED CROSSINGS

Figure 2 shows the expectation value of the x component of the angular momentum calculated by assuming a deformation $\beta=0.25$ of a well deformed nucleus. The situation with few protons and neutrons occupying the $h_{11/2}$ shell is studied. The alignment of a pair of neutrons with the axis of rotation (x), caused by the Coriolis force, shows up as the steplike rise of the angular momentum from small values to about 10. It corresponds to the crossing of g band with the neutron s band, where the inflection point defines the crossing frequency. It is seen that the presence of the odd proton delays the alignment of the neutron pair. As demonstrated by Fig. 1 both in the $h_{11/2}$ and in the $i_{13/2}$ shells a similar delay of the crossing frequency is seen for the combination ($Z=1$, $N=2,4,6$). The estimated number of protons and neutrons in the high-j shell, denoted by Z_j and N_j, must be compared with Z and N of the calculations, respectively.

Figure 3 demonstrates that for near symmetric filling of the $g_{9/2}$ shell ($Z=2$, $N=2$) and ($N=3$, $Z=2$) the crossing frequency in the odd-A nuclei lies below the first crossing in the even-even nuclei ($Z=N=2$), which corresponds to the simultaneous alignment of protons and neutrons [17]. This inversion of the order of the band crossings seems to be seen in the Br and Kr isotopes, as illustrated by Fig. 1. The figure also shows that the inverse ordering for near symmetric filling as compared to asymmetric filling can be attributed to a delay of the crossing in the even-even $N=Z$ nuclei as compared to the ones with $N\neq Z$ [6], which is stronger than the delay in the odd-Z system.

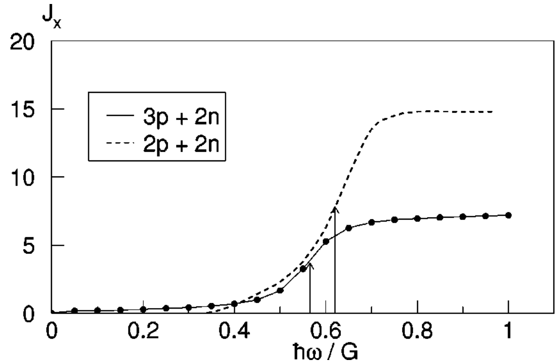

FIG. 3. Angular momentum J_x as a function of the rotational frequency for particles in a $h_{9/2}$ shell with deformation $\beta=0.25$. The case $Z=2$, $N=3$ is identical. The arrows indicate the crossing frequencies. For odd-Z a constant of 11/2 is subtracted from J_x.

IV. SYSTEMATICS OF THE CROSSING FREQUENCY

The features discussed in the previous section reflect a systematic tendency that is illustrated in Figs. 4 and 5, showing $J_x(\omega)$ and $\mathcal{J}^{(2)}(\omega)=dJ_x/d\omega$ for the $f_{7/2}$ shell. (We have chosen $f_{7/2}$ shell since the dimensions of the matrices to be diagonalized are lower and a systematic study is possible.) The alignments show up as peaks in $\mathcal{J}^{(2)}$. We are interested in the alignment of a pair of $f_{7/2}$ neutrons, which is displayed in lower panels of the figures for the pure neutron $N=2$ and $N=4$ cases. If there were no $p-n$ correlations, the frequency at which these alignments appear would not change with the number of protons. However, this is not seen. Instead, the critical frequency for the neutron alignment first grows with the number of added protons. The shift reaches its maximum when $Z=N$ and then decreases when Z becomes larger than N. Of course, the role of N and Z could be exchanged.

