PHYSICAL REVIEW C VOLUME 58, NUMBER 6 DECEMBER 1998

Axially symmetric $B=2$ solution in the chiral quark soliton model

N. Sawado and S. Oryu
Department of Physics, Faculty of Science and Technology, Science University of Tokyo, Noda, Chiba 278-8510, Japan

(Received 17 July 1998)

The baryon-number-two ($B=2$) solution based on the SU(2) chiral quark soliton model (χQSM) is solved numerically, including fully the sea quark degrees of freedom. We confirm that the axially symmetric meson configurations yield the energy minimum for the $B=2$ state in the χQSM when taking into account quark dynamics. Due to the axially symmetric meson fields, six valence quarks occupy the lowest energy level, consistent with the Pauli exclusion principle. The minimal-energy of the entire system is obtained within the framework of a symmetric ansatz. The fermion determinant with axially symmetric meson fields is calculated by diagonalizing the corresponding Dirac Hamiltonian in a nonperturbative way, using a cylindrical Dirac basis. The baryon number density, calculated with quark fields corresponding to a soliton, is toroidal in shape. We also calculate the mean radius of the toroid from the quark fields. These results are closely related to Skyrme model calculations based upon pion degrees of freedom. Our model calculations clarify the underlying dynamical structure of the baryons at the quark level. [S0556-2813(98)50112-4]

PACS number(s): 24.85.+p, 13.75.Cs, 12.39.Fe, 12.39.Ki

Although QCD is generally accepted as the underlying theory of the strong interaction, most low- and medium-energy nuclear phenomenology may be successfully described in terms of the hadronic degrees of freedom. In the case of the deuteron, the simplest nuclear system, similar approaches exist [1,2]. Recent investigations for deuteron photodisintegration and deep inelastic scattering of leptons by nuclei suggest the necessity of including quark degrees of freedom [3–8]. For this reason it was suggested that nuclear theory should be reformulated, taking into account the underlying theory. However, QCD has difficulty in describing low- and medium-energy nuclear phenomenology because the coupling constants become extremely large at these energy scales, and it is desirable to formulate an effective, tractable theory for the strong interaction.

The most important feature of QCD in the low energy region is chiral symmetry and its spontaneous breakdown, or the appearance of Goldstone bosons. A simple quark model that incorporates the above features is the chiral quark soliton model (χQSM) [9–13], which is characterized by the following formulas:

$$Z=\int \mathcal{D}\pi\mathcal{D}\psi\mathcal{D}\psi^{\dagger}\,\exp\left[i\int d^4x\,\overline{\psi}(i\partial\!\!\!/-MU^{\gamma_5})\psi\right],\qquad(1)$$

with $U^{\gamma_5}(x)=e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_\pi}$.

For the case of baryon number one ($B=1$), the model was solved numerically in the Hartree approximation with a hedgehog ansatz: $\boldsymbol{\pi}(x)=\hat{\boldsymbol{r}}F(r)$ [9–13]. The profile function $F(r)$ varies between the topological boundary conditions $F(0)=-\pi$ and $F(\infty)=0$. A soliton solution with three valence quarks was obtained. The solution was identified as the nucleon (and Δ) after projecting onto good spin-isospin states.

In the same way, the hedgehog ansatz in which the profile function varies from $F(0)=-2\pi$ to $F(\infty)=0$ gives the baryon number two ($B=2$). But the minimal energy of this configuration was as large as three times the $B=1$ mass [9]. Therefore, the ansatz yields no ‘‘bound state’’ of the $B=2$ system.

On the other hand, in the Skyrme model the minimal energy configuration for $B=2$ has axial symmetry [14–19]. If we choose the z axis as the symmetry axis, meson fields U with a winding number m are given in Manton's conjecture [14],

$$U=\cos F(\rho,z)+i\boldsymbol{\tau}\cdot\hat{\boldsymbol{n}}\sin F(\rho,z),\qquad(2)$$

where

$$\hat{\boldsymbol{n}}=(\sin\Theta(\rho,z)\cos m\varphi,\sin\Theta(\rho,z)\sin m\varphi,\cos\Theta(\rho,z)).\qquad(3)$$

The functions $F(\rho,z)$ and $\Theta(\rho,z)$, the profile functions, are determined by minimizing the classical mass derived from the Skyrme Lagrangian. The calculation was done by Braaten and Carson [19]. They obtained toroidal configurations which were classically stable. The ground state of this solution had the quantum numbers of the deuteron; however, the calculated static properties were not always consistent with the deuteron [19].

