Emission angle dependence of fission fragment spin in 12C, 16O, and 19F1**232Th reactions**

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The average total spins of fission fragments were measured in ¹²C, ¹⁶O, and ¹⁹F+²³²Th reactions at near and above barrier energies for fragment emission angles parallel and perpendicular to the beam direction. The fragment spins for perpendicular emissions are observed to be higher compared to that for forward-backward emissions indicating the importance of the tilting mode (*K* degree) of spin excitations in the fission process. The observed angle dependence of fragment spin could be explained within the statistical model, only if one assumes that the collective modes (other than the tilting mode) also depend on the emission angle of the fragments. It is shown that in this way, the results on both the fragment angular distributions and fragment spins can be explained consistently using the same *K* distribution at the fission saddle point. $[$ S0556-2813(98)50208-7]

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The study of the spin distribution in fission fragments provides important information on the mechanism of spin generation and the excitation of collective degrees of freedom in fission processes. The total spin of the final fragments in the fission process is largely determined by the excitation of various angular momentum bearing modes, such as wriggling, bending, twisting, and tilting $[1,2]$. In heavy ion induced fission reactions, where the compound nucleus is populated with an initial spin distribution, a part of the initial angular momentum also gets transferred as the spin of the fission fragments. The total spin acquired by the fission fragments thus arises from the above two different contributions. The excitation of angular momentum bearing collective modes influences the final fragment spins in two ways: (i) it enhances the fragment spin over that of the rigid rotation predicted by the simple concept of rolling and sticking, and (ii) it introduces a randomly oriented component of angular momentum, which results in misalignment of the fragment spin in the plane perpendicular to the fission axis. These effects have been largely demonstrated by earlier studies $\lceil 3 - \rceil$ 7] with the measurements of the total fragment spin for a large number of systems. While the existence of these modes has been well established, there still does not exist a clear understanding of the relative importance of their contributions to the fragment spin as a function of energy and angular momentum of the fissioning system.

Among the various angular momentum bearing modes, the tilting mode $(K$ degree) has been the most extensively studied, because of its role in determining the angular distribution of the fission fragments. The excitation of the tilting mode also determines the angle dependence of the fragment spin distributions in the fission process $[8]$.

Recently, Schmitt *et al.* [9,10] carried out a number of measurements of fragment spin as a function of emission angles in a variety of heavy ion induced fission reactions. These studies have revealed that the fragment spin is dependent on the emission angle as expected from the statistical transition state model, but the angle dependence is quite weak as compared to the statistical model predictions. It has not been possible to consistently explain the angular variation of total fragment spin and the fragment angular distributions within the framework of the transition state model, using the same *K* distribution. The statistical model calculations carried out by Schmitt *et al.* implicitly assume the collective spin contributions to be thermally and independently excited. However, the coupling of these collective modes to the angle dependent tilting mode could significantly affect their excitations. Also, in several recent experiments $[11-$ 15], it has been shown that in certain heavy ion induced fusion-fission reactions, the fragment angular anisotropies are anomalously large at near barrier and sub-barrier energies, which has been interpreted to imply that the *K* distributions are much narrower compared to the rotating liquid drop model calculation. The narrowing of *K* distribution should also reflect on the angle dependence of fission fragment spins.

In order to investigate the effect of coupling between the tilting and the collective modes and their role in determining the angle dependence of fragment spins, we have carried out measurements of total fragment spins for fragment emissions along 90° and 165° with respect to the beam direction over a wide bombarding energy range in ¹²C, ¹⁶O, and ¹⁹F+²³²Th reactions using the gamma ray multiplicity technique. The results have been analyzed within the framework of the statistical transition state model to consistently explain the fission fragment angular distribution and the angle dependence of fragment spins for these systems.

The experiments were carried out using ¹²C, ¹⁶O, and ¹⁹F beams from the 14 MV BARC-TIFR pelletron accelerator facility at Mumbai. A self supporting 232 Th target of 1.8 mg/cm² thickness was placed at the center of a 20 cm diameter by 9.0 cm height scattering chamber. The gamma multiplicity setup consisted of 15 hexagonal $(57 \text{ mm} \times 63 \text{ mm})$ BGO scintillators mounted in a closely packed geometry around the scattering chamber. The total solid angle covered by all the BGO detectors was about 40% of 4π solid angle. The details of the BGO detector setup has been described in an earlier work $[16]$. Figure 1 shows the schematic diagram of the experimental setup for the measurements. A pair of totally depleted surface barrier detectors (ΔE) of thickness 17 μ m were mounted at 90° and 165° with respect to the beam direction to detect the fission frag-

