

Role of octupole-octupole interaction in neutron-rich $^{108-114}\text{Pd}$ isotopes

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The yrast spectra, quadrupole moments, quadrupole deformation parameter β_2 , $B(E2)$ transition probabilities and occupation probabilities for each angular momentum state are calculated for $^{108-114}\text{Pd}$ isotopes by carrying out cranked Hartree-Bogoliubov (CHB) calculations. These calculations have been performed by employing pairing-plus-quadrupole-quadrupole and also octupole-octupole interaction operating in a reasonably large valence space outside ^{76}Sr core. The calculations show a marked improvement in agreement with experiments when the octupole-octupole interaction term is included in the Hamiltonian. [S0556-2813(99)00401-X]

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I. INTRODUCTION

The study of ground-state properties of even mass palladium isotopes has been the subject of a large number of experimental studies [1–18]. In recent years, the experimental techniques such as Coulomb excitation and γ -ray spectroscopy have extended the new level schemes of some of these nuclei up to 16^+ . In contrast to the large scale effort that has been made on the experimental side, only a few theoretical models [19–22] have been proposed to explain the character of yrast spectra in these nuclei. The earliest phenomenological attempt at understanding the observed levels in the Pd region have had limited success [20]. Apart from the earlier studies in the framework of the variable moment of inertia (VMI) model, an attempt was also made by Smith and Valkov [21] to explain the observed features of the yrast bands in Pd isotopes by invoking instability towards asymmetric deformation at sufficiently high ($J^\pi > 8^+$) angular momenta. Sometime back Stachel *et al.* [22] attempted a study of the experimental excitation energies and $E2$ transition probabilities of the neutron-rich Pd isotopes in the framework of interacting boson model (IBA-I). It has been established by them that palladium isotopes follow the $\text{SU}(5) \rightarrow \text{O}(6)$ transition. The mechanism of this shape transition is not very clear from their calculation. Sometime back Aysto *et al.* [23] suggested the importance of triaxiality in neutron rich nuclei in the mass region $A \sim 108$. Shannon *et al.* [24] have also recently reported the occurrence of axially asymmetric shapes in neutron rich nuclei in the mass region $A \sim 100$. Ostensibly, one feels motivated to examine the degree to which nuclear low-lying states in $^{108-114}\text{Pd}$ could adopt axially asymmetric shapes. Such a possibility prompts us to ask a number of questions such as, how good will the nonaxial calculations for the mass chain of $^{108-114}\text{Pd}$ be? Secondly, how reliable are the calculations, performed with pairing plus quadrupole-quadrupole interaction? Is it enough to take only the quadrupole-quadrupole term of the potential energy expansion or it becomes important to incorporate a higher order potential energy term, in the form of octupole-octupole interaction in the two body interaction? This has been the inspiring factor for us to execute the present work.

In this paper, we have carried out a study of yrast states, quadrupole moments, quadrupole deformation parameter β_2 , $B(E2)$ transition probabilities, and occupation probabilities of $^{108-114}\text{Pd}$ isotopes in cranking framework. To this end, we have employed the cranked Hartree-Bogoliubov (CHB) framework in conjunction with the HB ansatz. These calculations have been performed by employing (1) a pairing-plus-quadrupole-quadrupole interaction (we shall denote it by PPQ) and (2) by including a higher order octupole-octupole energy term (we shall denote it by PPQO) operating in a reasonably large valence space spanned by $3s_{1/2}$, $2p_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$, and $1h_{11/2}$ orbits for protons as well as neutrons. The nucleus ^{76}Sr has been considered as an inert core.

In Sec. III, a comparison of the results obtained in the CHB framework with PPQ and PPQO are presented. The results on intrinsic quadrupole moments, exhibit a number of interesting features.

(i) In PPQ model of interaction, nonaxiality is seen to increase with angular momentum as we move up the yrast states for $^{108-114}\text{Pd}$.

(ii) In the above model, nonaxiality increases from ^{108}Pd to ^{110}Pd and then decreases with neutron number.

