Sum rule for the backward spin polarizability of the nucleon from a backward dispersion relation

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A new sum rule for γ_π, the backward spin polarizability of the nucleon, is derived from a backward-angle dispersion relation. Taking into account single-pion and multipion photoproduction in the s channel up to the energy $\omega_{max} = 1.5$ GeV and resonances in the t channel with mass below 1.5 GeV, it is found for the proton and neutron that $[\gamma_\pi]_p = -39.5 \pm 2.4$ and $[\gamma_\pi]_n = 52.5 \pm 2.4$, respectively, in units of 10^{-4} fm^4. [S0556-2813(99)02202-5]

I. INTRODUCTION

With recent progress in developing and using effective field theories, a practical knowledge of various low-energy parameters of hadrons and their interactions is of high current interest. Among such parameters are γ_i, the four spin polarizabilities of the nucleon, which characterize the spin-dependent response of the internal degrees of freedom to external soft electromagnetic fields [1–5]. Two linear combinations of γ_i have an especially transparent meaning. These are the forward and backward spin polarizabilities γ and γ_π, which are defined as the coefficients of $i\omega^3 \boldsymbol{\sigma} \cdot \boldsymbol{e}'^* \times \boldsymbol{e}$ in the structure-dependent (non-Born) part of the low-energy forward or backward Compton scattering amplitude. From the Gell-Mann–Goldberger–Thirring dispersion relation, one can predict the forward spin polarizability γ through the total photoabsorption cross sections with polarized beam and target with total helicities 1/2 and 3/2 [6]:

$$\gamma = \int_{\omega_0}^{\infty} (\sigma_{1/2} - \sigma_{3/2}) \frac{d\omega}{4\pi^2 \omega^3}. \tag{1}$$

Using more complicated dispersion relations, two other γ's can also be found [4,5]. However, this approach is not sufficient to predict the backward spin polarizability which is particularly sensitive to that part of the high-energy behavior of the spin-dependent Compton scattering amplitude driven by the invariant amplitude A_2 [5,7]. A dispersion estimate for γ_π obtained in Refs. [4,5] using an unsubtracted fixed-t dispersion relation for A_2 was based on the strong assumption that the high-energy asymptotics of A_2 are entirely determined by π^0 exchange [7], whereas other possible exchanges with heavier mesons or few-pion states are negligible. Although not well justified, this estimate still gives a result close to that obtained in the framework of leading order chiral perturbation theory (ChPT), with the Δ isobar included through the small-energy-scale expansion [3]. In particular, it was found for the proton

$$[\gamma_\pi]_p = -36.7\,[3],\ -34.3\,[4],\ -37.2\,[5] \tag{2}$$

(hereafter the units used for γ's are 10^{-4} fm^4). In all these calculations, the magnitude of γ_π is dominated by the contribution to nucleon Compton scattering from the t-channel π^0 exchange which yields

$$\gamma_\pi^{(\pi^0)} = \frac{g_{\pi NN}(0) F_{\pi\gamma\gamma}(0)}{2\pi m_{\pi^0}^2 m} \tau_3 = (-45.0 \pm 1.6)\tau_3, \tag{3}$$

where m is the nucleon mass and τ_3 is equal to 1 or -1 for the proton or neutron, respectively. Experimental $\pi^0\gamma\gamma$ and πNN couplings extrapolated to $t=0$ were taken to obtain the numerical value of $\gamma_\pi^{(\pi^0)}$. In the framework of ChPT, $\gamma_\pi^{(\pi^0)} = -e^2 g_A \tau_3 / (8\pi^3 f_\pi^2 m_{\pi^0}^2) = -45.3\tau_3$.

