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Multipion correlations in high energy collisions

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Any-order pion inclusive distributions for a chaotic source in high energy collisions are given, which can be used in both the theory and experiment to analyze the any-order pion interferometry. Multipion correlation effects on the two-pion and three-pion interferometry are discussed. [S0556-2813(98)50107-0]

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Hanbury-Brown and Twiss [1] were the first to apply the Bose-Einstein (BE) correlations to measure the size of distant stars. The method was first applied to particle physics by Goldhaber *et al.* (GGLP) [2] in 1959. Since then the size of the interaction region has been measured by numerous experiments in high energy collisions using different types of particles. The two-pion BE correlation is widely used in high energy collisions to provide the information for the space-time structure, degree of coherence, and dynamics of the region where the pions were produced [3]. Ultrarelativistic hadronic collisions provide the environment for creating dozens of pions [4–7]; therefore, one must take into account the effects of multipion BE correlations in those processes. The bosonic nature of the pion should affect the single and i-pion spectra and distort the $i(i\geqslant 2)$-pion correlation function; thus, it is very interesting to analyze the multipion BE correlation effects on i-pion interferometry [8–15]. On the other hand, one also wants to study higher-order pion interferometry directly to see what additional information can be extracted from higher-order pion interferometry. Now this aspect has aroused great interest among physicists [6,7,13,16–18]. Unfortunately all present analyses of multipion correlations are based on pure ''multipion interferometry formulas'' without the consideration of the higher-order pion correlation effects on the lower-order pion interferometry [6,7,13,16–18]. Thus we urgently need new multipion interferometry formulas which can be used in both the theory and experiment to analyze the any-order pion interferometry. This is the main aim of this Rapid Communication. In this Rapid Communication, considering multipion BE correlations, we derive new multipion correlations formulas which can be used to analyze the any-order pion interferometry. Those new multipion correlations formulas are structurally similar to the previous formulas but with modified source functions [see Eq. (21) for details]. This warrants the validity of the formulas used in earlier studies of the higher-order pion interferometry. Although we only study multipion BE correlations in this Rapid Communication, the results presented here are also held for kaon if the final state interactions are neglected.

The general definition of the ''pure'' n-pion correlation function $C_n(\mathbf{p_1},\ldots,\mathbf{p_n})$ is

$$C_n(\mathbf{p_1},\ldots,\mathbf{p_n})=\frac{P_n(\mathbf{p_1},\ldots,\mathbf{p_n})}{\prod_{i=1}^{n} P_1(\mathbf{p_i})}, \tag{1}$$

where $P_n(\mathbf{p_1},\ldots,\mathbf{p_n})$ is the probability of observing n pions with momenta $\{\mathbf{p_i}\}$ all in the same n-pion event. The n-pion momentum probability distribution $P_n(\mathbf{p_1},\ldots,\mathbf{p_n})$ can be expressed as [9,11]

$$P_n(\mathbf{p_1},\ldots,\mathbf{p_n})=\sum_{\sigma} \rho_{1,\sigma(1)}\rho_{2,\sigma(2)}\cdots\rho_{n,\sigma(n)}, \tag{2}$$

with

$$\rho_{i,j}=\rho(\mathbf{p_i},\mathbf{p_j})=\int d^4x g_w\left(x,\frac{(p_i+p_j)}{2}\right)e^{i(p_i-p_j)\cdot x}. \tag{3}$$

Here $\sigma(i)$ denotes the ith element of a permutation of the sequence $1,2,3,\ldots,n$, and the sum over σ denotes the sum over all $n!$ permutations of this sequence. $g_w(Y,k)$ can be explained as the probability of finding a pion at point Y with momentum k which is defined as [11]

$$g_w(Y,k)=\int d^4y j^*(Y+y/2)j(Y-y/2)\exp(-ik\cdot y), \tag{4}$$

with

$$\int g_w(x,k)d^4xd\vec{k}=n_0. \tag{5}$$

Where $j(x)$ is the current of the pion, n_0 is the mean pion multiplicity without the BE correlation [10,11]. From Eq. (1) and Eq. (2), the pure n-pion correlation functions can be expressed as [16]

$$C_n(\mathbf{p_1},\ldots,\mathbf{p_n})=\sum_{\sigma}\prod_{j=1}^{n}\frac{\rho_{j,\sigma j}}{\rho_{j,j}}. \tag{6}$$

The above correlation functions are widely used in both experiment and theory to analyze the multipion interferometry. But in high energy experiments, the pion multiplicity is so large that we must take into account the multipion correlations effects on lower-order pion interferometry.