These conclusions get modified when the PPQO model of interaction is used and the following features are observed.

(i) The results on intrinsic quadrupole moments in this model, predict a decrease of nonaxiality with neutron number.

(ii) In case of $^{108-110}\text{Pd}$, the degree of nonaxiality decreases as we move up the yrast states, whereas in the case of $^{112-114}\text{Pd}$, its value shows an increase, as we go up the yrast states.

A comparison of the results of $B(E2)$ values and yrast spectra shows that a marked improvement in agreement with the experimental values is obtained by carrying out calculations with the PPQO interaction. Besides this, a critical analysis of the results on occupation probabilities for protons, leads one to conclude that the quasideformed nature of Pd isotopes is intricately related to the $(2p_{1/2})_\pi$ subshell closure. The results indicate that the $(2p_{1/2})_\pi$ subshell closure takes place only when calculations are performed by including the octupole-octupole term in the Hamiltonian as without

TABLE I. Comparison of the experimental and calculated yrast levels using PPQ model of interaction in $^{108-114}\text{Pd}$ isotopes. Here $\langle Q_0^2 \rangle_\pi$, $\langle Q_2^2 \rangle_\pi$, $\langle Q_2^2 \rangle_\nu$ give the contribution of the protons (neutrons) to the components of quadrupole moment operator. The seventh column gives the value of quadrupole deformation parameter $(\beta_2)_{\text{Th}}$. The quadrupole moments have been calculated in units of b^2 , where $b = \sqrt{\hbar/m\omega}$ is the oscillator parameter.

| Nucleus | J^π | $\langle Q_0^2 \rangle_\pi$ | $\langle Q_2^2 \rangle_\pi$ | $\langle Q_2^2 \rangle_\nu$ | $\langle Q_2^2 \rangle_\nu$ | $(\beta_2)_{\text{Th}}$ |
|-------------------|---------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-------------------------|
| ^{108}Pd | 2^+ | 36.06 | 0.087 | 47.44 | 0.456 | 0.301 |
| | 4^+ | 36.02 | 0.105 | 47.27 | 0.654 | |
| | 6^+ | 35.99 | 0.145 | 47.13 | 0.914 | |
| | 8^+ | 35.78 | 0.981 | 44.63 | 6.804 | |
| | 10^+ | 35.70 | 0.990 | 44.35 | 6.969 | |
| | 12^+ | 35.59 | 0.971 | 44.03 | 6.933 | |
| | 14^+ | 35.44 | 0.928 | 43.68 | 6.724 | |
| ^{110}Pd | 2^+ | 36.16 | 0.462 | 48.46 | 2.712 | 0.282 |
| | 4^+ | 36.11 | 0.467 | 48.33 | 2.942 | |
| | 6^+ | 36.04 | 0.519 | 48.12 | 3.267 | |
| | 8^+ | 35.94 | 0.551 | 47.82 | 3.648 | |
| | 10^+ | 35.81 | 0.610 | 47.45 | 4.218 | |
| | 12^+ | 35.74 | 0.652 | 47.23 | 4.585 | |
| ^{112}Pd | 2^+ | 36.00 | 0.303 | 46.71 | 1.855 | 0.261 |
| | 4^+ | 35.98 | 0.705 | 46.69 | 4.704 | |
| | 6^+ | 35.91 | 0.733 | 46.46 | 4.973 | |
| | 8^+ | 35.81 | 0.759 | 46.16 | 5.271 | |
| | 10^+ | 35.68 | 0.808 | 45.76 | 5.715 | |
| ^{114}Pd | 2^+ | 35.85 | 0.025 | 45.16 | 0.103 | 0.241 |
| | 4^+ | 35.78 | 0.021 | 44.94 | 0.144 | |
| | 6^+ | 35.69 | 0.021 | 44.67 | 0.231 | |
| | 8^+ | 35.57 | 0.026 | 44.37 | 0.378 | |
| | 10^+ | 35.41 | 0.040 | 44.00 | 0.589 | |
| | 12^+ | 35.22 | 0.058 | 43.53 | 0.856 | |
| | 14^+ | 35.00 | 0.085 | 42.97 | 1.197 | |
| | 16^+ | 34.75 | 0.119 | 42.33 | 1.586 | |

this term in the potential energy $Z=40$ core remains polarized and leads to disagreement with the experimental results. Thus, the calculations indicate that the inclusion of octupole-octupole interaction energy term in the potential energy is very important for getting good agreement with experiments for Pd isotopes.