Recently, these theoretical predictions have been challenged by the first experimental estimate of the backward spin polarizability of the proton [8] which was (indirectly) obtained from a simultaneous analysis of pion photoproduction and unpolarized Compton scattering data:

$$[\gamma_\pi]_p = -27.1 \pm 3.4, \tag{4}$$

where statistical, systematic, and model-dependent errors were added in quadrature. Isolating the well-defined contribution (3) with errors already included in Eq. (4), one can infer the non-π^0 part of the backward spin polarizability which is of the most theoretical interest:

$$[\gamma_\pi^{(\text{non-}\pi^0)}]_p = 17.9 \pm 3.4. \tag{5}$$

On the other hand, all the cited theoretical approaches yield roughly one half of Eq. (5), with $\simeq +4$ coming from non-resonant pion production and another $\simeq +4$ from the Δ-resonance excitation. Therefore, the experimental finding (5) suggests that there is another missing and very large positive contribution to γ_π. In the framework of ChPT, this missing contribution might be due to next-to-leading-order effects, especially in the related counterterms. In the framework of fixed-t dispersion relations, the missing contribution could be sought in the high-energy part of the poorly con-

vergent dispersion relation for the amplitude A_2 or in heavy-meson exchanges which might contribute to $A_2(s,t)$ at large s and $t=0$.

Neither an extension of the ChPT calculations to higher orders nor a more exact treatment of the high-energy behavior of A_2 offer much promise to resolve this issue, since both approaches are technically difficult and suffer from badly controlled uncertainties. Therefore we develop here a different approach based on a backward-angle dispersion relation which is manifestly free from the convergence problem. This approach is very similar to that proposed for determining the difference of dipole electric and magnetic polarizabilities of the nucleon [9]. We derive a sum rule for γ_π with well-defined s- and t-channel contributions and then use it to predict the backward spin polarizability of the nucleon.

II. SUM RULE

We start by recalling the form of dispersion relations for nucleon Compton scattering amplitudes at the fixed scattering angle $\theta = \pi$ [9,10]. At this specific angle, the corresponding Mandelstam variables obey two constraints $s+u+t = 2m^2$ and $su=m^4$, and the structure of physical cuts in the complex plane of s is particularly simple. When $\theta = \pi$, we have for any cross-even analytical function $A(s,u,t) = A(u,s,t)$ which vanishes at high s and has singularities only at physical thresholds in s, u, or t channels:

$$\text{Re}\,A(s,u,t) = A^{\text{pole}}(s,u,t)$$
$$+ \frac{1}{\pi} \text{P} \int_{s_0}^\infty \left(\frac{1}{s'-s} + \frac{1}{s'-u} - \frac{1}{s'} \right)$$
$$\times \text{Im}_s A(s',u',t')\, ds'$$
$$+ \frac{1}{\pi} \text{P} \int_{t_0}^\infty \text{Im}_t A(s',u',t') \frac{dt'}{t'-t}, \qquad (6)$$

where $s'+u'+t'=2m^2$ and $s'u'=m^4$. Im_s and Im_t denote the imaginary parts of the amplitude in the s and t channels, which start at the thresholds $s_0=(m+m_\pi)^2$ and $t_0=m_\pi^2$, respectively.

We apply the backward dispersion relation (6) to the function

$$A \equiv A_2 + \left(1 - \frac{t}{4m^2} \right) A_5, \qquad (7)$$

where $A_i(s,u,t)$ are cross-even invariant amplitudes of Compton scattering [5,7] free from kinematical singularities and constraints. The function A determines the spin-dependent part of the Compton scattering amplitude T_{fi} in the backward kinematics, which has the following form in the lab frame [5]:

$$\frac{1}{8\pi m}[T_{fi}]_{\theta=\pi} = -\frac{\omega\omega'}{2\pi} \sqrt{1-\frac{t}{4m^2}} e'^* \cdot e \left(A_1 - \frac{t}{4m^2} A_5 \right)$$
$$- \frac{\omega\omega'}{2\pi m} \sqrt{\omega\omega'}\, i\boldsymbol{\sigma} \cdot e'^* \times e$$
$$\times \left[A_2 + \left(1 - \frac{t}{4m^2} \right) A_5 \right]. \qquad (8)$$

Here ω and $\omega' = \omega(1+2\omega/m)^{-1}$ are energies of the initial and final photons, so that $s=m^2+2m\omega$, $u=m^2-2m\omega'$, and $t=-4\omega\omega'$.