In this paper, we investigate the $B=2$ soliton with axially symmetric meson fields in the χQSM. First, we introduce the one particle Dirac Hamiltonian $H(U^{\gamma_5})$ and a complete set of single-quark states as the eigenstates of this Hamiltonian, given by

$$i\partial\!\!\!/-MU^{\gamma_5}=\beta(i\partial_t-H(U^{\gamma_5})),$$
$$H(U^{\gamma_5})=-i\boldsymbol{\alpha}\cdot\nabla+\beta MU^{\gamma_5},\qquad(4)$$

and

$$H(U^{\gamma_5})\phi_\mu(\boldsymbol{x})=E_\mu\phi_\mu(\boldsymbol{x}).\qquad(5)$$

Using these eigenvalues, the total energy of the system can be written as

RAPID COMMUNICATIONS

$$E_{\rm static}[U]=N_c E_v^{(1)}[U]+N_c E_v^{(2)}[U] + E_{\rm field}[U]-E_{\rm field}[U=1], \tag{6}$$

where

$$E_v^{(i)}=n_0^{(i)}E_0^{(i)}, \tag{7}$$

$$E_{\rm field}=N_c\sum_\nu \left(\mathcal{N}_\nu\left|E_\nu\right|+\frac{\Lambda}{\sqrt{4\pi}}\exp\left[-\left(\frac{E_\nu}{\Lambda}\right)^2\right]\right), \tag{8}$$

with

$$\mathcal{N}_\nu=-\frac{1}{\sqrt{4\pi}}\Gamma\left(\frac{1}{2},\left(\frac{E_\nu}{\Lambda}\right)^2\right). \tag{9}$$

The $E_v^{(i)}$ and $E_{\rm field}$ stand for the valence quark contribution to the energy for ith baryon and the sea quark contribution to the total energy, respectively. Here, n_0 is the occupation number of the valence quark; that is, n_0 is 0 or 1. $E_{\rm field}$ is evaluated by the familiar proper-time regularization scheme [20]. Λ is the cutoff parameter.

The Dirac Hamiltonian H with axially symmetric meson fields U commutes with the operator $K_3=L_3+\frac{1}{2}\sigma_3+\frac{1}{2}m\tau_3$. For $m=2$, H also commutes with the operator $\mathcal{P}=\beta\cdot\tau_3$. Due to the symmetries of the meson field configuration, the above operator $\beta\cdot\tau_3$ works as the parity operator. (For $m=1$, the parity operator is given by a conventional form $\mathcal{P}=\beta$.) L_3, σ_3, and τ_3 are the third-component of the orbital angular momentum, the spin angular momentum, and the isospin operator of the quark, respectively. K_3 is often called the third-component of the grand spin operator. Consequently, the eigenstates of H are specified by the magnitude of K_3 and the parity $\pi=\pm$. As L_3 is an integer and σ_3 and τ_3 are ± 1, so the possible values of K_3 are $\pm\frac{1}{2}$, $\pm\frac{3}{2}$, $\pm\frac{5}{2}\cdots$ for $m=2$. H also commutes with the ''time-reversal'' operator $\mathcal{T}=i\gamma_1\gamma_3\cdot i\tau_1\tau_3\mathcal{C}$, where $\mathcal{C}$ is the charge conjugation operator. By virtue of this invariant, we see that the states of $+K_3$ and $-K_3$ are degenerate in energy [21,22]. According to the Kahana and Ripka [23], we begin with investigating the spectrum of quark orbits as a function of the ''soliton size'' X (see Fig. 1). We find that only the $K_3=\pm\frac{1}{2}^+$ states dive into the negative-energy region as X increases. Therefore one concludes that the lowest-lying axially symmetric $B=2$ configuration is obtained by putting three valence quarks each in the first two positive energy states $K_3=\pm\frac{1}{2}^+$. In that case, one immediately finds that six valence quarks are all degenerate in energy. As a result, in our $B=2$ system each baryon has equal classical mass. The degeneracy of the baryon in our system is a distinct feature of choosing axial symmetry as the symmetry of the meson fields.