II. CALCULATIONAL DETAILS

The one and two-body parts of the Hamiltonian. The spherical single particle energies (SPEs) that we have employed are (in MeV) $(2p_{1/2}) = -1.3$, $(1g_{9/2}) = -1.0$, $(2d_{5/2}) = 5.4$, $(3s_{1/2}) = 6.4$, $(2d_{3/2}) = 7.9$, $(1g_{7/2}) = 8.4$, and $(1h_{11/2}) = 9.4$. This set of input SPEs is exactly the same as that employed in a number of successful shell model calculations in $A \sim 100$ nuclei by Vergados and Kuo [25] as well as by Federman and Pittel [26] except for a reduction of the $1g_{9/2}$ energy by about 1 MeV.

The two-body effective interactions that we have employed is of pairing-plus-quadrupole-quadrupole (PPQ) type [27]. The pairing part can be written as

TABLE II. Comparison of the experimental and calculated yrast levels using PPQO model of interaction in $^{108-114}\text{Pd}$ isotopes. Here $\langle Q_0^2 \rangle_\pi$, $\langle Q_2^2 \rangle_\pi$, $\langle Q_2^2 \rangle_\nu$ give the contribution of the protons (neutrons) to the components of quadrupole moment operator. The seventh column gives the value of quadrupole deformation parameter $(\beta_2)_{\text{Th}}$. The quadrupole moments have been calculated in units of b^2 , where $b = \sqrt{\hbar/m\omega}$ is the oscillator parameter.

| Nucleus | J^π | $\langle Q_0^2 \rangle_\pi$ | $\langle Q_2^2 \rangle_\pi$ | $\langle Q_2^2 \rangle_\nu$ | $\langle Q_2^2 \rangle_\nu$ | $(\beta_2)_{\text{Th}}$ |
|-------------------|---------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-------------------------|
| ^{108}Pd | 2^+ | 24.05 | 3.39 | 42.18 | 14.34 | 0.254 |
| | 4^+ | 24.04 | 3.34 | 42.21 | 13.99 | 0.254 |
| | 6^+ | 24.02 | 3.27 | 42.23 | 13.49 | 0.252 |
| | 8^+ | 24.99 | 3.18 | 42.22 | 12.88 | 0.251 |
| | 10^+ | 23.96 | 3.07 | 42.14 | 12.19 | 0.249 |
| | 12^+ | 23.90 | 2.93 | 42.00 | 11.44 | 0.247 |
| | 14^+ | 23.84 | 2.84 | 41.76 | 10.84 | 0.245 |
| ^{110}Pd | 2^+ | 24.10 | 3.77 | 39.05 | 16.30 | 0.239 |
| | 4^+ | 24.14 | 3.66 | 39.42 | 15.48 | 0.233 |
| | 6^+ | 24.18 | 3.55 | 39.79 | 14.65 | 0.237 |
| | 8^+ | 24.09 | 3.55 | 39.63 | 14.36 | 0.236 |
| | 10^+ | 24.07 | 3.46 | 39.71 | 13.60 | 0.234 |
| | 12^+ | 24.05 | 3.35 | 39.79 | 12.60 | 0.233 |
| ^{112}Pd | 2^+ | 23.84 | 2.35 | 44.68 | 8.65 | 0.210 |
| | 4^+ | 23.85 | 2.37 | 44.49 | 8.80 | 0.211 |
| | 6^+ | 23.86 | 2.40 | 44.16 | 9.07 | 0.212 |
| | 8^+ | 23.76 | 2.46 | 43.71 | 9.43 | 0.211 |
| | 10^+ | 23.64 | 2.52 | 43.15 | 9.82 | 0.211 |
| ^{114}Pd | 2^+ | 22.88 | 0.27 | 42.44 | 0.47 | 0.18 |
| | 4^+ | 22.86 | 0.30 | 42.26 | 0.58 | 0.18 |
| | 6^+ | 22.81 | 0.35 | 42.65 | 1.90 | 0.18 |
| | 8^+ | 22.78 | 0.46 | 41.57 | 1.08 | 0.18 |
| | 10^+ | 22.65 | 0.61 | 41.02 | 1.55 | 0.178 |
| | 12^+ | 22.50 | 0.76 | 40.22 | 2.06 | 0.177 |
| | 14^+ | 22.34 | 0.91 | 39.81 | 2.57 | 0.175 |
| | 16^+ | 21.87 | 2.14 | 37.16 | 8.13 | 0.176 |