Assuming for the amplitude T_{fi} a standard Regge behavior $\sim s^{\alpha_R(u)}$ for high s and fixed u, we have

$$A(s,u,t) \sim s^{\alpha_R(0)-3/2} \quad \text{when} \quad s\to\infty, \quad u\to 0. \qquad (9)$$

Here $\alpha_R(0) = \frac{3}{2} - \alpha' m_\Delta^2 \approx 0.13$ for the leading Regge trajectory, which is that of the $\Delta(1232)$ resonance with the slope $\alpha' \approx 0.9$ GeV^{-2}. Such a high-energy behavior of A guarantees convergence of the dispersion relation (6).

In the Born approximation, which is determined by the electric charge $q=(\tau_3+1)/2$ and the anomalous magnetic moment κ of the nucleon, the amplitude A becomes a (double) pole function of s,

$$A^{\text{Born}}(s) = me^2 \frac{\kappa^2+4q\kappa+2q^2}{(s-m^2)(u-m^2)} \qquad (10)$$

(here $e^2 \approx 4\pi/137$ and $su=m^4$). It vanishes at high s and hence coincides with A^{pole} in the dispersion relation (6). Correspondingly, the integrals in Eq. (6) give the non-Born part $A^{\text{NB}}=A-A^{\text{Born}}$ of the function A. When $s=u=m^2$ and $t=0$, these integrals determine the backward spin polarizability of the nucleon

$$\gamma_\pi \equiv -\frac{A^{\text{NB}}(m^2,m^2,0)}{2\pi m} = \gamma_\pi^s + \gamma_\pi^t, \qquad (11)$$

where

$$\gamma_\pi^s = -\frac{1}{2\pi^2 m} \int_{s_0}^\infty \frac{s'+m^2}{s'-m^2} \text{Im}_s A(s',u',t') \frac{ds'}{s'}, \qquad (12)$$

$$\gamma_\pi^t = -\frac{1}{2\pi^2 m} \int_{t_0}^\infty \text{Im}_t A(s',u',t') \frac{dt'}{t'}. \qquad (13)$$

We will refer to Eqs. (12) and (13) as s- and t-channel contributions, though actually γ_π^s includes contributions from both s and u channels. Applying unitarity and using the well-known formalism of helicity amplitudes, one can express $\text{Im}_s A$ in Eq. (12) in terms of the photoabsorption cross sections σ_λ^n with specific total helicity λ of the beam and target and with relative parity $P_n=\pm 1$ of the final state n with respect to the target, resulting in

$$\gamma_\pi = \int_{\omega_0}^\infty \sqrt{1 + \frac{2\omega}{m}} \left(1 + \frac{\omega}{m}\right)$$

$$\times \sum_n P_n [\sigma_{3/2}^n(\omega) - \sigma_{1/2}^n(\omega)] \frac{d\omega}{4\pi^2 \omega^3} + \gamma_\pi^t. \quad (14)$$

This sum rule for the backward spin polarizability of hadronic spin-1/2 systems is our main result.

The s-channel contribution to γ_π is given by a manifestly convergent integral of spin-dependent partial cross sections which are bound by the total cross section $\Sigma_n(\sigma_{3/2}^n + \sigma_{1/2}^n) = 2\sigma_{tot}$. The integrand in Eq. (14) has a parity structure similar to that found for the spin-independent part of T_{fi} [9]:

$$\Delta\sigma \equiv \sum_n P_n(\sigma_{3/2}^n - \sigma_{1/2}^n) = \{\sigma_{1/2}(\Delta P = \text{yes}) - \sigma_{1/2}(\Delta P = \text{no})\}$$

$$- \{1/2 \to 3/2\}. \quad (15)$$

In comparison with Eq. (1), photoexcitations n which carry the same parity as the nucleon (i.e., $\Delta P = \text{no}$) contribute to the backward spin polarizability with an opposite sign.

