Consistent description of NN and πN interactions using the solitary boson exchange potential

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A unified description of NN and πN elastic scattering is presented in the framework of the one solitary boson exchange potential (OSBEP). This model already successfully applied to analyze NN scattering is now extended to describe πN scattering while also improving its accuracy in the NN domain. We demonstrate the importance of regularization of πN scattering amplitudes involving Δ isobars and derivative meson-nucleon couplings, as this model always yields finite amplitudes without recourse to phenomenological form factors. We find an empirical scaling relation of the meson self-interaction coupling constants consistent with that previously found in the study of NN scattering. Finally, we demonstrate that the OSBEP model does not contradict the soft-pion theorems of πN scattering. [S0556-2813(98)00907-8]

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I. INTRODUCTION

In low- and medium-energy hadron-hadron physics, the NN and πN interactions play a most significant role in our understanding of the dynamics of strong interactions. In that energy regime, specific effects of quantum chromodynamics (QCD) are hidden in effective degrees of freedom such as baryon and meson fields. There are a number of models which address this problem, of which there exist QCD inspired approaches whose aim is to connect phenomena to fundamental QCD features, such as chiral symmetry. With this approach, NN [1–4] and πN [5,6] interactions have been sought with means that led to fundamental insights such as the πN low-energy theorems. But to date, none of these QCD inspired models give an accuracy in description of experimental data as that provided by phenomenological boson exchange [7–13] or inversion potential investigations [14–17]. It seems that chiral symmetry is not a dominant factor in the NN and πN scattering observables up to elastic threshold [18]. Nonetheless, even a chiral symmetry breaking phenomenological approach should be based on concepts which eventually connect to a chiral symmetry maintaining model. As our analysis will show, chiral symmetry is restored in the boson exchange model in the limit $m_\pi \to 0$.

A useful exercise is to interpolate between the QCD inspired and the accurate phenomenological hadron-hadron interaction models. We do so using the one solitary boson exchange potential (OSBEP) in application to NN scattering [19]. The basic idea is to parametrize the effects of chiral symmetry via nonlinear terms in the meson Lagrangian with a structure equivalent to the linear σ model. In contrast to that linear σ model, we do not impose symmetry conditions on coupling constants and masses. Rather we take these entities as free parameters so breaking chiral symmetry to some extent. Then, we solve the decoupled nonlinear meson field equations analytically and quantize the quasiclassical solutions, so defining *solitary mesons*, to obtain the propagator. Proper normalization of the meson fields then ensures that all self-energy diagrams remain finite. A notable result of this approach is that the coupling constants and masses obey an empirical scaling law; one which is similar to the symmetry constraints of chiral models. Our hope is that this scaling law can be related to some underlying symmetry eventually.

The OSBEP model has been determined successfully in np and pp interactions [20]. Thus we now seek its extension to provide a consistent model for both NN and πN interactions. To do so, we must include the Δ isobar specifically and use a chiral symmetry conserving pseudovector (PV) πNN meson-baryon coupling rather than the pseudoscalar (PS) coupling used previously [20]. As a consequence, the proper normalization in the OSBEP model has to be adjusted to provide finite results for the self-energy diagrams involving Δ isobars and derivative meson-baryon vertices. The refit of the parameters which enter the NN potential even yields an improvement to the quality of the original model fit. But the major achievement is that we now have a unified framework for both NN and πN interactions.

Consistent application of a potential model in NN as well as πN interactions has long been an unresolved puzzle. The major concern is that the πNN form factor differed in analyses of these systems. The NN data demand a rather hard cutoff mass (e.g., $\Lambda_{\pi NN}=1.7$ GeV in the Bonn-B potential [21]), and that value cannot be reconciled with the much softer cutoff necessary to fit πN data below 1 GeV. Schütz *et al.* [13] conjecture the reason to be that the form factors depend on all momenta in the external legs of the vertices. However, Holzwarth and Machleidt [22] state that it is impossible to describe NN and πN interactions consistently if one uses an analytical parametrization such as the monopole form of most boson exchange models, and instead propose a Skyrme-model form factor which might be appropriate for both systems. However, to our knowledge, no potential model involving a form factor parametrization exists to date that not only sensibly describes both systems but also gives a sufficiently accurate fit to scattering data.

As indicated above, we have found a unified NN and πN potential built upon the OSBEP model for the form factors. Since all self-energy diagrams are regularized *by construction* in this model, it is corollary that finite scattering amplitudes result for both scattering systems. More important, fits to NN as well as πN data have given a consistent parameter set which provides an agreement with the data as good as found using the best conventional and separate NN or πN models. A notable feature of our result is the low number of parameters we have to specify. There are no adjustable cutoff masses in our model and many possible parameters of the meson nonlinearities are not as they are interrelated by a simple scaling relation which leaves the pion self-interaction coupling constant only as adjustable parameter. Besides that, the only parameters we have, therefore, are the meson-meson and meson-baryon coupling constants which are not fixed by experiment or symmetry relations.

After we give an overview of the main concepts of the OSBEP model in Sec. II, we show in Sec. III how the proper normalization serves to yield finite scattering amplitudes. We also introduce the scaling law for the masses and self-interaction coupling constants in Sec. III. After that, we briefly sketch the application in NN scattering in Sec. IV. Finally, in Sec. V, we present the πN solitary boson exchange potential and in Sec. VI we compare the results for NN and πN phase shifts given by this unified model with those obtained using conventional models and empirical phase shift analyses. Additionally, we address the problem of chiral symmetry by calculating scattering lengths as function of the pion mass and show that our model does not contradict the Weinberg-Tomozawa relations.

II. THE OSBEP MODEL

Motivated by the linear σ-model approach [23], we assume that nonlinear self-interactions for each meson field ($\beta=\pi,\eta,\rho,\omega,\sigma,\delta$) entering the boson exchange potential lead to meson Lagrangians

$$\mathcal{L}_\beta=\frac{1}{2}(\partial_\mu\Phi_\beta\partial^\mu\Phi_\beta-m_\beta^2\Phi_\beta^2)-\frac{\lambda_1^\beta}{2p+2}\Phi_\beta^{2p+2}-\frac{\lambda_2^\beta}{4p+2}\Phi_\beta^{4p+2}+\mathcal{L}_{\mathrm{int}}. \tag{2.1}$$

Note that by choosing proper values for λ_1^β, λ_2^β, and p, one can retain the structure of the linear σ model. For convenience, spin and isospin indices are dropped. The Lagrangian $\mathcal{L}_{\mathrm{int}}$ contains meson-meson as well as meson-baryon vertices which enter the NN and πN scattering amplitudes and will be discussed below.

The main assumption now is that as $t\rightarrow\pm\infty$, the meson fields only decouple from *external* sources and thus the nonlinear self-interaction current has to appear in the field equation for each Fourier component

$$\partial_\mu\partial^\mu\Phi_\beta(x,k)+m_\beta^2\Phi_\beta(x,k)+\lambda_1^\beta\Phi_\beta^{2p+1}(x,k)+\lambda_2^\beta\Phi^{4p+1}(x,k)=0. \tag{2.2}$$

Quasiclassical solutions can be obtained by the method of base functions [24]. Essentially, one makes the ansatz $\Phi=\Phi(\varphi)$, where φ are free wave solutions of the Klein-Gordon equation. This reduces Eq. (2.2) to an ordinary differential equation which is solved by direct integration and the solutions can be expressed as a power series in φ. As a naive quantization rule, inserting free wave operators for $\varphi(x,k)$ gives the *solitary meson fields* with

$$\Phi_\beta(x,k)=\sum_{n=0}^{\infty}C_n^{1/2p}(w_\beta)b_\beta^n\varphi_\beta^{2pn+1}(x,k), \tag{2.3}$$

where

$$\varphi_\beta(x,k)\equiv\frac{1}{\sqrt{2\omega_kVD_k^{(\beta)}}}a_\beta(k)e^{-ikx}. \tag{2.4}$$

Here, V is the volume of the system and $\omega_k=(\vec{k}^2+m_\beta^2)^{1/2}$. Note that we have used a factor $1/\sqrt{D_k^{(\beta)}}$, a Lorentz-invariant function of k, which is of use in the proper normalization of the solitary meson fields that we give later in Sec. III. The coefficients $C_n^a(x)$ are Gegenbauer polynomials and w_β and b_β are functions of the coupling constants and the order p of the self-interaction, namely,

$$w_\beta=\frac{1}{b_\beta}\frac{\lambda_1^\beta}{4(p+1)m_\beta^2}, \tag{2.5}$$

where

$$b_\beta=\sqrt{\left(\frac{\lambda_1^\beta}{4(p+1)m_\beta^2}\right)^2-\frac{\lambda_2^\beta}{4(2p+1)m_\beta^2}}. \tag{2.6}$$

After solution of the field equations the interactions between mesons and baryons are treated perturbatively, and we assume that $\mathcal{L}_{\mathrm{int}}$ contains the following couplings.