$$V_p = (G/4) \sum_{ij} S_i S_j a_i \bar{a}_i \bar{a}_j a_j,$$

where i denotes the quantum numbers $(nljm)$. The state \bar{i} is same as i but with the sign of m reversed. Here S_i is the phase factor $(-1)^{j-m}$.

The q - q part of the interaction is given by

$$V_{qq} = \chi/2 \sum_{ijkl} \sum_\nu \langle i | q_\nu^2 | k \rangle \langle j | q_{-\nu}^2 | l \rangle (-1)^\nu a_i a_j a_l a_k,$$

where the operator q_ν^2 is given by

$$q_\nu^2 = (16\pi/5)^{1/2} r^2 Y_\nu^2(\theta, \phi).$$

The strengths for the like particle (nn) as well as neutron-proton (np) interaction were taken as

$$\chi_{nn} (= \chi_{pp}) = -0.0122 \text{ MeV } b^{-4}$$

$$\chi_{np} = -0.030 \text{ MeV } b^{-4}.$$

TABLE III. Comparison of experimental $B(E2;0^+ \rightarrow 2^+)$ values with the values calculated with PPQ interaction (i.e., corresponding to Th 1) and PPQO interaction (i.e., corresponding to Th 2) in $^{108-114}\text{Pd}$ isotopes. The value of the e_{eff} has been varied around the empirical value of Z/A . The $B(E2)$ values are in units of $e^2 b^2$.

| Nucleus | $B(E2;0^+ \rightarrow 2^+)$ | | | | | | |
|-------------------|-----------------------------|------|------|--------------------|------|------|----------|
| | Th 1 (PPQ) | | | Th 2 (PPQO) | | | (expt.) |
| | $e_{\text{eff}} =$ | | | $e_{\text{eff}} =$ | | | |
| 0.40 | 0.45 | 0.50 | 0.40 | 0.45 | 0.50 | | |
| ^{108}Pd | 1.13 | 1.27 | 1.42 | 0.60 | 0.68 | 0.77 | 0.76(40) |
| ^{110}Pd | 1.15 | 1.31 | 1.46 | 0.57 | 0.65 | 0.73 | 0.87(40) |
| ^{112}Pd | 1.14 | 1.28 | 1.43 | 0.63 | 0.72 | 0.81 | 0.63(10) |
| ^{114}Pd | 1.13 | 1.27 | 1.42 | 0.58 | 0.67 | 0.75 | 0.34(10) |

Here $b(= \sqrt{\hbar/m\omega})$ is the oscillator parameter.

These values for the strengths of the q - q interactions employed for Pd isotopes compare favorably with the ones employed in earlier axial calculations in mass $A \sim 100$ nuclei [28].

The strength for the pairing interaction was fixed through the approximate relation $G = 18 - 21/A$. The calculation of the energy states has been done by using cranked Hartree-Bogoliubov framework. This formalism has been discussed in detail by Goodman [29].

