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# Backward elastic $p$-$^3$He scattering and high momentum components of the $^3$He wave function

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It is shown that because of a dominance of $np$-pair transfer mechanisms of backward elastic $p$-$^3$He scattering for incident proton kinetic energies $T_p>1$ GeV the cross section of this process is defined mainly by the values of the Faddeev component of the wave function of the $^3$He nucleus, $\varphi^{23}(\mathbf{q}_{23},\mathbf{p}_1)$, at high relative momenta $q_{23}>0.6$ GeV/$c$ of the $NN$ pair in the $^1S_0$ state and at low spectator momenta $p_1\sim 0-0.2$ GeV/$c$. [S0556-2813(98)50907-7]

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The cross section of backward elastic $p$-$^3$He scattering at the kinetic energy of incident proton $T_p>1$ GeV displays three remarkable peculiarities [1,2]: (i) In the Born approximation only one mechanism of the process $p$-$^3\mathrm{He}\rightarrow{}^3\mathrm{He}p$ dominates, it is the so-called sequential transfer (ST) of the noninteracting *np* pair. The contribution from the mechanisms of nonsequential transfer (NST), interacting *np*-pair transfer (IPT) and deuteron exchange is negligible. The heavy particle stripping mechanism was also investigated in Refs. [3–5] and found to be important at back angles for $T_p\leqslant 0.6$ GeV. However the phenomenological $^3$He wave functions restricted to the two-body configuration, which does not permit ST mechanisms, were used in that analysis. (ii) The most important role in the Faddeev wave function $\varphi^{23}(\mathbf{q}_{23},\mathbf{p}_1)$ of $^3$He plays the channel with the orbital momentum $L=0$, spin $S=0$, isotopic spin $T=1$ of two nucleons with numbers 2 and 3 and the orbital momentum $l=0$ of nucleon spectators with number 1 (it corresponds to $\nu=1$ in the notation of Ref. [6]). If this channel is excluded from the full wave function $\Psi=\varphi^{23}+\varphi^{31}+\varphi^{12}$, the cross section falls by several orders of magnitude. (iii) Rescatterings in the initial and final states decrease the cross section at $\theta_{\mathrm{c.m.}}=180°$ considerably in comparison with the Born approximation and make it agree satisfactorily with the available experimental data [7] for $T_p>0.9$ GeV.

Because of this evident connection between the structure of the $^3$He nucleus and the dominating mechanism one can hope to obtain information about high momentum components of the $^3$He wave function from the cross section of the $p$-$^3\mathrm{He}\rightarrow{}^3\mathrm{He}p$ process. However, in Refs. [1,2] it was mentioned that the $D$ components of the $^3$He wave function are of surprisingly minor importance in the process under discussion at $T_p>1$ GeV. Moreover, relativistic effects estimated in Ref. [2] at $T_p\sim 1$ GeV by means of substituting the relativistic arguments into the $^3$He wave function instead of the nonrelativistic ones give rather small contributions to the cross section. For this reason, in Refs. [1,2] it was concluded that the sensitivity of the $p$-$^3\mathrm{He}\rightarrow{}^3\mathrm{He}p$ cross section to the high momentum components of the $^3$He wave function is rather weak in spite of high momenta transferred at $T_p>1$ GeV. Moreover, as was found in [7], the role of the triangular diagram of one-pion exchange (OPE) with the subprocess $pd\rightarrow{}^3\mathrm{He}\pi^0$ related to the $\Delta$- and double $\Delta$-excitation is in qualitative agreement with the absolute value of the experimental cross section at $T_p>0.5$ GeV.

In the present work it is shown that the absolute value of the $p$-$^3\mathrm{He}\rightarrow{}^3\mathrm{He}p$ cross section at $\theta_{\mathrm{c.m.}}=180°$ and $T_p>1$ GeV is directly related to the high momentum components of the Faddeev $S$-wave function of $^3$He, $\varphi^{23}(\mathbf{q}_{23},\mathbf{p}_1)$, respecting the relative momentum $\mathbf{q}_{23}$ whereas rather low values of the ''spectator'' momentum $\mathbf{p}_1$ are involved in the amplitude of this process. It is shown also, that because of rescatterings in the initial and final states the contribution of the OPE mechanism is one order of magnitude lower in comparison with the experimental data.