Scalar meson-baryon coupling ($\beta=\sigma,\delta$):

$$\mathcal{L}_{mbb}^{(s)}(x)=-g_\beta:\bar{\Psi}(x)\Psi(x):\tilde{\Phi}_\beta(x). \tag{2.7}$$

Pseudovector meson-baryon coupling ($\beta=\pi,\eta$):

$$\mathcal{L}_{mbb}^{(pv)}(x)=\frac{g_\beta}{2M}:\bar{\Psi}(x)\gamma_5\gamma_\mu\Psi(x):\partial^\mu\tilde{\Phi}_\beta(x). \tag{2.8}$$

Vector meson-baryon coupling ($\beta=\rho,\omega$):

$$\mathcal{L}_{mbb}^{(v)}(x)=g_\beta:\bar{\Psi}(x)\left[\gamma_\mu\tilde{\Phi}_\beta^\mu(x)+\frac{\kappa g_\beta}{2M}\sigma_{\mu\nu}\partial^\mu\tilde{\Phi}_\beta^\nu(x)\right]\Psi(x):, \tag{2.9}$$

where

$$\sigma_{\mu\nu}=\frac{i}{2}[\gamma_\mu,\gamma_\nu].$$

$\pi N\Delta$ coupling:

$$\mathcal{L}_{\pi N\Delta}(x)=\frac{g_{\pi N\Delta}}{m_\pi}\bar{\Psi}(x)\vec{T}[x_\Delta\gamma^\mu\gamma_\nu\Psi_\Delta^\nu(x)+\Psi_\Delta^\mu(x)]\partial_\mu\vec{\tilde{\Phi}}_\pi(x)+\mathrm{H.c.} \tag{2.10}$$

$\sigma\pi\pi$ coupling:

$$\mathcal{L}_{\sigma\pi\pi}(x)=\frac{g_{\sigma\pi\pi}}{2m_\pi}\tilde{\Phi}_\sigma(x)\partial_\mu\vec{\tilde{\Phi}}_\pi(x)\partial^\mu\vec{\tilde{\Phi}}_\pi(x). \tag{2.11}$$

$\rho\pi\pi$ coupling:

$$\mathcal{L}_{\rho\pi\pi}(x)=g_{\rho\pi\pi}\vec{\tilde{\Phi}}_\rho^\mu(x)[\vec{\tilde{\Phi}}_\pi(x)\times\partial_\mu\vec{\tilde{\Phi}}_\pi(x)]. \tag{2.12}$$

In these couplings $\Psi(x)$ are nucleon isospinors and, for isovector mesons, the operator $\tilde{\Phi}_\beta(x)$ has to be replaced by $\vec{\tau}\vec{\tilde{\Phi}}_\beta(x)$. To avoid double counting, the vertex operator is a weighted projection of the sum over all Fourier components of the solitary meson field given in Eq. (2.3), i.e.,

$$\tilde{\Phi}_\beta(x)=\sum_{N,N',\vec{k}}\frac{1}{\sqrt{N'!}}|N',k\rangle\langle N',k|\times[\Phi_\beta(x,k)+\Phi_\beta^\dagger(x,k)]|N,k\rangle\langle N,k|\frac{1}{\sqrt{N!}}. \tag{2.13}$$

Since the propagator is used as the probability for a solitary meson to move between the interaction vertices x and y, we must define it using the fields defined in Eq. (2.13) by

$$iP_\beta(x-y)=\langle 0|T\tilde{\Phi}(x)\tilde{\Phi}(y)|0\rangle. \tag{2.14}$$

Inserting Eq. (2.3) into Eq. (2.13) one obtains the momentum space amplitude [19]

$$iP_\beta(k^2,m_\beta)=\sum_{n=0}^{\infty}[C_n^{1/2p}(w_\beta)]^2\frac{b_\beta^{2n}}{(2V)^{2pn}}\times\frac{(2pn+1)^{2pn-2}}{D_{k,n}^{(\beta)2pn+1}(\vec{k}^2+M_{n,\beta}^2)^{pn}}i\Delta_F(k^2,M_{n,\beta}), \tag{2.15}$$

with the Feynman propagator being

$$i\Delta_F(k^2,M_{n,\beta})=\frac{i}{k^2-M_{n,\beta}^2}, \tag{2.16}$$

and a mass spectrum given by

$$M_{n,\beta}=(2pn+1)m_\beta.$$

The normalization, $D_{k,n}^{(\beta)}$ in the propagator Eq. (2.15), is obtained from the normalization $D_k^{(\beta)}$ in Eq. (2.4) by substituting

$$k^\mu\to\frac{1}{2pn+1}k^\mu.$$

It is useful at this point to introduce dimensionless coupling constants α_β, α_1^β, and α_2^β, respectively,

$$\alpha_\beta=\frac{b_\beta}{(2m_\beta V)^p},$$

$$\alpha_1^\beta=\frac{\lambda_1^\beta}{4(p+1)m_\beta^2(2m_\beta V)^p},$$

$$\alpha_2^\beta=\frac{\lambda_2^\beta}{4(2p+1)m_\beta^2(2m_\beta V)^{2p}}. \tag{2.17}$$

The final amplitude, which we define to be the *solitary meson propagator,* then is

$$iP_\beta(k^2,m_\beta)=\sum_{n=0}^{\infty}[C_n^{1/2p}(w_\beta)]^2\times\frac{(m_\beta^p\alpha_\beta)^{2n}(2pn+1)^{2pn-2}}{D_{k,n}^{(\beta)2pn+1}(\vec{k}^2+M_{n,\beta}^2)^{pn}}i\Delta_F(k^2,M_{n,\beta}), \tag{2.18}$$

with

$$w_\beta=\frac{\alpha_1^\beta}{\sqrt{\alpha_1^{\beta 2}-\alpha_2^\beta}}. \tag{2.19}$$

For $p=1/2$, one gets the amplitude for scalar fields while with $p=1$ the amplitude is that for pseudoscalar particles. Vector mesons require p to be 1 and each term of the sum is multiplied with a Minkowski tensor,

$$f_n^{\mu\nu}=\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{M_{n,v}^2}\right). \tag{2.20}$$

III. PROPER NORMALIZATION

The proper normalization constant in Eq. (2.18) is now fixed by physical boundary conditions. First, we impose the constraints familiar from renormalization theory, i.e., the propagator has to have a pole of residue i at the on-shell point, $k^2=m_\beta^2$. In addition, we assume that (i) all amplitudes are Lorentz invariant, (ii) $D_k^{(\beta)}$ is dimensionless and larger than unity, (iii) the fields vanish when the interaction vanishes, and, most important, (iv) all self-energy diagrams are finite. This leads to the ansatz

$$D_k^{(\beta)}=\left\{1+\left[\left(\frac{1}{\alpha_1^\beta 4(p+1)(2m_\beta)^p}\right)^{2/p}+\left(\frac{1}{\alpha_2^\beta 4(2p+1)(2m_\beta)^{2p}}\right)^{1/p}\right]\times\left(\sqrt{\vec{k}^2+m_\beta^2}-k_0\right)^2\right\}^{N_{pn}^{(\beta)}}. \tag{3.1}$$