The parameters of the octupole-octupole term were calculated from a relation suggested by Bohr and Mottelson [30]. According to them, the approximate magnitude of these coupling constants for isospin $T=0$ is given by

$$\chi_\lambda = \left(\frac{4\pi}{2\lambda + 1} \right) \frac{M\omega_0^2}{A \langle r^{2\lambda-2} \rangle} \text{ for } \lambda = 0, 1, 2, \dots,$$

and the parameters for the $T=1$ case are approximately half the magnitude of their $T=0$ counterparts. The parameters of this term were, therefore, fixed as

$$\chi_{pp3}(= \chi_{nn3}) = -0.00366 \text{ MeV } b^{-6}$$

and

$$\chi_{np3} = -0.00690 \text{ MeV } b^{-6}.$$

TABLE IV. The values of occupation probabilities V_j^2 [normalized to $(2j+1)$] without the inclusion of octupole-octupole interaction (i.e., PPQ interaction).

| Nucleus | (a) for protons | | | | | | |
|-------------------|------------------|-----------|-----------|-----------|-----------|-----------|------------|
| | $s_{1/2}$ | $p_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ | $g_{7/2}$ | $g_{9/2}$ | $h_{11/2}$ |
| ^{108}Pd | 0.248 | 0.0 | 0.211 | 0.199 | 0.062 | 0.497 | 0.0 |
| ^{110}Pd | 0.247 | 0.0 | 0.213 | 0.201 | 0.064 | 0.494 | 0.0 |
| ^{112}Pd | 0.249 | 0.0 | 0.209 | 0.198 | 0.061 | 0.498 | 0.0 |
| ^{114}Pd | 0.252 | 0.0 | 0.207 | 0.196 | 0.060 | 0.501 | 0.0 |
| Nucleus | (b) for neutrons | | | | | | |
| | $s_{1/2}$ | $p_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ | $g_{7/2}$ | $g_{9/2}$ | $h_{11/2}$ |
| ^{108}Pd | 0.397 | 0.998 | 0.360 | 0.442 | 0.326 | 0.948 | 0.419 |
| ^{110}Pd | 0.394 | 1.0 | 0.356 | 0.530 | 0.367 | 0.970 | 0.498 |
| ^{112}Pd | 0.476 | 1.0 | 0.445 | 0.607 | 0.476 | 0.977 | 0.505 |
| ^{114}Pd | 0.50 | 1.0 | 0.470 | 0.663 | 0.601 | 0.986 | 0.539 |

III. RESULTS AND DISCUSSION

In Tables I and II, the results of CHB calculations, using PPQ and PPQO models of two-body interaction, respectively, on intrinsic quadrupole moments and quadrupole deformation parameter (β_2) are presented. The values for the two components of quadrupole moments $\langle Q_0^2 \rangle$ and $\langle Q_2^2 \rangle$ are presented separately for protons and neutrons. The values of quadrupole deformation parameter $(\beta_2)_{\text{Th}}$ are calculated from the values of intrinsic quadrupole moments by using the standard formulas suggested by Bohr [31]. The values of $(\beta_2)_{\text{Th}}$ with PPQ interaction turn out to be 0.301, 0.282, 0.261, 0.241, for $^{108-114}\text{Pd}$, respectively, and the values with the inclusion of octupole-octupole interaction (i.e., PPQO interaction) are 0.254, 0.239, 0.210, 0.18. The values suggested by Raman *et al.* [32] are 0.234(6), 0.257(6), 0.216(18), and 0.156(24) for $^{108-114}\text{Pd}$, respectively. The values of $(\beta_2)_{\text{Th}}$ calculated with PPQO interaction are in better agreement with the experimental values as suggested by Raman *et al.* [32].

