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Faddeev-type calculation of ηd threshold scattering

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The scattering length for the η-meson collision with deuteron is calculated on the basis of rigorous few-body equations (AGS) for various ηN input. The results obtained strongly support the existence of a resonance or quasibound state close to the ηd threshold.
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The production of η mesons and their collisions with nuclei have been studied experimentally and theoretically with increasing interest during the last years. To a large extent this is motivated by the fundamental questions of charge-symmetry breaking and the breakdown of the Okubo-Zweig-Iizuka rule. Another relevant question concerns the possible formation of η-nucleus quasibound states.

In many respects the η meson is similar to the π^0 meson despite being four times heavier. Both are neutral and spinless, have almost the same lifetime ($\sim 10^{-18}$ sec), and are the only mesons that have a high probability of pure radiative decay, that is, their quarks can annihilate into on-shell photons. However, when they are involved in nuclear reactions they behave rather differently. The S_{11}-resonance $N^*(1535)$, for instance, is formed in both πN and ηN systems, but at different collision energies,

$$E^{res}_{\pi N}(S_{11}) = 1535 \ \text{MeV} - m_N - m_\pi \approx 458 \ \text{MeV},$$

$$E^{res}_{\eta N}(S_{11}) = 1535 \ \text{MeV} - m_N - m_\eta \approx 49 \ \text{MeV}.$$

Thus, due to the large mass of the η meson (547.45 MeV), this resonance is very close to the ηN threshold. Furthermore it is very broad, with $\Gamma \approx 150$ MeV, covering the whole low energy ηN region. As a result the interaction of nucleons with η mesons in this region, where the S-wave interaction dominates, is much stronger than with pions.

After its creation the $N^*(1535)$ resonance decays into ηN and πN channels with equally high probabilities [1]

$$N^*(1535) \rightarrow \begin{cases} N + \eta & (35-55\%), \\ N + \pi & (35-55\%), \\ \text{other decays} & (\leqslant 10\%), \end{cases} \qquad (1)$$

which indicates that the ηN and πN interactions are to be treated by a coupled channel analysis. The resulting ηN interaction, obtained in this way, turned out to be attractive [2]. This raises the question of whether the attraction is strong enough to support η-nucleus quasibound states. Let us recall in this context that, because of their short lifetime, η mesons can only be observed in final states of certain nuclear reactions. Within nuclei they are considered to undergo multiple absorption and production processes via the S_{11} resonance, with a final transition into pions. Such quasibound states therefore would be of considerable interest for studying η-meson properties in more detail.

For the calculation of these states various model treatments were employed, among them the optical potential method [3–6], the Green's function method [7], and the modified multiple scattering theory [8]. Calculations, based on the exact Alt-Grassberger-Sandhas (AGS) equations [9], for the ηd system were also made in Ref. [10] in the early 1990s.

The predictions concerning the possibility of η-mesic nucleus formation are very diverse. One obvious reason for such a diversity is the poor knowledge of the ηN forces. Another reason comes from the differences among the employed approximations, some of which might be detrimental in view of the resonant character of the ηN dynamics and the delicacy of the quasibound state problem. As was shown in Ref. [10] this problem cannot be adequately addressed by a meson-nucleus optical model or any low-order perturbation theory.

Among the approximate approaches the few-body dynamics of the η-nucleus systems was most explicitly treated in our previous calculations [11–15] based on the finite-rank approximation (FRA) of the nuclear Hamiltonians. The shortcoming of these calculations is the neglect of excitations of the nuclear ground states. This appears justified in the $\eta ^4$He and possibly in the η-triton (^{3}He) case, but is quite questionable in η-deuteron collisions.

In the present paper we therefore treat the η-deuteron system on the basis of the exact few-body equations (AGS). Both the NN and ηN amplitudes entering them are chosen in separable form, which reduces the dimension of these equations to one. The same ηN amplitude has been used in the FRA calculations. This allows us to compare our present calculations of the ηd scattering length with the previous approximate results, i.e., to examine the effect of the neglect of nuclear excitations employed in the FRA. It turns out that the discrepancies are not large for most of the ηN parameter sets. This indicates that the conclusions drawn in our previous investigation [11–15] were already fairly reliable and should be even more reliable in the less sensitive η-triton or $\eta ^4$He cases.