For $n\pi$ events, considering the n-pion correlations effect, the i-pion correlation function can be defined as [11]

$$C_i^n(\mathbf{p_1},\ldots,\mathbf{p_i})=\frac{P_i^n(\mathbf{p_1},\ldots,\mathbf{p_i})}{\prod_{j=1}^{i} P_1^n(\mathbf{p_j})}, \tag{7}$$

where $P_i^n(\mathbf{p_1},\ldots,\mathbf{p_i})$ is the the normalized modified i-pion inclusive distribution in n-pion events that can be expressed as

$$P_i^n(\mathbf{p_1},\ldots,\mathbf{p_i})=\frac{\int\prod_{j=i+1}^{n}d\mathbf{p_j}P_n(\mathbf{p_1},\ldots,\mathbf{p_n})}{\int\prod_{j=1}^{n}d\mathbf{p_j}P_n(\mathbf{p_1},\ldots,\mathbf{p_n})}. \tag{8}$$

Now we define the function $G_i(\mathbf{p},\mathbf{q})$ as [10,11]

$$G_i(\mathbf{p},\mathbf{q})=\int\rho(\mathbf{p},\mathbf{p_1})d\mathbf{p_1}\rho(\mathbf{p_1},\mathbf{p_2})d\mathbf{p_2}\cdots\rho(\mathbf{p_{i-2}},\mathbf{p_{i-1}})d\mathbf{p_{i-1}}\rho(\mathbf{p_{i-1}},\mathbf{q}). \tag{9}$$

From the expression of $P_n(\mathbf{p_1},\ldots,\mathbf{p_n})$ [Eq. (2)], the one-pion to three-pion inclusive distribution can be expressed as [11]

$$P_1^n(\mathbf{p})=\frac{1}{n}\frac{1}{\omega(n)}\sum_{i=1}^{n}G_i(\mathbf{p},\mathbf{p})\cdot\omega(n-i), \tag{10}$$

$$P_2^n(\mathbf{p_1},\vec{\mathbf{p}}_2)=\frac{1}{n(n-1)}\frac{1}{\omega(n)}\sum_{i=2}^{n}\left[\sum_{m=1}^{i-1}G_m(\mathbf{p_1},\mathbf{p_1})\cdot G_{i-m}(\mathbf{p_2},\mathbf{p_2})+G_m(\mathbf{p_1},\mathbf{p_2})\cdot G_{i-m}(\mathbf{p_2},\mathbf{p_1})\right]\omega(n-i), \tag{11}$$

$$\begin{aligned}P_3^n(\mathbf{p_1},\vec{\mathbf{p}}_2,\vec{\mathbf{p}}_3)=&\frac{1}{n(n-1)(n-2)}\frac{1}{\omega(n)}\sum_{i=3}^{n}\left[\sum_{m=1}^{i-2}\sum_{k=1}^{i-m-1}G_m(\mathbf{p_1},\mathbf{p_1})\cdot G_k(\mathbf{p_2},\mathbf{p_2})\cdot G_{i-m-k}(\mathbf{p_3},\mathbf{p_3})\right.\\&+G_m(\mathbf{p_1},\mathbf{p_2})\cdot G_k(\mathbf{p_2},\mathbf{p_1})\cdot G_{i-m-k}(\mathbf{p_3},\mathbf{p_3})+G_m(\mathbf{p_2},\mathbf{p_3})\cdot G_k(\mathbf{p_3},\mathbf{p_2})\cdot G_{i-m-k}(\mathbf{p_1},\mathbf{p_1})\\&+G_m(\mathbf{p_3},\mathbf{p_1})\cdot G_k(\mathbf{p_1},\mathbf{p_3})\cdot G_{i-m-k}(\mathbf{p_2},\mathbf{p_2})+G_m(\mathbf{p_1},\mathbf{p_2})\cdot G_k(\mathbf{p_2},\mathbf{p_3})\cdot G_{i-m-k}(\mathbf{p_3},\mathbf{p_1})\\&\left.+G_m(\mathbf{p_1},\mathbf{p_3})\cdot G_k(\mathbf{p_3},\mathbf{p_2})\cdot G_{i-m-k}(\mathbf{p_2},\mathbf{p_1})\right]\omega(n-i),\end{aligned} \tag{12}$$