In the case that there is only one nonlinear term in the Lagrangian ($\alpha_2^\beta\equiv 0$), the term in Eq. (3.1) containing α_2^β is to

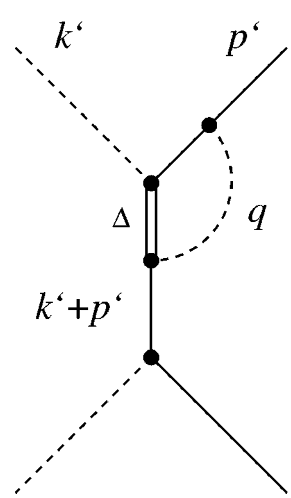

FIG. 1. $\pi N\Delta$ vertex correction.

be deleted. The exponent $N_{pn}^{(\beta)}$ can be chosen for each meson type to yield finite scattering amplitudes. At this point, it is crucial to note that assuming a momentum-dependent normalization $D_k^{(\beta)}$ for the fields in Eq. (2.3) *always* gives finite self-scattering amplitudes for all interactions. Even in cases where the interaction is nonrenormalizable in standard models, such as with massive spin-1 ρ and ω mesons, our method can be applied. On the other hand, an energy-dependent normalization affects the canonical commutation relations in coordinate and momentum space. The equal-time commutator of the field operator and its conjugate momentum is no longer a δ function $\delta(\vec{x}-\vec{x}')$ in coordinate space, as in conventional field theoretical models, but approaches a *finite* value for $\vec{x}=\vec{x}'$ and vanishes otherwise. This can be interpreted as a finite particle size due to the self-interaction. Another important point of the model is that causality is preserved since the equal-time commutators of the fields remain unchanged, i.e.,

$$[\Phi_\beta(\vec{x},t;k),\Phi_\beta(\vec{x}',t;k)]=[\tilde{\Phi}_\beta(\vec{x},t),\tilde{\Phi}_\beta(\vec{x}',t)]=0.$$

To determine $N_{pn}^{(\beta)}$ one has to consider the most divergent self-energy amplitude for each type of meson. For scalar and vector mesons, this is the first correction to the two-point function, Eq. (2.14). Whereas for scalar mesons it is sufficient to choose

$$D_k^{(s)}=\mathcal{O}(k^2)\Rightarrow N_{pn}^{(s)}=1,$$

for vector mesons one has to use

$$D_k^{(v)}=\mathcal{O}(k^4)\Rightarrow N_{pn}^{(v)}=2,$$

due to the additional momentum dependence in Eq. (2.20). For the pion however, the $\pi N\Delta$ vertex correction (see Fig. 1) is the most divergent amplitude. The combination of derivative coupling in Eq. (2.10) and the Δ propagator which grows linearly with momentum, requires a strong normalization for the pion. Therefore we have to choose

$$D_k^{(ps)}=\mathcal{O}(k^6)\Rightarrow N_{pn}^{(ps)}=3,$$

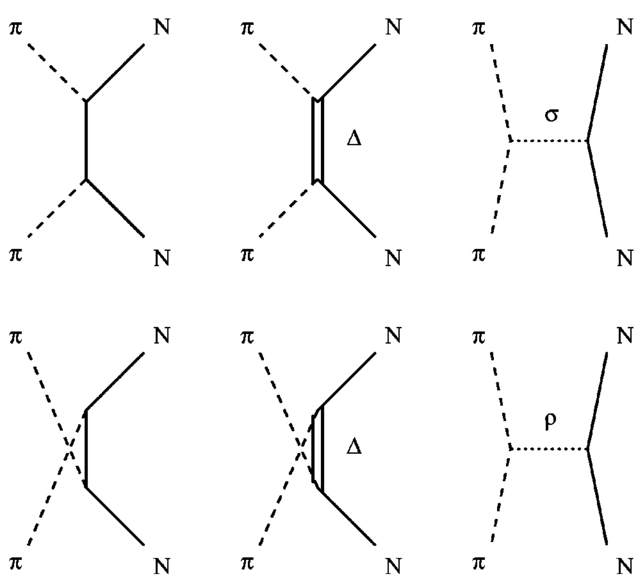

FIG. 2. Feynman-diagrams for the πN interaction.

TABLE I. Meson masses and proper normalizations associated with OSBEP.

β	π	η	ρ	ω	σ	δ
m_β [MeV]	138.03[a]	548.8	769	782.6	550[b]	983
$N_{pn}^{(\beta)}$	3	3	2	2	1	1

[a]For the *pp* potential, we used the neutral pion mass $m_{\pi_0}=134.9764$ MeV.

[b]For the $T=0$ *np* potential, we adopted $m_\sigma=720$ MeV from the Bonn-B potential.

to obtain finite results for all self-energy diagrams in *NN and* πN interactions. The meson masses and normalizations used are listed in Table I. Note that in our former work [19,20], $N_{pn}^{(ps)}=1$ sufficed since we used PS coupling for the πNN and ηNN vertex and the Δ isobar was not treated. This modification now demands a refit of the parameters entering the *NN* potential to maintain (or improve upon) the accuracy of fits to data.

IV. APPLICATION TO *NN* INTERACTIONS

The concept of the *NN* solitary boson exchange potential is very similar to the Bonn-B OBEP [21]. It has been described in detail in Refs. [19] and [20]. The inclusion of the Δ isobar and the PV coupling for the πNN and ηNN vertices do not significantly change the actual form of the potential. The Δ intermediate states do not contribute in the one boson exchange approximation and the PV coupling on-shell is identical to, and off-shell is very similar to the PS coupling when the potential is evaluated in the Blankenbecler-Sugar (BbS) reduction of the Bethe-Salpeter equation [9]. Negative energy states do not contribute. We note also that Machleidt [21] has shown that by slightly changing the parameters, it is possible to obtain equally good results for the Bonn-B potential using either the PS or PV coupling. We confirm this result and obtain fits of similar quality with both PS and PV coupling. The most important modification in the *NN* potential is to change the proper normalization exponent $N_{pn}^{(\beta)}$ of the pseudoscalar mesons (π and η). The question arises whether the empirical scaling relation for the self-interaction coupling constants and masses, found in a previous comparison of the solitary meson propagator to the Bonn-B form factors [19], remains valid. Using the strong normalization, $N_{pn}^{(\beta)}=3$ for π and η, a similar analysis indicates that the scaling relation generalizes to

$$\frac{\alpha_\beta}{\sqrt{N_{pn}^{(\beta)}}}=\frac{\alpha_\pi}{\sqrt{N_{pn}^{(\pi)}}}\left(\frac{m_\pi}{m_\beta}\right)^p, \tag{4.1}$$

and thus still serves to minimize the number of parameters we need to specify. Note that we simplified the model by setting $\alpha_2^\beta=0$ so that $\alpha_\beta=\alpha_1^\beta$ is the only self-interaction parameter for all mesons. In the linear σ model, which motivated our ansatz for the nonlinear terms in the meson Lagrangian, chiral symmetry also demands $\alpha_2^\beta=0$ for pseudoscalar and vector mesons. On the other hand, this does not apply for scalar mesons. However, the scalar σ meson in the potential model serves as a parametrization of two-pion exchange. It is not a fundamental particle as considered in the σ model. The second scalar meson, δ only contributes a little.

V. THE πN SOLITARY BOSON EXCHANGE POTENTIAL

The structure of the πN boson exchange potential was adopted from the work of Pearce and Jennings [11]. The only changes arise for the form factors which can be dropped due to the proper normalization of the solitary meson fields and the three-dimensional reduction of the scattering equation to account for solitary mesons in the intermediate πN states.