Now we come to the discussion of intrinsic quadrupole moments $\langle Q_2^2 \rangle_\pi$ and $\langle Q_2^2 \rangle_\nu$. When we employ the PPQ interaction, these two values are seen to increase as we move up the yrast states for $^{108-114}\text{Pd}$. The values of these quantities are a measure of the degree of nonaxiality present in a nucleus. As their value is increasing with angular momen-

TABLE V. The values of occupation probabilities V_j^2 [normalized to $(2j+1)$] with the inclusion of octupole-octupole interaction (i.e., PPQO interaction).

| (a) for protons | | | | | | | |
|-------------------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| Nucleus | $s_{1/2}$ | $p_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ | $g_{7/2}$ | $g_{9/2}$ | $h_{11/2}$ |
| ^{108}Pd | 0.107 | 1.0 | 0.028 | 0.154 | 0.010 | 0.467 | 0.0 |
| ^{110}Pd | 0.117 | 1.0 | 0.030 | 0.157 | 0.010 | 0.462 | 0.0 |
| ^{112}Pd | 0.079 | 1.0 | 0.022 | 0.147 | 0.009 | 0.480 | 0.0 |
| ^{114}Pd | 0.041 | 1.0 | 0.011 | 0.131 | 0.007 | 0.503 | 0.0 |
| (b) for neutrons | | | | | | | |
| Nucleus | $s_{1/2}$ | $p_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ | $g_{7/2}$ | $g_{9/2}$ | $h_{11/2}$ |
| ^{108}Pd | 0.450 | 1.0 | 0.386 | 0.533 | 0.331 | 0.971 | 0.333 |
| ^{110}Pd | 0.568 | 1.0 | 0.396 | 0.678 | 0.429 | 0.979 | 0.333 |
| ^{112}Pd | 0.515 | 1.0 | 0.394 | 0.665 | 0.442 | 0.987 | 0.50 |
| ^{114}Pd | 0.574 | 1.0 | 0.435 | 0.805 | 0.538 | 0.997 | 0.50 |

tum, the degree of nonaxiality present in the nucleus increases as we move along the yrast states. Secondly, the values of $\langle Q_2^2 \rangle_\pi$ and $\langle Q_2^2 \rangle_\nu$ increase from $^{108-110}\text{Pd}$ and then decrease with neutron number, which means that the degree of nonaxiality first increases from $^{108-110}\text{Pd}$ and then decreases as we go from $^{110-114}\text{Pd}$ with neutron number.

These results are found to get modified when calculations are carried out with the inclusion of octupole-octupole interaction. Here values of $\langle Q_2^2 \rangle_{\pi,\nu}$ decrease with neutron number for $^{108-114}\text{Pd}$, so the degree of nonaxiality decreases with neutron number. Further the values of $\langle Q_2^2 \rangle_{\pi,\nu}$ decrease as we move along the yrast states for $^{108,110}\text{Pd}$ whereas its values increase for $^{112,114}\text{Pd}$ showing thereby a decrease in the degree of nonaxiality for $^{108,110}\text{Pd}$ and an increase in the degree of nonaxiality for $^{112,114}\text{Pd}$ as we move along the yrast states.

From the above discussion, it is seen that the CHB wave function with PPQO interaction reproduces the experimental data on quadrupole moments and β_2 values more satisfactorily than the CHB wave function obtained with PPQ interaction. Another test to check the reliability and accuracy of the CHB wave function is to determine how accurately the experimental values of $B(E2; 0^+ \rightarrow 2^+)$ transition probabilities are reproduced. In order to examine this, we have calculated the $B(E2; 0_1^+ \rightarrow 2_1^+)$ for $^{108-114}\text{Pd}$, both with PPQ and PPQO interactions. It has been shown that the intrinsic electric quadrupole moments are related to $B(E2; J_i^+ \rightarrow J_f^+)$ by

$$B(E2; J_i^+ \rightarrow J_f^+) = \left(\frac{5}{16\pi} \right) \left(\begin{matrix} J^+ & 2 & J^+ \\ 0^i & 0 & 0^f \end{matrix} \right)^2 \times (e_\pi \langle Q_0^2 \rangle_\pi + e_\nu \langle Q_0^2 \rangle_\nu)^2,$$

