Anomalous magnetic moment of quarks

Pedro J. de A. Bicudo, J. Emílio F. T. Ribeiro, and Rui Fernandes
Departamento de Física and Centro de Física das Interacções Fundamentais, Edifício Ciência, Instituto Superior Técnico, Avenida Rovisco Pais, 1096 Lisboa, Portugal

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In the case of massless current quarks we find that the breaking of chiral symmetry usually triggers the generation of an anomalous magnetic moment for the quarks. We show that the kernel of the Ward identity for the vector vertex yields an important contribution. We compute the anomalous magnetic moment in several quark models. The results show that it is hard to escape a measurable anomalous magnetic moment for the quarks in the case of spontaneous chiral symmetry breaking. [S0556-2813(99)00202-2]

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I. INTRODUCTION

Theoretically, the various hadronic electromagnetic form factors are usually described in terms of pole dominance together with contributions arising from virtual mesonic exchanges [1]. A third contribution to the electromagnetic form factors should come from the quark microscopic interaction itself, in close analogy with QED. It is clear that these three scenarios should not be independent but just three different aspects of the same model. This desideratum can be achieved, at least qualitatively, in terms of a quark field theory displaying spontaneous breaking of chiral symmetry (SχSB). In such a description any hadron, when seen from the trivial vacuum Fock space, appears as a collection of an infinite number of quark-antiquark pairs together with the appropriate valence quarks. It happens that the contributions of this quark sea can be summarized in terms of a new set of valence quasiquarks which now carry the information on the details of the physical vacuum through a modified propagator [2]. In this fashion we recover the simplicity of the constituent quark picture. It is the role of the Ward identities to ensure charge conservation throughout this process. And this they do at the expenses of the quark magnetic moment which, in general, becomes nonzero. As will be shown in this paper, to maintain, throughout the process of SχSB, a zero anomalous magnetic moment for the quarks constitutes the exception rather than the rule and is just the consequence of particular choices for the Lagrangian. However, the BCS diagonalization of the Hamiltonian (mass gap equation) does not preclude quark pair creation or annihilation processes from occurring. In fact it sets the strength of mesonic contributions for such physical processes as decay widths and meson-nucleon interactions [3], among others. The counterparts of these processes, when seen from the point of view of photon coupling, are precisely pole dominance and mesonic cloud contributions for the electromagnetic form factors. The objective of this paper is to set up the general formalism for the evaluation of electromagnetic form factors in the presence of SχSB and to use it to evaluate the u and d anomalous magnetic moments for various models.

In the Pauli notation for fermions with charge e_f, the electromagnetic current up to first order in the photon momentum q_ν is

$$j^\mu = e_f \bar{u}\Gamma^\mu u = e_f \bar{u}\left[\gamma^\mu + i\frac{\sigma^{\mu\nu}}{2M}q_\nu a_f\right]u,$$

$$\mu_f = \mu_0[e_f](1+a_f), \quad \mu_0[e_f] = \frac{e_f\hbar}{2Mc}, \tag{1}$$

where a_f stands for the anomalous part of the magnetic moment μ_f and M is the particle mass. Of course, as usual, for neutral charged fermions μ_f is given by $\mu_0[e]a_f$ where $-e$ is the electron charge. The magnetic moment of ground state hadrons is measured experimentally. For instance we have for the proton and neutron $a_p=1.79$ and $a_n=-1.91$. In the constituent quark model for light hadrons we have

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d), \quad \mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$
$$\Rightarrow \mu_u = 1.852\mu_{0p}, \quad \mu_d = -0.972\mu_{0p}, \tag{2}$$

and the quark magnetic moments are nearly proportional to the charges $e_u = \frac{2}{3}e$, $e_d = -\frac{1}{3}e$, which suggests that the gyromagnetic factor $2(1+a)$ is nearly flavor independent. The quantity which can be measured is $M/(1+a)$. For quark flavors u and d we have

$$M_u \simeq (1+a_u)338\ \text{MeV}, \quad M_d \simeq (1+a_d)322\ \text{MeV}. \tag{3}$$

The constituent quark model can be applied to fit the hadron spectrum, with a confining interaction, a hyperfine interaction, and a zero point energy [4]. The required parameters are of the order of $\alpha_s = 0.974$, $M_u \simeq M_d = 420$ MeV which would suggest a sizable a of the order of 0.15–0.3. It is also clear that we will need $a_d - a_u \simeq 0.05$ in order to recover the isospin symmetry.