On the other hand, if we adopt the hedgehog ansatz for the $B=2$ system, one finds that the resultant Hamiltonian commutes with the grand spin operator $\mathbf{K}$ and the parity operator $\mathcal{P}$. The magnitude of the grand spin operator K has the values of 0, 1, 2, 3,...; then, if we first put three valence quarks on the state $K=0^+$, the next three quarks must be placed in higher energy states. If the second strong level $K=0^-$ is occupied by the next three valence quarks, the total energy is about 4260 MeV [9]. If we do not restrict the problem to ''Skyrme-like configuration'' [24], one can place the quarks into $K=1^+$ or $K=1^-$. Unfortunately, there is no thorough analysis which adopts these configurations within the framework of the χQSM. In the σ-model calculation, their total energy is about 2620 MeV for the $K=0^+,1^+$ configuration [24]. In any case, the resulting total energy is much larger than twice the mass with isolated baryon. As a result, we confirm that the energy of the axially symmetric $K_3=\pm\frac{1}{2}^+$ configuration is lower than that of all possible $B=2$ hedgehog configurations. Finally we conclude that the lowest-lying $B=2$ state has axial symmetry, and is obtained by putting six valence quarks in the degenerated states $K_3=\pm\frac{1}{2}^+$.

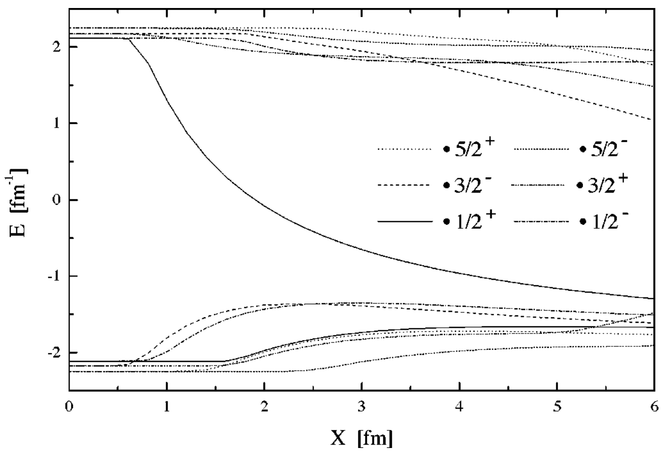

FIG. 1. Spectrum of the quark orbits are illustrated as a function of the ''soliton size'' X. Profile functions are given by $F(\rho,z)=-\pi+\pi(\rho^2+z^2)^{1/2}/X$, and $\Theta(\rho,z)=\tan^{-1}(\rho/z)$. The orbits are labeled by K_3^π.

The meson field configuration that minimizes the total energy $E_{\rm static}[U]$ is determined by the following extremum conditions:

$$\frac{\delta}{\delta F(\rho,z)}E_{\rm static}[U]=0, \quad \frac{\delta}{\delta\Theta(\rho,z)}E_{\rm static}[U]=0. \tag{10}$$

By using the explicit form of $E_{\rm static}[U]$, this yields the following equations of motion for the profile functions $F(\rho,z)$ and $\Theta(\rho,z)$,

$$R_{12}(\rho,z)\cos\Theta(\rho,z)=R_3(\rho,z)\sin\Theta(\rho,z), \tag{11}$$

$$S(\rho,z)\cos F(\rho,z)=P(\rho,z)\sin F(\rho,z), \tag{12}$$

and

$$P(\rho,z)=R_{12}(\rho,z)\sin\Theta(\rho,z)+R_3(\rho,z)\cos\Theta(\rho,z), \tag{13}$$

where

$$R_{12}(\rho,z)=2R_{12v}(\rho,z)+R_{120}(\rho,z), \tag{14}$$

$$R_3(\rho,z)=2R_{3v}(\rho,z)+R_{30}(\rho,z), \tag{15}$$

$$P(\rho,z)=2P_v(\rho,z)+P_0(\rho,z), \tag{16}$$

where subscripts v and 0 denote the contributions from the valence quarks and the sea quarks, respectively. The explicit forms of R_{12}, R_3, and S are

$$R_{12v}(\rho,z)=n_0\int d\varphi\overline{\phi}_0(\rho,\varphi,z)i\gamma_5 \times(\tau_1\cos\varphi+\tau_2\sin\varphi)\phi_0(\rho,\varphi,z),$$

