Elastic scattering of pions from the three-nucleon system

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We examine the scattering of charged pions from the trinucleon system at a pion energy of 180 MeV. The motivation for this study is the structure seen in the experimental angular distribution of back-angle scattering for π^+-^{3}He and π^--^{3}H but for neither π^--^{3}He nor π^+-^{3}H. We consider the addition of a double spin flip term to an optical model treatment and find that, though the contribution of this term is non-negligible at large angles for π^+-^{3}He and π^--^{3}H, it does not reproduce the structure seen in the experiment. [S0556-2813(99)02003-8]

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I. INTRODUCTION

The measurement of the differential cross section for the elastic scattering of $\pi^{\pm}$ from ^{3}H and ^{3}He has recently been extended into the backward hemisphere by Matthews *et al.* [1]. For all four cases, at an incident pion energy of 180 MeV, there is a slight dip in the cross section at ~130° lab (or 5.8 fm^{-2} momentum transfer), but for π^--^{3}H and π^+-^{3}He there is also a very distinctive rise in the cross section at larger angles. Moreover, the authors stated that, due to the resolution of the spectrometer, the dip may be even deeper and sharper. They compared experimental data to scattering models by Kamalov, Tiator, and Bennhold [2] and by Gibbs and Gibson [3]. For π^--^{3}H and π^+-^{3}He both models agree reasonably well with the cross section up to 130°, but both fail to predict the distinctive rise in the cross section at large angles. For π^+-^{3}H and π^--^{3}He both models give acceptable agreement at all angles. Figure 1 compares the experimental data [1,4] (squares) to a theoretical curve from Ref. [3] (dashed line).

One is thus led to consider mechanisms that would give a significant rise in the cross section in the backward hemisphere for both π^--^{3}H and π^+-^{3}He, but not for π^+-^{3}H or π^--^{3}He. Similar rises in large-angle cross sections have been seen for π^+ scattering from ^{12}C and ^{16}O [5]. In the three-nucleon case the dependence of this effect on the charge of the pion and on the target may give an indication of the cause.

The pion kinetic energy, $T_\pi = 180$ MeV, is near the peak of the P_{33} resonance for π-nucleon scattering. At this energy the elastic scattering cross sections are in the ratio $\sigma(\pi^+p)/\sigma(\pi^+n) \approx \sigma(\pi^-n)/\sigma(\pi^-p) \approx 9$. Therefore, we expect π-nucleus scattering at $T_\pi = 180$ MeV to be dominated by the π^+-p and π^--n amplitudes. We use the convention that even-nucleon reactions refer to π^+-^{3}He and π^--^{3}H reactions (which have two dominant scattering centers), and odd-nucleon reactions refer to π^--^{3}He and π^+-^{3}H reactions (which only have one dominant scattering center at this energy). We also use the nomenclature of Schiff [6] where an ''even'' or an ''odd'' nucleon corresponds, respectively, to one of the like nucleons or to the unlike nucleon (the neutron in ^{3}He or the proton in ^{3}H).

Qualitatively we can understand the first dip (around 90°) as a minimum in the spin-independent amplitude due to the p-wave dominance of the pion-nucleon phase shifts. For this intuitive view, consider the single-scattering impulse approximation, where the basic dependence of the amplitude is given by the sum of the relevant πN amplitudes multiplied by a form factor. For π-nucleon scattering, the amplitude consists of two incoherent terms: a non-spin-flip term, $f(\theta)$, and a spin-flip term, $g(\theta)$. Near the peak of the P_{33} resonance $f(\theta) \sim \cos(\theta)$ (plus an s-wave contribution which shifts the minimum slightly away from 90°), and $g(\theta) \sim \sin(\theta)$.

In the approximation that the trinucleon system exists in a pure s state (which we use throughout this paper), the ampli-

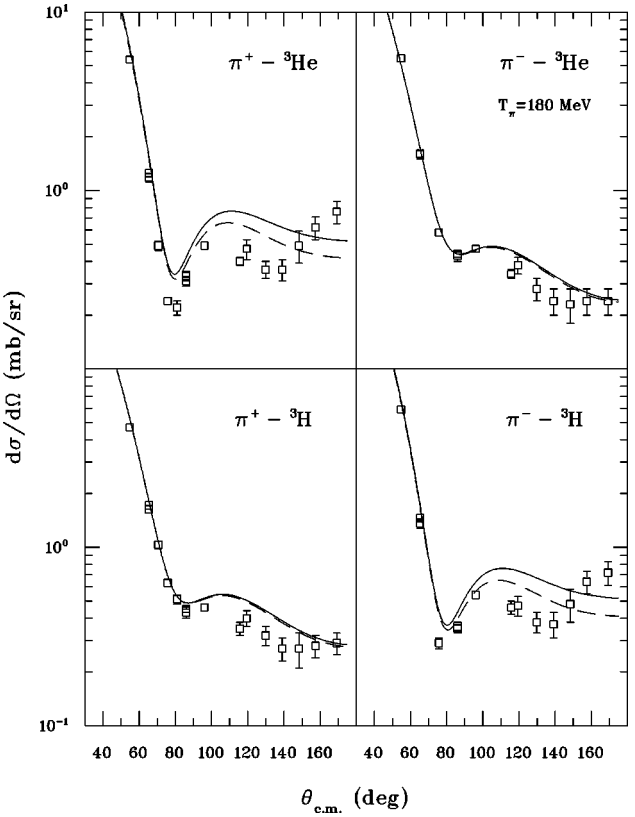

FIG. 1. Comparison of experimental data from Refs. [1,4] (squares) to a prediction from the optical model of Ref. [3] with DSF term (solid line) and without DSF term (dashed line).

tude $g(\theta)$ arises entirely from the interaction of the pion with the odd nucleon since, in first order, the spin-flip amplitudes cancel from the even nucleons. For π^--^{3}He this interaction is strong, thus near 90° c.m. the minima in $|f(\theta)|^2$ is significantly filled in by $|g(\theta)|^2$. For the even-nucleon interaction π^+-^{3}He, π^+-p has the largest amplitude and the interaction with the odd particle is weak so that the filling of the minimum is considerably less. An analogous argument follows for $\pi^\pm$-^{3}H.

