Relativistic mean field approach and the pseudospin symmetry

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Based on the relativistic mean field (RMF) approach the existence of the broken pseudospin symmetry is investigated. Both spherical RMF and constrained deformed RMF calculations are carried out employing realistic Lagrangian parameters for spherical and for deformed sample nuclei. The quasidegenerate pseudospin doublets are confirmed to exist near the Fermi surface for both spherical and deformed nuclei. [S0556-2813(98)51107-7]

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Pseudospin symmetry was discovered in nuclear physics nearly 30 years ago [1–3]. The recent claim [4] that pseudospin symmetry may arise because of the near equality in magnitude of attractive scalar and repulsive vector fields in relativistic mean theory has revived the activity related to the understanding of the origin of this symmetry in real nuclei. The concept of pseudospin symmetry [1,2] is based on the experimental observation of the existence of quasidegenerate doublets of normal parity orbitals $(n,l,j=l+\frac{1}{2})$ and $(n-1,\ l+2,\ j=l+\frac{3}{2})$ such as $(4s_{1/2},3d_{3/2})$, $(3d_{5/2},2g_{7/2})$, etc., in the same major shell. Since for spherical systems the quantum numbers j^{π} are conserved, the pseudospin angular momenta $(\tilde{l},\tilde{s}=1/2)$ satisfy $\tilde{j}=j=\tilde{l}\pm\frac{1}{2}$.

In order to interpret this near degenerate pair of $j=l+1/2$ and $j=l+3/2$ states as pseudospin doublets corresponding to $\tilde{m}_s=\pm 1/2$, $\tilde{l}$ has to be $l+1$. It then follows for the major oscillator quantum number: $\tilde{N}=N-1$, for the radial quantum number $\tilde{n}=(\tilde{N}-\tilde{l})-1$ and for the parity $\tilde{\pi}=-\pi$. For zero pseudospin orbit splitting, the pseudospin multiplet will be degenerate. Thus the pair of orbitals $(4s_{1/2},3d_{3/2})$ and $(3d_{5/2},2g_{7/2})$ can be viewed as the $(2\tilde{p}_{1/2},2\tilde{p}_{3/2})$ and $(1\tilde{f}_{5/2},1\tilde{f}_{7/2})$ pseudospin doublets. The symmetry can also be investigated in deformed nuclei. In the asymptotic Nilsson scheme one finds the pseudospin quantum numbers $(\tilde{N}=N-1,\ \tilde{n}_3=n_3,\ \tilde{\Lambda}=\Lambda+1$, and $\tilde{\Omega}=\Omega)$. Therefore, the Nilsson orbitals $[N,n_3,\Lambda,\Omega=\Lambda+1/2]$ and $[N,n_3,\Lambda+2,\Omega=\Lambda+3/2]$ can be viewed as the pseudospin-orbit doublets $[\tilde{N},\tilde{n}_3,\tilde{\Lambda},\tilde{\Omega}=\tilde{\Lambda}\pm 1/2]$ [5].

Apart from the rather formal relabeling of quantum numbers various proposals for an explicit transformation from the normal scheme to the pseudospin scheme have been made in the last 20 years and several nuclear properties have been investigated in this scheme [6–9]. However, the origin of pseudospin symmetry remained unknown until the recent observation of Ginocchio [4,10], where for the first time the origin of this symmetry is claimed to be revealed as due to the near equality in magnitude of the attractive scalar and repulsive vector fields in relativistic theories. Here in this Rapid Communication we follow this idea and investigate to what extent the pseudospin symmetry is broken for realistic cases. For this purpose we concentrate on spherical as well as on deformed nuclei and we use the framework of relativistic mean field (RMF) theory [11]. It has been shown that this phenomenological approach is very successful in describing the ground state nuclear properties of spherical, deformed nuclei, and also for nuclei far away from the beta stability line (see, for example, [12–14]).