with

$$\omega(n)=\frac{1}{n!}\int\prod_{k=1}^{n}d\mathbf{p_k}P_n(\mathbf{p_1},\ldots,\mathbf{p_n})=\frac{1}{n}\sum_{i=1}^{n}C(i)\omega(n-i),C(i)=\int G_i(\mathbf{p},\mathbf{p})d\mathbf{p}. \tag{13}$$

Here $\omega(n)$ is the pion multiplicity distribution probability.

Similar expressions can be given for $i(i\leqslant n)$ pion inclusive distributions. From the above method the i-pion correlation function can be calculated for n-pion events. Experimentally, one usually mixes all events to analyze the two-pion and higher-order pion interferometry. Then the i-pion correlation function can be expressed as [19,20]

$$C_i^{\phi}(\mathbf{p_1},\ldots,\mathbf{p_i})=\frac{N_i(\mathbf{p_1},\ldots,\mathbf{p_i})}{\prod_{j=1}^{i}N_1(\mathbf{p_j})}. \tag{14}$$

Here the i-pion inclusive distribution, $N_i(\mathbf{p_1},\ldots,\mathbf{p_i})$, can be expressed as

$$N_i(\mathbf{p_1},\ldots,\mathbf{p_i})=\frac{\sum_{n=i}^{\infty}\omega(n)\cdot n(n-1)\cdots(n-i+1)P_i^n(\mathbf{p_1},\ldots,\mathbf{p_i})}{\sum_{n}\omega(n)}, \tag{15}$$

with

$$\int N_i(\mathbf{p_1},\ldots,\mathbf{p_i})\prod_{j=1}^{i}d\mathbf{p_j}=\langle n(n-1)\cdots(n-i+1)\rangle. \tag{16}$$

Then the one-pion to three-pion inclusive distribution reads

$$N_1(\mathbf{p})=\sum_{i=1}^{\infty}G_i(\mathbf{p},\mathbf{p}), \tag{17}$$

$$N_2(\mathbf{p_1},\mathbf{p_2})=\sum_{i=1}^{\infty} G_i(\mathbf{p_1},\mathbf{p_1})\sum_{j=1}^{\infty} G_j(\mathbf{p_2},\mathbf{p_2})+\sum_{i=1}^{\infty} G_i(\mathbf{p_1},\mathbf{p_2})\sum_{j=1}^{\infty} G_j(\mathbf{p_2},\mathbf{p_1}), \tag{18}$$