where e_π and e_ν are effective charges for protons and neutrons. The effective charges have been chosen such that $e_\pi = 1 + e_{\text{eff}}$ and $e_\nu = e_{\text{eff}}$. The values of e_{eff} have been fixed through an empirical relation $e_{\text{eff}} = Z/A$ as suggested by Motelson [33]. We have also examined the sensitivity of the $B(E2)$ results to small changes in the value of e_{eff} around the value of Z/A . The results presented in Table III, indicate that, with the inclusion of octupole-octupole term, a good agreement for the observed values of $B(E2; 0^+ \rightarrow 2^+)$ tran-

sition probabilities [32] has been obtained for the set of $^{108-114}\text{Pd}$ isotopes. For example, in ^{108}Pd the experimental value of $B(E2; 0^+ \rightarrow 2^+)$ is 0.760(40) and the value corresponding to Th 1 (i.e., with PPQ interaction) is 1.42 whereas the value corresponding to Th 2 (i.e., with PPQO interaction) is 0.77 when $e_{\text{eff}} = 0.50$. Similarly for ^{110}Pd , the experimental value is 0.870(40) and that corresponding to Th 1, is 1.46, whereas the value corresponding to Th 2, is 0.73, for the same value of effective charge.

From the low-lying energy systematics of Pd isotopes, what comes out is that these nuclei are quasideformed with $E_{4_1^+}/E_{2_1^+}$ values in the range of 2.50. We know that for a deformed nucleus $E_{4_1^+}/E_{2_1^+}$ value should be around 3.33 and its value for a spherical nucleus is 2.0. From the results presented in Tables I and II, we note that the calculated results for $\langle Q_0^2 \rangle_{\pi,\nu}$ for Pd isotopes without the inclusion of octupole interaction energy term are larger than those with the inclusion of octupole interaction energy term. Since the quadrupole moment values are directly proportional to the deformation parameter, this means that the calculations carried out without octupole interaction, predict Pd isotopes to be considerably more deformed than they actually are observed to be. The calculations performed with inclusion of octupole interaction are seen to produce much smaller values of $\langle Q_0^2 \rangle_{\pi,\nu}$ for Pd isotopes, meaning thereby that they are less deformed than predicted by calculations made without octupole interaction. Thus, the results obtained with inclusion of octupole interaction, are closer to experiments.

In order to understand the mechanism for the quasideformed nature and absence of emergence of large deformations in $^{108-114}\text{Pd}$ isotopes, it is important to critically examine the results on occupation probabilities in the light of the logic advocated by Sugita and Arima [34] for the build up of large deformations in the Zr and Mo isotopes. They suggested that one of the main factors responsible for producing large deformations in mass region $A \sim 100$ at neutron number $N > 60$ is the promotion of protons from $p_{1/2}$ orbit to $g_{9/2}$ orbit resulting in producing holes in $(p_{1/2})_\pi$ orbit and occupation of down sloping components of $(g_{9/2})$ Nilsson orbit.

In Tables IV and V, the results on occupation probabilities using PPQ and PPQO models of two body interaction,