The η-deuteron scattering length is the value of the elastic scattering amplitude

$$f(\mathbf{p}_1',\mathbf{p}_1;z)=-(2\pi)^2\mu_1\langle\mathbf{p}_1';\psi_d|U_{11}(z)|\mathbf{p}_1;\psi_d\rangle \qquad (2)$$

at zero collision energy. Here the subscript 1 labels the $\eta(NN)$ partition and the η-deuteron channel whose asymptotic states are normalized as

$$\langle\mathbf{p}_1';\psi_d|\mathbf{p}_1;\psi_d\rangle=\delta(\mathbf{p}_1'-\mathbf{p}_1).$$

The transition operator U_{11} obeys the system of AGS equations

$$U_{ij}(z)=(1-\delta_{ij})g_0^{-1}(z)+\sum_{k=1}^{3}(1-\delta_{ik})t_k(z)g_0(z)U_{kj}(z),$$
$$i,j=1,2,3, \qquad (3)$$

where g_0 is the free Green's function in the three-body space, and t_i the two-body T matrix for the ith pair $(t_1=t_{NN})$. For both $t_{\eta N}$ and t_{NN} we used one-term separable forms

$$t_i(z)=|\chi_i\rangle\tau_i(z)\langle\chi_i|. \qquad (4)$$

For the NN subsystem Eq. (4) implies that the asymptotic wave function is related to the form-factor $|\chi_1\rangle$ according to

$$|\mathbf{p}_1;\psi_d\rangle=g_0(z)|\chi_1\rangle|\mathbf{p}_1\rangle, \qquad (5)$$

at $z=p_1^2/2\mu_1+E_d$ with E_d being the deuteron energy. Due to Eqs. (4) and (5) the scattering amplitude (2) can be rewritten as

$$f(\mathbf{p}_1',\mathbf{p}_1;z)=-(2\pi)^2\mu_1\langle\mathbf{p}_1'|X_{11}(z)|\mathbf{p}_1\rangle, \qquad (6)$$

where the operators X_{ij}, defined as

$$X_{ij}(z)=\langle\chi_i|g_0(z)U_{ij}(z)g_0(z)|\chi_j\rangle,$$

obey the system of equations

$$X_{ij}(z)=Z_{ij}(z)+\sum_{k=1}^{3}Z_{ik}(z)\tau_k\left(z-\frac{p_k^2}{2\mu_k}\right)X_{kj}(z) \qquad (7)$$

with

$$Z_{ij}(z)=(1-\delta_{ij})\langle\chi_i|g_0(z)|\chi_j\rangle.$$

The identity of the nucleons implies that $X_{31}=X_{21}$, $\tau_3=\tau_2$, and $Z_{31}=Z_{21}$, which reduces the system (7) to two coupled equations:

$$\begin{aligned}X_{11}(z)&=2Z_{21}(z)\tau_2\left(z-\frac{p_2^2}{2\mu_2}\right)X_{21}(z),\\ X_{21}(z)&=Z_{21}(z)+Z_{21}(z)\tau_1\left(z-\frac{p_1^2}{2\mu_1}\right)X_{11}(z)+Z_{23}(z)\tau_2\left(z-\frac{p_2^2}{2\mu_2}\right)X_{21}(z).\end{aligned} \qquad (8)$$

Eventually, after making the S-wave projection of the matrix elements $\langle\mathbf{p}_i'|X_{ij}|\mathbf{p}_j\rangle$ and $\langle\mathbf{p}_i'|Z_{ij}|\mathbf{p}_j\rangle$, we end up with one-dimensional integral equations that can be solved numerically by replacing the integrals by Gaussian sums.

The S-wave separable nucleon-nucleon and η-nucleon T matrices of the form (4) were adopted from Refs. [16] and [11]. However the parameters originally proposed in Ref. [16] for the T matrix

$$t_{NN}(p',p;z)=\frac{1}{4\pi}v(p')\frac{A(z)}{1-A(z)B(z)}v(p),$$

$$v(p)=\frac{\gamma}{\beta^2+p^2},$$

$$A(z)=-\mathrm{tgh}\left(1-\frac{z}{E_c}\right),$$

$$B(z)=\int_0^\infty\frac{p^2v^2(p)}{z-p^2/m_N+i\varepsilon}\,dp,$$

were slightly modified to $E_c=0.816$ fm^{-1}, $\beta=1.604$ fm^{-1}, and $\gamma^2=1.883$ fm^{-2}, which correspond to more recent val-

TABLE I. Comparison of ηd scattering lengths (in fm), obtained using the AGS and FRA methods, for nine combinations of the parameters of the ηN potential.