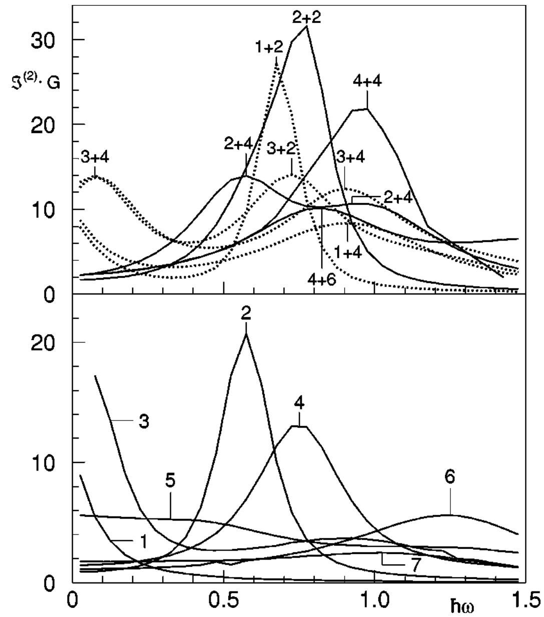

FIG. 5. Dynamic moment of inertia $\mathcal{J}^{(2)}$ as a function of the rotational frequency for particles in a $f_{7/2}$ shell with deformation $\beta=0.25$. Upper panel: The particle numbers are quoted in the form $Z+N$. Full lines: even-A; dotted: odd-A. Lower panel: Only one kind of particle, the number of which is quoted.

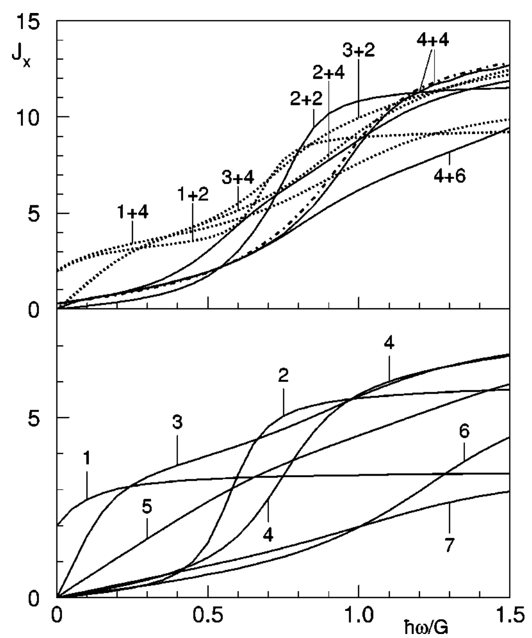

FIG. 4. Angular momentum J_x as a function of the rotational frequency for particles in a $f_{7/2}$ shell with deformation $\beta=0.25$. Upper panel: The particle numbers are quoted in the form $Z+N$. Full lines: even-A; dotted: odd-A; dashed dotted: only $T=1$ part of the δ force. Lower panel: Only one kind of particle, the number of which is quoted.

Qualitatively, one may ascribe the progressive delay of the crossing to $p-n$ correlations that disfavor the simultaneous alignment of protons and neutrons (e.g., the presence of $p-n$ pairs with a low J), generated by the attractive $p-n$ interaction. The more protons are added the stronger the correlations become, until $Z=N$. As discussed in [18], the character of the $p-n$ interaction between the aligned protons and neutrons changes from attractive to repulsive when Z exceeds N, because the character of the aligned configuration changes from particlelike to holelike. This shifts the $p-n$ correlations to the highly excited states, whereas the yrast states are not much modified by the $p-n$ interaction.

The shift of the crossing frequency is a consequence of the $T=1$ part of the δ interaction. The dashed dotted line in Fig. 4 shows a calculation where the $T=0$ part of the δ interaction has been removed. The function $J_x(\omega)$ almost coincides with the one calculated with the full force. The question if the $p-n$ correlations that cause the shifts belong to the particle-hole channel or the particle-particle channel (or to both) cannot be decided on the basis of the shell model calculations. For this, comparisons with mean field calculations are necessary, which will be addressed in a forthcoming paper. We state here only the existence of strong $p-n$ correlations of the $T=1$ type between nucleons in the same j-shell, which cause a delay of the first band crossing.

