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Noncoplanarity effects in proton-proton bremsstrahlung

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Noncoplanarity in proton-proton bremsstrahlung is investigated. Significant effects are observed for certain photon polar angles, ψ_γ. Such noncoplanarity effects, not of dynamical origin, are possibly responsible for past disagreements between theory and experiment. The Harvard noncoplanar coordinate system, which avoids kinematic singularities in the cross section, is used in our calculations and is recommended for use in the analysis of experimental data. Alternative methods of presenting cross sections are discussed. [S0556-2813(98)50110-0]

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During the last three decades proton-proton bremsstrahlung *(ppγ)* has been studied with great interest experimentally and theoretically, primarily because of its promise as a probe of the off-shell properties of the nucleon-nucleon interaction. Because sensitivity to off-shell effects increases as the bombarding energy is increased and the scattering angle of the two final-state protons is decreased, recent $pp\gamma$ experiments [1,2] measured cross sections and analyzing powers at energies between 190 MeV and the pion production threshold and for small scattering angles; additional experiments have gone above the threshold for pion production [3,4]; measurements reported in Refs. [5, 6] were only for cross sections. The cross section measurements [1], which showed unexpected large discrepancies with theoretical expectations, were particularly exciting. The discrepancies remain unresolved and challenged experimentalists to perform more precise measurements and theorists to improve their model calculations.

Various theoretical avenues have been explored in the effort to resolve differences with the data. Because the $pp\gamma$ process involves two identical protons, the leading electric dipole terms cancel and no photon emission results from single-meson exchange between the two protons at the tree level. Therefore, most theorists have focused their attention upon higher order effects and corrections such as rescattering terms, relativistic spin corrections, negative energy states, Δ-isobar admixtures, electromagnetic form factors, higher order exchange currents, and the difference between pseudoscalar and pseudovector πN couplings. Estimates for contributions from some of these effects have been significant, especially at extreme (forward and backward) photon angles. However, the magnitude of the overall correction has not resolved the discrepancy between theory and experiment. The discrepancy in angular dependence of the cross section for asymmetric proton angles has been a particular problem.

The primary purpose of this Rapid Communication is to report on the noncoplanarity aspect of proton-proton bremsstrahlung, a significant effect which has not been systematically investigated.

The first successful measurement of $pp\gamma$ noncoplanar cross sections was performed by the Harvard group [7] some 30 years ago. The experimental arrangement has since become known as the ''Harvard geometry.'' The two outgoing protons and the photon in the final state have momenta denoted by $\vec{p}_1'$, $\vec{p}_2'$, and $\vec{K}$, respectively. In the conventional spherical coordinate system, these three momenta are expressed in terms of the polar angle θ and the azimuthal angle ϕ as

$$\begin{aligned}\vec{p}_i' &= \vec{p}_i'(p_i', \theta_i, \phi_i), \quad (i=1,2)\\ &\equiv p_i' \sin\theta_i \cos\phi_i \hat{e}_x + p_i' \sin\theta_i \sin\phi_i \hat{e}_y\\ &\quad + p_i' \cos\theta_i \hat{e}_z, \end{aligned} \tag{1a}$$

$$\begin{aligned}\vec{K} &= \vec{K}(K, \theta_\gamma, \phi_\gamma)\\ &\equiv K \sin\theta_\gamma \cos\phi_\gamma \hat{e}_x + K \sin\theta_\gamma \sin\phi_\gamma \hat{e}_y\\ &\quad + K \cos\theta_\gamma \hat{e}_z. \end{aligned} \tag{1b}$$

The Harvard noncoplanar coordinate system is instead defined by introducing two new angles, $\overline{\theta}$ and $\overline{\phi}$. (See the Appendix of Ref. [8] for the definitions.) By using these new angles, one can write $\vec{p}_1'$, $\vec{p}_2'$, and $\vec{K}$ as

$$\begin{aligned}\vec{p}_1' &= \vec{p}_1'(p_1', \overline{\theta}_1, \overline{\phi}_1)\\ &\equiv p_1' \cos\overline{\phi}_1 \sin\overline{\theta}_1 \hat{e}_x\\ &\quad + p_1' \cos\overline{\phi}_1 \cos\overline{\theta}_1 \hat{e}_z + p_1' \sin\overline{\phi}_1 \hat{e}_y, \end{aligned} \tag{2a}$$