Using the Lagrangians in Eqs. (2.7)–(2.12), the diagrams in Fig. 2 can be evaluated using standard Feynman rules, attaching a factor $1/\sqrt{D_k^{(\pi)}}$ [Eq. (3.1) with $N_{pn}^{(\pi)}=3$] to each vertex with an external pion of momentum k, and replacing the standard Feynman propagator in the σ- and ρ-exchange diagrams by the solitary meson propagator, Eq. (2.18) for the σ and ρ mesons, respectively. To describe a self-interacting pion in the intermediate state one has to modify the two-particle propagator of the Bethe-Salpeter (BS) scattering equation

$$\mathcal{T}(p'_\mu,p_\mu,s)=\mathcal{V}(p'_\mu,p_\mu,s)+\int\frac{d^4q}{(2\pi)^4}\mathcal{V}(p'_\mu,q_\mu,s)\mathcal{G}(q_\mu,s)\mathcal{T}(q_\mu,p_\mu,s), \tag{5.1}$$

where p_μ, q_μ, and p'_μ are the momenta of the incoming, intermediate, and outgoing nucleon, respectively. The incoming particles are on their mass shell, i.e.,

$$p_0=\sqrt{\vec{p}^2+M^2}\equiv\epsilon_N \quad\text{and}\quad k_0=\sqrt{\vec{k}^2+m_\pi^2}\equiv\epsilon_\pi .$$

In the center of mass (c.m.) system one gets

$$s=(p_\mu+k_\mu)^2=(p'_\mu+k'_\mu)^2=(\epsilon_N+\epsilon_\pi)^2,$$

and the pion momenta will be omitted since

$$k_\mu=(\sqrt{s}-p_0,-\vec{p}) \quad\text{and}\quad k'_\mu=(\sqrt{s}-p'_0,-\vec{p}\,').$$

The BS propagator then becomes

$$\mathcal{G}(q_\mu,s)=iP_\pi(p_\mu+k_\mu-q_\mu)S_F(q_\mu). \tag{5.2}$$

It is important to note that in Eq. (5.2) the solitary meson propagator is used for the intermediate pions instead of the Feynman propagator. Due to the proper normalization, $iP_\pi(k_\mu)$ now carries *by construction* a sufficiently strong decay with increasing momentum to regularize all diagrams so that phenomenological form factors are not needed.

In the model of Pearce and Jennings [11], there are two different reduction schemes for the four-dimensional equation, Eq. (5.1). We use the ''smooth-propagator'' formalism since it has the correct one-body limit [25]. The Blankenbecler-Sugar reduction does not have this property. While this is not a major problem for equal-mass systems such as NN scattering, it may cause problems in a study of πN scattering. In conventional models, the reduction is performed using the substitution [11]

$$i\Delta_F(p_\mu+k_\mu-q_\mu)S_F(q_\mu)\to\delta(q_0-\epsilon_N)\mathcal{G}_{sm}^{\text{lin}}(\vec{q},s),$$

where

$$\mathcal{G}_{sm}^{\text{lin}}(\vec{q},s)=\frac{2\pi}{\sqrt{s}}\,\frac{\gamma_0\epsilon_N-\vec{\gamma}\vec{q}+M}{\vec{p}^2-\vec{q}^2+i\epsilon}. \tag{5.3}$$

This propagator is transformed to describe solitary mesons simply by setting

$$iP_\pi(k_\mu)\equiv i\Delta_F(k_\mu)F_\pi(k_\mu)=\frac{i}{k^2-m_\pi^2}F_\pi(k_\mu),$$

and from Eq. (2.18), one gets

$$F_\pi(k_0;|\vec{k}|)=\sum_{n=0}^{\infty}\frac{(m_\pi\alpha_\pi)^{2n}(2n+1)^{2n-2}}{D_{k,n}^{(\pi)2pn+1}[\vec{k}^2+(2n+1)^2m_\pi^2]^n}\times\frac{k^2-m_\pi^2}{k^2-(2n+1)^2m_\pi^2}. \tag{5.4}$$

Recall that the proper normalization constant was designed to yield in $iP_\pi(k_\mu)$, a pole with residue i at $k^2=m_\pi^2$. Thus $F_\pi(k_0;|\vec{k}|)=1$ at the pion pole. The reduction of the Bethe-Salpeter equation, Eq. (5.1), for solitary mesons can now be performed in analogy to the development of Eq. (5.3) by the substitution

$$iP_\pi(p_\mu+k_\mu-q_\mu)S_F(q_\mu)\to\delta(q_0-\epsilon_N)\mathcal{G}_{sm}(\vec{q},s),$$

where

$$\mathcal{G}_{sm}(\vec{q},s)=\frac{2\pi}{\sqrt{s}}F_\pi(\epsilon_\pi;|\vec{q}|)\,\frac{\gamma_0\epsilon_N-\vec{\gamma}\vec{q}+M}{\vec{p}^2-\vec{q}^2+i\epsilon}. \tag{5.5}$$

Inserting Eq. (5.5) into the Bethe-Salpeter equation, Eq. (5.1) and performing a partial wave decomposition [11], the one-dimensional scattering equation for the partial wave T matrix results (p denotes $|\vec{p}|$ and ℓ stands for $\{L,T,J\}$)

$$T_\ell(p',p,s)=V_\ell(p',p,s)+\int_0^\infty q^2dqV_\ell(p',q,s)G_{sm}(q,s)T_\ell(q,p,s), \tag{5.6}$$

where

$$G_{sm}(q,s)=\frac{M}{(2\pi)^3\sqrt{s}}\,\frac{F_\pi(\epsilon_\pi;q)}{p^2-q^2+i\epsilon}. \tag{5.7}$$

Explicit forms for the pseudopotentials, V_ℓ, corresponding to the Feynman amplitudes in Fig. 2, evaluated with the model of Pearce and Jennings, are listed in Ref. [11]. The OSBEP pseudopotentials then are obtained by replacing the form factors with $1/\sqrt{D_k^{(\pi)}}$ for each pion leg of momentum k^μ and by substituting the Feynman propagators with the solitary meson propagators in the σ- and ρ-exchange amplitudes. Phase shifts are then calculated from the on-shell T matrix on defining the density of states by

TABLE II. The optimal parameter values of our OSBEP model. The parameters influencing phase shift calculations for the NN and πN scattering systems are indicated.

Name	Value	NN	πN
$g_\pi^2/4\pi$	13.75 (fixed)	x	x
α_π	0.7471	x	x
κ_ρ	3.3982	x	x
$g_\eta^2/4\pi$	0.0745	x	
$g_\rho^2/4\pi$	1.6725	x	
$g_\omega^2/4\pi$	22.499	x	
$g_{\sigma_0}^2/4\pi$	12.2415	x	
$g_{\sigma_1}^2/4\pi$	8.9523 (np)	x	
$g_{\sigma_1}^2/4\pi$	8.8461 (pp)	x	
$g_\delta^2/4\pi$	1.4172	x	
$g_{\rho\pi\pi}g_\rho/4\pi$	5.7047		x
$g_{\sigma\pi\pi}g_\sigma/4\pi$	-0.7434		x
$g_{\pi N\Delta}^2/4\pi$	0.213954 (fixed)		x
x_Δ	-0.1829		x

$$\mathrm{disc}G_{sm}(q,s)=-\frac{2\pi i}{p^2}\rho(p)\delta(p-q),$$

and with Eq. (5.7) to have

$$\rho(p)=\frac{pM}{(2\pi)^3 2\sqrt{s}}F_\pi(\epsilon_\pi;q),$$

so that defining

$$\tau_\ell(p)=-\pi\rho(p)T_\ell(p,p)$$

the phase shifts can be specified by

$$\delta_\ell(p)=\arctan\frac{\mathrm{Im}\tau_\ell(p)}{\mathrm{Re}\tau_\ell(p)}.$$

VI. RESULTS

We calculated the NN and πN phase shifts separately and compared the results with the latest single-energy phase shift analyses; SM97 [26] for NN and SM95 [27] for πN scattering, respectively. Since there are no phenomenological form factors in our model and the scaling law relates all meson nonlinearities to the pion self-interaction coupling constant α_π, that constant and the meson-baryon and meson-meson coupling constants were the only parameters we adjusted to achieve fits to data. Of these parameters, the tensor-vector ratio κ and the pion self-interaction coupling constant α_π are involved with both potentials. Hence those two play a crucial role in the determination of our optimal parameter set of values. We noticed that, when the πN data alone are considered, a rather low value of α_π (around 0.4) is favored. Alone, the NN system is much better described with a value of α_π of about 0.7. However, this larger value can be reconciled with the πN data. To do so one must set the value of κ as low as possible without losing much accuracy in fits to the NN data.