It has been suggested [7] that the rise in the cross section at large angles might be due to the interaction of the incident pion with both of the like nucleons, flipping the spin of each so that the spin of the pair is conserved. That is, for $\pi^\pm$-^{3}He, the incident pion sequentially induces a single spin flip of each proton, thereby leaving the final pair with spin 0. In the common optical model treatment, a potential is constructed from the single scattering amplitude of the type discussed previously, and then the nuclear amplitude is obtained from the solution with this potential in a wave equation. Thus a spin projection change is absent from this type of treatment and must be calculated separately. For this reason double spin flip (DSF) scattering is not directly included in current optical models. The DSF, whose amplitude is coherent with $f(\theta)$, should clearly have more effect on the cross section of the even-nucleon reactions than of the odd-nucleon reactions. As each spin-flip amplitude preferentially leads to scattering around 90°, the two scatterings will lead to a forward-backward peaked angular distribution of the scattered particles. The forward part will likely be much smaller than the amplitude from the first order optical potential, but at large angles the two contributions may be comparable.

Franco [8] has investigated multiple spin flip effects for π-^{4}He scattering in the Glauber approximation. However, the Glauber approximation is not applicable in the large angle scattering region where the DSF is expected to be of significance.

We treat the DSF as a second order correction to the scattering model of Ref. [3] and calculate it in the distorted wave impulse approximation (DWIA). This calculation is presented in Sec. III, after briefly reviewing the scattering model of Ref. [3] in Sec. II. The results are discussed in Sec. IV, and conclusions are presented in Sec. V.

II. BASIC SCATTERING MODEL

In the case of moderately heavy nuclei the optical model can be extended to include, by direct calculation, a number of effects including finite range and medium modification of the pion-nucleon scattering amplitudes [9]. The medium corrections can be understood, in an approximate sense, in terms of a shift in the energy at which the phase shifts are to be evaluated and a transformation from the pion-nucleon to the pion-nucleus frame (the so-called ''angle transform''). The effect of the finite range of the pion-nucleon interaction is included by assuming plausible forms for the off-shell t-matrix for the πN interaction. A number of studies of the pion-nucleon interaction have attempted to determine the ranges (there is a different range for each spin-isospin partial wave). For a recent analysis using local potentials see Ref. [10]. These ranges can also be treated as phenomenological and fitted to the data [11].

Another important element in pion scattering theory is due the possibility of true absorption of the pion, converting its mass into energy, as opposed to the usual ''optical model absorption'' where the incident particle is simply removed from the beam by inelastic scattering. A number of attempts have been made to include this effect from fundamentals [12,13]. In the present calculations we adopt the method used in Ref. [3] of including an imaginary term in the potential proportional to the square of the density. Approximate values of the parameter can be estimated from fits to heavier nuclei (see Refs. [11,14]).

The optical model contains a long known [15] correction due to the fact that, since the t-matrix is used to describe each individual scattering, no nucleon can be struck twice successively. Thus, in a multiple scattering picture, there can be A scatterings the first time but only $A-1$ for each subsequent order. If all scatterings are of the same strength, this correction can be made by solving a wave equation with a potential having an overall strength of $A-1$, instead of A, and then multiplying the resulting t-matrix by the factor $A/(A-1)$ (the KMT factor). For an optical model in which there are only three nucleons involved, this effect is of considerably greater importance than for a heavier nucleus where this factor is close to unity. We do not attempt to do anything beyond what was used in Ref. [3]. The forward cross section is normally the most reliably calculated in scattering theories and it was noted in Ref. [3] that a treatment of this correction involving factors of the order described above was necessary to obtain agreement with the data at forward angles.

We have carried out a search over variations of the scattering parameters used in Table I of Ref. [3]. No substantial rise in the backward direction was seen. In order to focus on the DSF contribution to the cross section, we have fixed the values at energy shift=0 MeV, angle transform parameter=1, ρ^2 coefficient=8.8 fm^4, and the off shell ranges (s and p) at 600 MeV/c. For a discussion of the corrections which come in for scattering of pions from few-nucleons systems, see Ref. [18].

For the KMT factor, the calculated value [Eq. (A4) of Ref. [3]] is used. We emphasize that a factor of this size is necessary in order to get agreement in the forward direction. If a factor of unity is used the cross section forward of 90° is shifted toward smaller angles. However, see the discussion in the conclusion on this point.

The spin-flip amplitude, $g(\theta)$, is calculated in the distorted wave Born approximation, where the distortion is due to scattering from the even nucleons. The appropriate potential to use in calculating these distorted waves is unclear. The KMT treatment for the elastic scattering from two particles indicates that the potential should contain a factor of 1/2. Using only the wave functions from this calculation, however, no correction to the amplitude is ever made. Thus it is not clear if a factor of 1/2, unity, or an intermediate value should be used. The use of 1/2 gave favorable results when calculating the odd-nucleon interaction cross sections in Ref. [3] and we use the same value here.

In calculating the DSF amplitude the distortion is taken to be from the odd nucleon and one of the even nucleons. We use a factor of unity here. This is consistent with the study of single charge exchange [16] (in which the distortion is also

due to one strong and one weak interaction), for which the results were best when the full optical potential was used.