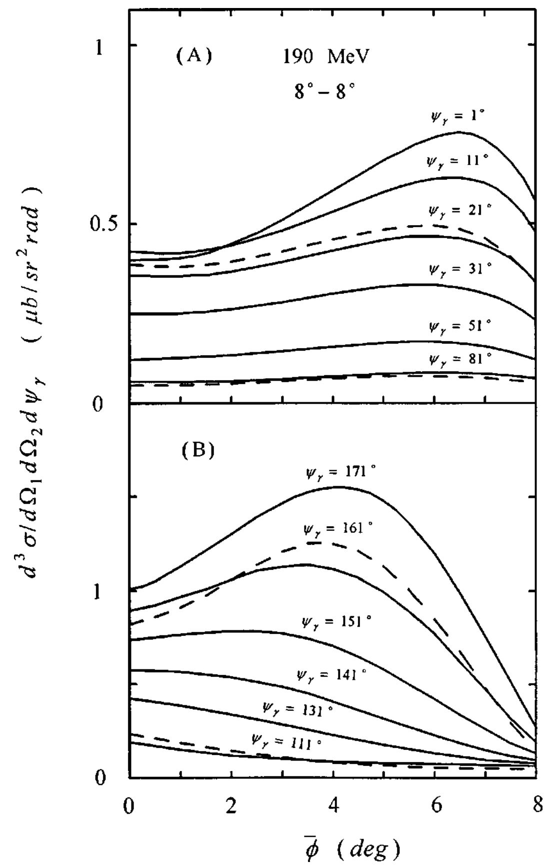

FIG. 1. Noncoplanar $pp\gamma$ cross sections $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ as a function of the noncoplanarity angle $\overline{\phi}$ at an incident energy of 190 MeV for $\overline{\theta}_1=\overline{\theta}_2=8°$ and various ψ_γ. The solid curves were calculated using a *TuTts* amplitude [11,12], while the dashed curves represent results from an OBE model [10].

$$\vec{p}_2'=\vec{p}_2'(p_2',\overline{\theta}_2,\overline{\phi}_2)$$
$$\equiv -p_2'\cos\,\overline{\phi}_2\sin\,\overline{\theta}_2\hat{e}_x$$
$$+p_2'\cos\,\overline{\phi}_2\cos\,\overline{\theta}_2\hat{e}_z+p_2'\sin\,\overline{\phi}_2\hat{e}_y, \qquad (2b)$$

$$\vec{K}=\vec{K}(K,\overline{\theta}_\gamma,\overline{\phi}_\gamma)$$
$$\equiv K\,\cos\,\overline{\phi}_\gamma\sin\,\overline{\theta}_\gamma\hat{e}_x+K\,\cos\,\overline{\phi}_\gamma$$
$$\times\cos\,\overline{\theta}_\gamma\hat{e}_z-K\,\sin\,\overline{\phi}_\gamma\hat{e}_y. \qquad (2c)$$

There is another important photon polar angle ψ_γ employed in the Harvard noncoplanar coordinate system. It is defined as follows: There are the three noncoplanarity angles $\overline{\phi}_i$, $i=1,2,\gamma$. The angle $\overline{\phi}=(\overline{\phi}_1+\overline{\phi}_2)/2$ has a kinematically allowed limit $\overline{\phi}_{\max}$, termed the maximum noncoplanarity angle. The corresponding photon in this limiting case is called ''the limiting gamma ray''; its momentum is $\vec{q}=\vec{q}(q,\overline{\theta}_0,\overline{\phi}_0)$, which is given by Eq. (2c) with $(K,\overline{\theta}_\gamma,\overline{\phi}_\gamma)$ replaced by $(q,\overline{\theta}_0,\overline{\phi}_0)$, respectively. To define ψ_γ, one constructs a new photon momentum $\vec{K}'=\vec{K}'(K',\psi_\gamma,0)$ from $\vec{K}$ and $\vec{q}$, $\vec{K}'=\vec{K}-\alpha\vec{q}$, where the scalar α is chosen such that $\vec{K}'$ lies in the XZ reference plane and has a polar angle equivalent to ψ_γ.