$$\begin{aligned}N_3(\mathbf{p_1},\mathbf{p_2},\mathbf{p_3})=&\sum_{i=1}^{\infty} G_i(\mathbf{p_1},\mathbf{p_1})\sum_{j=1}^{\infty} G_j(\mathbf{p_2},\mathbf{p_2})\sum_{k=1}^{\infty} G_k(\mathbf{p_3},\mathbf{p_3})+\sum_{i=1}^{\infty} G_i(\mathbf{p_1},\mathbf{p_2})\sum_{j=1}^{\infty} G_j(\mathbf{p_2},\mathbf{p_1})\sum_{k=1}^{\infty} G_k(\mathbf{p_3},\mathbf{p_3})\\&+\sum_{i=1}^{\infty} G_i(\mathbf{p_3},\mathbf{p_1})\sum_{j=1}^{\infty} G_j(\mathbf{p_1},\mathbf{p_3})\sum_{k=1}^{\infty} G_k(\mathbf{p_2},\mathbf{p_2})+\sum_{i=1}^{\infty} G_i(\mathbf{p_2},\mathbf{p_3})\sum_{j=1}^{\infty} G_j(\mathbf{p_3},\mathbf{p_2})\sum_{k=1}^{\infty} G_k(\mathbf{p_1},\mathbf{p_1})\\&+\sum_{i=1}^{\infty} G_i(\mathbf{p_1},\mathbf{p_2})\sum_{j=1}^{\infty} G_j(\mathbf{p_2},\mathbf{p_3})\sum_{k=1}^{\infty} G_k(\mathbf{p_3},\mathbf{p_1})+\sum_{i=1}^{\infty} G_i(\mathbf{p_1},\mathbf{p_3})\sum_{j=1}^{\infty} G_j(\mathbf{p_3},\mathbf{p_2})\sum_{k=1}^{\infty} G_k(\mathbf{p_2},\mathbf{p_1}).\end{aligned} \tag{19}$$

A similar expression for the $i(i>3)$ pion inclusive distribution can be given. Following Refs. [14,15,21], we define the following function:

$$H_{ij}=H(\mathbf{p_i},\mathbf{p_j})=\sum_{k=1}^{\infty} G_k(\mathbf{p_i},\mathbf{p_j}). \tag{20}$$

Then the n-pion inclusive distribution can be expressed as

$$N_n(\mathbf{p_1},\dots,\mathbf{p_n})=\sum_{\sigma} H_{1\sigma(1)}H_{2,\sigma(2)}\cdots H_{n,\sigma(n)}\,. \tag{21}$$

Here $\sigma(i)$ denotes the ith element of a permutation of the sequence 1,2,3,...,n, and the sum over σ denotes the sum over all $n!$ permutations of this sequence. One of the interesting things about Eq. (21) is that it is very similar to Eq. (2). The only difference is that Eq. (2) only contains the first term of H_{ij} $(\rho(\mathbf{p_i},\mathbf{p_j})=G_1(\mathbf{p_i},\mathbf{p_j}))$. With the help of Eq. (21), the general form of the i-pion correlation function [Eq. (14)] can be re-expressed as [16]

$$C_i^{\phi}(\mathbf{p_1},\dots,\mathbf{p_i})=\sum_{\sigma}\prod_{j=1}^{i}\frac{H_{j,\sigma(j)}}{H_{j,j}}. \tag{22}$$

In the derivation of Eq. (22), we mention nothing about the structure of the source, so our results are in principle independent of the detail form of the source, i.e., one can use any kind of source function (which may contain resonance and flow) to study higher-order pion interferometry. Assuming that $H_{ij}=|H_{ij}|\exp(i\phi_{ij})$, it is clear that two-pion interferometry does not depend on the phase ϕ_{ij} which exists in higher-order pion interferometry. So higher-order interferometry can be used to extract the information for the phase [17]. If the source distribution function is symmetric in the coordinate space we have $\phi_{ij}=0$. Then multipion correlations contain the same information as two-pion interferometry does. In the following, we will use a simple model to study the multi-pion correlations effects on two-pion and three-pion interferometry. Similar to Refs. [9–11], we assume the source distribution function $g(x,p)$ as