The remainder of the paper is organized as follows. In Sec. II we develop the full electromagnetic vertex Γ_μ in SχSB theories, in Sec. III we apply the formalism to examples of quark models, and in Sec. IV we conclude.

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II. FULL ELECTROMAGNETIC VERTEX Γ_μ IN SχSB THEORIES

The Ward identity

$$
\begin{aligned}
& i q_\mu S(p+q/2)\Gamma^\mu S(p-q/2)\\
&\quad = S(p+q/2) - S(p-q/2) \Leftrightarrow q_\mu \Gamma^\mu\\
&\quad = iS^{-1}(p+q/2) - iS^{-1}(p-q/2)
\end{aligned}
\tag{4}
$$

is obeyed both by the bare vertex Γ_0^μ and by the Bethe-Salpeter vertex Γ^μ [5]. We will show that in the limit of small momentum q, this identity has the following solution for the vertex:

$$
\Gamma^\mu(p,q) = i\frac{\partial}{\partial p_\mu}S^{-1}(p) + q_\nu \mathcal{T}^{\nu\mu}(p) + o(q^2), \tag{5}
$$

where $q_\nu \mathcal{T}^{\nu\mu}(p)$ is defined as the kernel which is not determined by the Ward identity,

$$
q_\mu[q_\nu \mathcal{T}^{\nu\mu}(p)] = 0. \tag{6}
$$

The Ward identity ensures that charge conservation survives renormalization. However, it does not constrain the kernel, which is a signature of the renormalization. In particular the kernel contributes to the anomalous magnetic moment of fermions.

This can clearly be seen in QED where the infrared and ultraviolet divergences can be removed from the photon propagator,

$$
\frac{i}{(p'-p)^2} \to \frac{i}{(p'-p)^2-\lambda^2} - \frac{i}{(p'-p)^2-\Lambda^2}. \tag{7}
$$

The vertex is given by

$$
\Gamma^\mu = \Gamma_o^\mu - i\partial^\mu\Sigma + q_\nu \mathcal{T}_o^{\nu\mu} \tag{8}
$$

and, up to first order in α, the contributions from the self-energy and the kernel to the anomalous magnetic moment are, respectively,

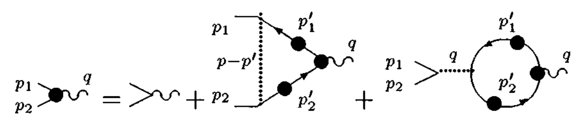
$$
\left(-1-2\ln\frac{\lambda}{M}\right)\frac{\alpha}{2\pi}, \quad \left(2+2\ln\frac{\lambda}{M}\right)\frac{\alpha}{2\pi}. \tag{9}
$$

In the case of actual QED, where $\lambda\to 0, \Lambda\to\infty$, they are both infrared divergent but their sum is finite: $\alpha/2\pi$.

As for QCD, there has been a considerable effort on how to derive quark models by integrating out, under various approximations, the gluonic degrees of freedom. An interesting and promising approach is provided by the cumulant expansion of the interaction term of the QCD Lagrangian [6]. A nonlocal Nambu–Jona-Lasinio- (NJL-) type Lagrangian is obtained when we retain only bilocal correlators. This is essentially the same approximation as was used by Cahill and Roberts [7]. In this approximation one neglects triple and higher order gluon vertices involving quarks but considers full gluon propagators. A recent work along these lines on the spectroscopy of heavy-light quark mesonic sector has just been completed [8]. For those physical processes where the hadronic size can be put to zero this approximation should be the same as large N_c [9].