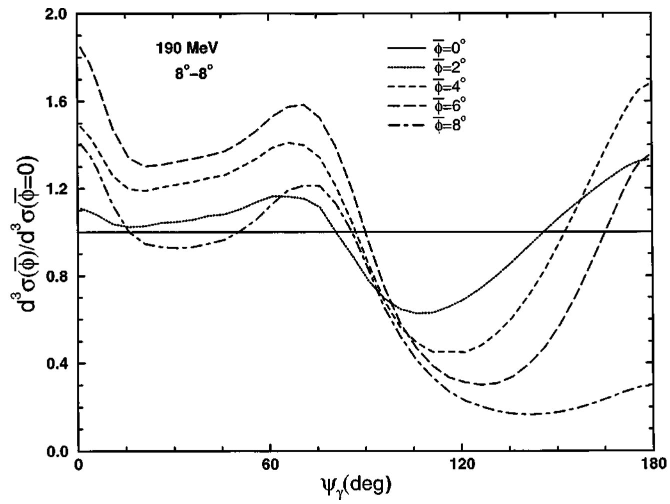

FIG. 2. The cross section ratio $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma/(d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)_{\overline{\phi}=0}$ as a function of ψ_γ at an incident energy of 190 MeV for $\overline{\theta}_1=\overline{\theta}_2=8°$ and $\overline{\phi}=0°,2°,4°,6°,8°$. Calculations were made using a *TuTts* amplitude [11,12].

In terms of the spherical coordinate system we can choose the set $(\theta_1,\phi_1,\theta_2,\phi_2,\theta_\gamma)$ to be independent variables and express the differential cross section in the form $d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$. For the coplanar case, this cross section is well behaved when plotted as a function of θ_γ over the entire range from 0° to 360°. However, this form is *not* recommended for the noncoplanar situation, because the cross section exhibits kinematic singularities [9] when plotted as a function of θ_γ; the range of allowed θ_γ no longer extends from 0° to 360°.

The purpose of introducing the more complex Harvard noncoplanar coordinate system is to remove the kinematic singularities. Using that coordinate system we can choose the set $(\overline{\theta}_1,\overline{\phi}_1,\overline{\theta}_2,\overline{\phi}_2,\psi_\gamma)$ to be the independent kinematic variables and express the cross section in the form $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$. There are three advantages to expressing the cross section in this form: (i) $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ reduces to $d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$ as ψ_γ reduces to θ_γ in the coplanar case; (ii) the angle ψ_γ always runs through 2π, so that the cross section can be defined as a function of ψ_γ for the full $0°\leqslant\psi_\gamma\leqslant 360°$ range; (iii) $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ is free of kinematic singularities. Therefore, we have investigated the noncoplanar $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ cross section as a function of ψ_γ, the noncoplanarity angle $\overline{\phi}$, and the bombarding energy.

Most theorists have calculated coplanar cross sections of the form $d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$. Because the measured cross sections involve contributions from noncoplanar events, the experimental cross sections must be ''corrected'' in order to compare with these theoretical predictions. Historically one has calculated the double differential cross section (often referred to in the literature as the integrated cross section),

$$d^2\sigma/d\Omega_1 d\Omega_2=\int (d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)d\psi_\gamma, \qquad (3)$$

as a function of the noncoplanarity angle $\overline{\phi}$ and has used the result to determine a constant correction factor for converting the experimental (noncoplanar) cross sections into a coplanar

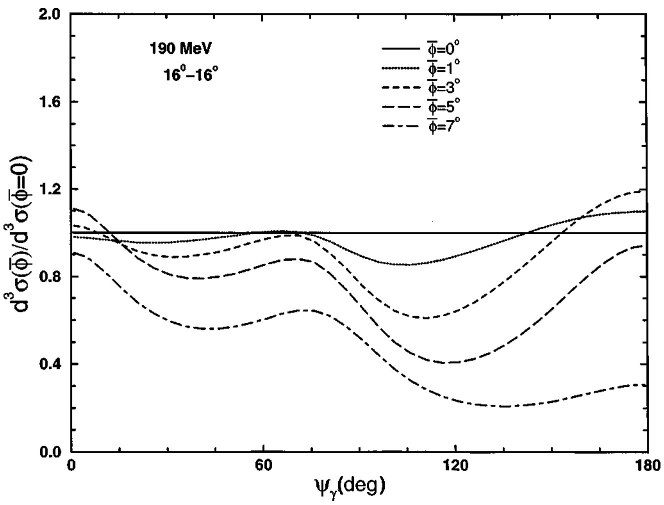

FIG. 3. The cross section ratio $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma/(d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)_{\bar{\phi}=0}$ as a function of ψ_γ at an incident energy of 190 MeV for $\bar{\theta}_1=\bar{\theta}_2=16°$ and $\bar{\phi}=0°,1°,3°,5°,7°$. Calculations were made using a *TuTts* amplitude [11,12].