III. DOUBLE SPIN FLIP CALCULATION

Let $\mathbf{k}$ and $\mathbf{k}'$ be the pion's initial and final center of mass momenta, respectively, and $|\mathbf{k}|=|\mathbf{k}'|=k, \hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'=\cos\theta$. Let the two even nucleons have coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$, spin operators $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_2}$, and wave function $\chi(\mathbf{r}_1,\mathbf{r}_2)$. The pion's initial and final distorted wave functions are then $\Psi^{(+)}(\mathbf{k},\mathbf{r}_1)$ and $\Psi^{(-)*}(\mathbf{k}',\mathbf{r}_2)$. Assuming closure over the intermediate states and plane wave propagation of the pion in the intermediate state, the double scattering amplitude is given by

$$F(\mathbf{k},\mathbf{k}')=\frac{1}{2\pi^2}\int d\mathbf{r}_1\, d\mathbf{r}_2\, d\mathbf{q}\chi^*(\mathbf{r}_1,\mathbf{r}_2) \times[f_2(\mathbf{q},\mathbf{k}_2)\Psi^{(-)*}(\mathbf{k}',\mathbf{r}_2)]e^{i\mathbf{q}\cdot\mathbf{r}_2}P(q) \times e^{-i\mathbf{q}\cdot\mathbf{r}_1}[f_1(\mathbf{k}_1,\mathbf{q})\Psi^{(+)}(\mathbf{k},\mathbf{r}_1)]\chi(\mathbf{r}_1,\mathbf{r}_2), \quad (1)$$

where the pion propagator is

$$P(q)=\frac{1}{q^2-k^2}, \quad (2)$$

the pion-nucleon spin flip amplitude is

$$f_i(\mathbf{q}_1,\mathbf{q}_2)=i\lambda v(q_1)v(q_2)\boldsymbol{\sigma_i}\cdot(\mathbf{q}_1\times\mathbf{q}_2), \quad (3)$$

and the off-shell form factor is

$$v(q)=\frac{\alpha^2+k^2}{\alpha^2+q^2}. \quad (4)$$

The quantity α describes the range of the pion-nucleon interaction, which we take to be 600 MeV/c, and λ is derived from the πN phase shifts [17]. Since f_i is an operator in nucleon spin space, we must compute the expectation value of the operator

$$O=(\boldsymbol{\sigma_1}\cdot\mathbf{k}_1\times\mathbf{q})(\boldsymbol{\sigma_2}\cdot\mathbf{q}\times\mathbf{k}_2). \quad (5)$$

For a pure singlet case [19]

$$\langle O\rangle_{S=0}=-(\mathbf{k}_1\times\mathbf{q})\cdot(\mathbf{q}\times\mathbf{k}_2) =-q^2[(\mathbf{k}_1\cdot\hat{\mathbf{q}})(\mathbf{k}_2\cdot\hat{\mathbf{q}})-\mathbf{k}_1\cdot\mathbf{k}_2]. \quad (6)$$

We write

$$|\chi(\mathbf{r}_1,\mathbf{r}_2)|^2=\rho(r_1)\rho(r_2) \quad (7)$$

for the target in its ground state, so that the DSF scattering amplitude is now

$$F(\mathbf{k},\mathbf{k}')=\frac{\lambda^2}{2\pi^2}\int d\mathbf{r}_1\, d\mathbf{r}_2\, d\mathbf{q}q^2v^2(q)P(q) \times e^{i\mathbf{q}\cdot\mathbf{r}_2}\rho(r_2)e^{-i\mathbf{q}\cdot\mathbf{r}_1}\rho(r_1)v(k_1)v(k_2) \times[(\mathbf{k}_1\cdot\hat{\mathbf{q}})(\mathbf{k}_2\cdot\hat{\mathbf{q}})-\mathbf{k}_1\cdot\mathbf{k}_2] \times\Psi^{(-)*}(\mathbf{k}',\mathbf{r}_2)\Psi^{(+)}(\mathbf{k},\mathbf{r}_1). \quad (8)$$

As the momenta are operators on the distorted waves, we set

$$\mathbf{k}_j\equiv -i\nabla_j, \quad (9)$$

and define

$$\psi(\mathbf{k},\mathbf{r}_1)\equiv v(k_1)\Psi^{(+)}(\mathbf{k},\mathbf{r}_1), \quad \psi(\mathbf{k}',\mathbf{r}_2)\equiv v(k_2)\Psi^{(-)*}(\mathbf{k}',\mathbf{r}_2). \quad (10)$$

We use a partial wave expansion for the pion wave function

$$\Psi^{(+)}(\mathbf{k},\mathbf{r}_1)=4\pi\sum_{lm} i^l Y_{lm}(\hat{\mathbf{k}})Y^*_{lm}(\hat{\mathbf{r}}_1)\Psi_l(kr_1), \quad (11)$$

so that, by use of a double Fourier transform and orthonormality of the spherical harmonics,

$$\psi(\mathbf{k},\mathbf{r}_1)=\frac{1}{(2\pi)^3}\frac{\alpha^2+k^2}{\alpha^2-\nabla_1^2}\int d\boldsymbol{\kappa}e^{-i\boldsymbol{\kappa}\cdot\mathbf{r}_1}\Psi^{(+)}(\mathbf{k},\boldsymbol{\kappa}) \quad (12)$$

$$=\frac{2}{\pi}(\alpha^2+k^2)4\pi\sum_{lm} i^l Y_{lm}(\hat{\mathbf{k}})Y^*_{lm}(\hat{\mathbf{r}}_1) \times\int dr\, d\kappa\frac{r^2\kappa^2 j_l(\kappa r_1)j_l(\kappa r)\Psi_l(kr)}{\alpha^2+\kappa^2}, \quad (13)$$

where we identify

$$\psi_l(kr_i)=\frac{2}{\pi}(\alpha^2+k^2)\int dr d\kappa\frac{r^2\kappa^2 j_l(\kappa r_i)j_l(\kappa r)\Psi_l(kr)}{\alpha^2+\kappa^2} \quad (14)$$

$$=\int dr r^2 G_l(r,r_i)\Psi_l(kr), \quad (15)$$

and where contour integration gives

$$G_l(r,r')\equiv\frac{2}{\pi}(\alpha^2+k^2)\int_0^\infty d\kappa\frac{\kappa^2 j_l(\kappa r')j_l(\kappa r)}{\alpha^2+\kappa^2} \quad (16)$$

$$=-\alpha(\alpha^2+k^2)h_l^{(+)}(i\alpha r_>)j_l(i\alpha r_<) \quad (17)$$

for $r_>$ the greater of r and r'. In a similar fashion

$$\psi(\mathbf{k}',\mathbf{r}_2)=4\pi\sum_{lm}(-i)^l Y_{lm}(\hat{\mathbf{k}}')Y^*_{lm}(\hat{\mathbf{r}}_2)\psi_l(kr_2), \quad (18)$$

as $\Psi^{(-)*}(\mathbf{k},\mathbf{r})=\Psi^{(+)}(-\mathbf{k},\mathbf{r})$.