FIG. 1. Schematic diagram of the experimental setup.

ments from the various reactions. Coincidence between the ΔE signals and/or of all the BGO detectors was used as the trigger for the experiment. The threshold for each BGO detector was set at about 160 keV using γ -ray calibration sources. The timing signals of all the BGO detectors and the energy signals of the two ΔE detectors were recorded in list mode for further offline analysis. The efficiency of the BGO detector setup was determined using standard sources of $137Cs$ and $60Co$. The experimental data were sorted out to determine the γ -ray fold distributions. The experimentally measured fold distributions were converted to multiplicity distributions using the formalism described by Van der Werf [17]. A brief description of the analysis procedure for the determination of various moments of gamma ray multiplicity distributions from the measured fold distribution is as follows.

If *N* is the number of detectors employed, each having a solid angle Ω , and *M* is the multiplicity of the γ rays emitted in the event, then the probability of *p* detectors firing in coincidence $(p \text{ fold coincidence})$ is given by

$$
P_{Np}^{M} = {N \choose p} \sum_{k=0}^{p} (-1)^{p-k} {p \choose k} [1 - (N-k)\Omega]^{M}.
$$
 (1)

The fold distribution corresponding to a multiplicity distribution $G(M)$ is obtained as

$$
Q_N(p) = \sum_{M=0}^{M_{\text{max}}} P_{Np}^M(\Omega) G(M), p = 0, 1, 2, ..., N.
$$
 (2)

This can be written in terms of factorial moments such as

$$
Q_N(p) = \sum_{m=0}^M \left\langle \binom{M}{m} m! \right\rangle A_{N,pm}(\Omega), \tag{3}
$$

where

$$
A_{N,pm} = \frac{(-1)^m}{m!} {N \choose p} \sum_{k=0}^p (-1)^{p-k} {p \choose k} (N-k)^m \Omega^m \quad (4)
$$

and the angular brackets imply the average taken over the multiplicity distribution $G(M)$. Inverting Eq. (3) , we obtain the factorial moments of the multiplicity distribution as

$$
\left\langle \binom{M}{m} m! \right\rangle = \sum_{p=0}^{N} A_{N,pm}^{-1}(\Omega) Q_N(p).
$$
 (5)

 A_{pm}^{-1} is the response matrix and is dependent on the detector solid angle and the number of detectors employed in the multiplicity setup. Eq. (5) shows explicitly the expansion of factorial moments in terms of the fold probabilities. These factorial moments can be related in a straightforward way to different moments

$$
\left\langle \binom{M}{m} m! \right\rangle = \left\langle M(M-1) \dots (M-m+1) \right\rangle. \tag{6}
$$

The average total fission fragment spin was determined from the average gamma ray multiplicity using the relation

$$
\langle S_T \rangle = 2(M_\gamma - \alpha) + \beta M_n, \qquad (7)
$$

where α is the total number of statistical γ rays, M_n is the average number of neutrons emitted from the fragments, and β is the average spin removed by the emitted neutrons. The values of $\alpha=5$ and $\beta=0.5$ were choosen as those widely adopted in the literature $[18,19]$. The results on the neutron multiplicities were taken from the systematics of the measurements available from earlier experiments $[20]$. Figure 2 shows the results on the variation of average fragment spins with a bombarding energy for the three reactions studied for fragment emissions at 90° and 165° with respect to the beam. It is observed that the average total fragment spins for fragments emitted along 90° to the beam are higher than those emitted along 165°. The dependence is stronger at higher energies and gets weaker as one approaches the barrier energy. The present results for the $16O+232$ Th system, extrapolated to higher energies, are in agreement with the results of Schmitt *et al.* [10] measured at $E_{lab} = 120$ MeV. At sub-barrier energies the angle dependence of the fragment spin appears to vanish completely for all the three systems studied.

According to the transition state model, the angular distribution of fission fragments are determined by the fluctuations in the orientation of the fission axis (tilting mode) with respect to the total angular momentum vector I [21]. The width K_0^2 of the *K* distribution determines the angular anisotropy of fission fragments. The width K_0^2 is given as K_0^2 $T/\hbar^2(1/J_{\parallel}-1/J_{\perp})$, where J_{\parallel} and J_{\perp} are the respective moments of inertia for rotations parallel and perpendicular to the symmetry axis at the saddle. In heavy ion induced fusion reactions, total spin *I* lies in a plane perpendicular to the beam axis and the fluctuation or the tilting of the fission axis with respect to *I* impresses a component of the total angular momentum *I*, along the fission axis. The fragments emitted along the beam direction result from $K=0$ quantum states. However, the fragments emitted along 90° to the beam result from all possible *K* states, ranging from $K = -I$ to $K = I$ and hence bears spin components from $K=-I$ to $K=I$. The spins induced in the fragments are, thus, expected to be dependent on their direction of emission; the spin being maximum for fragments emitted along the 90° direction to the beam. This of course, is not the only mode of spin inducement in fission fragments. The total spin acquired by the fission fragments results from both the rigid rotation and the