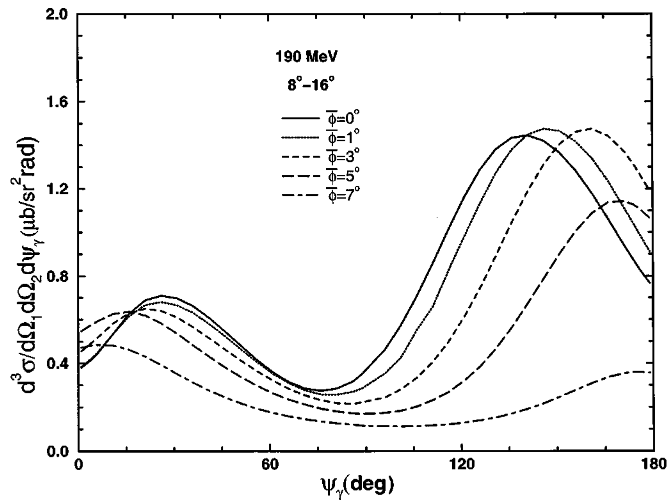

FIG. 4. Noncoplanar cross sections $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ as a function of ψ_γ at 190 MeV for $(\bar{\theta}_1,\bar{\theta}_2)=(8°,16°)$ and $\bar{\phi}=0°,1°,3°,5°,7°$. Calculations were made using a *TuTts* amplitude [11,12].

result. As was shown in Refs. [8,10], this double differential cross section $d^2\sigma/d\Omega_1 d\Omega_2$ is typically a monotonically decreasing function of $\bar{\phi}$. This feature is rather general, as it is fairly independent of the theoretical model used and this has been confirmed by other experiments. The correction factor obtained via this procedure is a constant. That is, one relates the experimental noncoplanar cross section $(d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)_{\rm exp}$ at ψ_γ to an ''experimental'' coplanar cross section $(d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma)_{\rm exp}$ at θ_γ as follows:

$$(d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma)_{\rm exp}=C[(d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)_{\rm exp}]_{\psi_\gamma=\theta_\gamma}, \tag{4}$$

where C is the constant correction factor. It is in this approximation that $(d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma)_{\rm exp}$ has been obtained for comparison with theoretical calculations of $d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$.

We point out that $(d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma)_{\rm exp}$, obtained in this manner from Eq. (4), is not an actual coplanar ($\bar{\phi}=0°$) cross section. Therefore, it should not be used to compare with theoretical coplanar cross section calculations. The reason can be seen from the result of our investigation on the dependence of the noncoplanar cross section $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ upon $\bar{\phi}$ for the entire range of ψ_γ. In Fig. 1 we plot various noncoplanar curves for $\bar{\theta}_1=\bar{\theta}_2=8°$ at an incident energy of 190 MeV, the energy of the high statistics KVI experiment [2]. Each curve represents the noncoplanar cross section as a function of $\bar{\phi}$ for a given ψ_γ. For this case one has $\bar{\phi}_{\rm max}=8.46°$. In order to demonstrate that the shape of these noncoplanar curves is fairly independent of the theoretical approach, we have used both a *TuTts* soft-photon approximation [11,12] and a one-boson-exchange model [10] in our calculations. The noncoplanarity effect (the dependence of $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ upon $\bar{\phi}$) differs markedly as a function of ψ_γ. In particular, for a given ψ_γ the cross section $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ is not necessarily a monotonically decreasing function of $\bar{\phi}$. Any noncoplanarity correction factor will not be a constant in the angular range $0° \leqslant \psi_\gamma \leqslant 180°$. For example, we see from Fig. 1 that the noncoplanarity effect is significant in the regions $\psi_\gamma<15°$ and $\psi_\gamma>150°$, while it is quite small around $\psi_\gamma=90°$. Because of this significant angular dependence of the noncoplanarity, a better procedure to use in approximating the ''experimental'' coplanar cross section is

$$(d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma)_{\rm exp}=[C(\psi_\gamma)(d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)_{\rm exp}]_{\psi_\gamma=\theta_\gamma}, \tag{5}$$

where $C(\psi_\gamma)$ is an angular dependent correction factor. For a given ψ_γ, the curves shown in Fig. 1 can be used to estimate $C(\psi_\gamma)$.