The RMF starts with a Lagrangian density describing the nucleons as Dirac spinors ψ, of mass m, interacting via the meson (σ-, ω-, and ρ-) and the electromagnetic fields. The standard Lagrangian density used in the RMF theory is written as [13]

$$\begin{aligned}\mathcal{L}=&\bar{\psi}(\gamma(i\partial-g_\omega\omega-g_\rho\vec{\rho}\vec{\tau}-eA)-m-g_\sigma\sigma)\psi\\&+\frac{1}{2}(\partial\sigma)^2-U(\sigma)-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+\frac{1}{2}m_\omega^2\omega^2\\&-\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu}+\frac{1}{2}m_\rho^2\vec{\rho}^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned}\tag{1}$$

It includes a nonlinear self-interaction $U(\sigma)$ of the σ field

$$U(\sigma)=\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4,\tag{2}$$

which takes into account in a phenomenological way the density dependence of the parameters of the model. $m_\sigma(g_\sigma)$, $m_\omega(g_\omega)$, $m_\rho(g_\rho)$ are the respective meson masses (coupling constants) and g_2 and g_3 are the coupling strengths of the nonlinear sigma field $U(\sigma)$.

It is straightforward to write the coupled baryon spinor and the mesons mean field equations. Starting from the Dirac equation for the single nucleon radial wave function with the spherical attractive scalar $(S=-g_\sigma\sigma)$ and the repulsive vector $(V=g_\omega\omega)$ potentials and following the standard procedure, by eliminating the small components (g_i), the large components (f_i) obey the following second order differential equation:

$$\begin{aligned}&\left\{-\nabla^2-\frac{S'+V'}{2m-E-(S+V)}\left(\frac{\partial}{\partial r}+\frac{\kappa_i+1}{r}\right)\right\}f_i\\&\quad=-[2m-E-(S+V)][E-(S-V)]f_i.\end{aligned}\tag{3}$$

Here the eigenvalues denoted by κ_i, of the operator $-\beta(\boldsymbol{\Sigma}\cdot\mathbf{L}+1)$, are given by

$$\kappa_i = \mp\left(j_i + \frac{1}{2}\right) \quad \text{for} \quad j_i = l_i \pm \frac{1}{2}, \tag{4}$$

and S' (V') are the derivatives of the potentials S (V) with respect to r. The binding energy $E \geqslant 0$ is measured with respect to the nucleon mass M in natural units $\hbar = c = 1$.

On the other hand eliminating the large component f_i we have for the small component g_i the following second order differential equation:

$$\left\{-\nabla^2 - \frac{S'-V'}{E-(S-V)}\left(\frac{\partial}{\partial r} - \frac{\kappa_i - 1}{r}\right)\right\} g_i = [2m - E - (S+V)][E-(S-V)]g_i . \tag{5}$$

For the case of equal strengths, $S = V$, Eq. (5) reduces to

$$-\nabla^2 g_i + E(S+V)g_i = E(2m-E)g_i . \tag{6}$$

Clearly Eq. (6) has an energy dependent potential ($E(V+S)$) and has the eigenvalue $E(2m-E)$. After scaling the radial variable $r = x/(\sqrt{E})$, the potential has a complicated ($\sqrt{E}$) dependence, i.e., $S(x/\sqrt{E}) + V(x/\sqrt{E})$. In such a situation Eq. (6) is no longer a normal Schrödinger eigenvalue equation. Further, it is obvious that in this equation all solutions with ''bound'' states in the Fermi sea with $E \geqslant 0$ are shifted to one degenerate eigenvalue with $E = 0$, which, in fact, is not bound. The corresponding wave functions are not normalizable. This indeed is a nonphysical situation. This equation is the same as Eq. (3) of Ref. [4] in the scaled variable x when written in terms of the partial waves and using the relation $l(l+1) = \kappa(\kappa-1)$. Here l, the angular momentum of the lower component g_i is identified with the pseudospin angular momentum ($\tilde{l}$). This is the pseudospin symmetry limit of Ref. [4], where the doublets $j = \tilde{l} \pm 1/2$ with the same $\tilde{l}$ are degenerate. However, in this limit only the Dirac sea states exist, and no Dirac valence bound states, and therefore it contradicts reality. According to these considerations in all realistic situations the pseudospin symmetry must be broken. Therefore the question arises, to which extent is it broken in real nuclei? So far only the spherical case has been investigated for square well potentials [4] and for spherical solutions of the RMF equations [15,16].

