Gauge-invariant tree-level photoproduction amplitudes with form factors

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We show how the gauge-invariance formulation given by Haberzettl is implemented in practice for photoproduction amplitudes at tree level with form factors describing composite nucleons. We demonstrate that, in contrast to Ohta's gauge-invariance prescription, this formalism allows electric current contributions to be multiplied by a form factor, i.e., it does not require that they be treated like bare currents. While different in detail, this nevertheless lends support to previous *ad hoc* approaches which multiply the Born amplitudes by an overall form factor. Numerical results for kaon photoproduction off the nucleon are given. They show that the gauge procedure by Haberzettl leads to much improved χ^2 values compared to Ohta's prescription. [S0556-2813(98)51007-2]

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The question of gauge invariance is one of the central issues in dynamical descriptions of how photons interact with hadronic systems [1–4]. While there is usually no problem in finding definitive answers at the level of tree diagrams with bare, pointlike particles, the problem rapidly becomes very complicated once one attempts to incorporate the electromagnetic interaction consistently within the full complexity of a strongly interacting hadronic system [4]. As a case in point, as is well known, even the tree-level amplitude for pion photoproduction off the nucleon is not gauge invariant if one employs hadronic πNN form factors to account for the fact that nucleons are composite objects, and not point-like.

In order to restore gauge invariance in these situations, one needs to construct additional current contributions beyond the usual Feynman diagrams to cancel the gaugeviolating terms. One of the most widely used methods to this end is due to Ohta [2]. For pion photoproduction off the nucleon at the level of the Born amplitude, Ohta's prescription amounts to dropping all strong-interaction form factors for all gauge-violating electric current contributions [3]. In other words, gauge invariance is regained by treating the offending terms exactly as in the bare case, thus losing any effect because of the compositeness of the nucleons. This undesireable situation is sometimes remedied in an ad hoc fashion by multiplying the gauge-invariant bare amplitude by an overall form factor taken to simulate the average effect of the fact that nucleons are not pointlike [5]. Within Ohta's scheme, however, there is no foundation for such recipes [3].

Recently, Haberzettl [4] has put forward a comprehensive treatment of gauge invariance in meson photoproduction. This includes a prescription for restoring gauge invariance in situations when one cannot, for whatever reason, handle the full complexity of the problem and therefore must resort to some approximations. It is the purpose of the present paper to provide a detailed comparison of this approach with Ohta's. While the general *Ansatz* in Ref. [4] was quite different from Ohta's, we will show that both approaches can be understood as different ways of taking the limit of the vanish-

ing photon momentum. The way this limit is treated in Ref. [4] will introduce more flexibility in how form factors can be retained for the terms where they are replaced by constants in Ohta's prescription. Although different in detail, this finding actually lends support to approaches which multiply the Born amplitude by an overall form factor.

We will use the reaction $\gamma p \rightarrow n \pi^+$ with pseudoscalar coupling for the πNN vertex as a simple example to elucidate the main features of the present investigation, similar to the discussion of Ohta's approach [2] in Ref. [3]. Using different, or more general, couplings for the vertex would not add anything essential to the following discussion; it would only complicate the presentation.

For bare nucleons, the tree-level amplitude (see Fig. 1) for $\gamma p \rightarrow n \pi^+$ for a pure pseudoscalar coupling is given as (see [3], and references therein)

$$\boldsymbol{\epsilon} \cdot \boldsymbol{M}_{fi} = \sum_{j=1}^{4} A_j \overline{\boldsymbol{u}}_n (\boldsymbol{\epsilon}_{\mu} \boldsymbol{M}_j^{\mu}) \boldsymbol{u}_p \,, \tag{1}$$

which represents an expansion based on operators

$$M_1^{\mu} = -\gamma_5 \gamma^{\mu} \, k, \qquad (2a)$$

$$M_2^{\mu} = 2 \gamma_5(p^{\mu} k \cdot p' - p'^{\mu} k \cdot p), \qquad (2b)$$

$$M_3^{\mu} = \gamma_5(\gamma^{\mu} \ k \cdot p - p^{\mu} \ k), \qquad (2c)$$

$$M_{4}^{\mu} = \gamma_{5}(\gamma^{\mu} \ k \cdot p' - p'^{\mu} \ k), \qquad (2d)$$

with coefficient functions

$$A_1 = \frac{ge}{s - m^2} (1 + \kappa_p) + \frac{ge}{u - m^2} \kappa_n, \qquad (3a)$$

$$A_2 = \frac{2ge}{(s-m^2)(t-\mu^2)},$$
 (3b)