Therefore we hold the view that such quark models are appropriate to study electromagnetic properties of hadrons, even for light quarks, provided we have small enough photon momenta and the physics of chiral symmetry breaking is treated correctly. Therefore, at this stage, rather than focusing on a specific example of the NJL Lagrangian, we will study the static electromagnetic properties of a wide class of quark effective quartic interactions.

In quark models with dynamical SχSB, the vector vertex Γ^μ is a solution of the Bethe-Salpeter equation



$$
p_1 = p + \frac{q}{2},\ p_2 = p - \frac{q}{2},\ p_1' = p' + \frac{q}{2},\ p_2' = p' - \frac{q}{2} \tag{10}
$$

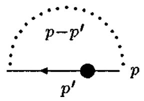
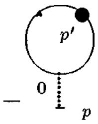
where the strong interaction, which is described by a dotted line in the diagrams, is iterated to all orders in the Bethe-Salpeter equation. As usual the solid circles represent the full vertex and quark propagator. This equation can be written

$$
\begin{aligned}
\Gamma^\mu(p,q) = \Gamma_0^\mu - i\int \frac{d^4p'}{(2\pi)^4} & V(p'-p,p'+p,q)\Omega_a S(p_1')\\
& \times \Gamma^\mu\left(p',\frac{q}{2}\right) S(p_2')\Omega_a\\
& - V(q,p'+p,-p'+p)\Omega_a\\
& \times \mathrm{tr}\{S(p_1')\Gamma^\mu(p',q)S(p_2')\Omega_a\},
\end{aligned}
\tag{11}
$$

where the -1 factor from the fermion loop was included in the tadpole term. The momentum dependence of the potential is only assumed to conserve the total momentum, and in this case it depends on three-momenta. The Dirac, flavor, and color structure of the interaction is determined by the Ω_a matrices. In order to have dynamical SχSB, we require this structure to be chiral invariant. Substituting the Ward identity in the ladder Bethe-Salpeter equation for the vertex we get

$$
\begin{aligned}
& iS^{-1}(p_1) - iS^{-1}(p_2)\\
&\quad = iS_0^{-1}(p_1) - iS_0^{-1}(p_2)\\
&\qquad - \int \frac{d^4p'}{(2\pi)^4} V(p'-p,p'+p,q)\Omega_a[S(p_1')-S(p_2')]\Omega_a\\
&\qquad - V(q,p'+p,-p'+p)\Omega_a \mathrm{tr}\{[S(p_1')-S(p_2')]\Omega_a\}.
\end{aligned}
\tag{12}
$$

For particular cases of the potential $V(p_1'-p_1, p_1'+p_2, p_1'-p_2')$ we recover the BCS mass gap equation

$$\mathcal{S}^{-1}(p) = \mathcal{S}_0^{-1}(p) \; - $$  $$-$$  (13)

provided that either the rainbow diagram vanishes, or if it does not, then we must have

$$V(p'-p,p'+p,q)=V(p'-p,p'+p,0), \tag{14}$$

and that either the tadpole diagram vanishes, or if it does not, then we must have

$$V(q,p'+p,-p'+p)=V(0,p'+p,-p'+p). \tag{15}$$

Equation (13) can be written

$$iS^{-1}(p)=iS_0^{-1}(p) - \int \frac{d^4p'}{(2\pi)^4} V(p'-p,p'+p,0)\Omega_a S(p')\Omega_a - V(0,p'+p,-p'+p)\Omega_a \mathrm{tr}\{S(p')\Omega_a\}. \tag{16}$$