In the Born approximation the amplitude of transfer of two nucleons with numbers 2 and 3 in the process $0+\{123\}\rightarrow 1+\{023\}$ (*et id.* $p$-$^3\mathrm{He}\rightarrow{}^3\mathrm{He}p$) can be written as [1,2]

$$T_B=6(2\pi)^{-3}\int d^3q_{23}L_{23}(q_{23},p_1)\chi^+_{p'}(1)\times\{\varphi_f^{23^+}(0;23)\varphi_i^{31}(2;31)+\varphi_f^{02^+}(3;02)\varphi_i^{31}(2;31)+\varphi_f^{30^+}(2;30)\varphi_i^{31}(2;31)\}\chi_p(0), \tag{1}$$

where $\varphi^{ij}(k;ij)=\varphi^{ij}(\mathbf{q}_{ij},\mathbf{p}_k)$ is the Faddeev component of the wave function of the bound state $\{ijk\}$, $\chi_p(\chi_{p'})$ is the spin-isotopic spin wave function of the incident (final) proton; $L_{23}=\varepsilon+\mathbf{q}_{23}^2/m+3\mathbf{p}_1^2/4m$, $m$ is the nucleon mass, $\varepsilon$ is the $^3$He binding energy. The subscripts $i$ and $f$ in Eq. (1) refer to the initial and final nucleus, respectively. The terms $\varphi_f^{23^+}\varphi_i^{31}$, $\varphi_f^{02^+}\varphi_i^{31}$, $\varphi_f^{30^+}\varphi_i^{31}$ correspond to the IPT, ST, and NST mechanisms, respectively. In the explicit form the ST mechanism has the following structure of arguments:

$$\varphi_f^{02^+}\varphi_i^{31}=\varphi_f^{02^+}\left(\mathbf{q}_{02}=-\tfrac{1}{2}\mathbf{q}_{23}-\tfrac{3}{4}\mathbf{Q}_0,\mathbf{p}_3=\mathbf{q}_{23}-\tfrac{1}{2}\mathbf{Q}_0\right)\times\varphi_i^{31}\left(\mathbf{q}_{31}=-\tfrac{1}{2}\mathbf{q}_{23}+\tfrac{3}{4}\mathbf{Q}_1,\mathbf{p}_2=-\mathbf{q}_{23}-\tfrac{1}{2}\mathbf{Q}_1\right), \tag{2}$$

where $\mathbf{Q}_0$ ($\mathbf{Q}_1$) is the momentum of incident (final) proton in the center-of-mass system (c.m.s) of the final (initial) nucleus $^3$He. As was noted in [2], at the scattering angle $\theta_{\mathrm{c.m.}}=180°$ two of four momenta in Eq. (2) can simultaneously become equal to zero at an integration over $\mathbf{q}_{23}$. On the contrary, in the corresponding formulas for the IPT and NST

 