$$R_{120}(\rho,z)=\sum_\nu \mathcal{N}_\nu\,\mathrm{sgn}(E_\nu)\int d\varphi\overline{\phi}_\nu(\rho,\varphi,z)i\gamma_5 \times(\tau_1\cos\varphi+\tau_2\sin\varphi)\phi_\nu(\rho,\varphi,z),$$

$$R_{3v}(\rho,z)=n_0\int d\varphi\overline{\phi}_0(\rho,\varphi,z)i\gamma_5\tau_3\phi_0(\rho,\varphi,z), \quad (17)$$

$$R_{30}(\rho,z)=\sum_\nu \mathcal{N}_\nu\mathrm{sgn}(E_\nu)\int d\varphi\overline{\phi}_\nu \times(\rho,\varphi,z)i\gamma_5\tau_3\phi_\nu(\rho,\varphi,z), \quad (18)$$

$$S_v(\rho,z)=n_0\int d\varphi\overline{\phi}_0(\rho,\varphi,z)\phi_0(\rho,\varphi,z), \quad (19)$$

$$S_0(\rho,z)=\sum_\nu \mathcal{N}_\nu\mathrm{sgn}(E_\nu)\int d\varphi\overline{\phi}_\nu(\rho,\varphi,z)\phi_\nu(\rho,\varphi,z). \quad (20)$$

In order to evaluate Eqs. (11) and (12) numerically, the following procedures were employed. First, we start from the initial functions $F_0(\rho,z)$ and $\Theta_0(\rho,z)$ that satisfy the boundary conditions given by Braaten and Carson [19]:

$$F(\rho,z)\to 0 \quad \text{as} \quad \rho^2+z^2\to\infty, \quad (21)$$

$$F(0,0)=-\pi, \quad \Theta(0,z)=\begin{cases}0, & z>0\\ \pi, & z<0\end{cases}. \quad (22)$$

We solve the one particle Dirac equation using the above functions $F_0(\rho,z)$ and $\Theta_0(\rho,z)$. Second, $R_{12}(\rho,z)$, $R_3(\rho,z)$, and $S(\rho,z)$ are calculated from the resultant eigenvalues and eigenfunctions; $\Theta(\rho,z)$ is given by Eq. (11). Third, the function $F(\rho,z)$ is obtained on the basis of Eqs. (12) and (13). Then, new iterates of $F(\rho,z)$ and $\Theta(\rho,z)$ are obtained by resolving the Dirac equation. This procedure is continued until self-consistency is attained.

Before reporting our results, we provide some comments on our numerical calculations. (i) Numerical calculations were performed for several values for the constituent quark mass M, from 350 MeV to 1000 MeV. The proper-time cutoff parameter Λ was not a free parameter, but was determined so as to reproduce the pion decay constant $f_\pi=93$ MeV [25]. (ii) We chose the initial functions $F_0(\rho,z)=-\pi+\pi\sqrt{\rho^2+z^2}/R$, with $R=1.0$, and $\Theta_0(\rho,z)=\tan^{-1}(\rho/z)$. We tried several forms of the initial functions and confirmed that the final result was independent of these choices. (iii) Diagonalization of the Dirac Hamiltonian was done following the method of Kahana and Ripka [23]. They used a discretized plane wave basis in a spherical box with radius D, which was defined by their grand spin and the parity. As previously stated, since our Dirac Hamiltonian had axially symmetric property, the grand spin **K** was no longer a good quantum number of the eigenstates. In our case, since the third component of the grand spin K_3 and the parity $\pi=\pm$ were only the good quantum number of the states, then we modified the Kahana-Ripka basis into those which were defined by the K_3 and the parity $\pi=\pm$. The new basis are the eigenstates of the free Hamiltonian in a cylindrical box with height $2\times D_z$ and radius D_ρ. Based on these, we are enabled to diagonalize our Dirac Hamiltonian.