Thus Eq. (8) in the DWIA may be written

$$F(\mathbf{k},\mathbf{k}')=\frac{\lambda^2}{2\pi^2}\int d\mathbf{r}_1\,d\mathbf{r}_2\,d\mathbf{q}q^2v^2(q)P(q)\times e^{i\mathbf{q}\cdot\mathbf{r}_2}\rho(r_2)e^{-i\mathbf{q}\cdot\mathbf{r}_1}\rho(r_1)\times[\nabla_1\cdot\nabla_2-(\hat{\mathbf{q}}\cdot\nabla_1)(\hat{\mathbf{q}}\cdot\nabla_2)]\times\psi(\mathbf{k},\mathbf{r}_1)\psi(\mathbf{k}',\mathbf{r}_2). \quad (19)$$

The DSF amplitude is twice the value given by Eq. (19), as the process may proceed in two time orders.

A. Evaluation: First technique

While Eq. (19) may be calculated directly, it is more easily evaluated by integration by parts on $\mathbf{r}_1$ and $\mathbf{r}_2$. We find

$$F(\mathbf{k},\mathbf{k}')=-\frac{\lambda^2}{2\pi^2}\int d\mathbf{r}_1\,d\mathbf{r}_2\,d\mathbf{q}q^2v^2(q)P(q)\psi(\mathbf{k},\mathbf{r}_1)\times e^{-i\mathbf{q}\cdot\mathbf{r}_1}\psi(\mathbf{k}',\mathbf{r}_2)e^{i\mathbf{q}\cdot\mathbf{r}_2}\rho'(r_1)\rho'(r_2)\times[\hat{\mathbf{r}}_1\cdot\hat{\mathbf{r}}_2-(\hat{\mathbf{q}}\cdot\hat{\mathbf{r}}_1)(\hat{\mathbf{q}}\cdot\hat{\mathbf{r}}_2)]. \quad (20)$$

Let us consider each term in brackets separately, and label them F_1 and F_2, respectively.

The first term is calculated by expanding the wave functions, exponentials, and $\hat{\mathbf{r}}_1\cdot\hat{\mathbf{r}}_2$ in partial waves. We find

$$F_1(\mathbf{k},\mathbf{k}')=-\frac{\lambda^2}{2\pi^2}(4\pi)^3\sum_{Ll}(2L+1)\times(C^{000}_{1Ll})^2P_L(\cos\theta)I_{L,L,l,l}(r_1,r_2), \quad (21)$$

where

$$I_{L_1,L_2,l_1,l_2}(r_1,r_2)\equiv\int dr_1\,dr_2\,r_1^2r_2^2\rho'(r_1)\rho'(r_2)\times\psi_{L_1}(kr_1)\psi_{L_2}(kr_2)\eta_{l_1,l_2}(r_1,r_2), \quad (22)$$

and

$$\eta_{l_1,l_2}(r_1,r_2)\equiv\int_0^\infty dqq^4v^2(q)P(q)j_{l_1}(qr_1)j_{l_2}(qr_2)=\frac{i\pi}{2}k^3j_{l_<}(kr_<)h^{(+)}_{l_>}(kr_>)+\frac{\pi}{4}\alpha(\alpha^2+3k^2)j_{l_<}(i\alpha r_<)h^{(+)}_{l_>}(i\alpha r_>)+\frac{i\pi}{4}\alpha^2(\alpha^2+k^2)[r_<j'_{l_<}(i\alpha r_<)h^{(+)}_{l_>}(i\alpha r_>)+r_>j_{l_<}(i\alpha r_<)h^{(+)\prime}_{l_>}(i\alpha r_>)] \quad (23)$$

provided $l_<<(l_>+3)$ where $r_>$ is the greater of r_1 and r_2 and $l_>$ is $r_>$'s respective index. To calculate the second term of Eq. (20), we write

$$\hat{\mathbf{q}}\cdot\hat{\mathbf{r}}e^{i\hat{\mathbf{q}}\cdot\hat{\mathbf{r}}}=4\pi\sum_{lLM}i^lj_l(qr)(C^{000}_{1Ll})^2Y_{LM}(\hat{\mathbf{q}})Y^*_{LM}(\hat{\mathbf{r}}). \quad (24)$$

Expanding the functions as before, we find

$$F_2(\mathbf{k},\mathbf{k}')=\frac{\lambda^2}{2\pi^2}(4\pi)^3\sum_{l_1l_2L}i^{l_2-l_1}(2L+1)\times(C^{000}_{1Ll_1})^2(C^{000}_{1Ll_2})^2P_L(\cos\theta)I_{L,L,l_1,l_2}(r_1,r_2). \quad (25)$$

Thus we have

$$F(\mathbf{k},\mathbf{k}')=32\pi\lambda^2\sum_L(2L+1)P_L(\cos\theta)\times\Bigg\{\sum_{l_1\neq l_2}i^{l_2-l_1}(C^{000}_{1Ll_1})^2(C^{000}_{1Ll_2})^2I_{L,L,l_1,l_2}(r_1,r_2)+\sum_l[(C^{000}_{1Ll})^4-(C^{000}_{1Ll})^2]I_{L,L,l,l}(r_1,r_2)\Bigg\}. \quad (26)$$

We may further reduce this expression with Racah algebra,

$$F(\mathbf{k},\mathbf{k}')=32\pi\lambda^2\sum_L(2L+1)P_L(\cos\theta)\times\{-2(C^{0\,0}_{1\,L\,L-1})^2(C^{0\,0}_{1\,L\,L+1})^2I_{L,L,L-1,L+1}\times(r_1,r_2)+[(C^{0\,0}_{1\,L\,L-1})^4-(C^{0\,0}_{1\,L\,L-1})^2]\times I_{L,L,L-1,L-1}(r_1,r_2)+[(C^{0\,0}_{1\,L\,L+1})^4-(C^{0\,0}_{1\,L\,L+1})^2]I_{L,L,L+1,L+1}(r_1,r_2)\}. \quad (27)$$

As the first two indices on the integral term, *I*, are the indices for the pion's wave function, the angular momentum of the pion is conserved as it must be for the elastic interaction with a spin-zero system.