The t-channel contribution (13) cannot be recast through directly observable quantities because a large part of the integration region is unphysical, $t < 4m^2$. Nevertheless the following general remarks provide guidance for the nature of this contribution. The key point is that the imaginary part in Eq. (13) is determined by intermediate states of even total angular momentum J, charge parity $C = +1$, isospin $I \leq 2$ ($I \leq 1$ for the nucleon target), and ordinary parity $P = -1$, i.e.,

$$J^{PC} = 0^{-+}, \ 2^{-+}, \ 4^{-+}, \ldots, \quad I = 0, 1, 2. \quad (16)$$

The property $P = -1$ can be easily seen from the relation $A = 2T_5/t$ between the amplitude A and the amplitude T_5 of Refs. [5,7] taken at the backward angle. The amplitude T_5 contributes to the full Compton scattering amplitude as $T_{fi} \sim \bar{u}' \gamma_5 u T_5$, and the coefficient of T_5 changes sign when the P operator is applied to the nucleons. The constraint for J follows from the Landau-Yang theorem [11] which states, in particular, that any two-photon system of negative parity has an even J.

Among the lightest hadronic states which have the correct quantum numbers (16) to contribute to $\text{Im}_t A$ are the pseudoscalar mesons π^0, η, and η'. Any two-body systems of pseudoscalar mesons such as $\pi\pi$, $K\bar{K}$, $\pi\eta$, or $\pi\eta'$ are strictly excluded by parity and the evenness of J. The 3π continuum is allowed. Due to $C = +1$ and $I \leq 2$, it necessarily carries isospin $I^G = 1^-$ and is produced in the reaction $2\gamma \to 3\pi$ through anomalous Wess-Zumino-Witten vertices. The 3π continuum appears partly in the form of quasi-two-particle states such as $\pi^0\sigma$ with even orbital momentum l or $(\pi\rho)_{I=1}$ with odd l and even J, and partly in the form of broad resonances such as $\pi(1300)$. Furthermore, there is a 4π continuum, and so on.
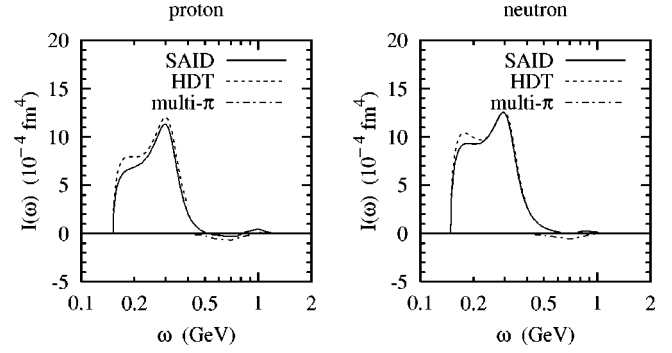

FIG. 1. The s-channel integrand of $\gamma_\pi^s = \int_{\omega_0}^\infty I(\omega) \, d\omega/\omega$, $I(\omega) = \sqrt{1 + 2\omega/m}(1 + \omega/m)\Delta\sigma/4\pi^2\omega^2$ for the proton and neutron. Solid lines: the contribution of $\gamma N \to \pi N$ with the SAID multipoles [13]. Dashed lines: the same with the HDT multipoles [14]. Dashed-dotted lines: the contribution of multipion states.

III. SATURATION OF THE SUM RULE

The dominating contribution to γ_π^s comes from single-pion photoproduction $\gamma N \to \pi N$, which yields the cross section

$$\Delta\sigma^{\pi N} = 8\pi \frac{q^*}{\omega^*} \sum_{l=0}^\infty (-)^l (l+1) \left\{ |A_{l+}|^2 - |A_{(l+1)-}|^2 \right.$$