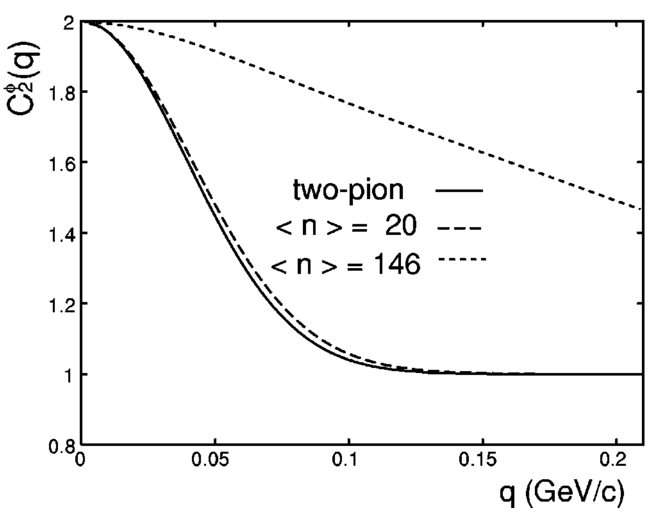

FIG. 1. Multipion correlation effects on two-pion interferometry. The solid line corresponds to the result of the pure two-pion interferometry. The dashed line and dotted line correspond to $\langle n\rangle=20{,}146$, respectively. The input value of R and Δ are 5 fm and 0.25 GeV.

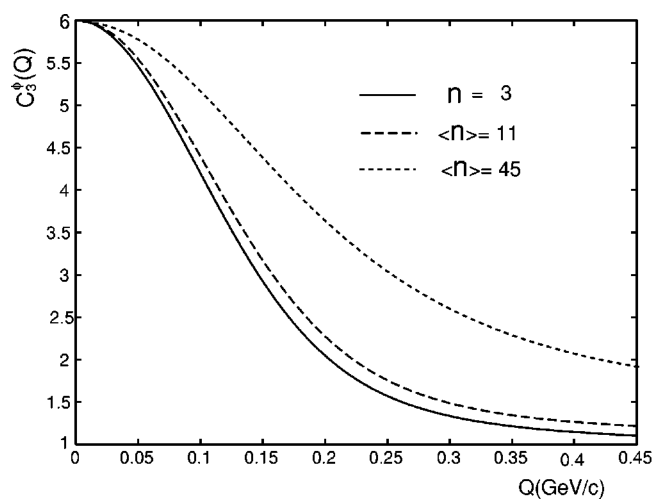

FIG. 2. Multipion correlation effects on three-pion interferometry. The solid line corresponds to the pure three-pion interferometry. The dashed line and dotted line correspond to $\langle n\rangle=11{,}45$, respectively. The input value of R and Δ are 3 fm and 0.36 GeV.

$$g(x,p)=n_0\cdot\left(\frac{1}{\pi R^2}\right)^{3/2}\exp(-r^2/R^2)\left(\frac{1}{\pi\Delta^2}\right)^{3/2}\times\exp(-p^2/\Delta^2). \tag{23}$$

Here R and Δ are parameters which represent the radius of the chaotic source and the momentum range of pions, respectively. Using Eq. (9), Eq. (14), and Eq. (15), one can calculate the any-order pion correlation function for any kind of source distribution [22]. Csörgő and Zimáyi have found analytical solutions for the above special source distribution [14,15] which are quoted here:

$$G_n(\mathbf{p_1},\mathbf{p_2})=j_n\exp\left\{-\frac{b_n}{2}[(\gamma_+^{n/2}\mathbf{p_1}-\gamma_-^{n/2}\mathbf{p_2})^2+(\gamma_+^{n/2}\mathbf{p_2}-\gamma_-^{n/2}\mathbf{p_1})^2]\right\}, \tag{24}$$

$$j_n=n_0^n\left(\frac{b_n}{\pi}\right)^{3/2},\quad b_n=\frac{1}{\Delta^2}\frac{\gamma_+-\gamma_-}{\gamma_+^n-\gamma_-^n} \tag{25}$$

with

$$\gamma_\pm=\frac{1}{2}(1+x\pm\sqrt{1+2x}),\quad x=R^2\Delta^2/2. \tag{26}$$

Using this solution, one can calculate the any-order pion interferometry for the above source distribution. The analytical solution has the advantage over the previous method [10,11] in that it can be used in theory analyses.