In Figs. 2 and 3, we present results for the profile functions $F(\rho,z)$, $\Theta(\rho,z)$ with $M=400$ MeV. In Figs. 4 and 5 we display the baryon number density. Figure 4 shows the contribution from the valence quark and Fig. 5 from the sea quark. From Figs. 4 and 5 it is found that the baryon number density has a toroidal shape. This result is consistent with other chiral invariant models using axially symmetric meson fields, such as the Skyrme model [19] and a naive quark meson model which involved six valence quarks and a pion cloud [23]. The classical soliton energies corresponding to various values of M are given in Table I. As M increases, the valence quark contribution rapidly decreases, while that of the sea quark grows rapidly. Around $M\sim 650$ MeV, the valence level crosses zero energy and dives into the negative-energy region. For $M>650$ MeV, the systems are dominated by the sea quark contribution. As for the total energy, which is the sum of the valence quark and the sea quark contribution, there is essentially no noticeable change as M increases. This is a characteristic behavior of our solution. The increase in the total energy value for large M is perhaps due to the omission of the higher wave numbers from our basis.

Here, one could consider the classical ‘‘binding energy,’’ which is given by

$$-E_{\mathrm{bound}}=E_{\mathrm{static}}[U]-2M_{B=1,\mathrm{hedgehog}}. \quad (23)$$

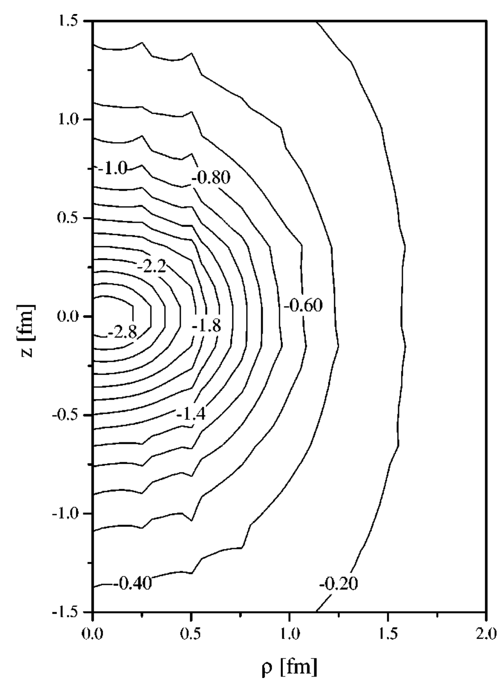

FIG. 2. Contour plot of the self-consistent profile function $F(\rho,z)$, with $M=400$ MeV.

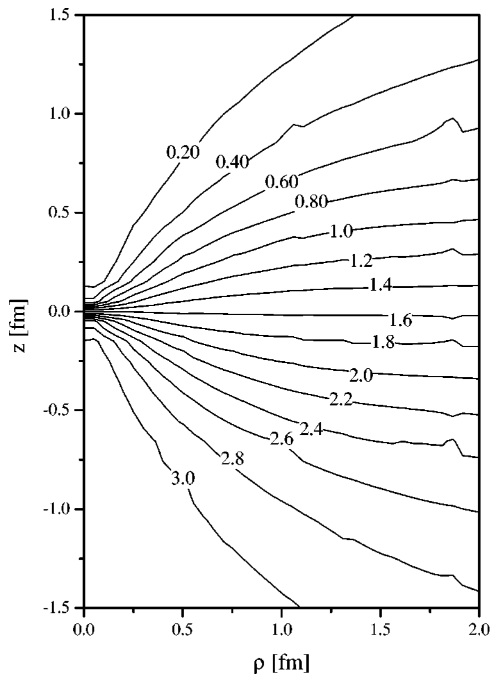

FIG. 3. Contour plot of the self-consistent profile function $\Theta(\rho,z)$, with $M=400$ MeV.

The classical nucleon energy with the hedgehog ansatz $M_{B=1,\text{hedgehog}}$ has been calculated by various authors [9–12], in which the values are chosen around 1200 MeV. Thus, the classical binding energy E_{bound} is about 70 MeV at $M=400$ MeV. This is close to the Skyrme model result [19], and superior to the quark meson model value [23] which is 219 MeV.

The mean radius of the toroid for the quark distribution is estimated by

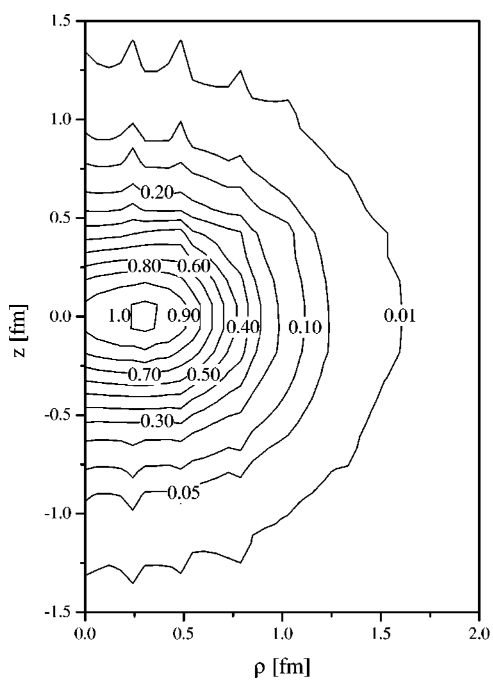

FIG. 4. Baryon number density from valence quark contribution, with $M=400$ MeV.