$$\left. - \frac{l(l+2)}{4} (|B_{l+}|^2 - |B_{(l+1)-}|^2) \right\}. \quad (17)$$

Here a sum over channels with charged and neutral pions is implied, q^* and ω^* are the pion and photon momenta in the c.m. frame, and $A_{l\pm}$ and $B_{l\pm}$ are the standard Walker photoproduction multipoles [12]. The latter are taken from the computer code SAID, solution SP97K [13], and for large angular momenta $j = l \pm \frac{1}{2} \geq \frac{9}{2}$ from the one-pion-exchange diagram (Ref. [7], Appendix B).[1] In Fig. 1 we have plotted the integrand of Eq. (14) found with the SAID multipoles. The integrand is clearly peaked at low energies and practically vanishes above ~ 0.5 GeV, thus supporting a convergence of the backward dispersion relation. In order to show possible uncertainties in evaluating $\Delta\sigma$, we also present in Fig. 1 results found with the E_{0+}, M_{1-}, E_{1+}, M_{1+} multipoles of Ref. [14] (HDT), which were obtained using fixed-t dispersion relations and several coupling parameters adjusted to fit recent experimental data from Mainz and Bonn. In the HDT set of multipoles, A_{0+} ($= E_{0+}$) for charged pions near the pion threshold is larger than that in the SAID set and is closer to predictions of low-energy theorems. The difference between results found with the SAID and HDT sets yields an estimate of experimental uncertainties in the s-channel integral.

The evaluation of other (mainly multipion) contributions to γ_π^s has been done in the framework of the simple model

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[1]Formula (B12) for f_4 in that appendix contains a mistake which is not present in the computer code GNGN used to get numerical results in Ref. [7]. Instead of the pion velocity v, the ratio of the c.m. momenta q^*/ω^* should stand there.

TABLE I. The backward spin polarizability of the nucleon, Eq. (14). Shown separately are s-channel integrals from single-pion photoproduction with SAID [13] and HDT [14] multipoles, and from multipion production. The t-channel integral includes both the pole contributions of low-lying pseudoscalar mesons and their radial excitations (see the text).

10^{-4} fm^4	proton	neutron
s-channel, $\gamma N \to \pi N$		
SAID, 150–500 MeV	7.29	9.22
(HDT, 150–500 MeV)	(8.35)	(9.63)
SAID, 500–1500 MeV	0.02	0.13
s-channel, $\gamma N \to (\geq 2\pi) N$	-0.28	-0.23
t-channel		
π^0	-45.0 ± 1.6	$+45.0 \pm 1.6$
η	-1.00	-1.00
η'	-0.57	-0.57
total non-π^0	5.5 ± 1.8	7.5 ± 1.8
total	-39.5 ± 2.4	52.5 ± 2.4
experiment [8]	-27.1 ± 3.4	

of Ref. [7]. This model takes into account inelastic decays of πN resonances to the channels $2\pi N$, $3\pi N$, ηN, etc., and an incoherent nonresonant background consisting of a $\gamma N \to \pi \Delta$ contribution (calculated in the Born approximation) and an additional $E1(j = 3/2)$ contribution which was adjusted to reproduce the total photoabsorption cross section. The s-wave photoproduction of $\pi \Delta$ yields a visible negative contribution to $\Delta \sigma$ near the two-pion threshold. The whole integrand related with the multipion states is depicted in Fig. 1. It is relatively small and evidently appears to converge.

Even though the multipoles $A_{l\pm}$ and $B_{l\pm}$ are known up to energies $\omega \sim 2$ GeV, the computation of $\Delta \sigma$ due to multipion channels becomes very model dependent when $\omega \gtrsim 1$ GeV. For this reason we cut integrations of all the partial cross sections in Eq. (14) at $\omega_{\max} = 1.5$ GeV. The corresponding integrals are given in Table I. They change very little when a lower cutoff, such as $\omega_{\max} = 1$ GeV, is chosen. Uncertainties in our predictions for γ_π^s come mainly from the πN channel which results in errors $\simeq \pm 1$, whereas errors in the multipion contribution are estimated to be $\simeq \pm 0.1$.

Next we consider an evaluation of Eq. (13). Since the 2π contribution to $\text{Im}_t A$ is forbidden and since the phase space for 3π at low energies is small, one can expect a strong dominance of γ_π^t by the lightest pseudoscalar meson π^0 and, to a lesser extent, by η and η'. In the mass region $\sqrt{t} \lesssim 1.5$ GeV, these mesons and their "radial excitations" $\pi(1300)$, $\eta(1295)$, $\eta(1440)$ with $J^{PC} = 0^{-+}$ are the only t-channel resonances having the allowed quantum numbers (16).