A delay of the onset of the band crossing for $N=Z=36$ has recently been observed in the Kr isotopes [6] (cf. Fig. 1). It is suggested in the paper that the delay may be a consequence of $T=0$ pair correlations. As already mentioned, the dashed dotted line in Fig. 4 shows a calculation where we have switched off the $T=0$ part of the interaction. It is seen that the results are almost the same as for the full force, i.e., the $T=0$ correlations do not influence the alignment significantly in the present model analysis. It remains to be seen whether this conclusion will hold in a realistic study. It is consistent with mean field studies of Ref. [19], who find that the frequency of the (AB) crossing is sensitive to the $T=1$ but not to the $T=0$ pair field.

Comparing in Figs. 4 and 5 the alignment of the odd-proton in the $Z=3$ system with the one in the $Z=3$, $N=2,\ 4$, one notices also the the alignment of an odd-proton is hindered by the presence of neutrons in the same j-shell.

In order to study the deformation dependence of the discussed effects we have also performed the calculations for spherical shape [$\kappa=0$ in Eq. (1)]. In this case, the angular momentum is conserved and the alignment plots like Figs. 2 and 3 become real step functions, because the eigenstates of Eq. (1) with different angular momenta J cross sharply as functions of ω. In the spherical case there are no rotational bands and the crossing frequency has no immediate meaning. Nevertheless, the comparison with the case of substantial deformation is instructive, because it permits us to guess what can be expected for weakly deformed nuclei where band crossings are still observed.

Figure 6 shows the crossing frequencies for the spherical case. For most combinations of Z and N the odd-proton delays the alignment of the first pair in the even-neutron system. However, the N dependence of the shift is different from the case of large deformation. As in the case of large deformation, the first crossing in the even-even systems occurs later for $N=Z$ than for $Z\neq N$.

It has already been pointed out in a previous work [17] that for even-even $N=Z$ nuclei the first band crossing has the character of a double-alignment. However, the calculations were done in a truncated basis, whereas in the present work, they have been carried out with no approximations. In addition, it is demonstrated that it is $T=1$ component of the two-body $n-p$ interaction which is responsible for the particle-number dependence of the band crossing. In a previous work [17] such break-up of the $n-p$ interaction was not carried out.

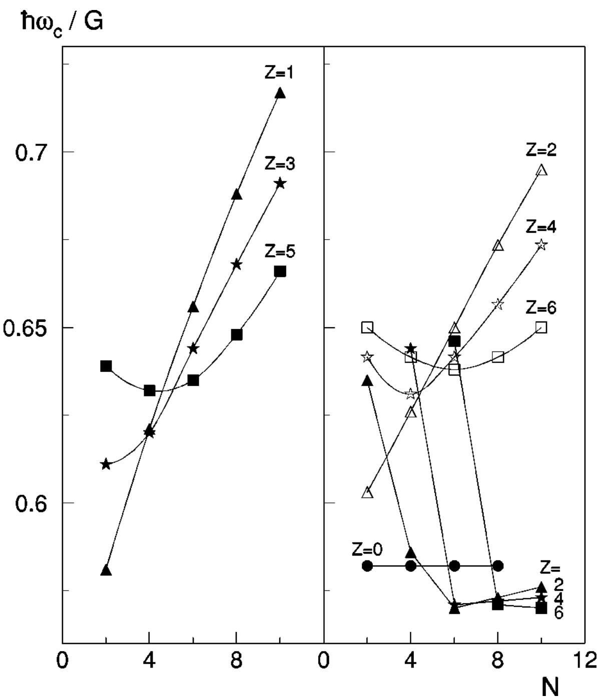

FIG. 6. Crossing frequencies of the lowest aligned configurations of particles in a spherical $h_{11/2}$ shell. The neutron number N is even. Left panel: Alignment $J=11/2\rightarrow 31/2$ in odd Z nuclei. Right panel: Alignment $J=0\rightarrow 10$ (full symbols) and $J=0\rightarrow 20$ (open symbols) in even Z nuclei.