TABLE III. Bare and renormalized values for nucleon and Δ masses and coupling constants. The bare values are used in the pseudopotentials for the nucleon and Δ pole diagrams in the P_{11} and P_{33} channel, respectively.

	M [MeV]	$g_\pi^2/4\pi$	M_Δ [MeV]	$g_{\pi N\Delta}^2/4\pi$
bare	1346.51	1.8687	1027.80	0.0437
dressed	938.926	13.75	1232	0.2139

TABLE IV. χ^2/datum for the OSBEP and several potential models. Data and χ^2 values for the Nijm93 and Paris potential were taken from SAID [35].

Model	No. of parameters	np[a]	pp[b]	Total
OSBEP	8	2.9	6.7	4.1
Nijm93	15	5.6	2.2	4.5
Bonn-B	15	12.1	5.8[c]	10.1
Paris	≈ 60	12.5	2.3	9.2

[a]Energy bin 1–300 MeV (2713 data points).
[b]Energy bin 1–300 MeV (1292 data points).
[c]pp version $g_{\sigma_1}^2/4\pi=8.8235$, see Ref. [20].

We emphasize a good fit of the NN phase shifts as they are determined more accurate than are the πN phases and stay from a larger database. Therefore, first we adjusted the parameters of the model to find a fit to the NN data. It turned out to be even better than in our earlier work [20]. Then, we used the remaining parameters in a πN analysis to perform a fit with respect to the SM95 phase shift analysis [27]. We used those in preference to the Karlsruhe-Helsinki phases [28] as the SM95 data have associated error bars which allow us to make a weighted fit. The ultimate parameter set values are listed in Table II. From those values note that the πNN coupling constant is smaller than the value of $g_\pi^2/4\pi=14.4$ previously used. The first indication that such should be so came from a Nijmegen analysis [29] which suggests $f_{\pi NN}^2=0.0745$ and thus $g_\pi^2/4\pi=13.79$ when our values for the pion and nucleon masses are used. Also, Arndt and coworkers with their analysis of πN scattering [30] have deduced a similar value. We confirmed that Arndt result in an independent analysis [31] and so we fixed the πNN coupling constant to that value, viz.

$$\frac{g_\pi^2}{4\pi}=13.75. \tag{6.1}$$

TABLE V. The properties of the deuteron.

	Bonn-B [21]	OSBEP	Exp.	Ref.
E_B [MeV]	2.2246	2.22459	2.22458900(22)	[36]
μ_d	0.8514[a]	0.8456[a]	0.857406(1)	[37]
Q_d [fm^2]	0.2783[a]	0.2728[a]	0.2859(3)	[38]
A_S [fm$^{-1/2}$]	0.8860	0.8788	0.8802(20)	[38]
D/S	0.0264	0.0256	0.0256(4)	[39]
r_{rms} [fm]	1.9688	1.9554	1.9627(38)	[38]
P_D [%]	4.99	6.00		

[a]Meson exchange current contributions not included.

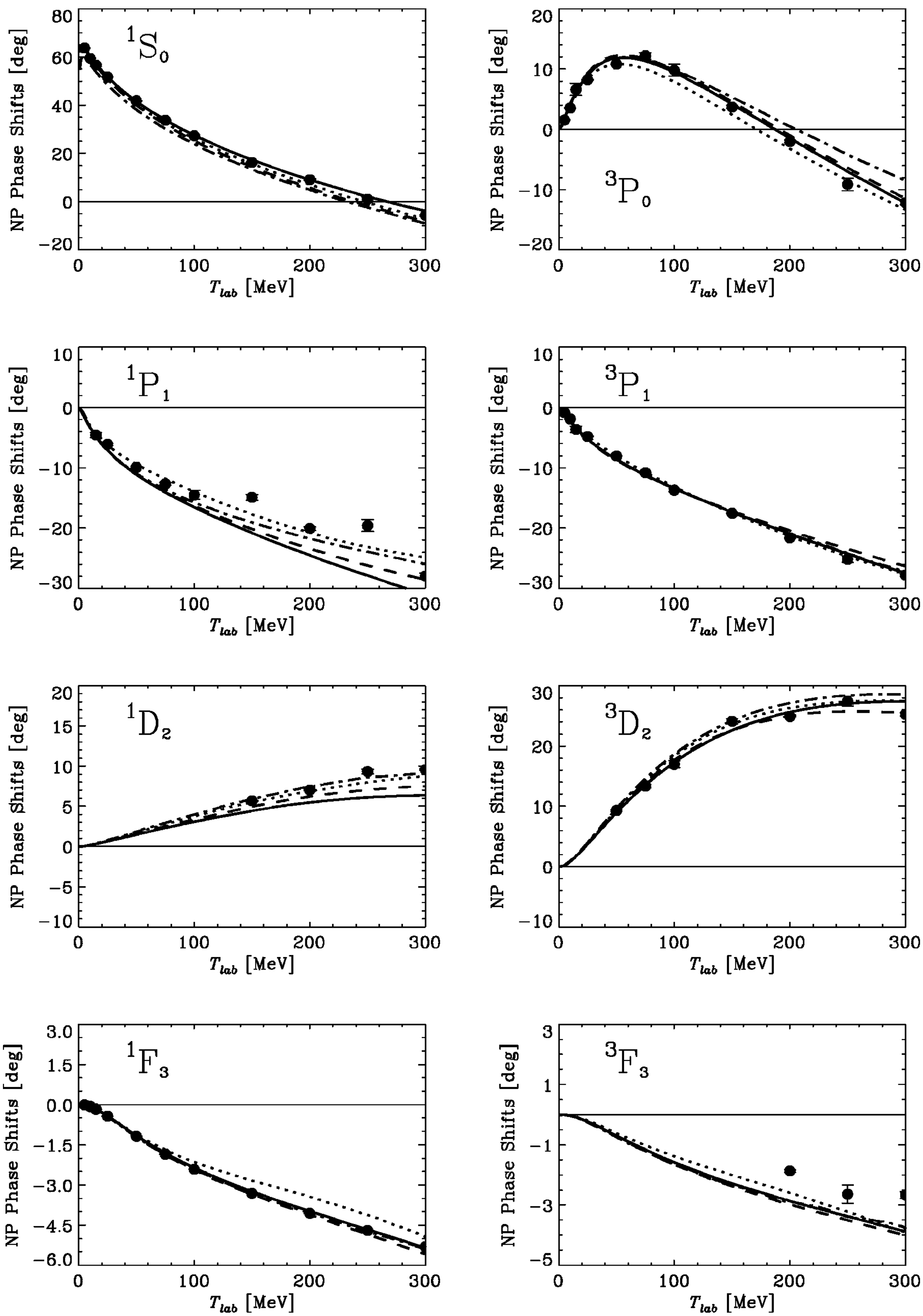

FIG. 3. *np* phase shifts. The Arndt SM97 [35] phase shifts (circles) are compared with the phase shifts calculated using the Nijm93 [7] (dotted), Bonn-B [21] (dashed), Paris [8] (dash-dotted) potentials, and with our OSBEP (solid).