	$\alpha=2.357$ (fm^{-1})	$\alpha=3.316$ (fm^{-1})	$\alpha=7.617$ (fm^{-1})	$a_{\eta N}$ (fm)
AGS	$0.71+i0.79$	$0.71+i0.84$	$0.71+i0.92$	$0.27+i0.22$
FRA	$0.66+i0.82$	$0.65+i0.85$	$0.62+i0.89$	
AGS	$0.79+i0.68$	$0.81+i0.73$	$0.83+i0.81$	$0.28+i0.19$
FRA	$0.75+i0.73$	$0.74+i0.76$	$0.72+i0.81$	
AGS	$1.81+i2.44$	$1.64+i2.99$	$0.75+i4.00$	$0.55+i0.30$
FRA	$1.53+i2.00$	$1.38+i2.15$	$1.14+i2.22$	

TABLE II. Results of the AGS and three different approximate calculations of $A_{\eta d}$ with $\alpha = 3.316$ fm^{-1}.

Ref.	ηN input $a_{\eta N}$ (fm)	Exact $A_{\eta d}$ (fm) AGS	Approximate $A_{\eta d}$ (fm) MST I [8]	MST II [8]	FRA
[20]	$0.25+i0.16$	$0.73+i0.56$	$0.66+i0.71$	$0.66+i0.58$	$0.65+i0.70$
[2]	$0.27+i0.22$	$0.71+i0.84$	$0.57+i0.97$	$0.64+i0.81$	$0.59+i0.96$
[21]	$0.291+i0.360$	$0.38+i1.36$	$0.17+i1.35$	$0.42+i1.25$	$0.21+i1.35$
[3]	$0.30+i0.30$	$0.61+i1.22$	$0.39+i1.28$	$0.58+i1.11$	$0.42+i1.27$
[21]	$0.430+i0.394$	$0.50+i2.07$	$0.14+i1.91$	$0.65+i1.73$	$0.24+i1.88$
[2]	$0.44+i0.30$	$1.15+i1.89$	$0.63+i1.93$	$1.01+i1.50$	$0.68+i1.86$
[20]	$0.46+i0.29$	$1.31+i1.99$	$0.72+i2.04$	$1.11+i1.54$	$0.76+i1.96$
[22]	$0.476+i0.279$	$1.49+i2.06$	$0.81+i2.15$	$1.22+i1.56$	$0.84+i2.05$
[23]	$0.51+i0.21$	$2.37+i1.77$	$1.48+i2.31$	$1.65+i1.39$	$1.38+i2.22$
[3]	$0.55+i0.30$	$1.64+i2.99$	$0.61+i2.73$	$1.40+i1.98$	$0.69+i2.51$
[21]	$0.579+i0.399$	$0.34+i3.31$	$-0.13+i2.64$	$0.93+i2.41$	$0.13+i2.52$
[24]	$0.62+i0.30$	$1.80+i4.30$	$0.36+i3.36$	$1.65+i2.41$	$0.55+i2.95$
[22]	$0.876+i0.274$	$-8.81+i4.30$	$-2.76+i4.24$	$2.42+i5.55$	$-0.67+i3.98$
[22]	$0.888+i0.274$	$-8.63+i3.49$	$-2.90+i4.12$	$2.37+i5.79$	$-0.73+i3.99$
[25]	$0.98+i0.37$	$-4.69+i1.59$	$-2.75+i2.77$	$-0.06+i6.20$	$-1.18+i3.59$

ues of the triplet NN scattering length, $a_{NN}=5.424$ fm, and the effective range $r_{NN}=1.759$ fm [17,18]. With these parameters the deuteron is bound at 2.205 MeV and has an rms radius $\sqrt{\langle r^2\rangle_d}=1.887$ fm.

Instead of treating ηN and πN as a two-channel system it is customary to describe the ηN interaction by a one-channel complex potential. The strength parameter λ of the corresponding T matrix,

$$t_{\eta N}(p',p;z)=\frac{\lambda}{(p'^2+\alpha^2)(z-E_0+i\Gamma/2)(p^2+\alpha^2)}, \quad (9)$$