The two-pion interferometry for the different mean multiplicity $\langle n\rangle$ is shown in Fig. 1. It is clear that multipion correlation makes the radius smaller. For the three-pion interferometry, we choose the variable $Q^2=(\mathbf{p_1}-\mathbf{p_2})^2+(\mathbf{p_2}-\mathbf{p_3})^2+(\mathbf{p_3}-\mathbf{p_1})^2=\mathbf{q_{12}}^2+\mathbf{q_{23}}^2+\mathbf{q_{31}}^2$ and integrate the other eight variables; then we have the three-pion correlation function $C_3^\phi(Q)$ as

$$C_3^\phi(Q)=\frac{\int N_3(\mathbf{p_1},\mathbf{p_2},\mathbf{p_3})\,\delta(Q^2-\mathbf{q_{12}}^2-\mathbf{q_{23}}^2-\mathbf{q_{31}}^2)\,d\mathbf{p_1}d\mathbf{p_2}d\mathbf{p_3}}{\int N_1(\mathbf{p_1})N_1(\mathbf{p_2})N_1(\mathbf{p_3})\,\delta(Q^2-\mathbf{q_{12}}^2-\mathbf{q_{23}}^2-\mathbf{q_{31}}^2)\,d\mathbf{p_1}d\mathbf{p_2}d\mathbf{p_3}}. \tag{27}$$

Three-pion interferometry for different $\langle n\rangle$ is shown in Fig. 2. It is clear that as $\langle n\rangle$ becomes larger, the deviations of the three-pion correlation from ''pure three-pion'' interferometry become larger.

In the following, we will discuss the effects of resonance, flow, and energy conservation on the above calculations. The effects of the resonance on pure two-pion interferometry is not a new subject that has been extensively studied by different authors [23]. Effects of the resonance on the higher-order pion interferometry based on pure multipion interferometry formulas have been recently studied by Csörgő in Ref. [16]. Because Eq. (21) is similar to Eq. (2), all analyses in Ref. [16] can be applied here. In principle, the resonance should not affect the multipion correlation formulas presented here, but because of the limited momentum resolution of the data we will have modified n-pion BE correlation functions as presented in Ref. [16]. The basic idea is that because of the limited resolution of the data, the contributions from the long lived resonance to the correlator are concentrated at a lower relative momentum region that is not resolved by the two-pion interferometry. Thus the interference term $(G_1(\mathbf{p_i},\mathbf{p_j}),i\neq j)$ now mainly contains the contributions from directly emitted pions and pions from short lived resonances. While the single particle spectrum $(G_1(\mathbf{p_i},\mathbf{p_i}))$ is not affected by the two-particle momentum resolution, the intercept of n-pion interferometry is smaller than the ideal value $n!$. Based on the pure two-pion interferometry formula, one has found that the flow makes the apparent radius derived from the two-pion interferometry smaller [24]. Because of the multipion BE correlation effects, it is expected that the apparent radius derived from two-pion interferometry will become much smaller. Considering the effects of the energy constraint on the multipion interferometry, I have derived a new multipion inclusive distribution according to the method presented in Ref. [25]. This new formula is similar to Eq. (21). If we integrated the energy from zero to infinity we will have Eq. (21) again. According to the definition of Eq. (14), in the calculation of Eq. (22), we actually mix all events (with different multiplicities and different energies). It means that in the experiment, one has already integrated all the possible energy of the pions, so it is not necessary for us to consider energy constraint effects here.

In conclusion, in this Rapid Communication, we have derived the i-pion inclusive distribution and i-pion correlation function for a chaotic source which can be used in experiment and theory to analyze multipion interferometry. For a simple model, multipion correlation effects on two-pion and three-pion interferometry were discussed. It was shown that for larger mean pion multiplicity, the deviations of the three-pion and two-pion correlation from the ''pure'' three-pion and two-pion correlation became larger. The effects of resonance, flow, and energy constraint on the multipion correlation formulas were discussed.

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[21] This definition can be found in the preprint version of Ref. [10] and also can be found in Refs. [14,15] where the author defined it as $G(i,j)$.
[22] The detailed information about the calculation can be found in Refs. [10,11,14,15], which originated from Pratt while derived in detail in Ref. [11]. On the other hand, Zajc gave Monte-Carlo methods [9] which enable us to study multipion correlation effects on the lower-order pion interferometry. To my best knowledge, the amount of calculable work for a real model will increase astronomically for all methods. So a method which enables us to quickly calculate multipion correlations is still a debate for physicists.
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