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$$A_3 = \frac{ge}{s - m^2} \frac{\kappa_p}{m},\tag{3c}$$

$$A_4 = \frac{ge}{u - m^2} \frac{\kappa_n}{m},\tag{3d}$$

where *m* and μ are the masses of the nucleon and the pion, respectively, *g* is the pseudoscalar πNN coupling constant, and *e* the elementary charge. The anomalous magnetic moments of the neutron and the proton are denoted here by κ_n and κ_p . The Mandelstam variables *s*, *u*, and *t* are given as (cf. Fig. 1)

$$s = (p+k)^2 = (p'+q)^2,$$
 (4a)

$$u = (p' - k)^2 = (p - q)^2,$$
 (4b)

$$t = (p - p')^2 = (q - k)^2,$$
 (4c)

i.e., $s+u+t=2m^2+\mu^2$ since all external particles are onshell. (For the present case, m'=m in Fig. 1.)

Obviously, since each of the operators M_i^{μ} is gauge invariant by itself, i.e., $k_{\mu} \cdot M_i^{\mu} = 0$, the total photoproduction current is also gauge invariant. This result is obtained only if the vertices are bare, without any form factors. Since the terms proportional to M_1^{μ} , M_3^{μ} , and M_4^{μ} arise from purely magnetic contributions—and therefore are always gauge invariant by themselves, irrespective of whether one uses form factors or not—the problematic term, as pointed out already in Ref. [3], clearly is A_2 which arises here from the sum of the electric contributions of the *s* and *t* channels.

If one now considers the nucleons as composite objects and introduces form factors for the hadronic vertices, the result for the first three diagrams of Fig. 1 is

$$\boldsymbol{\epsilon} \cdot \tilde{M}_{fi} = \sum_{j=1}^{4} \hat{A}_{j} \bar{u}_{n} (\boldsymbol{\epsilon}_{\mu} M_{j}^{\mu}) u_{p} + \boldsymbol{\epsilon} \cdot \tilde{M}_{\text{viol}}, \qquad (5)$$

with gauge-invariant contributions

$$\hat{A}_1 = \frac{ge}{s - m^2} (1 + \kappa_p) F_1 + \frac{ge}{u - m^2} \kappa_n F_2,$$
 (6a)

$$\hat{A}_2 = \frac{2ge}{(s-m^2)(t-\mu^2)}\hat{F},$$
 (6b)

$$\hat{A}_3 = \frac{ge}{s - m^2} \frac{\kappa_p}{m} F_1, \qquad (6c)$$

$$\hat{A}_4 = \frac{ge}{u - m^2} \frac{\kappa_n}{m} F_2, \qquad (6d)$$

and a gauge-violating term

$$\boldsymbol{\epsilon} \cdot \boldsymbol{\tilde{M}}_{\text{viol}} = -g e \boldsymbol{\bar{u}}_n \boldsymbol{\gamma}_5 \boldsymbol{\epsilon}_{\mu} \left[\frac{2p'^{\mu}}{s - m^2} (\hat{F} - F_1) + \frac{2q^{\mu}}{t - \mu^2} (\hat{F} - F_3) \right] \boldsymbol{u}_p.$$
(7)

The momentum dependence of the form factors appearing here can be read off Fig. 1, i.e.,

$$F_1 = F_1(s) = f[(p+k)^2, m'^2, \mu^2],$$
 (8a)

$$F_2 = F_2(u) = f[m^2, (p'-k)^2, \mu^2],$$
(8b)

$$F_3 = F_3(t) = f[m^2, m'^2, (p - p')^2], \qquad (8c)$$

(here, m' = m) where use is made of the fact that the form factor may always be written as a function of the squares of the four momenta of its three legs [cf. Eq. (18)] (which does *not* mean, however, that it may be taken as a function f(s,u,t) of the Mandelstam variables, as it is sometimes stated [3]). At this stage, \hat{F} appearing in Eqs. (6b) and (7) is undefined; it was introduced here to be able to isolate the gauge-violating current contribution in a form that makes the comparison with Eq. (1) easy. Clearly, the full amplitude $\epsilon \cdot \tilde{M}_{fi}$ does not depend on it since the sum of the \hat{F} contributions from Eq. (7) exactly cancels the \hat{A}_2 term.