Each of the pseudoscalar mesons M makes a contribution

$$A^{(M)}(t) = \frac{g_{MNN} F_{M\gamma\gamma}}{t - m_M^2} \tau_M \qquad (18)$$

to the amplitude (7) and, due to the identity

$$A^{(M)}(t) = \frac{1}{\pi} \int_{t_0}^{\infty} \text{Im}_t A^{(M)}(t') \frac{dt'}{t' - t - i\epsilon}, \qquad (19)$$

each makes a contribution

$$\gamma_\pi^{(M)} = -\frac{A^{(M)}(0)}{2\pi m} = \frac{g_{MNN} F_{M\gamma\gamma}}{2\pi m_M^2 m} \tau_M \qquad (20)$$

to the spin polarizability. Here, the couplings g_{MNN} and $F_{M\gamma\gamma}$ are those which stand in the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = i g_{MNN} \bar{\psi} \gamma_5 \tau_M M \psi + \frac{1}{8} F_{M\gamma\gamma} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} M, \qquad (21)$$

where $\gamma_5 = +(i/24)\epsilon^{\mu\nu\alpha\beta}\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta$, $F_{\mu\nu}$ is the electromagnetic field tensor, and the isospin factor τ_M is either 1 or τ_3 for isoscalar and isovector mesons, respectively. The radiative couplings $F_{M\gamma\gamma}$ can be fixed through the two-photon radiative widths $\Gamma_{M \to 2\gamma} = m_M^3 F_{M\gamma\gamma}^2 / 64\pi$, which are known experimentally. Alternatively, the ratios

$$\frac{F_{\eta\gamma\gamma}}{F_{\pi\gamma\gamma}} = \frac{\cos\theta_P}{\sqrt{3}} - \sqrt{\frac{8}{3}} \sin\theta_P \simeq 0.85 \quad (\text{exp.}: 0.95 \pm 0.06), \qquad (22a)$$

$$\frac{F_{\eta'\gamma\gamma}}{F_{\pi\gamma\gamma}} = \frac{\sin\theta_P}{\sqrt{3}} + \sqrt{\frac{8}{3}} \cos\theta_P \simeq 1.51 \quad (\text{exp.}: 1.24 \pm 0.07) \qquad (22b)$$

can be taken from the constituent quark model (CQM). Here $\theta_P \simeq -10.1°$ is the SU(3) octet-singlet mixing angle for pseudoscalar mesons. The strong couplings are reliably known only for pions, and one can use the value $g_{\pi NN}^2 / 4\pi = 13.75$ suggested by the VPI group [15] which lies between the two extremes advocated by the Nijmegen [16] and Uppsala [17] groups. The knowledge of strong couplings for η and η' is much more uncertain, and we use here predictions of the CQM

$$\frac{g_{\eta NN}}{g_{\pi NN}} = \frac{\sqrt{3}}{5} \cos\theta_P - \frac{\sqrt{6}}{5} \sin\theta_P \simeq 0.43, \qquad (23a)$$

$$\frac{g_{\eta' NN}}{g_{\pi NN}} = \frac{\sqrt{3}}{5} \sin\theta_P + \frac{\sqrt{6}}{5} \cos\theta_P \simeq 0.42. \qquad (23b)$$

The CQM value (23a) is $\simeq 2.5$ times higher than an estimate [18] obtained from data [19] on near-threshold η photoproduction from the proton. This higher ηNN coupling was found [18] to give, through the Born diagrams of $\gamma N \to \eta N$, too large p-wave photoproduction multipoles compared to those inferred from the data [19]. Note, however, that the intermediate nucleons in those Born diagrams propagate very far from the mass shell, and therefore the ηNN coupling which is effective there is certainly reduced by a nucleon off-shell form factor in comparison with its on-shell magnitude. That is why we suppose that the CQM prediction is not incompatible with the η-photoproduction data and use

the theoretical estimates (23) for on-shell nucleons. A cautious reader can take our estimate as an upper limit.