In the present Rapid Communication we investigate the broken pseudospin symmetry both for the spherical and deformed nuclei within the relativistic mean field approach. For our study, we choose ^{208}Pb as a representative of spherical nuclei and ^{154}Dy as a representative of deformed nuclei. We use in our calculations the Lagrangian parameter set NL3 [17] which successfully reproduces the ground state properties of nuclei, spread over the entire periodic table. The other parameter sets like NL1 and NLSH (see [14]) are expected to give almost identical results for these nuclei.

First, spherical RMF calculations in the coordinate space are carried out for ^{208}Pb. The calculated binding energy and the charge radius agree remarkably well with the experiment. The calculated single particle energies for the bound orbitals near the Fermi surface are shown in Fig. 1(a) for neutrons and protons. It is clear from the figure that the pairs of bound neutron valence orbitals ($2g_{7/2}$,$3d_{5/2}$) and ($1i_{11/2}$,$2g_{9/2}$) which correspond to pseudospin doublets ($2\tilde{f}_{7/2}$,$2\tilde{f}_{5/2}$) and

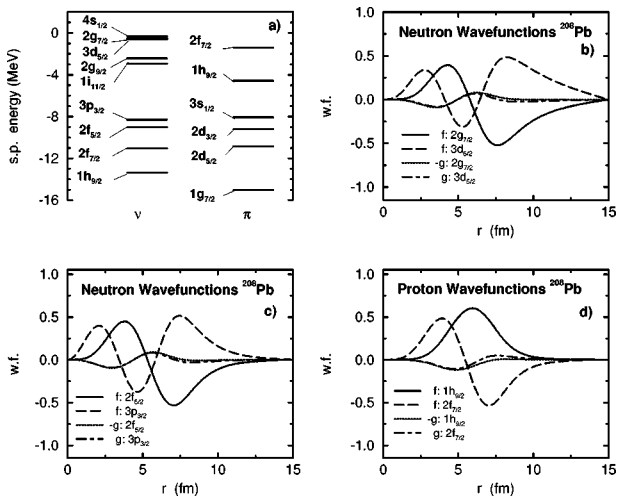

FIG. 1. Pseudospin splitting in the spherical nucleus ^{208}Pb: (a) single particle spectra in the vicinity of the Fermi surface for neutrons ν and protons (π) and large (f) and small (g) components of the Dirac wave functions for the pseudospin doublets $\nu 2\tilde{d}$ (b), $\nu 2\tilde{f}$ (c), and $\pi 2\tilde{g}$ (d).

($1\tilde{h}_{11/2}$,$1\tilde{h}_{9/2}$), respectively, are quasidegenerate, indicating only a small breaking of pseudospin symmetry. The same is more or less true for the pairs of neutron hole (($2f_{5/2}$,$3p_{3/2}$),($1h_{9/2}$,$2f_{7/2}$)), proton valence (particle) ($1h_{9/2}$,$2f_{7/2}$), and proton hole (($2d_{3/2}$,$3s_{1/2}$),($1_{7/2}$,$2d_{5/2}$)), orbitals forming the pseudospin doublets, but here the energy separation between the partners of the respective doublets is relatively larger. The larger the binding energy is, the larger the separation is. This indicates that the concept of the pseudospin symmetry becomes better and better for the orbitals as their energies approach closer and closer to the continuum. This is consistent with the results found in Ref. [4] for the square well potentials. In addition, the energy separation becomes larger, if the pseudoorbital angular momentum ($\tilde{l}$) increases. The dependence of the energy splitting of the pseudospin partners on the energy E and on the pseudoorbital angular momentum $\tilde{l}$ can easily be understood from Eq. (5). For a given pseudoorbital angular momentum $\tilde{l}$ the term in Eq. (5) which splits the pseudospin partners is

$$\frac{S'-V'}{(S-V)-E}\,\frac{\kappa_i}{r} . \tag{7}$$

It has the energy dependence ($E-(S-V)$) in the denominator. Now ($S-V$) is about 50 MeV. Bound states in the Fermi sea have a binding energy $E < 50$ MeV. For increasing binding energy E, i.e., going to more deeply bound states, the denominator decreases. This then results in a larger energy splitting between the pseudospin partners. For example, for the orbit $\tilde{l} = 3$ the energy splitting between the pseudospin partners ($1g_{7/2}$ and $2d_{5/2}$) will be relatively larger as compared to that between ($2g_{7/2}$ and $3d_{5/2}$). In addition, the bigger the value of $\tilde{l}$ is, the larger the splitting is. For instance, the energy splitting between the pseudospin partners ($1i_{11/2}$ and $2g_{9/2}$) corresponding to $\tilde{l} = 5$ is relatively larger as compared to that between the partners ($2g_{7/2}$ and $3d_{5/2}$) which