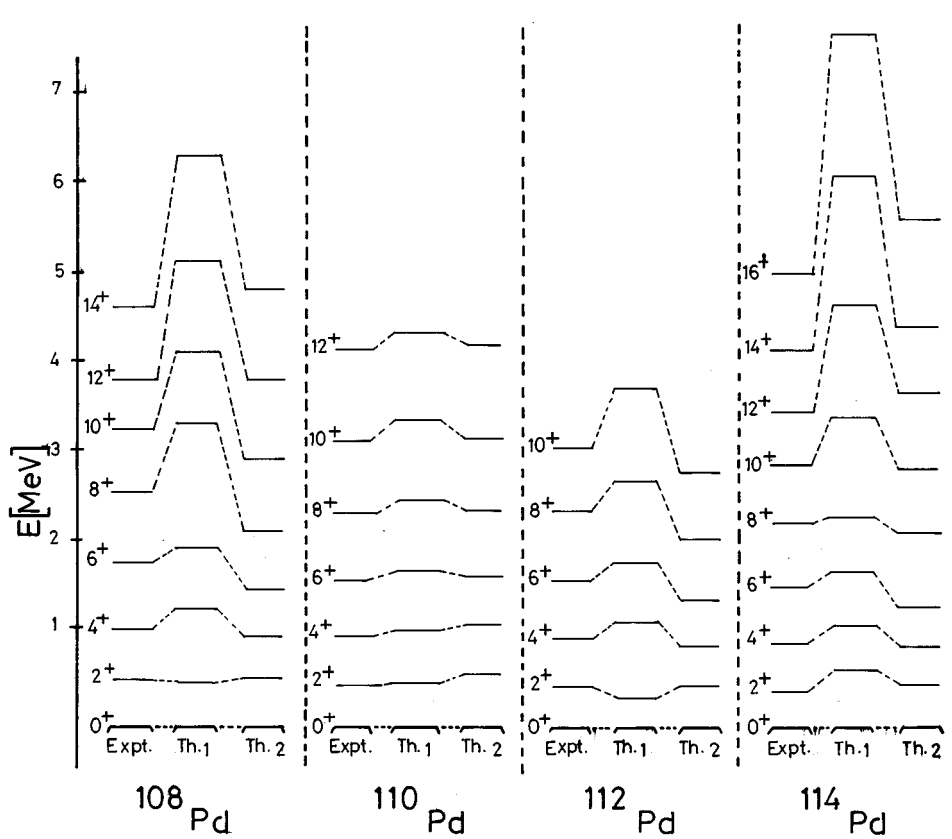


FIG. 1. Yrast spectra in $^{108-114}\text{Pd}$ isotopes. Th 1 and Th 2 are the calculated spectra corresponding to the PPQ and PPQO models of the two-body interaction, respectively. The experimental data is taken from Refs. [7,12,15,16,18,35].

respectively, are presented. From the occupation probabilities of protons presented in Table IV(a), it may be noted that there are two holes in $(p_{1/2})_{\pi}$ orbit and the $(g_{9/2})_{\pi}$ orbit is nearly half filled. In the light of the logic put forth by Sugita and Arima [34], this type of distribution of protons should predict large deformation for Pd isotopes which is contrary to what is observed experimentally. Now coming to the results of occupation probabilities of protons presented in Table V(a). It is found that $(p_{1/2})_{\pi}$ orbit is fully occupied in all the above mentioned Pd isotopes. This implies that the key factor responsible for producing large deformations in mass region $A \sim 100$ at $N > 60$ is absent in the distribution of proton occupation probabilities obtained with PPQO model of two body interaction, forbidding thereby the emergence of large deformations in these isotopes. Thus $(p_{1/2})_{\pi}$ subshell closure is responsible for the quasideformed nature of Pd isotopes.

The low-lying yrast spectra of $^{108-114}\text{Pd}$ isotopes has been displayed in Fig. 1. The spectra corresponding to Th 1 has

been obtained by using pairing plus quadrupole-quadrupole model (PPQ) for the two-body interaction whereas the spectra corresponding to Th 2 has been obtained by including a higher order octupole-octupole energy term in the two-body residual interaction (PPQO). It turns out from our calculations that corresponding to Th 2, the observed low-lying yrast spectra for $^{108-114}\text{Pd}$, respectively, is reasonably well reproduced. Thus, the inclusion of octupole-octupole interaction improves the agreement of calculated spectra with the experiments.

IV. CONCLUSIONS

From the results of our calculations, it turns out that the available experimental data can be explained reasonably well when the octupole-octupole interaction term is included in the two-body residual interaction. The quasideformed nature of $^{108-114}\text{Pd}$ is intricately connected with the $(p_{1/2})_{\pi}$ subshell closure or nonpolarizability of $Z=40$ core. The calculations predict a decrease of nonaxiality with neutron number.

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