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# Shears mechanism and particle-vibration coupling

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The shears mechanism, underlying the rotational-like behavior observed in the $M1$ bands in neutron-deficient Pb isotopes, is interpreted as a consequence of a residual interaction between the proton and neutron blades. By deriving the angle $\theta$ between the proton and neutron spin vectors $\vec{j}_\pi$ and $\vec{j}_\nu$ in the $^{198,199}$Pb bands, it is shown that the main ingredient of this effective force can be described by a $P_2(\theta)$ term with a strength of 400–600 keV. Such an interaction can be mediated through the core by particle-vibration coupling, and qualitative agreement with experimental estimates may indicate the possible role of this coupling in the appearance of shears bands. [S0556-2813(98)50308-1]

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Using the tilted axis cranking (TAC) model, S. Frauendorf [1] interpreted the rotational behavior of the $M1$ bands in neutron-deficient Pb isotopes as a coupling of $h_{9/2}$ and $i_{13/2}$ protons and $i_{13/2}$ neutron holes to a slightly deformed core. The total angular momentum is generated by aligning the proton and neutron spin vectors $\vec{j}_\pi$ and $\vec{j}_\nu$ in a way that resembles the closing of a pair of shears, hence the name usually given to these structures: *shears bands.* The vectors $\vec{j}_\pi$ and $\vec{j}_\nu$ make up the two blades of the shears. Recent GAMMASPHERE data on the lifetimes of states in the $M1$ bands in $^{198,199}$Pb isotopes [2] provided a very sensitive test of the shears mechanism; the derived $B(M1)$ values are in good agreement with the TAC predictions [1,2].

In a previous work [3] we presented a semiclassical analysis of the $B(M1)$ and $B(E2)$ values [2], based on a schematic model of the coupling of two long $j$ vectors $(\vec{j}_\pi,\vec{j}_\nu)$. With this simple assumption we showed that the picture of the shears mechanism is consistent with the experimental data, suggesting that the important degree of freedom is indeed the shears angle. Going one step further, if we know the level energies and the angle $\theta$ between $\vec{j}_\pi$ and $\vec{j}_\nu$, it is also possible to obtain information on the nature of an effective interaction, $V_{\pi\nu}$, between the proton and the neutron constituents of the blades (‘‘bladons’’). This idea follows the work of Anantaraman and Schiffer [4] in the analysis of two-nucleon spectra, but applied in our case to the two-‘‘bladon’’ spectra of the shears bands. Within the framework of this simple system this effective interaction is responsible for the splitting of the otherwise degenerate multiplet, and may give rise to a rotational-like behavior.

In this Rapid Communication we will discuss further the form of the residual interaction and interpret its nature as a consequence of particle-vibration coupling. We follow the nomenclature introduced in Fig. 1 and make the reasonable assumption that the interactions between the protons in the proton blade and the neutron holes in the (neutron-hole) blade are independent of the shears angle $\theta$. In this way, the energy needed to form each of the blades is part of the bandhead energy and the excitation energy along the band is given only by the change in potential energy caused by the recoupling of the angular momenta in the shears, i.e.,

$$V_{\pi\nu}(I(\theta))=E(I)-E_{\mathrm{bandhead}}. \tag{1}$$

As discussed in Ref. [3], the shears angle $\theta$ that corresponds to a given state in the band can be derived using the expression $\cos\theta=I^2-j_\pi^2-j_\nu^2/2j_\pi j_\nu$, once $j_\nu$ and $j_\pi$ are determined to reproduce the bandhead spins. A small effect due to the possible increasing contribution from the core is taken into account by decomposing the total spin as $I=I_{\mathrm{shears}}+R_{\mathrm{core}}$. We assume the linear relation $R_{\mathrm{core}}=(\Delta R/\Delta I)(I-I_{\mathrm{bandhead}})$ and determine $\Delta R/\Delta I$ from the difference between the maximum observed spin and the sum of $j_\pi$ and $j_\nu$, i.e., $\Delta R=I_{\mathrm{max}}-(j_\pi+j_\nu)$, over the spin range of the band, $\Delta I=I_{\mathrm{max}}-I_{\mathrm{bandhead}}$. Expressed as a fraction of the total spin, this contribution accounts for a maximum (at the top of each band) of ≈15% for band 2 in $^{199}$Pb and ≲5% for the other bands.