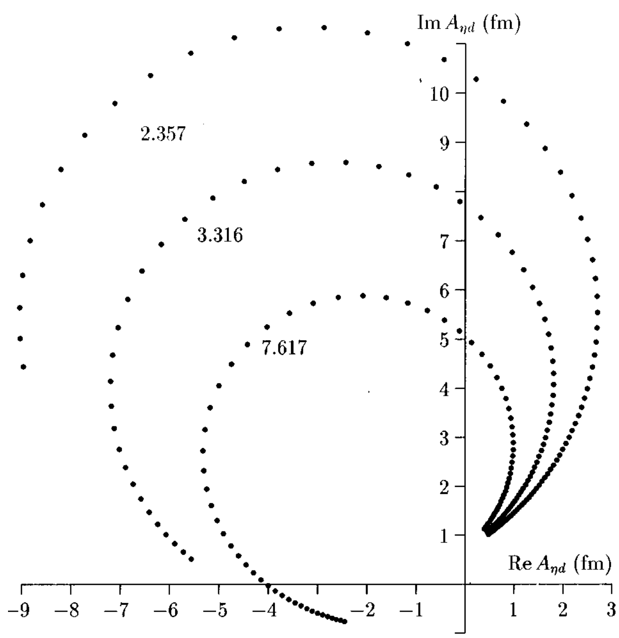

FIG. 1. The values of $A_{\eta d}$ calculated for Im $a_{\eta N}=0.30$ fm while Re $a_{\eta N}$ is changing from 0.25 fm to 1 fm with the step 0.01 fm. An increase of Re $a_{\eta N}$ moves the points in the anticlockwise direction along the curve trajectories that correspond to three choices of the range parameter α.

is chosen to reproduce the complex η-nucleon scattering length $a_{\eta N}$,

$$\lambda=\frac{\alpha^4(E_0-i\Gamma/2)}{(2\pi)^2\mu_{\eta N}}a_{\eta N}, \quad (10)$$

the imaginary part of which accounts for the flux losses into the πN channel. The range parameter α in Eq. (9) is fixed in a somewhat more complicated way (see Refs. [2,19]), while E_0 and Γ are the parameters of the S_{11} resonance [1],

$$E_0=1535 \text{ MeV}-(m_N+m_\eta), \quad \Gamma=150 \text{ MeV}.$$

The two-body scattering length $a_{\eta N}$, which defines the strength parameter λ via Eq. (10), is not accurately known. Different analyses [20] provided for $a_{\eta N}$ the values in the range

$$0.27 \text{ fm} \leqslant \text{Re } a_{\eta N} \leqslant 0.98 \text{ fm},$$

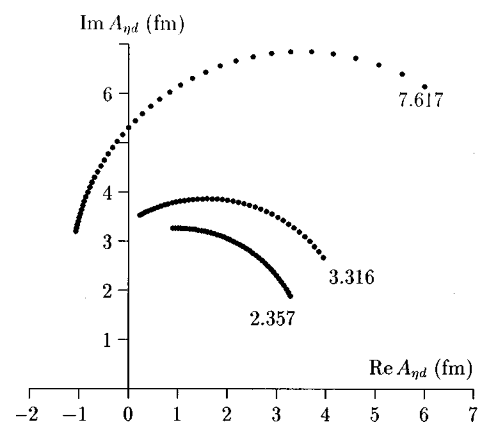

FIG. 2. The values of $A_{\eta d}$ calculated for Re $a_{\eta N}=0.60$ fm while Im $a_{\eta N}$ is changing from 0.2 fm to 0.4 fm with the step 0.005 fm. An increase of Im $a_{\eta N}$ moves the points in the anticlockwise direction along the curve trajectories that correspond to three choices of the range parameter α.

TABLE III. Convergence of the AGS and FSA results for decreasing sequence of the meson mass values. The parameters of the ηN potential are fixed by $a_{\eta N}=(0.75+i0.27)$ fm and $\alpha=2.357\ \text{fm}^{-1}$.

η mass	$A_{\eta d}^{\text{AGS}}$ (fm)	$A_{\eta d}^{\text{FSA}}$ (fm)
m_η	$3.941+i6.702$	$1.936+i3.162$
$m_\eta/2$	$1.548+i0.596$	$1.374+i0.856$
$m_\eta/3$	$0.891+i0.283$	$0.878+i0.439$
$m_\eta/4$	$0.629+i0.185$	$0.640+i0.292$
$m_\eta/5$	$0.487+i0.138$	$0.503+i0.218$
$m_\eta/10$	$0.230+i0.061$	$0.242+i0.095$
$m_\eta/20$	$0.113+i0.029$	$0.119+i0.045$
$m_\eta/30$	$0.075+i0.019$	$0.079+i0.029$
$m_\eta/40$	$0.056+i0.014$	$0.059+i0.022$
$m_\eta/50$	$0.045+i0.011$	$0.047+i0.017$

$$0.19\ \text{fm} \leq \text{Im}\ a_{\eta N} \leq 0.37\ \text{fm}. \tag{11}$$

The parameter α is also known with large uncertainty. Three different values are given in the literature, namely, $\alpha=2.357\ \text{fm}^{-1}$ [2], $\alpha=3.316\ \text{fm}^{-1}$ [19], and $\alpha=7.617\ \text{fm}^{-1}$ [2]. We therefore calculate the η-deuteron scattering length $A_{\eta d}$ for values of $a_{\eta N}$ and α covering these intervals. The results of our calculations are given in Tables I, and II, and also shown in Figs. 1 and 2.