Now we insert the expression (5) for Γ^μ in the Bethe-Salpeter equation (11), in order to find the kernel $q\mathcal{T}$ and expand it up to first order in q. The equation for the tensor $\mathcal{T}$, which is antisymmetric, is then

$$\mathcal{T}^{\nu\mu}=\mathcal{T}_0^{\nu\mu}-i\int \frac{d^4p}{(2\pi)^4} V\Omega_a(1-\mathrm{tr})\{S\mathcal{T}^{\nu\mu}S\Omega_a\},$$

$$\mathcal{T}_0^{\nu\mu}=-\frac{1}{2}\int \frac{d^4p}{(2\pi)^4} V\Omega_a(1-\mathrm{tr})\{\mathcal{J}^{\nu\mu}\Omega_a\},$$

$$\mathcal{J}^{\nu\mu}=\partial^\nu(S)S^{-1}\partial^\mu(S)-\partial^\mu(S)S^{-1}\partial^\nu(S). \tag{17}$$

This is a self-consistent forced linear integral equation. Let us consider a general quark propagator, solution of the mass gap equation, of the form

$$S(p^\mu)=\frac{iF(p)}{\not{p}-M(p)}, \tag{18}$$

where $p=\sqrt{p^\mu p_\mu}$. The integrand $\mathcal{J}^{\nu\mu}$ is, then,

$$\mathcal{J}^{\nu\mu}=-i\frac{F}{(p^2-M^2)^2}\left[\frac{1}{2}\{\not{p},[\gamma^\nu,\gamma^\mu]\}+M[\gamma^\nu,\gamma^\mu] - \frac{\dot{M}}{p}(p^\nu[\not{p},\gamma^\mu]-p^\mu[\not{p},\gamma^\nu])\right], \tag{19}$$

where the overdot denotes d/dp. In general, we find

$$\mathcal{T}^{\nu\mu}=t_1(p)\{\not{p},[\gamma^\nu,\gamma^\mu]\}+t_2(p)M[\gamma^\nu,\gamma^\mu]+t_3(p) \times(p^\nu[\not{p},\gamma^\mu]-p^\mu[\not{p},\gamma^\nu]). \tag{20}$$

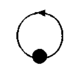
Up to $o(q^2)$ the electromagnetic current of the quark is, then,

$$j^\mu=\frac{e_f}{F}\bar{u}\left[\gamma^\mu-\frac{p^\mu}{p}\dot{M}-(\not{p}-M)\frac{p^\mu}{p}\frac{\dot{F}}{F}+Fq^\nu\mathcal{T}^{\nu\mu}\right]u$$

$$=\frac{e_f(1-\dot{M})}{F}\bar{u}\left[\gamma^\mu+a\left(i\frac{\sigma^{\mu\nu}}{2M}q_\nu\right)\right]u,$$

$$a=\frac{\dot{M}+4M^2F(2t_1+t_2)}{1-\dot{M}}, \tag{21}$$

where the mass shell condition $p=M$ was used together with the Gordon identities. The anomalous magnetic moment a turns out to be independent of t_3 and $\dot{F}$. However, the dependence on M is crucial in models where t_1 and t_2 are finite. In those models a can be thought as a measure of SχSB. The quark condensate $\langle\bar{q}q\rangle$ is also a functional of the dynamically generated mass,

$$\langle\bar{q}q\rangle = -$$  $$= -n_c\, tr\int\frac{d^4p}{(2\pi)^4}S(p) \tag{22}$$

where the trace sums colors with $n_c=3$, but the flavor is kept fixed. Thus, at the onset of the spontaneous χSB, we will obtain an implicit relation between a, $\langle\bar{q}q\rangle$, and the constituent quark mass, which were simultaneously vanishing before the occurrence of this phase transition and now become nonzero.