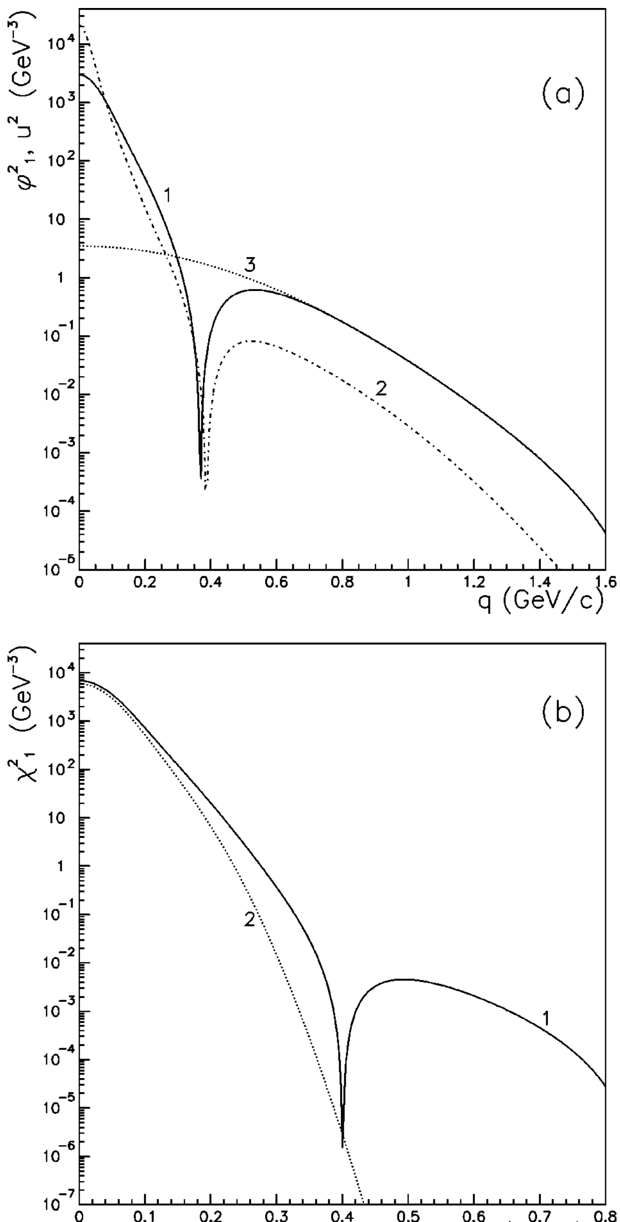


FIG. 1. The square of functions $\varphi_1(q)$, $\chi_1(q)$ from Ref. [13], the $S$ component of the deuteron wave function $u(q)$ from Ref. [14], and functions $\tilde{\varphi}_1(q)$ and $\tilde{\chi}_1(q)$ defined in the text. (a) curve 1: $\varphi_1^2(q)$; curve 2: $u^2(q)$; curve 3: $\tilde{\varphi}_1^2(q)$. (b) curve 1: $\chi_1^2(q)$; curve 2 $\tilde{\chi}_1^2(q)$.

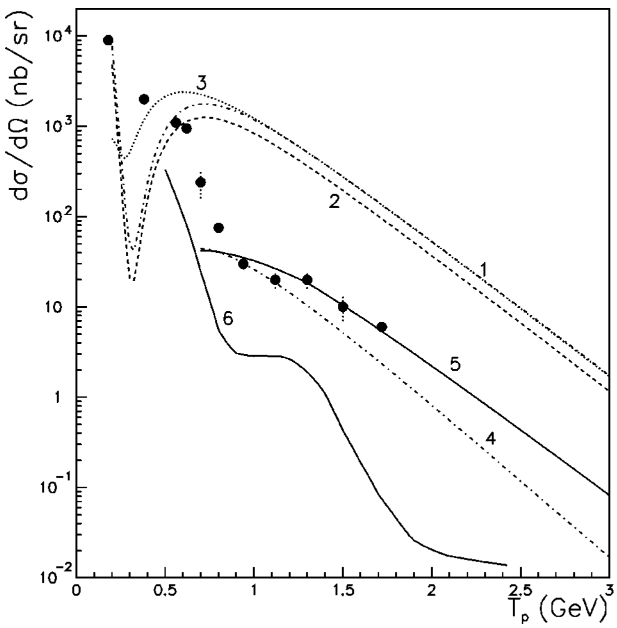


FIG. 2. The differential cross section of $p$-$^3$He elastic scattering at $\theta_{\rm c.m.}=180°$ as a function of incident proton kinetic energy $T_p$. Curves 1–4 show the results of calculations in the Born approximation for amplitude in Eq. (1): curve 1: with the $^3$He wave function from [13]; curve 2: with $\tilde{\varphi}_1(q_{23})$ instead of $\varphi_1(q_{23})$; curve 3: with $\tilde{\chi}_1(p_1)$ instead of $\chi_1(p_1)$; curve 4: with the deuteron w.f. $u(q_{23})$ instead of $\varphi_1(q_{23})$ and $\varphi_2(q_{23})$. The results obtained with the allowance for rescatterings in the initial and final states are shown by curves 5 and 6: curve 5: the *np*-transfer mechanism with the $^3$He wave function from [13]; curve 6: OPE. The experimental points are taken from Ref. [7].