Now, without a detailed dynamical treatment of the compositeness of the nucleons [4], *any* prescription for restoring gauge invariance amounts to introducing an additional contact current M_c^{μ} (generically depicted by the fourth diagram in Fig. 1), with on-shell matrix elements cancelling exactly the gauge-violating term (7), i.e.,

$$\bar{u}_n(\epsilon_\mu M_c^\mu) u_p = -\epsilon \cdot \tilde{M}_{\text{viol}}.$$
(9)

Apart from purely transverse components or terms proportional to k^{μ} , for the present example this contact current is essentially given by the term in the square brackets of Eq. (7) [2–4]. Adding this contact contribution to Eq. (5), one then obtains a gauge-invariant amplitude in analogy to Eq. (1),

$$\boldsymbol{\epsilon} \cdot \hat{\boldsymbol{M}}_{fi} = \sum_{j=1}^{4} \hat{A}_j \bar{\boldsymbol{u}}_n (\boldsymbol{\epsilon}_{\mu} \boldsymbol{M}_j^{\mu}) \boldsymbol{u}_p \,, \tag{10}$$

which *does* depend on \hat{F} now via \hat{A}_2 of Eq. (6b).

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Using the analytic continuation and minimal substitution, Ohta [2] finds that the required \hat{F} factor is constant,

Ohta:
$$\hat{F} = f(m^2, m'^2, \mu^2) = 1,$$
 (11)

determined by the normalization condition for the form factor in the nonphysical region where all three legs are on-shell [see Eq. (18)]. \hat{A}_2 thus reduces to A_2 of Eq. (3b), effectively freezing all degrees of freedom arising from the compositeness of the πNN vertex and treating it like a bare one for electric current contributions.

This determination of \hat{F} is sufficient to ensure that the additional contact-current contribution is singularity free at $s=m^2$ and $t=\mu^2$, for in this limit, both F_1 and F_3 become unity [cf. Eq. (18)],

$$F_1(s=m^2) = F_3(t=\mu^2) = 1.$$
(12)

In this limit, therefore, Eq. (7) reduces to nonsingular $\frac{0}{0}$ expressions. Note that in the present kinematics (where all external particles are on-shell) one has

$$s - m^2 = 2p \cdot k, \tag{13a}$$

$$u - m'^2 = -2p' \cdot k, \tag{13b}$$

$$t - \mu^2 = -2q \cdot k, \tag{13c}$$

and hence the limits in Eq. (12) correspond to the vanishing of the photon momentum. Therefore, *any* (reasonably behaved) subtraction function \hat{F} that becomes the unity for k = 0 is sufficient to restore gauge invariance without any unwanted singularities.

In Ref. [4], use is made of this freedom by allowing \hat{F} to be a function of the hadron momenta. The only functions available that have anything to do with the physics of the present problem are of course the form factors themselves. Haberzettl restores gauge invariance by constructing a contact current equivalent to choosing the subtraction function as

Choice A:
$$\hat{F} = F_3(t) = f[m^2, m'^2, (p-p')^2],$$
 (14)

which is the only function from those given in Eqs. (8a)-(8c) that does not depend explicitly on k to begin with. This, however, is an artifact, having taken both nucleon momenta as independent variables. Had we taken, for example, the pion momentum q as an independent variable instead of the final nucleon momentum p', we would have

$$F_1 = F_1(s) = f[(p+k)^2, m'^2, \mu^2], \qquad (15a)$$

$$F_2 = F_2(u) = f[m^2, (p-q)^2, \mu^2], \qquad (15b)$$

$$F_3 = F_3(t) = f[m^2, m'^2, (q-k)^2], \qquad (15c)$$

which, by the same reasoning, would point to choosing F_2 as the subtraction function. And if we choose (q,p',k) as the independent set, we would find F_1 . In other words, following Ref. [4], depending on the choice of variables, we can take any one of the three form factors as a subtraction function. In general, the subtraction vertex is the one whose single offshell leg is described in terms of the on-shell four momenta of the other two legs.