III. APPLICATION TO QUARK MODELS

We will now compute F, M, a, and $\langle\bar{q}q\rangle$ in particular models which are paradigmatic cases of chiral symmetry breaking and comply with the constraints of the Ward identity. The first two models, model I and model II, are simple models just used to introduce and exemplify the application of the formalism and to visualize properties of the main generic contributions to the quark anomalous magnetic moment. Their results are unphysical. Finally we will consider in model III a more elaborate case, already containing some physical features to be expected from QCD.

A. Simple models

Model I is the first original NJL model [10]. The Lagrangian of model I is

$$\mathcal{L}_{\mathrm{I}}=\bar{q}i\not{\partial}q+G[(\bar{q}q)^2-(\bar{q}\gamma_5q)^2], \tag{23}$$

where $\mathcal{L}_{\mathrm{I}}$ is specific to the case of one flavor, but its results are similar to the ones of flavor-symmetric $\mathrm{U}_A(n_f)$ extended NJL models. The equations will be solved in the momentum representation. As usual the integrals are done in Euclidean space. A momentum cutoff Λ is included in order that the integral in the loop momentum is finite. Since the cutoff cannot be ascribed to the potential which has to be constant in momentum space, it must be included in the propagator,

$$S(p)=\frac{iF(p)}{\not p-M+i\epsilon}, \quad F(p)\to\Theta_{\text{Euclidean}}(\Lambda-p). \tag{24}$$

With a constant potential and this momentum cutoff, the loops turn out to be constant, independent of the external momentum p. It is convenient to evaluate the integrals,

$$I_1=-\int\frac{d^4p}{(2\pi)^4}\frac{iF}{(p^2-M^2)}=\frac{1}{16\pi^2}\left[\Lambda^2-M^2\ln\left(1+\frac{\Lambda^2}{M^2}\right)\right],$$

$$I_2=i\int\frac{d^4p}{(2\pi)^4}\frac{F}{(p^2-M^2)^2}$$
$$=\frac{1}{16\pi^2}\left[-\frac{\Lambda^2}{\Lambda^2+M^2}+\ln\left(1+\frac{\Lambda^2}{M^2}\right)\right], \tag{25}$$

where the solid angle $2\pi^2$ is included. The mass gap equation is

$$\not p-M=\not p-2G\int\frac{d^4p}{(2\pi)^4}\frac{iF}{(p^2-M^2)}\times[(\not p+M)-\gamma_5(\not p+M)\gamma_5-\text{tr}\{\not p+M\}], \tag{26}$$

with the solutions $M=0$ or $1=8n_cGI_1(M,\Lambda)$. The parameters Λ and G are determined once the quark dynamical mass and the quark condensate are fixed. We now study the kernel in model I. Because the integrals are constant, the antisymmetric tensor $\mathcal{T}$ is independent of p. Thus $\mathcal{T}^{\nu\mu}$ has to be of the t_2 type, proportional to $[\gamma^\nu, \gamma^\mu]$. Including the structure factors Ω_a we find that the tadpolelike term vanishes since $\sigma^{\nu\mu}$ and $\sigma^{\nu\mu}\gamma_5$ have a null trace. In this case of model I the rainbow diagram also cancels since the structure $1\otimes 1-\gamma_5\otimes\gamma_5$ projects on the terms with an odd number of Dirac γ matrices, of type t_1 but $[\gamma^\nu, \gamma^\mu]$ is even. Thus model I produces no kernel for the vector vertex and no anomalous magnetic moment for the quark [11].

Model II is the second original NJL model [12]. The Lagrangian is

$$\mathcal{L}_{\text{II}}=\bar{q}i\not\partial q+G[(\bar{q}q)^2-(\bar{q}\gamma_5\vec{\tau}q)^2], \tag{27}$$

where $\mathcal{L}_{\text{II}}$ is used for two flavors u and d. It only has an $\text{SU}(2)_A$ symmetry and breaks $\text{U}(1)_A$ from the onset. Its results are similar to those of flavor-symmetric $\text{SU}_A(n_f)$ extended NJL models. The anzats for the propagator is that of Eq. (18), and model II only differs from model I in the algebra. The mass gap equation is changed since $\vec{\tau}\cdot\vec{\tau}=3$ in the fermion line. We get