mechanisms only one argument can be equal to zero while the other three have large values $\sim|\mathbf{Q}_1|=|\mathbf{Q}_0|$ This makes the ST term dominant in Eq. (1). Indeed, the ST mechanism takes place only if the channels with the isotopic spin $T=1$ of the pair of nucleons $\{ij\}$ are included in the component $\varphi^{ij}(ij;k)$ either in the initial or final state. It is the direct consequence of the fact that the ST diagram either starts with or ends in the *pp* interaction. The $^3$He wave function from Ref. [6] contains only one such channel ($\nu=1$), namely, with the $^1S_0$ state of the *NN* pair. In the *S*-wave approximation for the $^3$He wave function the cross section decreases by 5–6 orders of magnitude for $T_p>1$ GeV if the channel $\nu=1$ is excluded [8]. The channels with $\nu\neq1$ corresponding to the isotopic spin $T=0$ of the *NN* pair (in particular, the $D$ components) can enter the ST amplitude only in combination with the channel $\nu=1$. For this reason the role of those channels is not so significant.

An obvious modification of formalism of the triangular OPE diagram from Refs. [9,10] is used here for the OPE amplitude. The cross section of the $p$-$^3$He$\rightarrow$ $^3$He$p$ process is expressed through the cross section of the reaction $pd\rightarrow{}^3$He$\pi^0$, which is taken here from the experimental data [11]. The overlap integral of $^3$He and deuteron wave functions, $\langle{}^3\mathrm{He}|d,p\rangle$, is taken from [12]. Rescatterings in the initial and final states for the OPE mechanism are taken here in the line of work [2] on the basis of the Glauber-Sitenko approximation.

Numerical calculations for the $np$-pair transfer mechanism are performed here on the basis of the formalism described in [1,2] using the $^3$He wave function obtained in Ref. [6] from the solution of Faddeev equations in momentum space for the Reid soft core (RSC) interaction potential between nucleons in the $^1S_0$ and $^3S_1$-$^3D_1$ states. The separable analytical parametrization for the $^3$He wave function is used here which has the following form in terms of the notation [13]:

$$\phi_\nu=n_\nu\varphi_\nu(q_{23})\chi_\nu(p_1). \qquad (3)$$

The square of the functions $\varphi_\nu(q),\chi_\nu(q)$ and the $S$ component of the deuteron wave function, $u(q)$, for the RSC potential [14] are shown in Fig. 1. The calculated differential cross section is shown in Fig. 2 in comparison with the experimental data [7].

The numerical results demonstrate the following important features of the process in question. First, the ST mechanism involves the high momentum components of the functions $\varphi_\nu(q_{23})$ for the *S*-wave states. The $^3$He wave function in the channel $\nu=1$ is probed at high momenta $\mathbf{q}_{23}>0.6$ GeV when the cross section is measured at $T_p>1$ GeV. To show it, in Fig. 1(a) we present a part of the function $\varphi_1(q_{23})$, denoted as $\tilde{\varphi}_1$, which coincides with $\varphi_1(q_{23})$ for $q_{23}>0.6$ GeV/$c$ and differs considerably from it for smaller momenta $q_{23}<0.5$ GeV/$c$. In Fig. 1(b) we also show a part of the function $\chi_1(p_1)$, denoted as $\tilde{\chi}_1$, which is very close to the total function $\chi_1(p_1)$ at small spectator momenta $p_1\sim 0-0.2$ GeV/$c$ and is negligible for $p_1>0.2$ GeV/$c$. The cross section calculated with these two parts instead of the full functions $\varphi_1$ and $\chi_1$ is shown in Fig. 2 by curves 2 and 3, respectively. One can see that these curves are very close to the total result obtained with the full functions $\varphi_1(q_{23})$ and $\chi_1(p_1)$. In contrast, it can be shown that the cross section calculated with the complementary parts $\varphi_1-\tilde{\varphi}_1$ and $\chi_1-\tilde{\chi}_1$ is 5–6 orders of magnitude smaller.

Second, the above result also displays that the ST mechanism involves rather low ''spectator'' momenta $p_1\sim 0-0.2$ GeV/$c$ in the function $\chi_\nu(p_1)$, which makes this mechanism dominant. The qualitative explanation for these results follows. One can find from Eq. (2), that for $\mathbf{Q}_1=-\mathbf{Q}_0$ (*et id.* $\theta_{\rm c.m.}=180°$) the equations $\mathbf{q}_{31}=\mathbf{q}_{02}$ and $\mathbf{p}_2=-\mathbf{p}_3$ are satisfied. Consequently, the main contribution in the integral over $d\mathbf{q}_{23}$ in Eq. (1) gives the region $|\mathbf{p}_2|=|\mathbf{p}_3|\sim 0$, in which $|\mathbf{q}_{31}|=|\mathbf{q}_{02}|\sim Q_1$. On the contrary, the region of $|\mathbf{q}_{31}|=|\mathbf{q}_{02}|\sim 0$ corresponds to $|\mathbf{p}_2|=|\mathbf{p}_3|\sim 2Q_1$ and plays an insignificant role since for $T_p>1$ GeV the momentum $Q_1$ is large, $Q_1>0.6$ GeV.[1]