To make this noncoplanarity effect clear, we illustrate in Figs. 2 and 3 the calculated cross section ratio

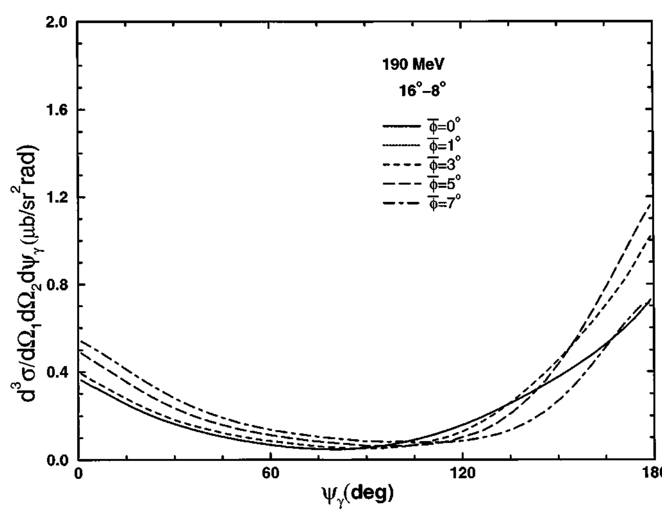

FIG. 5. Noncoplanar cross sections $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ as a function of ψ_γ at 190 MeV for $(\bar{\theta}_1,\bar{\theta}_2)=(16°,8°)$ and $\bar{\phi}=0°,1°,3°,5°,7°$. The curves for $\bar{\phi}=1°$ and $\bar{\phi}=0°$ are indistinguishable. Calculations were made using a *TuTts* amplitude [11,12].

$$\frac{d^3\sigma(\overline{\phi})}{d^3\sigma(\overline{\phi}=0)} \equiv \frac{d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma}{(d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma)_{\overline{\phi}=0}}, \quad (6)$$

as a function of ψ_γ for various $\overline{\phi}$. The maximum noncoplanarity angle $\overline{\phi}_{max}$ is 7.69° for $(\overline{\theta}_1, \overline{\theta}_2) = (16°, 16°)$ at 190 MeV. If the noncoplanarity effect is small, then the value of the ratio should be approximately unity. In Figs. 4 and 5 we show the actual cross section $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ as a function of ψ_γ for selected $\overline{\phi}$. These figures also demonstrate explicitly that the noncoplanarity effect is significant for some ψ_γ and that the correction factor $C(\psi_\gamma)$ varies with ψ_γ.

A comparison of Figs. 4 and 5 reveals, for the first time, that the noncoplanarity correction (in the range $0° \leqslant \psi_\gamma \leqslant 180°$) is much more important for the case in which $(\overline{\theta}_1, \overline{\theta}_2) = (8°, 16°)$ than for the case in which $(\overline{\theta}_1, \overline{\theta}_2) = (16°, 8°)$. Although it requires further study, one may anticipate better overall agreement between theory and experiment for cases with $\overline{\theta}_1 > \overline{\theta}_2$ than for cases with $\overline{\theta}_1 < \overline{\theta}_2$.

Even though Eq. (5) provides a significant improvement in estimating the coplanar cross section from the experimental data, obtaining $C(\psi_\gamma)$ represents a practical problem of some consequence. Furthermore, presenting ''data'' in this manner may leave unspecified the exact procedure used to obtain $C(\psi_\gamma)$. To avoid this uncertainty, experimentalists should preferably present their data as a function of $\overline{\phi}$, as was done by the Harvard group [7]. If it is not possible to measure noncoplanar cross sections as a function of $\overline{\phi}$, then $(d^3\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma)_{exp}$ should be quoted along with the experimental constraints, bins, efficiencies, etc. In that less than ideal situation, theorists could then compare with the experimental data by calculating the noncoplanar cross section $d^3\sigma/d\Omega_1 d\Omega_2 d\psi_\gamma$ as a function of $\overline{\phi}$ and then averaging over the published constraints to obtain the appropriate cross section for comparison. Alternatively, theorists must provide noncoplanar results with which experimentalists can ''Monte Carlo'' a proper comparison with the data.

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