$$\not p-M=\not p-2G\int\frac{d^4p}{(2\pi)^4}\frac{iF}{(p^2-M^2)}\times\left[(\not p+M)-2\frac{n_f^2-1}{n_f}\gamma_5(\not p+M)\gamma_5-\text{tr}\{\not p+M\}\right]\Rightarrow M=0 \quad \text{or}$$

$$1=2\left(2\frac{n_f^2-1}{n_f}-1+4n_fn_c\right)GI_1(M,\Lambda), \tag{28}$$

where n_f and n_c stand, respectively, for the number of flavors and colors. The rainbow diagram contributes in this case. The tadpole diagram contribution also changes. The preferred values for the parameters Λ and G are $\Lambda=1.65$ GeV and $G=1.23\ \text{GeV}^{-2}$ which yield $M=0.33$ GeV, $\langle\bar{q}q\rangle=-(0.25\ \text{GeV})^3$, and $f_\pi=0.09$ GeV. As in model I, the tadpole diagram will not contribute to the antisymmetric tensor which will be again of the t_2 type, $\mathcal{T}^{\nu\mu}=t_2[\gamma^\nu,\gamma^\mu]$. The first order term is a function of

$$\int\frac{d^4p}{(2\pi)^4}\mathcal{J}^{\nu\mu}=-I_2M[\gamma^\nu,\gamma^\mu]. \tag{29}$$

In order to evaluate the higher order terms, we calculate

$$\int\frac{d^4p}{(2\pi)^4}SM[\gamma^\nu,\gamma^\mu]S=-iM^2I_2q_\nu M[\gamma^\nu,\gamma^\mu]. \tag{30}$$

In this case we have two flavors with two different charges $e_u=\frac{2}{3}$, $e_d=-\frac{1}{3}$, and two anomalous magnetic moments a_f,

$$e_ut_u=GI_2\left[0\left(\frac{e_u}{2}-M^2e_ut_u\right)+(-2)\left(\frac{e_d}{2}-M^2e_dt_d\right)\right]$$
$$(u\leftrightarrow d). \tag{31}$$

The natural parameter is $2GM^2I_2(\Lambda,M)=0.004$. Inverting this equation we find the solution

$$a_u\simeq-2(2GM^2I_2)\frac{e_d}{e_u}=0.004 \quad\Rightarrow\quad M_u=339\ \text{MeV},$$

$$a_d\simeq-2(2GM^2I_2)\frac{e_u}{e_d}=0.016 \quad\Rightarrow\quad M_d=327\ \text{MeV}. \tag{32}$$

Although this effect is small, it has the right sign to correct the M_u and M_d inversion. If the tadpole term was removed from the mass gap equation, then the a_d-a_u would be bigger. This is possible, for instance, when the potential has a $\vec{\lambda}\cdot\vec{\lambda}$ dependence, λ being the Gell-Mann matrices. This will now be considered.

B. QCD-inspired nonlocal NJL model

Model III is the simplest QCD-inspired model. The Lagrangian is

$$\mathcal{L}_{\text{III}}(x)=\bar{q}(x)i\not\partial q(x)+\frac{1}{2}\bar{q}(x)\gamma^\alpha\frac{\vec{\lambda}}{2}q(x)\times\int d^4yV(x-y)\bar{q}(y)\gamma_\alpha\frac{\vec{\lambda}}{2}q(y). \tag{33}$$