The $\pi N\Delta$ coupling constant is then fixed by the quark-model relation [32]

$$\frac{g^2_{\pi N\Delta}}{4\pi}=\frac{72}{25}\left(\frac{m_\pi}{2M}\right)^2\frac{g^2_\pi}{4\pi}.$$

It should be noted that the large value of the pion self-interaction coupling constant (~ 0.7) can only be used in the πN potential if the $\pi N\Delta$ coupling constant is set to that quark model value. If one uses the value $g^2_{\pi N\Delta}/4\pi=0.36$, as chosen for most other πN potentials, the fit is much worse. Another important feature in Table II is the sign of the $\sigma\pi\pi$

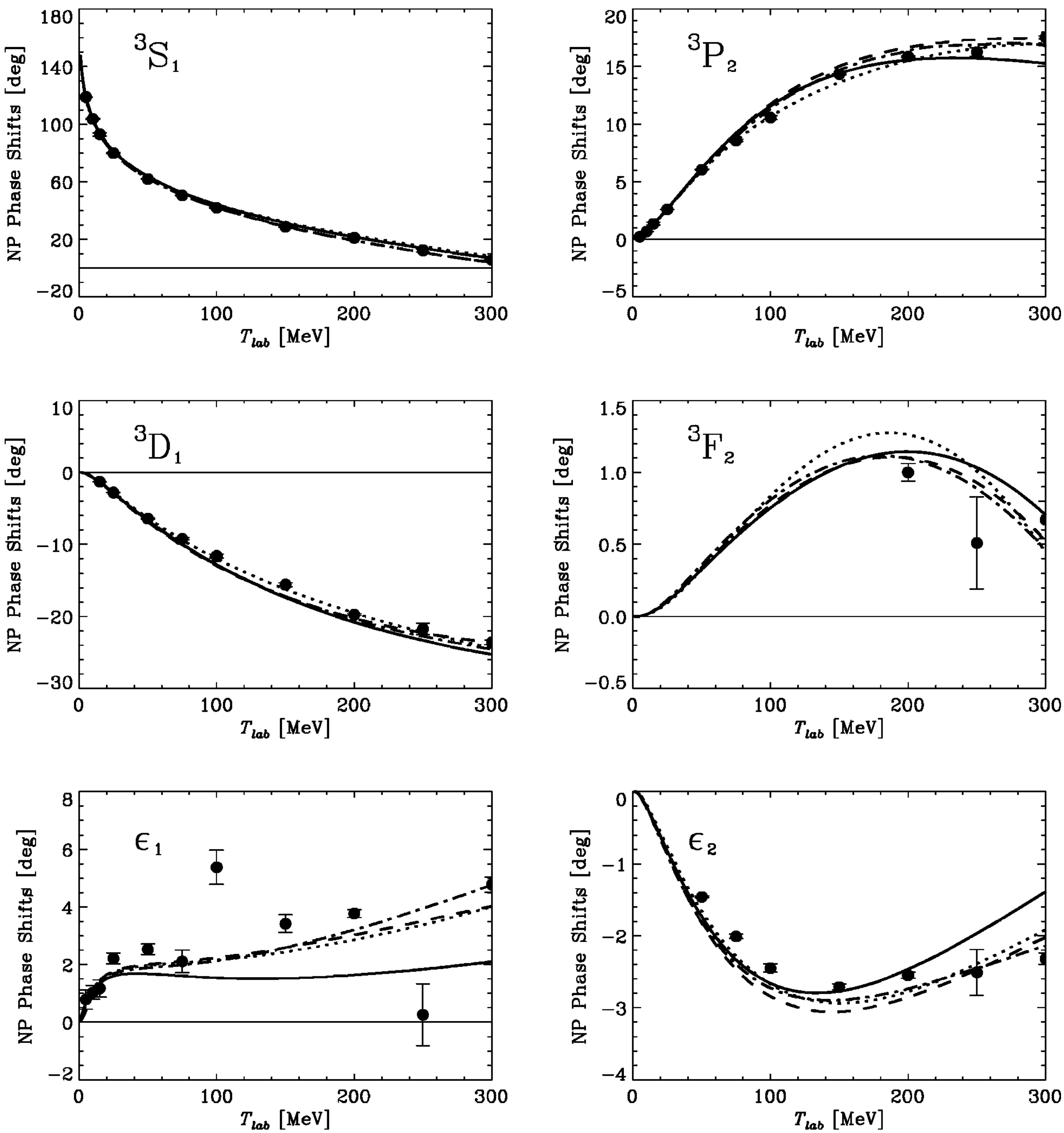

FIG. 4. SYM np phase shifts for the coupled $^3SD_1$ and $^3PF_2$ channels with notation as in Fig. 3.

coupling constant. In the work of Pearce and Jennings [11], this coupling is positive and very large ($g_{\sigma\pi\pi}g_\sigma/4\pi = 143.6$), which may be caused by the rather low cutoff mass (~ 500 MeV) they use in the form factor of the σNN and $\sigma\pi\pi$ vertices. Such a cutoff is very abrupt. Furthermore, using a model based on correlated two-pion exchange derived from dispersion relations, Schütz *et al.* [13] found the sign of the product $g_{\sigma\pi\pi}g_\sigma$ should be negative.

Since the Δ and nucleon pole diagrams are iterated in the πN scattering equation, one has to use the bare values for masses and coupling constants in the kernel of the integral equation for the P_{33} and P_{11} channels, respectively. In principle, these values are related to the physical ones by the renormalization procedure [33]. We simplified the model by finding the bare values that optimize the fit to the phase shifts in the relevant channels. First, we adjusted the other parameters to fit the phase shifts in the nonresonant channels. After that, there was but one bare mass and coupling constant for the nucleon and Δ which reproduced the phase shifts in the P_{11} and P_{33} channels, respectively. By this procedure, in principle the bare parameters were functions of the other parameters too. The results are given in Table III. The values of the parameters in Table II involved with the NN potential are very similar to those found using our original (pure NN) potential [20]. The proper normalizations of the π and η are the only features that vary, it is not surprising that the only significant change in the parameter values is that for the ηNN coupling constant the present result being considerably less than the former value of 0.702 [20] and that for the self-interaction coupling constant α_π, the present result