FIG. 2. Average total fragment spins for fragment emissions along $\theta_F = 90^\circ$ and $\theta_F = 165^\circ$ as a function of bombarding energy for different systems.

statistical excitation of the angular momentum bearing collective modes $[9,10]$. The rigid rotation spin corresponds to the component of *I* associated with the rotational motion perpendicular to the fission axis, the magnitude of which depends on the shape of the system. The resultant total frag-

FIG. 3. Width of *K* distribution (K_0^2) obtained from the fragment angular distribution measurements for ${}^{12}C+{}^{232}Th$ and $16O+232$ Th reactions. The experimental data were taken from various references as discussed in the text. The solid curves are the RLDM calculations.

ment spin can thus be expressed as $[8-10]$

$$
\langle S_T \rangle = \langle \sqrt{f^2 I_{CN}^2 + (1 - f^2) K^2 + S_{\text{coll}}^2} \rangle, \tag{8}
$$

where the angular bracket on the right hand side implies the average taken over *K* and *I* distributions using $W_{M,K}^I(\theta)$ exp($-K^2/2K_0^2$) as the weight factor and with the relevant *I* distribution. The first term corresponds to the contribution from the rigid rotation with *f* being the fraction of compound nucleus angular momentum dissipated into the fragment spin. The second term accounts for the angle dependent spin due to the excitation of the tilting mode of the fission axis relative to the total angular momentum vector, while the last term corresponds to the thermally excited collective modes. The average total spin for the fragment emission along the beam direction reduces to

$$
\langle S_T \rangle = \sqrt{f^2 \langle I \rangle_{CN}^2 + S_{\text{coll}}^2}.
$$
 (9)

In the existing statistical theory $[10,14,15]$, S_{coll} is assumed to be angle independent and the second term in Eq. (8) , therefore, solely determines the emission angle dependence of fragment spins and is governed by the K_0^2 parameter. The functional dependence of the mean squared projection of the total spin I , $\langle K^2 \rangle$ on the symmetry axis, for a fixed total spin *I* is given by

$$
\langle K^2 \rangle = \frac{\sum_{K=-I}^{I} K^2 W_{M=0,K}^{I}(\theta) \exp(-K^2/2K_0^2)}{\sum_{K=-I}^{I} W_{M=0,K}^{I}(\theta) \exp(-K^2/2K_0^2)}.
$$
 (10)

The above expression can be written in an analytic form after substituting for the angular yield $W_{M=0,K}^{I}(\theta)$:

$$
\langle K^2 \rangle = 0.5(I+0.5)^2 \sin^2 \theta [1 - I_1(\beta^2)/I_0(\beta^2)], \quad (11)
$$

FIG. 4. (a) Spin of fragments in collective modes, S_{coll} for fragment emission angles of 90° and 165° in $^{12}C+^{232}Th$ and $^{16}O+^{232}Th$ reactions. (b) Ratio of the collective spins for 90° and 165° emission angles in ${}^{12}C+{}^{232}Th$ and ${}^{16}O+{}^{232}Th$ reactions.

where $\beta = \alpha \sin(\theta)/\sqrt{2}$, and $\alpha = (I + 0.5)/(2K_0^2)^{1/2}$. *I*₀ and *I*₁ are the zeroth order and first order modified Bessels functions. One observes that the angle dependence of fragment spin is largely determined by the quantity α .

The experimentally observed values of fragment spins were analyzed within the framework of the above-mentioned statistical model. In Ref. $[22]$, we presented the analysis of the total spin of the fragments emitted along 165° to the beam direction, by using Eq. (9). The quantities f and S_{coll} were treated as parameters and their values were obtained by fitting the spin value for 165° data. [22]. The value of *f* determined from the fit is consistent with that expected for a two-spheroid configuration for the scission shape, in which the deformations of the fragments are fixed by requiring the Coulomb repulsion energy to be equal to the observed total kinetic energy. As reported earlier $[22]$, the fit yielded the value of $f \approx 0.2$ corresponding to fragment deformation of $C/R \approx 1.5$, where *C* is the semimajor axis of one of the spheroids and *R* is the radius of the equivalent sphere. The S_{coll} required to fit the data was expressed to be proportional to $A_{cn}^{5/6}T^{1/2}$. It is, however, shown that these values of *f* and S_{coll} cannot explain the fragment spins at 90° [using Eq. (8)], if one takes the *K* distributions derived from the fission fragment angular anisotropies. The width K_0^2 of the *K* distribution was obtained from the fragment angular anisotropies using the relation

$$
A = 1 + \frac{\langle I^2 \rangle}{4K_0^2}.
$$
 (12)