In the case of model III, the Dirac structure $\gamma^\mu \otimes \gamma_\mu$ is $U_A(n_f)$ chiral invariant. For $V(p)$ we will choose a color confining square well potential because of its calculational simplicity:

$$V(p'-p) = -G\Theta_{\text{Euclidean}}(\Lambda - |p'-p|). \tag{34}$$

The mass gap equation is

$$\frac{\not p - M}{F} = \not p - \frac{4i}{3}\int \frac{d^4p'}{(2\pi)^4} V(p'-p) F \frac{-2\not p' + 4M}{p'^2 - M^2}, \tag{35}$$

which includes the color factor of $\frac{4}{3}$. We now calculate the kernel. The first order term for the kernel is a functional of

$$\gamma^\alpha \mathcal{J}^{\nu\mu}\gamma_\alpha = iF\frac{\{\not p, [\gamma^\nu, \gamma^\mu]\}}{(p^2-M^2)^2}. \tag{36}$$

The terms with two gamma matrices, of the form $\sigma^{\mu\nu}$, are now canceled by the $\gamma^\alpha \otimes \gamma_\alpha$ of the interaction, and only the t_1-type term remains. Therefore this model differs from the previous ones insofar it covers the form factor t_1. The self-consistent equation for the antisymmetric tensor $\mathcal{T}$ will also close,

$$\gamma^\alpha S\{\not p, [\gamma^\nu, \gamma^\mu]\} S\gamma_\alpha = \frac{F^2(p^2+M^2)}{\frac{1}{2}(p^2-M^2)^2}\{\not p, [\gamma^\nu, \gamma^\mu]\}. \tag{37}$$

We get

$$\mathcal{T}^{\nu\mu} = t\{\not p, [\gamma^\nu, \gamma^\mu]\}$$

$$t(p) = t_0 - \frac{8}{3}i\int \frac{d^4p'}{(2\pi)^4} V(p'-p)\frac{p'\cdot p}{p^2} \times F^2(p')\frac{p'^2+M^2}{(p'^2-M^2)^2} t(p'),$$

$$t_0(p) = -\frac{2}{3}i\int \frac{d^4p'}{(2\pi)^4} V(p'-p)\frac{p'\cdot p}{p^2}\frac{F(p')}{(p'^2-M^2)^2}. \tag{38}$$

For the Euclidean integration it is convenient to evaluate the angular integrals,

$$I_3(p',p) = G\int_{-1}^{+1} dw\, \theta(\Lambda - \sqrt{p'^2+p^2-2wp'p}) = G(1+I_6)\theta(1-I_6)\theta(1+I_6) + 2G\theta(I_6-1) > 0,$$

$$I_4(p',p) = G\int_{-1}^{+1} dw\, w\, \theta(\Lambda - \sqrt{p'^2+p^2-2wp'p}) = G\frac{1-I_6^2}{2}\theta(1-I_6)\theta(1+I_6) > 0,$$

$$I_5 = \frac{\Lambda^2 - p'^2 - p^2}{2p'p}, \tag{39}$$

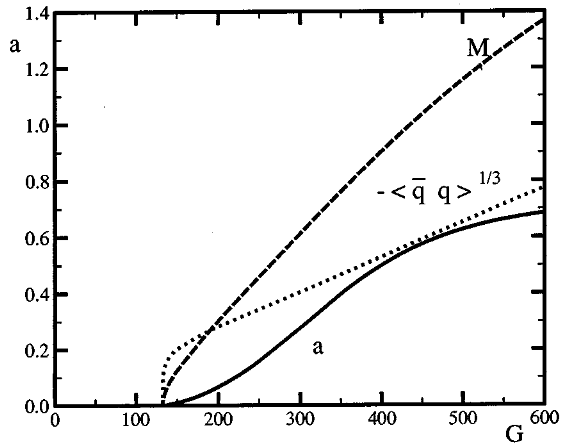


FIG. 1. The anomalous magnetic moment a (solid line) as a function of the adimensional coupling G for model III. The adimensional (in units of Λ) quantities, quark dynamical mass M (dashed line) and quark condensate (dotted line), are also shown.