V. CONCLUSIONS

The present simple model study indicates that the alignment of high-j nucleons with the rotational axis is sensitive to the $p-n$ interaction. For the important case of prolate deformation, the alignment of one kind of particles is delayed when the other kind of particles is present in the same j shell. The effect reaches its maximum for $N=Z$ and is a consequence of $T=1$ correlations between protons and neutrons. For $Z=N$, the crossing frequency in the even system is higher than for asymmetric filling. Similar trends are experimentally observed: The first backbend in ^{72}Kr with $Z=N$ seems to occur later than in the other even-even isotopes with $Z<N$ and the first backbend in even-neutron nuclei seems to be delayed by the presence of an odd-proton in the same j shell. Since in our model study the nucleons outside the intruder shell only give rise to a fixed external potential, the effects of the feedback from the intruder particles to remaining ones are absent. One important effect, previously discussed, is the change of deformation induced by the odd nucleon. It is quite likely that the observed shifts of the band crossing frequencies are a combination of these deformation changes and the correlations studied by the present model. By measuring the deformation and/or calculating it one may try to disentangle the two effects and probe the $p-n$ correlations. Finally, we would like to mention that while the results of the present model analysis seem to point into the right direction they should not be compared with the experimental data in a quantitative form. It would be quite interesting to study the effects in more realistic shell model configuration spaces.

[1] R. Bengtsson and S. Frauendorf, Nucl. Phys. **A327**, 139 (1979).
[2] S. Frauendorf, Phys. Scr. **24**, 349 (1981).
[3] J. Heese, N. Martin, C.J. Gross, W. Fieber, K.P. Lieb, A. Kuhnert, K.H. Maier, and X. Sun, Phys. Rev. C **41**, 1553 (1990).
[4] N. Martin, C.J. Gross, J. Heese, and K.P. Lieb, J. Phys. G **15**, L123 (1989).
[5] G.N. Sylvan, J.E. Purcell, J. Doring, J.W. Holcomb, G.D. Johns, T.D. Johnson, M.A. Riley, P.C. Womble, V.A. Wood, and S.L. Tabor, Phys. Rev. C **48**, 2252 (1993).
[6] G. de Angelis *et al.*, Phys. Lett. B **415**, 217 (1997).
[7] J. Heese, D.J. Blumenthal, A.A. Chisti, P. Chowdhury, B. Crowell, P.J. Ennis, C.J. Lister, and Ch. Winter, Phys. Rev. C **43**, R921 (1991).

[8] D.F. Winchell, M.S. Kaplan, J.X. Saladin, H. Takai, J.J. Kolata, and J. Dudek, Phys. Rev. C **40**, 2672 (1989).
[9] C. Gross *et al.*, Nucl. Phys. **A501**, 367 (1989).
[10] G.D. Johns, J. Doring, J.W. Holcomb, T.D. Johnson, M.A. Riley, G.N. Sylvan, P.C. Womble, V.A. Wood, and S.L. Tabor, Phys. Rev. C **50**, 2786 (1994).
[11] V.P. Janzen *et al.*, Phys. Rev. Lett. **70**, 1065 (1993).
[12] E.S. Paul, D.B. Fossan, K. Hauschild, I.M. Hibbert, H. Schnare, J.M. Sears, I. Thorsland, R. Wadsworth, A.N. Wilson, and J.N. Wilson, Phys. Rev. C **51**, R2857 (1995).
[13] F. Liden *et al.*, Nucl. Phys. **A550**, 365 (1992).
[14] R.A. Bark, R. Bengtsson, and H. Carlsson, Phys. Lett. B **339**, 11 (1994), and work quoted therein.
[15] R. Wyss and A. Johnson, *Proceedings of the International Conference on High Spin Physics*, Pittsburgh, 1990 (World Scientific, Singapore, 1991).
[16] W. Satula, R. Wyss, and F. Dönau, Nucl. Phys. **A565**, 573 (1993).
[17] J.A. Sheikh, N. Rowley, M.A. Nagarajan, and H.G. Price, Phys. Rev. Lett. **64**, 376 (1990).
[18] S. Frauendorf, J.A. Sheikh, and N. Rowley, Phys. Rev. C **50**, 196 (1994).
[19] K. Kaneko and Jing-je Zhang, Phys. Rev. C **57**, 1732 (1998).