Third, we have found numerically, that the contribution of the OPE mechanism without taking into account rescatterings is in agreement with the experimental data at $T_p=0.5-1.3$ GeV, but is by a factor $\sim 20-30$ smaller in comparison with the ST contribution in the Born approximation for $T_p>0.8$ GeV. After the allowance for rescatterings in the initial and final states the contribution of the OPE mechanism decreases by one order of magnitude and becomes considerably lower than the experimental data (Fig. 2). Probably, the cross section of the $p$-$^3\mathrm{He}\to{}^3\mathrm{He}p$ process for $T_p<1$ GeV is defined mainly by the multistep *pN*-scattering mechanisms discussed in Refs. [15,16] and heavy-particle stripping mechanism [3–5] also. We stress that the high momentum components of the functions $\varphi_\nu$ in Eq. (3) play the most important role in the competition between the OPE and ST mechanisms. One can see from Fig. 1(a), that the high momentum component in the function $\varphi_1(q)$ is richer in comparison with the deuteron wave function $u(q)$, especially for $q>0.5$ GeV/$c$. Actually, when substituting the *S* component of the deuteron wave function $u(q)$ into Eq. (3) instead of the function $\varphi_\nu(q_{23})$ for $\nu=1$ and 2 one finds the *np*-transfer cross section decreasing by a factor of $\sim 40$ (curve 4 in Fig. 2) and becoming comparable in absolute value with the OPE contribution in the Born approximation. Note in this connection that in the $pd\to dp$ process the contribution of the neutron exchange mechanism in the Born approximaion is not dominating [17] for $T_p>1$ GeV and comparable with the OPE mechanism [10,18].

As an additional test of the $np$-transfer mechanism the spin-spin correlation parameter $\Sigma$ is calculated here for the process with a polarized incident proton and a nucleus. This parameter is defined as

$$\Sigma=\frac{d\sigma(\uparrow\uparrow)/d\Omega-d\sigma(\uparrow\downarrow)/d\Omega}{d\sigma(\uparrow\uparrow)/d\Omega+d\sigma(\uparrow\downarrow)/d\Omega}, \tag{4}$$

where $d\sigma(\uparrow\uparrow)/d\Omega$ and $d\sigma(\uparrow\downarrow)/d\Omega$ are the cross sections for parallel and antiparallel spins of colliding particles, respectively. The numerical calculations with allowance for two channels $\nu=1$ and $\nu=2$ in the $^3$He wave function show that at $\theta_{\rm c.m.}=180°$ and $T_p\sim 1-2.5$ GeV the value $\Sigma$ is $\sim 0.1-0.15$ independently of the initial energy.

In conclusion, the remarkable sensitivity of the cross section of backward elastic $p$-$^3$He scattering to the high momentum components of the $^3$He wave function in the *S*-wave channel is found for energies above 1 GeV. The dominance of nucleon degrees of freedom is demonstrated. Since the mechanism of the *np*-pair transfer describes the available experimental data in the interval of incident energies 0.9–1.7 GeV satisfactorily, there is a reason to measure the cross section at higher energies in order to enlighten the validity of phenomenological *NN* potentials in describing the structure of lightest nuclei at high relative momenta of nucleons.

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[1] The replacement of the nonrelativistic momenta $Q_1^{nr}=Q_0^{nr}$ by the relativistic ones $Q_1^{\rm rel}=Q_0^{\rm rel}$ (where $Q^{\rm rel}<Q^{nr}$) practically does not change the ST cross section at energies $T_p=0.4-1.2$ GeV as was found in [2]. However, for energies $T_p>1$ GeV such a replacement becomes important and increases the cross section. Therefore, in a complete future analysis of this process one should take into account relativistic effects in a consistent way.

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