In Fig. 2(a) we plot the normalized effective interaction [4]

$$\frac{V(I(\theta))}{\overline{V}}=\frac{V(I(\theta))}{\Sigma_I(2I+1)V(I)/\Sigma_I(2I+1)} \tag{2}$$

derived from Eq. (1) for the bands in $^{198,199}$Pb as a function of $\theta$. If we restrict ourselves to spatial forces, we can expand the interaction in even multipoles as [5,6]

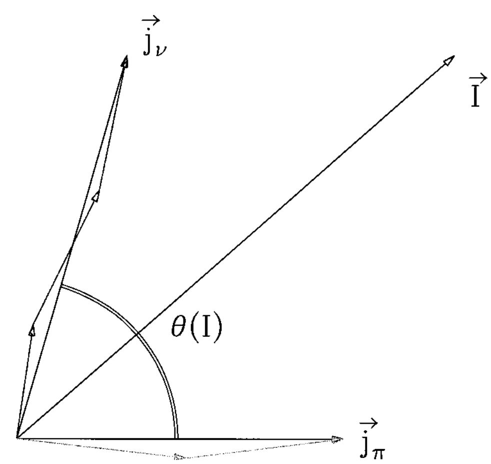


FIG. 1. Schematic drawing of the angular momentum coupling of neutron-hole and proton blades in a shears band. The small arrows represent the protons and neutrons constituting each blade.

 

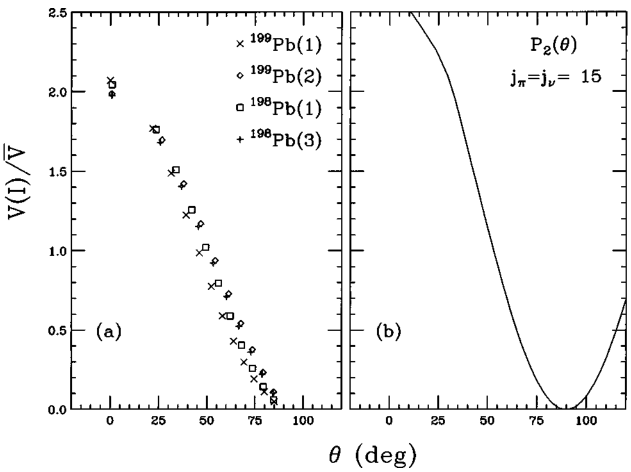


FIG. 2. Effective interaction between ''bladons'' as a function of the shears angle. The results derived from the excitation energies in the $M1$ bands in $^{198,199}$Pb (a) are compared with the expected dependence of a $P_2$ term for the case of $j_\pi = j_\nu = 15$ (b). Labels for the different bands follow those used in Ref. [2].

$$V_{\pi\nu}(\theta) = V_0 + V_2 P_2(\theta) + \cdots . \tag{3}$$

For comparison, the result expected for a $P_2$ term for $j_\nu = j_\pi = 15$ is shown in Fig. 2(b). Because we are dealing with the particle-hole channel this requires a positive $V_2$ in Eq. (3). The similarity between the two suggests to us that the main ingredient of this residual interaction between the protons and neutrons in the shears is therefore the long-range part of the nuclear force, represented by the $P_2$ term. We have previously shown in [3] that in fact a $P_2$ term can qualitatively explain the properties of these magnetic rotational bands. Experimentally, the strength of this interaction for the Pb region studied is found to be ~2.3 MeV, and taking into account ~2 protons and ~2–3 neutrons as constituents of the blades, we derive a value of 400–600 keV for a single proton, neutron-hole pair.

At this point, let us consider a spin-dependent force, represented, for example, by a $\vec{j}_\pi \cdot \vec{j}_\nu$ interaction. A $P_1$ term proportional to cos $\theta$ can now appear in Eq. (3) and naturally give rise to a rotational spectrum. However, since this term favors 0° or 180° coupling depending on its sign, one should realize that its contribution must be small based on the fact that the bandhead configuration is known to have perpendicular coupling [7]. Besides, the angles shown in Fig. 2 are derived under this assumption and explain the $B(M1)$ behavior rather well [3]. Likewise, a large contribution from a short-range component can be ruled out since, for example, a delta force cannot give rise to a rotational-like spectrum. While it remains as an intriguing question whether a spin-dependent term or a short-range term may manifest themselves in the shears mechanism, it is clear that the major role is played by the spatial long-range part of the force.

In order to understand this interaction, let us turn our attention to the theory of particle-vibration coupling [8]. The interaction Hamiltonian between a particle and a phonon of order λ arises from the variation in the single-particle potential $\delta V$ induced by the collective vibration. In a spherical nucleus, this coupling is usually given in the following form:

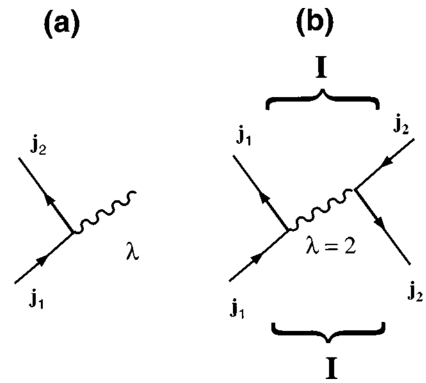


FIG. 3. Diagramatic representation of particle-vibration coupling. (a) First-order coupling. (b) Effective interaction arising from second-order coupling.