and the nonlinear integral mass gap equation for F and M, the integral for t_0, the linear integral equation for T, and the integral for $\langle \bar q q\rangle$ can be solved simultaneously:

$$F(p) = \left[1 + \int_0^\infty dp' \frac{I_5(p',p)}{6\pi^2 p}\frac{p'^4}{p'^2+M^2(p')}F(p')\right]^{-1},$$

$$M(p) = F(p)\int_0^\infty dp' \frac{I_4(p',p)}{3\pi^2}\frac{p'^3F(p')}{p'^2+M^2(p')}M(p'),$$

$$t_0(p) = \int_0^\infty dp' \frac{I_5(p',p)}{24\pi^2 p}\frac{p'^4F(p')}{[p'^2+M^2(p')]^2},$$

$$t(p) = t_0 - \int dp' \frac{I_5(p',p)}{6\pi^2 p} \times \frac{p'^4F(p')^2[p'^2-M^2(p')]}{[p'^2+M^2(p')]^2} t(p'),$$

$$\langle \bar q q\rangle = -\int_0^\infty dp \frac{3}{2\pi^2}\frac{p^3F(p)M(p)}{p^2+M^2(p)}. \tag{40}$$

The mass term has a trivial solution $M=0$ and another solution which breaks spontaneously chiral symmetry. A dimensional simplification occurs if we work in units of $\Lambda=1$. In this case the only parameter is G which is now adimensional. We find a critical value $G_c=132$ above which chiral symmetry occurs. In Fig. 1 we depict the values of M, $\langle \bar q q\rangle$, and a. We solve the integral equations numerically for F, M, and t with the Gauss iterative method and using the Gauss integration [13]. We find that at $p^2=-1$ these functions decrease by a factor of just $0.9 \to 0.7$. Since we cannot continue analytically the numerical solution, we use the approximation of nearly constant F, M, and t and compute the mass and the anomalous magnetic moment for $p=0$. The literature prefers a $\langle \bar q q\rangle = -(0.25\ \text{GeV})^3$. A dynamical quark mass $M=0.33$ GeV would correspond to $G=245\Lambda^{-2}$, $\Lambda=0.74$ GeV, and $a=0.15$. If we now consider a $M=(1+a)0.33$ GeV, then the lowest possible condensate is $\langle \bar{q}q\rangle=-(0.28\ \text{GeV})^3$ which corresponds to $G=300\Lambda^{-2}$, $\Lambda=0.69$ GeV, $M=0.42$ GeV, and $a=0.28$; see Fig. 1.

IV. CONCLUSIONS

NJL models I and II are the simplest models with chiral symmetry breaking. In NJL model I the anomalous magnetic moment a vanishes. In model II the U(1) breaking interaction yields a too small a, which nevertheless provides an example of an isospin dependence for a and, therefore, contributes to the $u-d$ mass inversion. The reason for the smallness of the anomalous magnetic moment stems from the presence of tadpole contributions, and were it not for this contribution, we would have obtained a much larger a. The tadpole contribution to M, if allowed, turns out to be very large due to a correspondingly large degeneracy factor (g=three colors $\times$ flavors $\times$ two spins). This is a general feature and not model dependent. This lowers the value of a. In models where this contribution is absent (due, for instance, to the traceless Gell-Mann matrices) it is possible to have a significant a. This is precisely the case of model III where a larger a is derived, compatible with the nonrelativistic constituent quark models. We also find that M, a, and $\langle \bar{q}q\rangle$ are functions of $(G-G_c)$ with critical exponents which are, respectively, 1, 2, and 1. The results for a obtained in these three models hint at a strong dependence of the quark anomalous magnetic moment on the details of the interaction and therefore calculation of the quark anomalous magnetic moment should constitute another stringent test for the realistic effective models of hadronic processes. The present work constitutes a first step in a more elaborate model unifying hadronic spectroscopy (including decay widths) and the electromagnetic form factors which have been shown to be consistent with the simple quark constituent picture precisely because of SχSB.

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