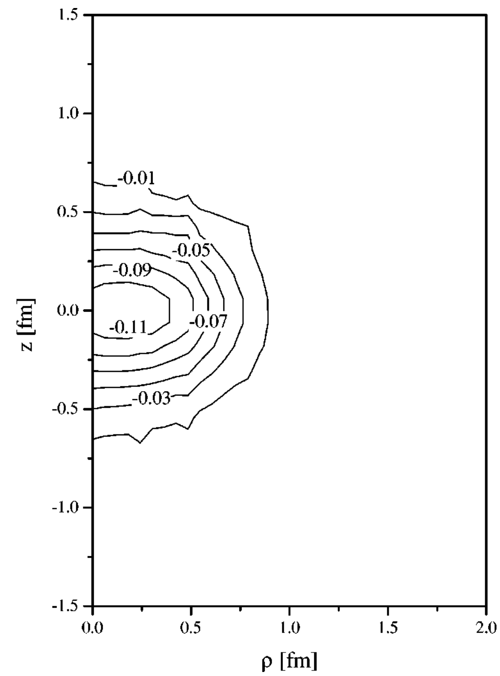

FIG. 5. Baryon number density from sea quark contribution, with $M=400$ MeV.

$$\langle\rho\rangle_v=\frac{1}{2}n_0\int \rho d\rho dz d\varphi \rho \phi_0^\dagger(\rho,\varphi,z)\phi_0(\rho,\varphi,z), \quad (24)$$

$$\langle\rho\rangle_0=\frac{1}{2}\sum_\nu \mathcal{N}_\nu \text{sgn}(E_\nu) \times\int \rho d\rho dz d\varphi \rho \phi_\nu^\dagger(\rho,\varphi,z)\phi_\nu(\rho,\varphi,z), \quad (25)$$

$$\langle\rho\rangle=\langle\rho\rangle_v+\langle\rho\rangle_0. \quad (26)$$

These values are also given in Table I and show a rapid decrease with increasing M. This is reasonable, because here M is regarded as the coupling constant between the quark and pion, so the larger M means a stronger quark-pion interaction. The stronger interaction may produce more compact solitons. At $M=400$ MeV, which may be a suitable choice, we obtained $\langle\rho\rangle=0.672$ fm, which is in qualitative agreement with the Skyrme model value 0.78 fm [19].

TABLE I. Classical mass spectrum (in MeV) and mean radius of toroid.

M	$6E_v$	E_{field}	E_{static}	$\langle\rho\rangle$ (fm)
350	1134	1189	2323	0.705
400	925	1397	2322	0.672
450	765	1569	2334	0.629
500	558	1760	2318	0.600
600	184	2153	2337	0.549
700		2384	2384	0.508
800		2466	2466	0.482
900		2589	2589	0.462
1000		2763	2763	0.447

In summary, we have obtained the axially symmetric $B=2$ soliton solution of the SU(2) χQSM. The solution was obtained in a self-consistent manner. The results are in qualitative agreement with those from the Skyrme model and other quark meson models. This suggests that these features are independent of the particular choice of chirally invariant model. Individualities of each model will become clearer after thorough investigations for various physical observables [26]. The most striking difference between our χQSM and the Skyrme model is the existence of quark degrees of freedom. From consideration of the single quark energy level, we confirm that the minimum energy configuration of $B=2$ is axially symmetric, while in the Skyrme model it is a conjecture. Since the χQSM includes the valence and the sea quark degrees of freedom, we can give theoretical support for nuclear medium effects such as the EMC effect in deep inelastic scattering experiments.