B. Evaluation: Second technique

Alternately, one may directly calculate Eq. (19). This calculation is straightforward, but lengthy. We use the covariant components of the operator $\boldsymbol{\nabla}$ in the spherical basis. For $\sigma=(-1,0,1)$,

$$\nabla_1\cdot\nabla_2=\sum_\sigma(-1)^\sigma\nabla^{(1)}_\sigma\nabla^{(2)}_{-\sigma}, \quad (28)$$

$$
\begin{aligned}
&(\hat{\mathbf{q}}\cdot\nabla_1)(\hat{\mathbf{q}}\cdot\nabla_2)\\
&\quad=\sqrt{4\pi}\sum_{\sigma\sigma'}\left[C^{000}_{1\,1\,0}C^{\sigma\sigma'0}_{1\,1\,\;0}Y_{00}(\hat{\mathbf{q}})\right.\\
&\qquad\left.+\frac{1}{\sqrt{5}}\sum_M(-1)^M C^{000}_{1\,1\,2}C^{\sigma\sigma' M}_{1\,1\;\;2}Y_{2M}(\hat{\mathbf{q}})\right]\nabla^{(1)}_{-\sigma}\nabla^{(2)}_{-\sigma'}\\
&\quad=\frac{1}{3}\nabla_1\cdot\nabla_2+\sqrt{\frac{8\pi}{15}}\sum_{\sigma\sigma' M}\\
&\qquad\times(-1)^M C^{\sigma\sigma' M}_{1\,1\;\;2}Y_{2M}(\hat{\mathbf{q}})\nabla^{(1)}_{-\sigma}\nabla^{(2)}_{-\sigma'}.
\end{aligned}
\tag{29}
$$

Let $F_S(\mathbf{k},\mathbf{k}')$ be the contribution to $F(\mathbf{k},\mathbf{k}')$ from Eq. (28) and let $F_D(\mathbf{k},\mathbf{k}')$ be the contribution from the second term in Eq. (29), so that

$$
F(\mathbf{k},\mathbf{k}')=\frac{2}{3}F_S(\mathbf{k},\mathbf{k}')+F_D(\mathbf{k},\mathbf{k}'). \tag{30}
$$

From Ref. [20] we have

$$
\begin{aligned}
&\nabla_\sigma Y_{\lambda\mu}(\hat{\mathbf{r}}_\mathbf{i})\psi_\lambda(kr_i)\\
&\quad=\sqrt{\frac{\lambda+1}{2\lambda+3}}C^{\mu\sigma\mu+\sigma}_{\lambda 1\;\lambda+1}F_\lambda(r_i)Y_{\lambda+1\mu+\sigma}(\hat{\mathbf{r}}_\mathbf{i})\\
&\qquad-\sqrt{\frac{\lambda}{2\lambda-1}}C^{\mu\sigma\mu+\sigma}_{\lambda 1\;\lambda-1}G_\lambda(r_i)Y_{\lambda-1\mu+\sigma}(\hat{\mathbf{r}}_\mathbf{i}),\\
&F_\lambda(r_i)\equiv\frac{d\psi_\lambda(kr_i)}{dr_i}-\frac{\lambda}{r_i}\psi_\lambda(kr_i),\\
&G_\lambda(r_i)\equiv\frac{d\psi_\lambda(kr_i)}{dr_i}+\frac{\lambda+1}{r_i}\psi_\lambda(kr_i).
\end{aligned}
\tag{31}
$$

If we proceed in a manner similar to that in Sec. III A, we find

$$
\begin{aligned}
F_S(\mathbf{k},\mathbf{k}')&=32\pi\lambda_f^2\sum_l P_l(\cos\theta)\\
&\quad\times\int dr_1dr_2r_1^2r_2^2\rho(r_1)\rho(r_2)\\
&\quad\times[(l+1)\eta_{l+1,l+1}(r_1,r_2)F_l(r_1)F_l(r_2)\\
&\quad+l\,\eta_{l-1,l-1}(r_1,r_2)G_l(r_1)G_l(r_2)]
\end{aligned}
\tag{32}
$$

and

$$
\begin{aligned}
F_D(\mathbf{k},\mathbf{k}')&=-32\pi\sqrt{\frac{2}{3}}\lambda_f^2\sum_l(-1)^l P_l(\cos\theta)\int dr_1dr_2r_1^2r_2^2\rho(r_1)\rho(r_2)\\
&\quad\times\left[(2l+3)(l+1)\eta_{l+1,l+1}(r_1,r_2)F_l(r_1)F_l(r_2)C^{0\;\;0}_{l+1\,l+1\;2}\begin{Bmatrix}l & l+1 & 1\\ 2 & 1 & l+1\end{Bmatrix}\right.\\
&\quad+l(2l-1)\eta_{l-1,l-1}(r_1,r_2)G_l(r_1)G_l(r_2)C^{0\;\;0}_{l-1\,l-1\;2}\begin{Bmatrix}l & l-1 & 1\\ 2 & 1 & l-1\end{Bmatrix}\\
&\quad\left.+2\sqrt{l(l+1)(2l-1)(2l+3)}\,\eta_{l-1,l+1}(r_1,r_2)G_l(r_1)F_l(r_2)C^{0\;\;0}_{l-1\,l+1\;2}\begin{Bmatrix}l & l-1 & 1\\ 2 & 1 & l+1\end{Bmatrix}\right],
\end{aligned}
\tag{33}
$$

where $\eta_{l_1,l_2}(r_1,r_2)$ is as previously defined.