In order to check our numerical procedure, we perform test calculations of the scattering length with decreasing values of the meson mass, and compare the results obtained with the corresponding scattering lengths given by

$$A_{\eta d}^{\text{FSA}}=-\frac{\mu}{\pi\alpha^4}\int_0^\infty\left\{\frac{\mu}{\pi\alpha^4 r}\left[r+\frac{e^{-\alpha r}}{2\alpha}(3+\alpha r)-\frac{3}{2\alpha}\right]-\frac{1}{\lambda}\left(E_0-\frac{i}{2}\Gamma\right)\right\}^{-1}|u_d(r)|^2dr,$$

where u_d is the radial wave function of the deuteron. This formula is easily derived in the fixed scatterer approximation (FSA). As it should be, the AGS and FSA results converge to each other when the target particles become much heavier than the incident one (see Table III).

In Table I we compare the present AGS calculations with our previous results obtained by means of the FRA [11]. In Table II we present $A_{\eta d}$ calculated with $\alpha=3.316\ \text{fm}^{-1}$ for various values of $a_{\eta N}$ given in the literature. For comparison, we show also the results of three different approximate calculations: that of Ref. [8] where two versions of the multiple scattering theory (MST) were used, and a new FRA calculation that we performed with the deuteron wave function (5). In contrast to our previous FRA calculations this wave function (which in the coordinate representation is of the Hulthen form) provides the same NN input as in the AGS calculations. The dependences of $A_{\eta d}$ on Re $a_{\eta N}$ and Im $a_{\eta N}$ are shown in Fig. 1 and Fig. 2.

The curves depicted in Fig. 1 are similar to Argand plots, though they represent the scattering amplitude as a function of the coupling constant instead of the collision energy. Despite this the circular movement of the points on these curves is of the same nature as in the genuine Argand plot. Indeed, to draw an Argand plot one moves the point on the energy axis from left to right in the vicinity of a resonance pole. It is clear that if we fix the energy instead and move the resonance pole itself from right to left, the behavior of the amplitude should be similar to the Argand circle. An increase of Re $a_{\eta N}$ makes the ηN interaction more attractive, which moves the ηd resonance poles towards negative energies, i.e., from right to left. The Argand-like shape of the curves in Fig. 1 implies therefore that at a certain value of Re $a_{\eta N}$ (within the interval from 0.25 fm to 1 fm) the resonance pole bypasses (from below) the point $E=0$ and becomes a quasibound pole.

It should be emphasized here that, in contrast to the genuine Argand plot, all the points depicted in Fig. 1 correspond to the same energy, $E=0$, and therefore the range in which the η-deuteron S-matrix pole moves on the energy plane, when Re $a_{\eta N}$ varies within its uncertainty interval, cannot be inferred from these circular curves. They, however, definitely indicate that such a pole exists and crosses the threshold line Re $E=0$. The positions of this pole for different values of Re $a_{\eta N}$ were explicitly calculated in a previous publication [15], where the FRA approximation was employed. Recent measurements of η production in the reaction $p+n\rightarrow d+\eta$ show a substantial enhancement of the cross section near threshold, as compared to what is expected from phase space analysis [26], implying the existence of such a pole.

As can be seen in Table II, both MST and FRA fail to give the correct $A_{\eta d}$ (especially its real part) in the case of strong ηN interaction (when Re $a_{\eta N}>0.5$ fm) while for small values of Re $a_{\eta N}$ these methods work reasonably well. Their failure in the case of strong two-body forces might be due to poor convergence of the multiple scattering series and to increased influence of the break-up channel.

To summarize, in the present work we perform exact AGS calculations for the ηd scattering length for various ηN input that include new data which appeared since the first calculation in 1991 [10]. The results obtained with these new data suggest strongly that a resonance or quasibound state could exist near the ηd threshold, in agreement with the prediction of Ref. [10].

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