The angular anisotropy data were taken from various experimentally measured angular distribution results discussed in the literature [11–13,15,23–25]. The $\langle I^2 \rangle$ values were obtained from the coupled channel calculations, which reproduce the experimentally measured fission excitation functions. The results on K_0^2 derived from the anisotropy data for both ¹²C and ¹⁶O+²³²Th reactions as a function of bombarding energy are shown in Fig. 3. The figure also shows the K_0^2 values obtained from the rotating liquid drop model (RLDM) calculation of Sierk $[26]$. It is shown that the experimentally measured K_0^2 values deviate significantly from the RLDM predicted values, at the near barrier energies. At energies much above the fusion barrier, the experimental K_0^2 values are in reasonable agreement with the RLDM calculations for both the systems. We have used the experimental K_0^2 values to determine the fragment spins at 90°.

Since the value of *f* is connected to the shape parameters at the scission, it is not expected to be different for 90° and 165° emission of fragments. This is supported largely by the fact that the mass and kinetic energy distributions of fragments do not differ with their angle of emission. We have, therefore, analyzed the fragment spin data at 90°, using the known values for f and K_0^2 and fitting to Eq. (8) to obtain $S_{\text{coll}}(90^{\circ})$ at different bombarding energies. It is observed that in order to explain the observed fragment spins at 90°, $S_{\text{coll}}(90^{\circ})$ is required to be substantially different from S_{coll} at 165° and has a bombarding energy dependence as shown in Fig. 4 for both the ${}^{12}C+\frac{232}{10}Th$ and ${}^{16}O+{}^{232}Th$ systems. The figure also shows the values of S_{coll} as derived from the fits to the 165° data. It can be seen that the values of $S_{\text{coll}}(165^\circ)$ follow the $T^{1/2}$ dependence as required by the statistical model [10]. However, the $S_{\text{coll}}(90^{\circ})$ values are strongly suppressed. This is brought out more explicitly in Fig. 4(b) by the ratio of $S_{\text{coll}}(90^\circ)/S_{\text{coll}}(165^\circ)$ as a function of bombarding energy for both ${}^{12}C+{}^{232}Th$ and ${}^{16}O+{}^{232}Th$ systems. The collective spin S_{coll} , is thus observed to be angle dependent and is suppressed for fragment emissions along 90° to the beam direction; the suppression factor being larger at higher bombarding energies. This appears to be a significant observation and consequential of the tilting degree of freedom. The statistical model assumes that the collective spins in all the degrees are independently excited, and the spin induced by each mode is determined by the amount of energy invested in exciting these modes. However, if the equilibration time for the tilting mode or *K* degree is larger than the time for other intrinsic collective modes, then the amount of energy locked in as the rotational energy may result in less energy being available for the inducement of other collective modes. The excitation of the *K* mode, thereby inhibits the excitation of other modes by a suppression factor which varies as $\exp(-\Delta E_{\rm rot}/T)$, where $\Delta E_{\rm rot}$ is the energy of the tilting mode. We have calculated this suppression factor by calculating $\Delta E_{\text{rot}}(K)$, averaged over *I* and *K* distributions for the 90° emission. The temperature *T* was assumed to be that at the saddle. The result of the calculation is shown by the solid curve in Fig. $4(b)$, which is observed to account for the reduction in fragment collective spins at a 90° emission angle as a function of bombarding energy for the two reactions. The excitation of the tilting mode, where the two fragments spin in the same preferential direction, when superimposed on the other collective modes (where the individual fragments spin in the opposite directions), acts to retard the spins in these modes.

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In the present work, we have studied the emission angle dependence of fragment spins for ¹²C, ¹⁶O, and ¹⁹F+²³²Th reactions at near and above barrier energies. The observed angle dependence is found to be much weaker than that predicted by the standard transition state model. The data were analyzed to determine the spins in the collective modes for fragment emissions in forward-backward and perpendicular directions. The experimental data on fragment spins require that the collective spin for fragment emissions along a 90° direction be suppresed over that for fragments emitted along forward-backward directions. The suppression of collective degrees of freedom is observed to increase with the bombarding energy, and may be understood on the basis that the spin excited in the collective modes such as bending, twisting, and wriggling is influenced by the excitation of the *K* degree.

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