IV. RESULTS

We performed calculations with the density of ^{3}He taken from the solution to the Faddeev equations [21]. The two treatments presented in the previous section were evaluated and agree to within the expected numerical accuracy. The first method depends on the numerical calculation of the derivatives of the three-body density while the second does not. The derivative operators act on the pion wave function in the second method while the first method has a more direct form to calculate, i.e., it is expressed as an expectation value of a *local* operator. It is easier to see how to include the δ-function effect, discussed shortly, with the second method.

Figure 1 shows the results of adding the DSF to the basic optical model for all four scattering cases. The dashed lines correspond to the optical model only and the solid lines include the DSF term. We see that the addition of the DSF term is indeed significant at large angles for the even nucleon cases. However, it does not give the structure seen in experiment.

Figure 2 compares the scattering amplitude of the DSF to that of the basic optical model for π^+-^{3}He. The dashed line represents $f(\theta)$ from the optical model only, the solid line the DSF amplitude, and the dash-dotted-line the DSF ampli-

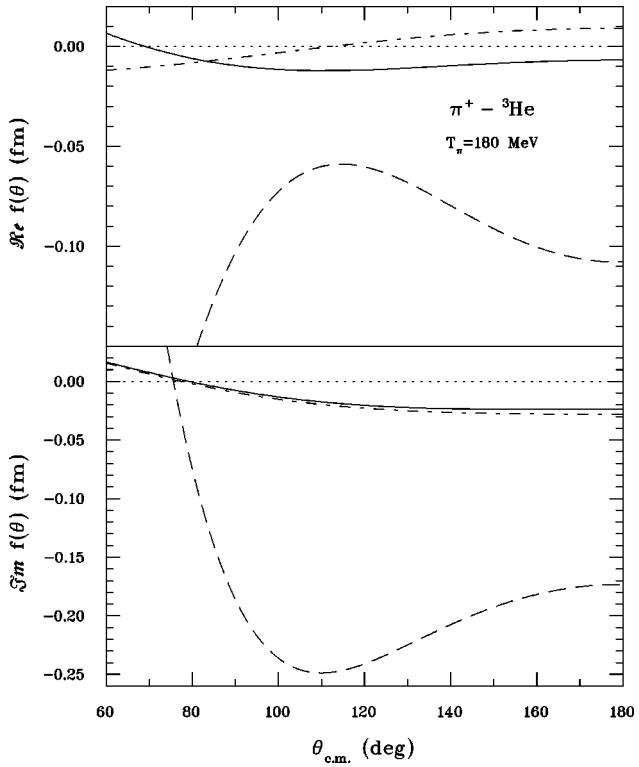

FIG. 2. Scattering amplitudes for π^+-^{3}He. The dashed line represents $f(\theta)$ from the optical model, the solid line the double spin-flip mechanism (solid line), and the dash-dotted line the DSF with the δ-function removed.

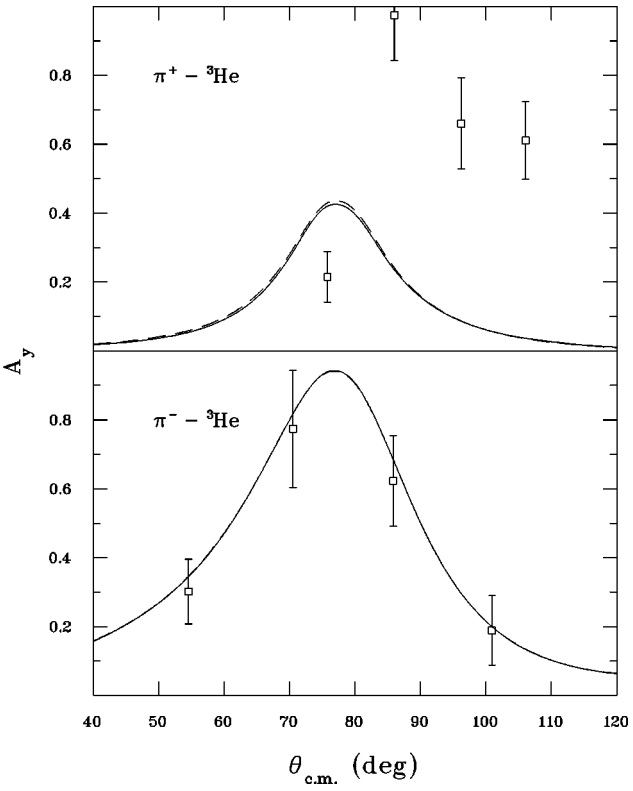

FIG. 3. Asymmetries for π^+ ^{3}He and π^- ^{3}He scattering from the optical model only (dashed line) and with the DSF term (solid line). The data are from Refs. [22] and [23]. For π^+ there are additional data points at forward angles which are negative (not shown).

tude with the δ function removed (discussed below). We see that in the region of the first minimum both the real and imaginary parts of the DSF amplitude are passing through zero, and thus have little effect in this region.

One may speculate that a cancellation between the two mechanisms might produce a minimum at 130° and that the present calculation does not have the correct phase. To check this possibility, we introduced an arbitrary phase difference between the two terms, but still found no case which gave the characteristic minima.

The asymmetries are shown in Fig. 3. Since polarization phenomena are typically sensitive to small corrections, one might expect important corrections from the DSF; but we note that its addition has little effect since the asymmetry is only large around the minimum in the cross section where the DSF amplitude vanishes.