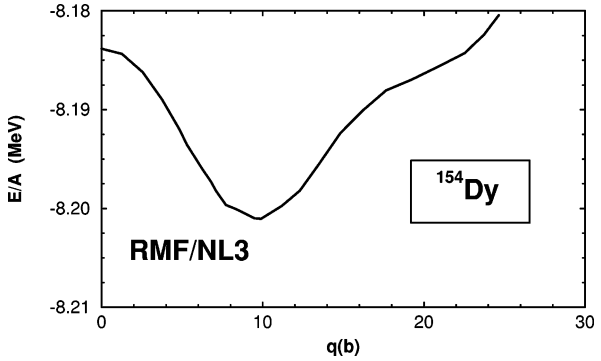

FIG. 2. Energy surface of the deformed nucleus ^{154}Dy as a function of the quadrupole moment q in units of barn.

correspond to $\tilde{l}=3$, in the same major shell. Interestingly, the sign of the energy splittings between the partners of the neutron valence doublets is opposite to that of the neutron hole, proton particle, and proton hole doublets.

The normalized single nucleon wave functions [both large (f) and small (g) components] are plotted for the pseudospin partners corresponding to the valence neutron pairs, the neutron hole pairs and valence proton pairs in Figs. 1(b), (c), and (d), respectively. The phase of the lower components (g) of one of the partners is reversed while plotting, in order to exemplify the differences in the lower components of the pseudospin partners. Clearly, the lower components are much smaller in magnitude as expected and are almost equal in magnitude. In the case of exact pseudospin symmetry, the

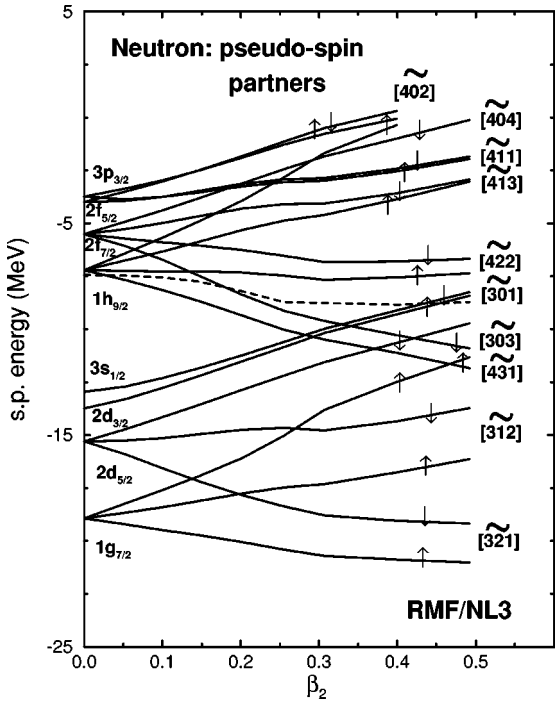

FIG. 3. Single particle energies of the deformed Dirac equation for the neutrons in the nucleus ^{154}Dy as a function of the quadrupole deformation parameter β_2. Asymptotic pseudospin quantum numbers are given and the pseudospin partners are indicated by arrows $\uparrow$ and $\downarrow$.