One may argue whether this dependence on the variable set should be allowed. From the point of view of minimal substitution, however, perhaps one should not find this surprising since technically speaking, one can only perform a minimal substitution in the variables which actually occur and hence the resulting current in general will reflect the underlying variable set. Ohta circumvented this problem by considering the vertex as a general function $f(p^2, p'^2, q^2)$ unconstrained by momentum conservation before performing the minimal substitutions. The resulting subtraction function (11) then corresponds to the nonphysical limit for taking all three variables to their mass-shell values. This prescription, thus, amounts to performing the infrared limit $k \rightarrow 0$ explicitly in the construction of the contact current, whereas in Ref. [4] the proper value for this limit is provided by the dynamics of the reaction by choosing the subtraction vertex as one with proper physical variables for its legs. (In Ref. [6], some formal problems associated with Ohta's nonphysical limit have been pointed out.)

In any case, within the gauge-invariance prescription of Ref. [4], it is possible to remove the dependence on the variable set by introducing a more "democratic" choice for \hat{F} , using a linear combination of the three limiting cases, namely,

Choice B:
$$\hat{F} = a_1 F_1(s) + a_2 F_2(u) + a_3 F_3(t)$$

= $\hat{F}(s, u, t)$, (16)

where $\hat{F}(s,u,t)$ is a short-hand notation for the preceding expressions. To ensure the correct limit for k=0, the coefficients need to add up to unity, $a_1+a_2+a_3=1$. The most democratic choice is $a_1=a_2=a_3=1/3$, of course. The previous choice A, in Eq. (14), is subsumed here with $a_1=a_2$ $=0, a_3=1$. In the subsequent applications, we will use this general form for \hat{F} and allow the coefficients a_i to be free parameters.

While the equations given above for pion photoproduction apply only at the tree level (in the spirit of Ref. [3]), recent models have gone much further [7–9] and have included the pion final-state interaction by iterating the full scattering equation. Such a treatment would go beyond the scope of the present paper. However, for kaon photoproducton, most recent computations [10–12] use tree-level diagrams only and adjust the coupling constants to reproduce the data. None of these calculations have included a hadronic form factor until now, even though preliminary results [13] indicate that the presence of such a form factor greatly influences the range of the extracted coupling constants. We therefore test here this particular implementation of gauge invariance by considering the two kaon photoproduction reactions $\gamma p \rightarrow \Lambda K^+$ and $\gamma p \rightarrow \Sigma^0 K^+$.

For both reactions, one can simply take over Eq. (6) and replace the pion by K^+ and the neutron by the respective hyperon. For $\gamma p \rightarrow \Lambda K^+$ one has

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$$\hat{A}_{\Lambda 1} = \frac{g_{K\Lambda N}e}{s - m^2} (1 + \kappa_p) F_{\Lambda 1}(s) + \frac{g_{K\Lambda N}e}{u - m_{\Lambda}^2} \kappa_{\Lambda} F_{\Lambda 2}(u),$$
(17a)

$$\hat{A}_{\Lambda 2} = \frac{2g_{K\Lambda N}e}{(s-m^2)(t-\mu^2)} \hat{F}_{\Lambda}(s,u,t),$$
(17b)

$$\hat{A}_{\Lambda 3} = \frac{g_{K\Lambda N}e}{s-m^2} \frac{\kappa_p}{m} F_{\Lambda 1}(s), \qquad (17c)$$

$$\hat{A}_{\Lambda 4} = \frac{g_{K\Lambda N}e}{u - m_{\Lambda}^2} \frac{\kappa_{\Lambda}}{m_{\Lambda}} F_{\Lambda 2}(u), \qquad (17d)$$

where F_{Λ} is the ΛKp form factor, with coupling constant $g_{K\Lambda N}$, and m_{Λ} is the Λ mass; κ_{Λ} is the corresponding anomalous magnetic moment. For the second reaction, $\gamma p \rightarrow \Sigma^0 K^+$, one replaces Λ by Σ .