We note that the CQM predictions for individual strong and electromagnetic couplings of η and η' depend on the mixing angle θ_P which is not perfectly known. In our estimates we use a standard choice $\theta_P = -10.1°$ derived from the nonet mass splitting, although experimental data on two-photon decays of these mesons are more compatible with a larger $\theta_P \simeq -20°$ [20]. However, a combined t-channel-exchange contribution of η and η' to Compton scattering is not so sensitive to θ_P. It would not depend on θ_P at all in the limit of equal masses of η's. Including form factors (see below), $\gamma_\pi^{(\eta)} + \gamma_\pi^{(\eta')}$ varies between -1.6 and -1.9 at $\theta_P = -10.1°$ and $\theta_P = -20°$, respectively.

Taking into account the radial excitations such as $\pi(1300)$, $\eta(1295)$, $\eta(1440)$, etc., involves a few more coupling constants. We avoid an explicit consideration of these couplings and assume instead that the radial excitations only renormalize the contributions (18) of the low-lying mesons $M = \pi$, η, η' by t-dependent form factors which effectively give a t dependence to the couplings in Eq. (18). Such a t dependence does not violate the validity of the dispersion relation (19) for the modified amplitudes $A^{(M)}(t)$, provided the form factors have singularities (poles or cuts) only at finite real $t > t_0$. These singularities also contribute to $\mathrm{Im}_t A^{(M)}$ and, in accordance with Eq. (19), result in replacing the original couplings in Eq. (20) by their magnitudes at $t = 0$, $g_{MNN} \to g_{MNN}(0)$, and $F_{M\gamma\gamma} \to F_{M\gamma\gamma}(0)$. Thus, we finally write

$$\gamma_\pi^t \simeq \sum_{M=\pi,\eta,\eta'} \frac{g_{MNN}(0)F_{M\gamma\gamma}(0)}{2\pi m_M^2 m} \tau_M. \qquad (24)$$

Using the experimental values of $g_{\pi NN}$ and $F_{\pi\gamma\gamma}$ known at $t = m_\pi^2$ [see Eq. (3) and Ref. [7]], reducing each of them by a monopole form factor $F(0)/F(m_\pi^2)$ with the cutoff $\Lambda \simeq 1$ GeV, and taking the CQM ratios (22), (23) to evaluate the couplings of η's at $t = 0$, we obtain the numbers given in Table I. As expected, the largest contribution comes from π^0 exchange, whereas η and η' introduce only a small correction. With our assumptions for the form factors, the radial excitations play a minor role, reducing for example the contribution from π^0 by only $1 - F^2(0)/F^2(m_\pi^2) \simeq 4\%$, thus

again supporting a convergence of the sum rule. A conservative estimate of uncertainties in γ_π^t is $\simeq \pm 1.6$ due to π^0, another $\simeq \pm 1$ due to η's, and an additional $\simeq \pm 1$ due to the form factors.

In total, gathering all the s- and t-channel pieces and combining uncertainties in quadrature, we obtain the non-π^0 part of the backward spin polarizability of the proton $[\gamma_\pi^{(\mathrm{non}-\pi^0)}]_p \simeq 5.5 \pm 1.8$. This number is only $\simeq \frac{1}{3}$ of the experimental estimate (5). Both for the proton and the neutron, $\gamma_\pi^{(\mathrm{non}-\pi^0)}$ is dominated by the s-channel contribution (12) from low energies $\omega \lesssim 500$ MeV, which is well under control, whereas the higher energies and the heavier t-channel exchanges contribute little (see Fig. 1 and Table I for more details). Therefore, it is difficult to reconcile our result with the experimental finding.

IV. CONCLUSIONS

Using the backward-angle dispersion relation, we derived a novel well-convergent sum rule for the backward spin polarizability. We applied it for an evaluation of γ_π for the nucleon and obtained results which were numerically close to both the naive dispersion estimates [4,5] based on an unsubtracted fixed-t dispersion relation for A_2 and the leading-order results of ChPT with the $\Delta(1232)$ included [3]. Our results support the physical conclusion inferred from the previous work that γ_π is strongly dominated by low-energy subprocesses of pion photoproduction, including the Δ-isobar excitation, and by π^0 exchange. They disagree, however, with the experimental finding of Ref. [8] which indicates a large additional contribution, perhaps from higher energies. Therefore, we suggest that the indirect estimate (4) be checked in further dedicated experiments with polarized particles. Some possibilities for that are sketched in Ref. [5].

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