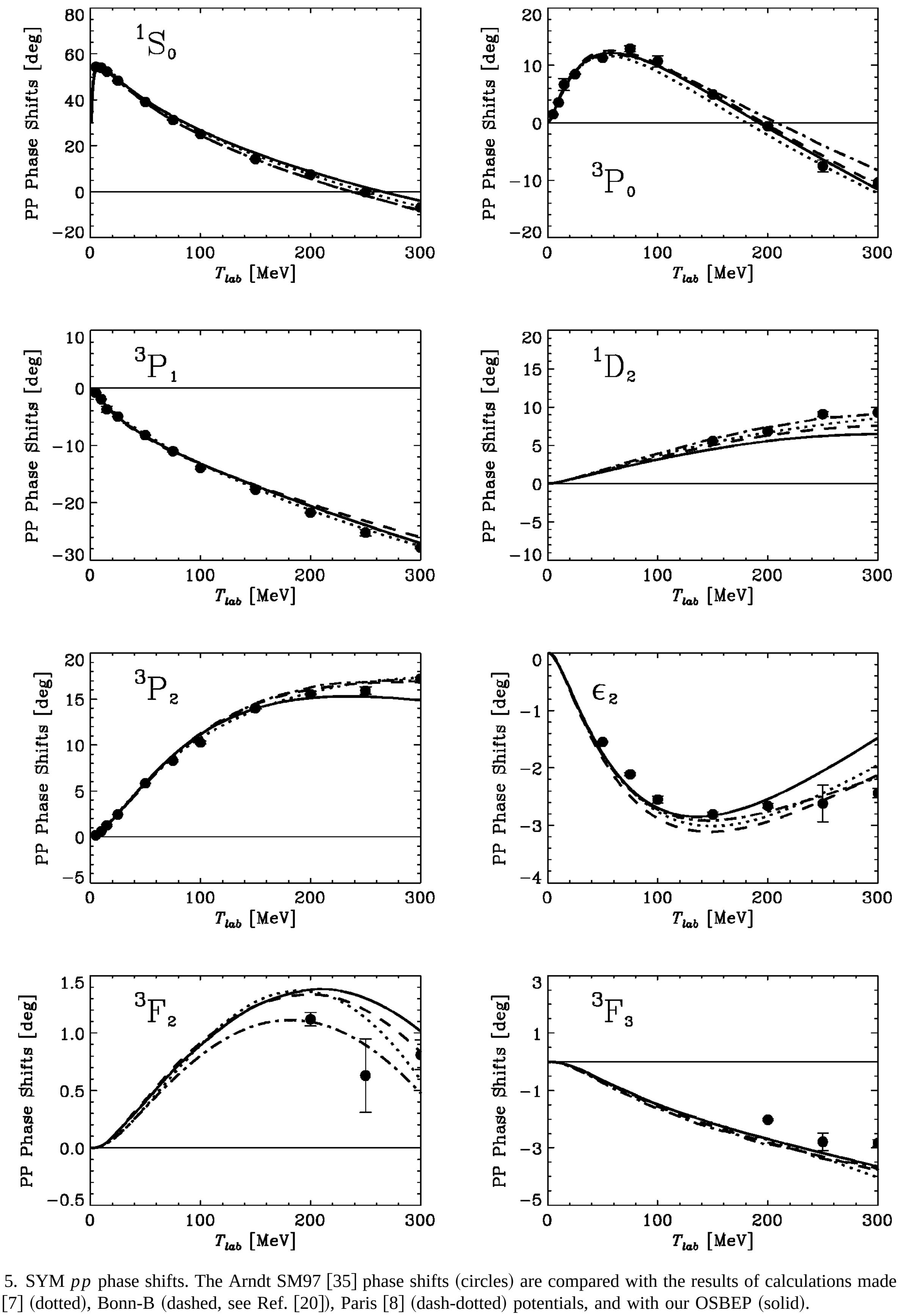

FIG. 5. SYM pp phase shifts. The Arndt SM97 [35] phase shifts (circles) are compared with the results of calculations made using the Nijm93 [7] (dotted), Bonn-B (dashed, see Ref. [20]), Paris [8] (dash-dotted) potentials, and with our OSBEP (solid).

being much larger than found with the fit using $N_{pn}^{(ps)}=1$ (there $\alpha_\pi=0.44065$ [20]). However, the generalized scaling law, Eq. (4.1), keeps the vector and scalar self-interaction coupling constants close to the values determined by the older fit.

We have used OSBEP to fit NN phase shifts (to 300

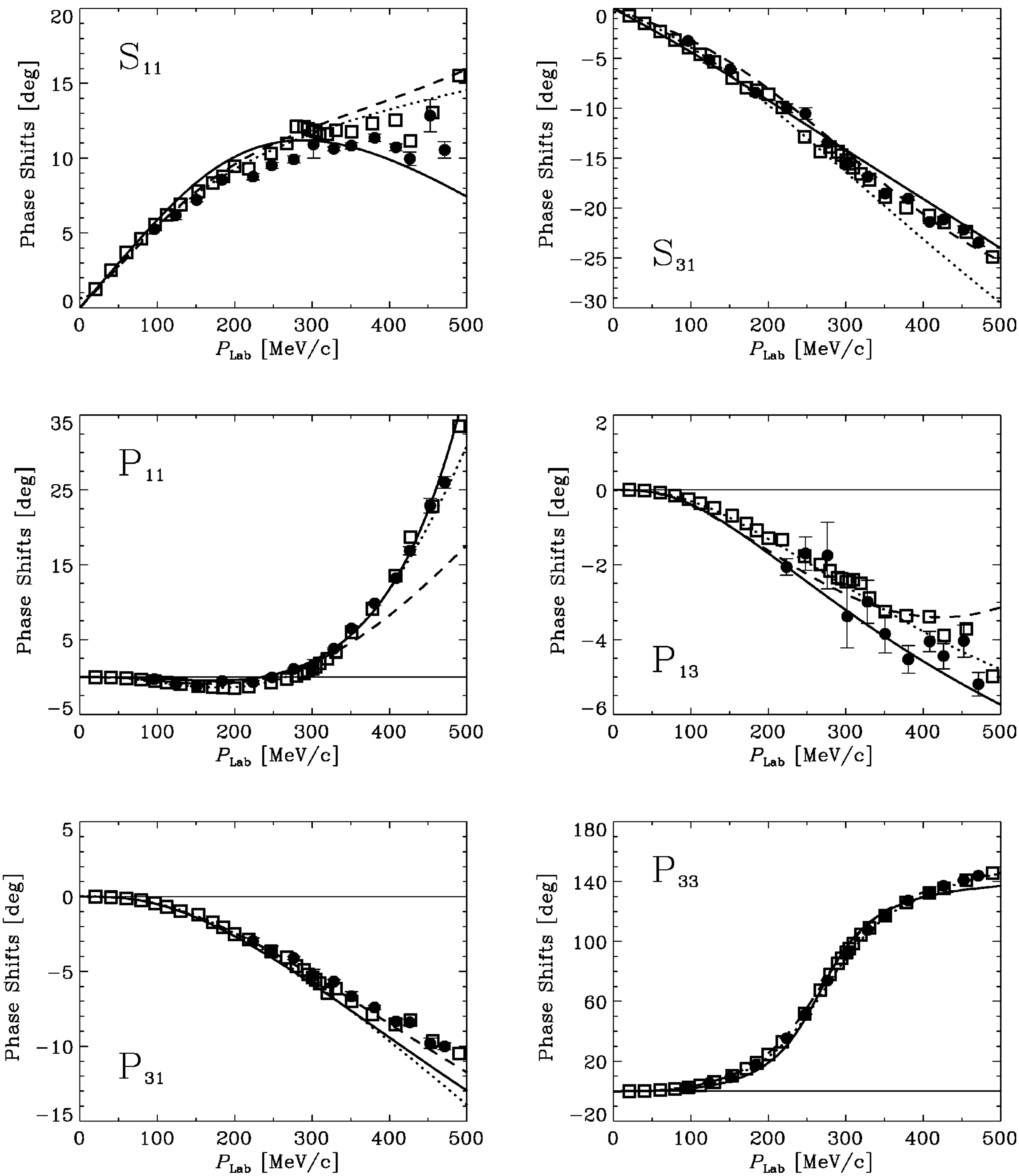

FIG. 6. πN phase shifts. The SM95 [27] (dots) and KH80 [28] (squares) phase shift analyses compared with results of calculations made by Pearce and Jennings [11] (dashed), Schütz *et al.* [13] (dotted), and with our OSBEP model (solid).

MeV) for numerous angular momentum channels and to fit πN phase shift data in all S and P channels to a momentum of 500 MeV/c. Excellent fits have been obtained as is evident from Table IV in which the χ^2/datum with respect to the world's NN database are listed in comparison to those found with standard models. A byproduct is that OSBEP yields excellent results for the properties of the deuteron. They are listed in Table V wherein comparison is made with the experimental values and with those associated with the Bonn-B force.

The phase shifts for diverse channels are compared with data and the predictions of standard models in Figs. 3–5 for NN scattering and in Fig. 6 for πN scattering. In Fig. 3 the np phase shifts for uncoupled channels (to $^3F_3$) are shown. The OSBEP results are as good as if not better than those of the standard models with rare exception. That is also the case with the coupled channels in Fig. 4. Finally, in Fig. 5, we show the pp phase shifts to which OSBEP does as well as the conventional potential calculations.

The S and P wave channel phase shifts for πN scattering as given by OSBEP and two other model calculations are compared with data in Fig. 6. The OSBEP results are again good and of a quality comparable to that found with the other model results.

However, while the OSBEP fit to the NN data is very satisfactory, providing at least the same quality as conventional models with a minimum number of the parameters, the πN fits could be further improved. Especially, in the S_{11} channel, inclusion of the $N^*(1535)$ resonance would contribute by increasing the value of the phase shifts at energies above 400 MeV [34]. We note also that the width of the Δ resonance in the P_{33} channel predicted by OSBEP is not as accurate as those found with the conventional models. The resonance is produced mainly from the background potential and not just from the Δ pole diagram alone and which reflects in the rather low value of the bare Δ mass listed in Table III. At the same time, the background potential has to compensate for the negligible effect of the $N^*(1535)$ in the S_{11} channel phase shifts. Inclusion of this resonance in the model would simultaneously improve the fit to the P_{33} channel data. Note that the πN data fit was performed using the SM95 phase shifts (dots) with their error bars as experimental input to a search. Thus, the OSBEP phases must deviate from the KH80 phase shifts values (squares) in the P_{13} channel.