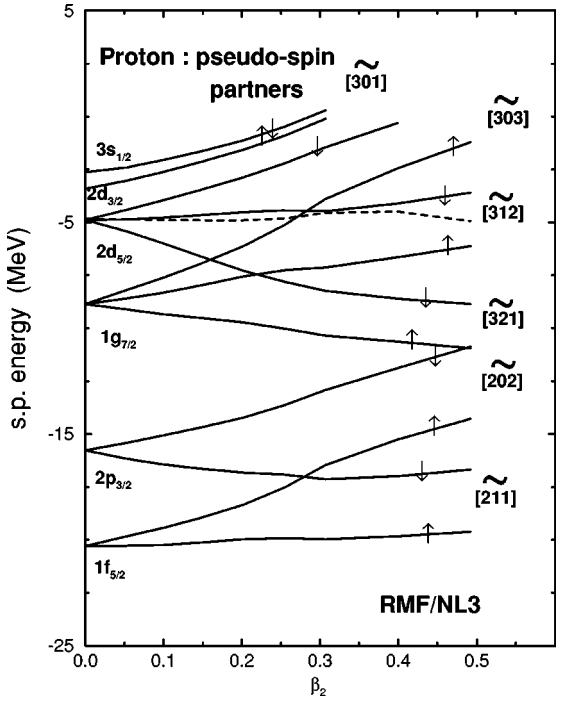

FIG. 4. Single particle energies for protons in ^{154}Dy, for details see Fig. 3.

lower component of the pseudospin partners should be identical (except for the phase). The very small differences between these g's, which mainly appear around the surface are negligible for the pseudospin partners having very small binding energies.

Next we consider deformed systems and impose constraints on the quadrupole moment. Constrained relativistic Hartree calculations have been carried out for the nucleus ^{154}Dy. The coupled differential equations for the spinors and the meson fields are given in Ref. [13]. They have been solved by expanding the spinors and the meson fields in terms of anisotropic oscillator wave functions. Numerical details are given in Refs. [13] and [18]. Pairing correlations are treated in the constant gap approximation and the Lagrangian parameter set NL3 [17] is used. The calculated potential energy surface is shown in Fig. 2. The value of the calculated ground state deformation parameter β_2 is 0.202 which is to be compared with 0.237, the corresponding experimental value. The calculated ground state binding energy 1262.95 MeV differs from the corresponding experimental value by merely 1.2 MeV.

The energies of the bound neutron pairs of orbitals corresponding to pseudospin doublets are plotted against the deformation β_2 ranging from 0.0–0.5 in Fig. 3. The asymptotic Nilsson quantum numbers $[N,n_3,\Lambda,\Omega]$ are good for large values of the deformation β_2. The pseudospin doublets $[\tilde{N},\tilde{n}_3,\tilde{\Lambda},\tilde{\Omega}=\tilde{\Lambda}\pm 1/2]$ [5] are indicated by $[\tilde{N},\tilde{n}_3,\tilde{\Lambda}]$ $\uparrow$ and $\downarrow$ in the figure. For zero deformation ($\beta_2=0$) the orbitals are indicated by the corresponding spherical states. The figure reveals the following:

(i) The energy splitting between the pseudospin partners is smaller for the valence orbitals and for the partners just below the Fermi surface.

(ii) This energy difference is relatively larger for the partners having larger pseudospin angular momentum ($\tilde{l}$).

(iii) In general, this separation stays almost constant and does not vary with deformation after reasonable value of β_2.

(iv) The energy difference between the $\downarrow$ and the $\uparrow$ partners always remains positive except for [$\tilde{\mathbf{4}}\tilde{\mathbf{0}}\mathbf{4}$], where there is crossing at around $\beta = 0.3$. Such a crossing is not very unusual, it has also been observed in Ref. [5].

These systematics are consistent with those observed in the spherical case above. A similar plot for the proton pseudospin doublets shown in Fig. 4 reveals identical systematics as those observed for the neutron case (Fig. 3). It is interesting to note that in Ref. [5] the energy difference between the valence neutron pseudospin partners is negative (opposite to ours) while it has the same sign as ours for protons. This may be because of the negative value obtained for V_{ls}, the strength of the pseudospin orbit interaction, from the Nilsson parametrization for $82 < N < 126$.

Similar calculations have also been carried out for other spherical and deformed nuclei and they show identical systematics. The conclusions presented here, are therefore rather general. In conclusion, it is shown in the relativistic mean field framework that quasidegenerate pseudospin doublets do exist near the Fermi surface for both spherical and deformed nuclei. The pseudospin symmetry is restored better and better as one moves closer to the continuum limit. These conclusions confirm the findings of Ginocchio [4,10].

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