The solutions obtained here were classical ones which have no definite spin, isospin quantum numbers corresponding to physical particles. Therefore the solutions should be quantized by projecting onto good spin, isospin states in order to estimate the energies, the mean square radius, and other static properties of the physical $B=2$ system. The quantization of our solution using the well-known cranking procedure in SU(2) is now in progress.

One of us (N.S.) is very grateful to Dr. S. Akiyama for many valuable discussions and comments during the early stages of this work.

[1] M. J. Zuilhof and J. A. Tjon, Phys. Rev. C **22**, 2369 (1980); E. Hummel and J. A. Tjon, *ibid.* **42**, 423 (1990); **49**, 21 (1994).
[2] L. P. Kaptari, A. Yu. Umnikov, S. G. Bondarenko, K. Yu. Kazakov, F. C. Khanna, and B. Kämpfer, Phys. Rev. C **54**, 986 (1996).
[3] S. J. Freedman *et al.*, Phys. Rev. C **48**, 1864 (1993); J. E. Belz *et al.*, Phys. Rev. Lett. **74**, 646 (1995); see also K. Maruyama *et al.*, RCNP Workshop on Quark Nuclear Physics with the Multi-GeV Photon Facility at SPring8 RCNP-P-134 (unpublished), p. 19.
[4] F. Gross and S. Luiti, Phys. Rev. C **45**, 1374 (1992).
[5] W. Melnitchouk, A. W. Schreiber, and A. W. Thomas, Phys. Rev. D **49**, 1183 (1994).
[6] S. A. Kulagin, G. Piller, and W. Weise, Phys. Rev. C **50**, 1154 (1994).
[7] A. Yu. Umnikov, F. C. Khanna, and L. P. Kaptari, Phys. Rev. C **53**, 377 (1996).
[8] C. E. Carlson and K. E. Lassila, Phys. Rev. C **51**, 364 (1995).
[9] D. I. Diakonov, V. Yu. Petrov, and P. V. Pobylitsa, Nucl. Phys. **B306**, 809 (1988).
[10] M. Wakamatsu and H. Yoshiki, Nucl. Phys. **A524**, 561 (1991).
[11] H. Reinhardt and R. Wünsch, Phys. Lett. B **215**, 577 (1988).
[12] Th. Meissner, F. Grümmer, and K. Goeke, Phys. Lett. B **227**, 296 (1989).
[13] H. Reinhardt, Nucl. Phys. **A503**, 825 (1989); for a recent review see R. Alkofer, H. Reinhardt, and H. Weigel, Phys. Rep. **265**, 139 (1996).
[14] N. S. Manton, Phys. Lett. B **192**, 177 (1987).
[15] A. Jackson, A. Jackson, and V. Pasquier, Nucl. Phys. **A432**, 567 (1985); J. Verbaarschot, T. Walhout, J. Wambach, and H. Wyld, *ibid.* **A468**, 520 (1987); A. J. Schramm, Y. Dothan, and L. C. Biedenharn, Phys. Lett. B **205**, 151 (1988); A. J. Schramm, Phys. Rev. C **37**, 1799 (1988).
[16] V. B. Kopeliovich and B. E. Stern, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 165 (1987) [JETP Lett. **45**, 203 (1987)]; J. Verbaarschot, Phys. Lett. B **195**, 235 (1987).
[17] H. Weigel, B. Schwesinger, and G. Holzwarth, Phys. Lett. B **168**, 321 (1986).
[18] A. Hosaka, M. Oka, and R. D. Amado, Nucl. Phys. **A530**, 507 (1991).
[19] E. Braaten and L. Carson, Phys. Rev. D **38**, 3525 (1988).
[20] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).
[21] D. Vautherin, Phys. Rev. C **7**, 296 (1973); D. Vautherin and D. M. Brink, *ibid.* **5**, 626 (1972).
[22] J. Segar, M. Sripriya, and M. S. Sriram, Phys. Lett. B **342**, 201 (1995).
[23] S. Kahana, G. Ripka, and V. Soni, Nucl. Phys. **A415**, 351 (1984); S. Kahana and G. Ripka, *ibid.* **A429**, 462 (1984).
[24] B. Blättel, J. Kunz, U. Mosel, and T. Reitz, Nucl. Phys. **A466**, 560 (1987).
[25] D. Ebert and H. Reinhardt, Nucl. Phys. **B271**, 188 (1986).
[26] M. Wakamatsu, Prog. Theor. Phys. **109**, 115 (1992).