It has long been known that there is a δ function in the s-wave (nucleon-nucleon) portion of the one-pion-exchange potential (see, e.g., Ref. [24] for a discussion of this δ function and its removal). This δ function also exists in the p-wave–p-wave part of pion double scattering [25,26], especially visible in double charge exchange [19,14].

If we refer back to Eq. (8), we see that the δ-function piece of the DSF amplitude results from the monopole portion of the expression $[(\mathbf{k}_1\cdot\hat{\mathbf{q}})(\mathbf{k}_2\cdot\hat{\mathbf{q}})-\mathbf{k}_1\cdot\mathbf{k}_2]$. In Sec. III B we explicitly expanded this expression in terms of the ∇ operator. Thus from the second treatment it is easy to see that the δ function should be removed from the $F_S(\mathbf{k},\mathbf{k}')$ term only. To do so, we make the replacement

$$\frac{q^2}{q^2-k^2}\rightarrow\left[\frac{q^2}{q^2-k^2}-1\right]=\frac{k^2}{q^2-k^2}, \tag{34}$$

which is equivalent to replacing q^4 with k^2q^2 in the integral expression for $\eta_{l_1,l_2}(r_1,r_2)$ in Eq. (23). The term $F_D(\mathbf{k},\mathbf{k}')$ remains unchanged.

As seen in Fig. 2, this correction essentially reverses the sign of the real part of the scattering amplitude, while having a small effect on the imaginary part. However, it is the imaginary portion of the scattering amplitude that dominates the cross section at large angles where the DSF is of significance. Thus the δ-function correction has a minimal effect on the scattering cross section, decreasing it by less than 4 percent. In principle this correction should also be made in the basic optical model, akin to the Lorentz-Lorenz effect [25,27] at resonance energies. However, such a consideration is beyond the scope of this paper.

It is known that the Faddeev densities, while representing the exact solution to the three-nucleon system expressed in terms of nucleon degrees of freedom, do not provide a completely accurate description of the electron-scattering cross sections. The problem is clearer at high momentum transfer near the first zero of the angular distribution [28]. The difference is often ascribed to meson exchange currents [29]. However, in order to explore the sensitivity of the results to the density used, we also performed calculations using the

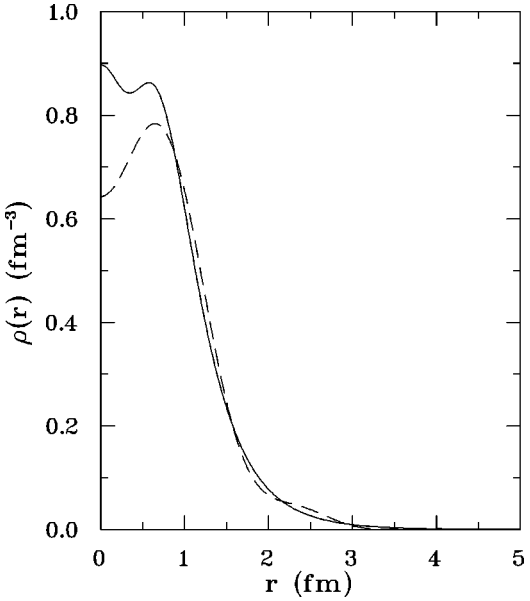

FIG. 4. Comparison of the Faddeev (solid line) and electron scattering densities (dashed line).

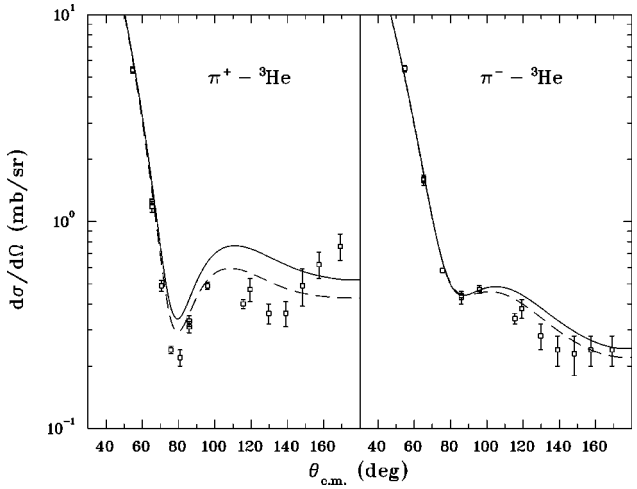

FIG. 5. Comparison of the results of the optical model with the Faddeev density (solid line) and the electron scattering density (dashed line). The radius for the electron scattering density has been rescaled the same rms value as the Faddeev density. If no such rescaling is done, the difference is considerably less.

charge densities of Ref. [30] corrected for the finite size of the proton using the proton charge parametrization from Ref. [31]. The two densities are compared in Fig. 4. Only the proton density was changed, the neutron density remaining that of the Faddeev calculations.

Figure 5 shows the comparison of the results of π^+-^{3}He scattering for the case in which the electron-scattering density was adjusted to have the same rms radius as the Faddeev density. The introduction of the electron scattering density at most shifts the cross section downward, and has little effect on the shape of the cross section at large scattering angles. For the calculation in which the two densities were used in their unmodified form the difference is even less. Thus it seems unlikely that the minimum is a form-factor effect, at least within the span of currently accepted functions. One can find a fit to the data by allowing an arbitrary form factor variation [32] but the density which results appears to be unphysical, having essentially no support for small values of the radial variable, i.e., it has a complete hole in the center.

V. CONCLUSIONS

We find that the double-spin-flip correction gives a non-negligible contribution to the large angle scattering cross section in the case of π^+-^{3}He. It should be included in any serious attempt to explain quantitatively the back-angle data on the three-nucleon system, and perhaps on other nuclei as well. However, because of the shape and phase of this coherent contribution, it does not appear that it can explain the dip and subsequent rise seen in the case of two strong scatters.