Clearly, a phenomenological description of the (γ, K) processes has to include resonance terms. However, the quality of the data has not yet permitted a clear identification of the relevant resonances in the reaction mechanism and, consequently, models with different resonances can all achieve a satisfactory description of the data [10-12]. These resonance terms are all gauge invariant independently and, therefore, do not depend on different prescriptions for restoring gauge invariance. For our empirical studies below we choose the same set of resonances as in Refs. [10,13], namely, the K^* in the t channel, and the $S_{11}(1650)$ and the $P_{11}(1710)$ states in the *s* channel. For Σ production, we also allow the $S_{31}(1900)$ and the $P_{31}(1910)$ state to contribute. We do not make any claims that this selection is unique or correct at the present time, but rather that it leads to a reasonable description of the (γ, K) processes and allows us to draw qualitative conclusions about the magnitude of the Born coupling constants. In the case of $p(\gamma, K^+)\Lambda$, separate coupled-channels analyses [14,15] found the $S_{11}(1650)$ and the $P_{11}(1710)$ states to play important roles. For simplicity, all resonances are multiplied here with the same hadronic form factor.

For the numerical evaluation of Eqs. (17), we choose covariant vertex parametrizations without any singularities on the real axis. For a baryon with mass m and four-momentum p decaying (virtually) into a baryon with mass m' and fourmomentum p' and a meson with mass μ and momentum p-p', the general vertex may be written as

$$F = f[p^2, p'^2, (p - p')^2], \qquad (18)$$

with the normalization $f(m^2, m'^2, \mu^2) = 1$. When applied to $\gamma p \rightarrow \Lambda K^+$ and $\gamma p \rightarrow \Sigma^0 K^+$, the masses *m* and μ appearing in Eq. (8) are always the nucleon and kaon masses, respectively, whereas $m' = m_{\Lambda}$ for the first and $m' = m_{\Sigma}$ for the second reaction. The vertex parametrization we employ here is of the form

$$f[p'^2, p^2, (p-p')^2] = \Lambda^4 / (\Lambda^4 + \eta^4), \qquad (19)$$

where Λ is some cutoff parameter, and

$$\eta^{4} = (p^{2} - m^{2})^{2} + (p'^{2} - m'^{2})^{2} + [(p - p')^{2} - \mu^{2}]^{2}.$$
(20)

In the nonrelativistic limit, this form reduces to the usual monopole form, depending on the squared three-momentum of the exchanged particle. For the three cases of Eq. (8), since two of the three vertex legs are always on-shell in the present applications, this translates into

$$F_1 = \Lambda^4 / [\Lambda^4 + (s - m^2)^2], \qquad (21a)$$

$$F_2 = \Lambda^4 / [\Lambda^4 + (u - m'^2)^2], \qquad (21b)$$

$$F_3 = \Lambda^4 / [\Lambda^4 + (t - \mu^2)^2], \qquad (21c)$$

which is, therefore, effectively the same as the form factors used in Ref. [16].

In the discussion of our numerical results, we focus our attention on the magnitude of the leading Born coupling constants $g_{K\Lambda N}$ and $g_{K\Sigma N}$. In contrast to the well-known πNN coupling constant, there are serious discrepancies between values for the KYN coupling constants extracted from electromagnetic reactions [12,13] and those from hadronic processes [17,18] which tend to be closer to accepted SU(3) values. If the leading coupling constants $g_{K\Lambda N}/\sqrt{4\pi}$ and $g_{K\Sigma N}/\sqrt{4\pi}$ are not allowed to vary freely and are fixed (close to what is obtained from hadronic reactions [18]) at reasonable SU(3) values of -3.8 and 1.2, respectively, the χ^2 per data point obtained in our model without hadronic form factors for the (γ, K) reactions comes out to be 55.8. If, on the other hand, the two couplings are allowed to vary freely, one obtains a much improved value of $\chi^2/N=3.33$ for $g_{K\Lambda N}/\sqrt{4\pi} = -1.89$ and $g_{K\Sigma N}/\sqrt{4\pi} = -0.37$. This clearly indicates that either there is a very large amount of SU(3) symmetry breaking or that important physics has been left out in the extraction of coupling constants from the (γ, K) processes. In this study, we advocate the second position and demonstrate that the inclusion of structure at the