$$H_{\rm int} = \delta V = -k_\lambda(r) \sum_\mu Y^*_{\lambda\mu}(\vartheta,\varphi)\alpha_{\lambda\mu}, \tag{4}$$

where $k_\lambda(r) = R_0\, \partial V/\partial r$ is the particle form factor, $\alpha_{\lambda\mu}$ is the phonon amplitude, and $r,\vartheta,\varphi$ represent the particle polar coordinates. The first-order coupling is usually represented by the graph in Fig. 3(a). To second order the particle-vibration coupling gives rise to an effective interaction that is represented for the particle-hole case by the graph in Fig. 3(b), and can be expressed as

$$V_\lambda = \frac{(2\lambda+1)}{4\pi C_\lambda} k_\lambda(r_1) k_\lambda(r_2) P_\lambda(\vartheta_{12}), \tag{5}$$

with $\vartheta_{12}$ being the angle between the position vectors of particles 1 and 2. This effective interaction is of a $P_\lambda$ type with a strength determined by the particle form factors $k_\lambda$ and the restoring force parameter $C_\lambda$, the latter of which is related to the amplitude and the energy of the phonon by $C_\lambda = (2\lambda+1)\hbar\omega_\lambda / |\langle 1 \| \alpha_\lambda \| 0\rangle|^2$. We now apply this result to the case of the quadrupole mode. To compare with the experimental data we have to evaluate the expectation value of this interaction potential in the angular momentum coupled state $|j_1 j_2 I\rangle$, which for large values of $j$ gives [9]

$$\langle j_1 j_2 I | V_2 | j_1 j_2 I\rangle = \left(\frac{1}{4}\right) \frac{5}{4\pi C_2} \langle k_2(1)\rangle \langle k_2(2)\rangle P_2(\theta). \tag{6}$$

Note that this is expressed in terms of the angle $\theta$, between the angular momentum vectors (i.e., the shears angle) which is the variable used in the experimental analysis; this transformation adds the factor 1/4 explicitly indicated in Eq. (6). (See Ref. [6].) We estimate $C_2 \approx 500$ MeV for the light Pb region from an extrapolation of the systematics of the restoring force parameters given in Ref. [8] (p. 529) for both Hg and Pb nuclei. Using $\langle k_2 \rangle \approx 50$ MeV [10] we obtain $V_2 \approx 500$ keV, to compare with the experimental estimate of 400–600 keV. Because of the uncertainty in the extrapolation of the $C_2$ coefficients, it is important to stress here that we should view this result as a qualitative agreement.

The particle-vibration coupling also gives rise to a renormalization of the electric charge, usually expressed in terms of a polarization charge

$$(e_{\rm pol})_\lambda = \frac{\langle k_\lambda \rangle}{\langle r^\lambda \rangle} \frac{3}{4\pi} \frac{ZeR^\lambda}{C_\lambda} \tag{7}$$

that, with the parameters above, can be estimated to be $(e_{\rm pol})_2 \approx 2$, not quite as large as we have estimated in Ref. [3] from the experimental $B(E2)$ values and may be an indication of a permanent deformation of the core. An interesting dimensionless parameter to consider is

$$f_\lambda = \left(\frac{2\lambda+1}{16\pi}\right)^{1/2} \left(\frac{\hbar\omega_\lambda}{2C_\lambda}\right)^{1/2} \frac{\langle k_\lambda \rangle}{\hbar\omega_\lambda}, \tag{8}$$

which measures the strength of the particle-vibration coupling with respect to the phonon energy and, in particular, to what extent the perturbation expansion is valid $(f_\lambda < 1)$. We find $f_2 \approx \frac{1}{2}$ [i.e., $\approx \frac{1}{4}$ for the relative magnitude of the second-order graph in Fig. 3(b)].

In summary, we have presented evidence that the shears mechanism proposed by Frauendorf can also be interpreted as an effective $P_2$ force between the neutron holes and the protons in the shears. Such an interaction can be mediated through the core by particle-vibration coupling acting in second order and estimates based on such a picture are in qualitative agreement with the experiment. We believe this may provide a complementary approach to the TAC model, which has the problem of requiring a deformed mean field for nuclei that are almost spherical. In our scenario we overcome this difficulty because the particle-phonon coupling is responsible for the splitting of the otherwise degenerate multiplet generated by the coupling of the ''bladons,'' and can give rise to the shears mechanism in spherical nuclei. We hope that this work will stimulate detailed particle-phonon coupling calculations which will be necessary to quantitatively confirm this idea.

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[10] See Ref. [8], p. 420. If we use, for example, the harmonic oscillator we get $k = M\omega_0^2 R_0^2 \approx 55$ MeV.