As one possible direction for further work to explain the effect, we observe that if the KMT factor is set to unity, while the agreement of the forward cross section with data is completely ruined, there is a slight dip at the appropriate back angle. This suggests than an improvement to the basic optical model may lead to the resolution of this issue. An approach which may offer hope is that of Garcilazo [33], but such a treatment involves a complete reformulation of the scattering theory.

The single energy shift used in the present calculations might also cause problems. In Ref. [9] it was found that the shift in energy was dependent on the pion-nucleus angular momentum which is an alternate expression of the results of earlier work in which it was a function of momentum transfer [34].

These two observations lead one to think that the next appropriate step might be a reexamination of the scattering theory to provide a more detailed representation of the physics.

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[1] S.K. Matthews *et al.*, Phys. Rev. C **51**, 2534 (1995); W.J. Briscoe *et al.*, Nucl. Phys. **A553**, 585c (1993).

[2] S.S. Kamalov, L. Tiator, and C. Bennhold, Phys. Rev. C **47**, 941 (1993).

[3] W.R. Gibbs and B.F. Gibson, Phys. Rev. C **43**, 1012 (1991).

[4] C. Pillai, D.B. Barlow, B.L. Berman, W.J. Briscoe, A. Mokhtari, B.M.K. Nefkens, and M.E. Sadler, Phys. Rev. C **43**, 1838 (1991); B.M.K. Nefkens, W.J. Briscoe, A.D. Eichon, D.H.

Fitzgerald, A. Mokhtari, J.A. Wightman, and M.E. Sadler, *ibid.* **41**, 2770 (1990).
[5] K.S. Dhuga *et al.*, Phys. Rev. C **35**, 1148 (1987).
[6] L.I. Schiff, Phys. Rev. **133**, B802 (1964).
[7] S.K. Matthews (private communication) to W.R. Gibbs.
[8] V. Franco, Phys. Rev. C **9**, 1690 (1974).
[9] W.B. Kaufmann and W.R. Gibbs, Phys. Rev. C **28**, 1286 (1983).
[10] W.R. Gibbs, Li Ai, and W.B. Kaufmann, Phys. Rev. C **57**, 784 (1998).
[11] M. Nuseirat, M.A.K. Lodhi, and W.R. Gibbs, Phys. Rev. C **58**, 314 (1998).
[12] H. Garcilazo and W.R. Gibbs, Nucl. Phys. **A381**, 487 (1982).
[13] E. Oset, M. Khankhasayev, J. Nieves, H. Sarafian, and M.J. Vicente-Vacas, Phys. Rev. C **46**, 2406 (1992).
[14] M. Nuseirat, M.O. El-Ghossain, M.A.K. Lodhi, W.R. Gibbs, and W.B. Kaufmann, Phys. Rev. C **58**, 2292 (1998).
[15] A.K. Kerman, H. McManus, and R.M. Thaler, Ann. Phys. (N.Y.) **8**, 551 (1959).
[16] M.L. Dowell, W. Fong, J.L. Matthews, H. Park, M. Wang, M.E. Yuly, E.R. Kinney, P.A.M. Gram, D.A. Roberts, and G.A. Rebka, Jr., Phys. Rev. C **51**, 1551 (1995).
[17] R.A. Arndt and R. Roper, SAID interactive dial-in program (unpublished).
[18] W.R. Gibbs, B.F. Gibson, A.T. Hess, G.J. Stephenson, and W.B. Kaufmann, Phys. Rev. C **13**, 2433 (1976); R. Mach, Nucl. Phys. **A205**, 56 (1973); R. Landau, Phys. Rev. C **15**, 2127 (1977); Ann. Phys. (N.Y.) **92**, 205 (1975); and Ref. [9].
[19] W.R. Gibbs, M. Elghossain, and W.B. Kaufmann, Phys. Rev. C **48**, 1546 (1993).
[20] D.A. Varshalovich, A.N. Moskalev, and V.K. Khersonskii, *Quantum Theory of Angular Momentum* (World Scientific, Singapore, 1988).
[21] J.L. Friar, B.F. Gibson, and G.L. Payne, Phys. Rev. C **35**, 1502 (1987); J.L. Friar, B.F. Gibson, C.R. Chen, and G.L. Payne, Phys. Lett. **161B**, 241 (1985).
[22] M.A. Espy *et al.*, Phys. Rev. Lett. **76**, 3667 (1996).
[23] M.A. Espy *et al.*, Phys. Rev. C **56**, 2607 (1997).
[24] W.R. Gibbs and B. Loiseau, Phys. Rev. C **50**, 2742 (1994).
[25] M. Ericson and T.E.O. Ericson, Ann. Phys. (N.Y.) **36**, 323 (1966).
[26] G. Baym and G.E. Brown, Nucl. Phys. **A247**, 395 (1975).
[27] J.M. Eisenberg, J. Hüfner, and E.J. Moniz, Phys. Lett. **47B**, 381 (1973).
[28] A. Amroun, V. Breton, J.M. Cavedon, B. Frois, D. Goutte, F.P. Juster, P. Leconte, J. Martino, Y. Mizuno, X.H. Phan, S.K. Platchkov, and I. Sick, Nucl. Phys. **A579**, 596 (1994).
[29] Ch. Hajduk, P.U. Sauer, and W. Strueve, Nucl. Phys. **A405**, 581 (1983); R. Schiavilla, V.R. Pandharipande, and R.B. Wiringa, *ibid.* **A449**, 219 (1986).
[30] J.S. McCarthy, I. Sick, and R.R. Whitney, Phys. Rev. C **15**, 1396 (1977).
[31] G. Höhler, E. Pietarinen, I. Sabba-Stefanescu, F. Borkowski, G.G. Simon, V.H. Walther, and R.D. Wendling, Nucl. Phys. **B114**, 505 (1976).
[32] Q. Zhao, Ph.D. thesis, New Mexico State University, 1997.
[33] H. Garcilazo, Phys. Lett. **82B**, 332 (1979).
[34] J. Maillet, J.P. Dedonder, and C. Schmit, Nucl. Phys. **A316**, 267 (1979) and references therein.