FIG. 2. Values of χ^2/N (where N is the number of data points) and cutoff parameter Λ for coupling constant values of $-g_{K\Lambda N}/\sqrt{4\pi}=3.4$, 3.8, and 4.2. The solid lines connect results obtained with Haberzettl's gauge formalism [4] and the dashed lines pertain to Ohta's [2] prescription.

hadronic vertex permits an adequate description of kaon photoproduction with couplings close to the SU(3) values, provided one uses the gauge procedure of Ref. [4].

The main numerical results of our investigation are summarized in Fig. 2. The upper panel shows χ^2 per data point as a function of $g_{K\Lambda N}/\sqrt{4\pi}$ for the two different gauge prescriptions by Ohta and Haberzettl. At a value of $g_{K\Lambda N}/\sqrt{4\pi} = -3.4$, the χ^2 obtained with Ohta's method is almost a factor of 2 larger, compared to the method by Haberzettl. With an increasing coupling constant the Ohta result rises sharply, leading to an unacceptably large χ^2 of 32.2 for $g_{K\Lambda N}/\sqrt{4\pi} = -4.2$. On the other hand, using the procedure of Ref. [4] keeps the χ^2 more or less constant. This dramatic difference between the two gauge prescriptions can easily be understood from Eq. (11) and the discussion following that equation. Ohta's method provides no possibility to suppress electric contributions since the form factor for this term is unity [cf. Eqs. (6b) and (11)]. In contrast, the method by Haberzettl allows for a hadronic form factor in this term as well.

The lower panel of Fig. 2 sheds additional light on the suppression mechanism. In the fits we performed the cutoff Λ of the form factor, cf. Eq. (19), was allowed to vary freely. In the case of Haberzettl's method, the cutoff decreases with increasing $K\Lambda N$ coupling constants, leaving the magnitude of the *effective* coupling, i.e., the coupling constant times form factor, roughly constant. Again, since Ohta's method does not involve form factors for electric contributions no such compensation is possible there, and as a consequence the cutoff remains insensitive to the coupling constant.

In obtaining Fig. 2 we have kept $g_{K\Sigma N}$ fixed at the value $g_{K\Sigma N}/\sqrt{4\pi}=1.2$. We have checked that varying the $K\Sigma N$ coupling between 1.0 and 1.4 leads only to very small changes. Furthermore, we allowed the coefficients a_i of Eq.

(16) to be free fit parameters. As it turns out, the fit only allows nonzero *s*- and *t*-channel contributions (i.e., a_2 is essentially zero), with a somewhat larger a_3 value (corresponding to an enhancement of the *t* channel), which of course is entirely consistent with the fact that Eq. (17b) contains only *s* and *t* channels.

We do not show the fitted resonance couplings here since we do not regard them as very realistic at this point. We emphasize again the qualitative nature of our findings, and clearly a more sophisticated calculation is required in order to obtain a quantitative description of the (γ, K) processes.

In summary, we have applied here the general gaugeinvariance restoration method proposed by Haberzettl to the specific example of pseudoscalar photoproduction at tree level. Using a phenomenological Born plus resonance model we have compared the procedures by Ohta [2] and Haberzettl 4 for kaon photoproduction. We found the latter to be superior since it can provide a reasonable description of the data using values for the leading couplings constants close to the SU(3) values. Such couplings cannot be accommodated in Ohta's method because of the absence of a hadronic form factor in the electric current contribution. The main purpose for measuring the meson photoproduction in the 1-2 GeV region is the study of resonances. In order to unambiguously separate the resonance from background contributions, it is imperative that background terms be able to account for hadronic structure while properly maintaining gauge invariance. As the present findings indicate, Ohta's prescription seems to be too restrictive in this respect, whereas the method put forward in Ref. [4] seems well capable of providing this facility. This favorable conclusion regarding Haberzettl's method is corroborated also by the findings of Ref. [19].

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