To test whether the model restores chiral symmetry in the limit $m_\pi \to 0$, we calculate the S-wave scattering lengths and compare them with the Weinberg-Tomozawa relations

$$a_+ = \frac{1}{3}(a_{S_{11}} + 2a_{S_{31}}) = \mathcal{O}(m_\pi^2)$$

and

$$a_- = \frac{1}{3}(a_{S_{11}} - a_{S_{31}}) = \mathcal{O}(m_\pi)$$

derived from the soft-pion theorems. The scattering lengths are plotted as a function of the pion mass in Fig. 7. It is obvious that both slopes follow the Weinberg-Tomozawa relations nicely and thus the model of solitary mesons does not contradict the soft-pion theorems.

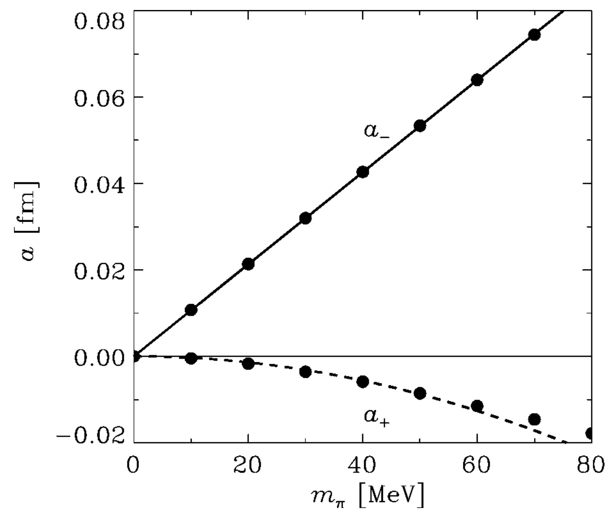

FIG. 7. S-wave scattering lengths from πN scattering calculations made using OSBEP as a function of the pion mass.

VII. SUMMARY AND OUTLOOK

In this work we have shown that the one solitary boson exchange potential OSBEP can be extended to describe simultaneously NN and πN scattering data. With this approach, we have no problem in having a consistent description of both systems. There is no incompatibility of the πNN form factor in particular. Since our solitary boson exchange method regularizes the self-energy diagrams *a priori*, the model enabled us to obtain consistently finite scattering amplitudes for NN as well as πN scattering. Additionally, we were able to retain the empirical scaling relation which already was successfully applied in a precise analysis of NN scattering alone. This relation serves to significantly reduce the number of parameters existent in our model below that required with all other methods. The model phase shifts agree very well with those found using the latest NN and πN phase shift analyses and, with the properties of the deuteron. The accuracy of the fits are comparable to those given by conventional potential models for NN and πN, respectively.

In the future we hope to apply this model in analyses of pion production processes. It is well known that a proper description of the very accurate data near threshold demands a NN final state interaction as well as a πN T matrix that are consistent with each other. The solitary boson exchange potential fulfills this need. Use of OSBEP to analyze $\pi\pi$ scattering is another interesting aim. It would be a serious test for this model to see if the dynamics of solitary mesons are compatible with such data and if the model can maintain the consistency we have found by studying the NN and πN systems.

Finally we note a need to perform a refined simultaneous fit to NN and πN and the calculation of πN scattering observables. Since the simultaneous fit to NN and πN data is very time consuming, the phase shifts shown here were obtained first by fitting the NN data and then by adjusting the remaining three parameters to fit the πN data. Therefore, the quality of fit to the NN phases is better than that to the πN ones. However, the accuracy of our results convince us that the solitary boson exchange potential works consistently for NN and πN interactions.

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[1] T. H. R. Skyrme, Nucl. Phys. **31**, 556 (1962).

[2] J. Wambach, in *Quantum Inversion Theory and Applications*, edited by H. V. von Geramb, Vol. 427 of Lecture Notes in Physics (Springer, New York, 1994).

[3] C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994).

[4] C. M. Shakin, Wei-Dong Sun, and J. Szweda, Phys. Rev. C **52**, 3353 (1995).

[5] S. Weinberg, Phys. Rev. Lett. **18**, 188 (1967).

[6] V. Bernard, N. Kaiser, and U. G. Meissner, Nucl. Phys. **A615**, 483 (1997).

[7] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D **17**, 768 (1978).

[8] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).

[9] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).

[10] C. Lee, S. N. Yang, and T.-S. H. Lee, J. Phys. G **17**, L131 (1991).

[11] B. C. Pearce and B. K. Jennings, Nucl. Phys. **A528**, 655 (1991).

[12] F. Gross and Y. Surya, Phys. Rev. C **47**, 703 (1993).

[13] C. Schütz, J. W. Durso, K. Holinde, and J. Speth, Phys. Rev. C **49**, 2671 (1994).

[14] H. Kohlhoff and H. V. von Geramb, in *Quantum Inversion Theory and Applications* [2].

[15] M. Sander, *Quanteninversion und Hadron-Hadron Wechselwirkungen* (Shaker Verlag, Aachen, 1997).

[16] C. Beck, Ph.D. thesis, University of Hamburg, 1995.

[17] M. Sander and H. V. von Geramb, Phys. Rev. C **56**, 1218 (1997).

[18] K. Holinde, in *Physics with GeV-Particle Beams*, edited by H. Machner and K. Sistemich (World Scientific, Singapore, 1995).

[19] L. Jäde and H. V. von Geramb, Phys. Rev. C **55**, 57 (1997).

[20] L. Jäde and H. V. von Geramb, Phys. Rev. C **57**, 496 (1998).

[21] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

[22] G. Holzwarth and R. Machleidt, Phys. Rev. C **55**, 1088 (1997).

[23] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York 1980).

[24] P. B. Burt, *Quantum Mechanics and Nonlinear Waves* (Harwood Academic, New York, 1981).

[25] E. D. Cooper and B. K. Jennings, Nucl. Phys. **A500**, 553 (1989).

[26] R. A. Arndt, C. H. Oh, I. J. Strakovsky, R. J. Workman, and F. Dohrmann, Phys. Rev. C **56**, 3005 (1997).

[27] R. A. Arndt, I. J. Strakovsky, R. J. Workman, and M. M. Pavan, Phys. Rev. C **52**, 2120 (1995).

[28] R. Koch and E. Pietarinen, Nucl. Phys. **A336**, 331 (1980).

[29] V. G. J. Stoks, R. Timmermans, and J. J. de Swart, Phys. Rev. C **47**, 512 (1993).

[30] R. A. Arndt, R. L. Workman, and M. M. Pavan, Phys. Rev. C **49**, 2729 (1994).

[31] M. Sander and H. V. von Geramb, Phys. Rev. C **56**, 1218 (1997).

[32] G. E. Brown and W. Weise, Phys. Rep. **22**, 279 (1975).

[33] B. C. Pearce and I. R. Afnan, Phys. Rev. C **34**, 991 (1986).

[34] C. Schütz, *Pion-Nukleon-Wechselwirkung und die Struktur von Nukleon-Resonanzen*, FZ Jülich report, 1995, p. 3130.

[35] Virginia Polytechnique Institute, SAID program, access via TELNET under clsaid.phys.vt.edu (login: said); german mirror at said-hh.desy.de (login: physics, password: quantum).

[36] G. L. Greene, E. G. Kessler, Jr., R. D. Deslattes, and H. Boerner, Phys. Rev. Lett. **56**, 819 (1986).

[37] I. Lindgren, in *Alpha-, Beta-, Gamma-Spectroscopy, Vol. II*, edited by K. Siegbahn (North-Holland, Amsterdam, 1965).

[38] T. E. O. Ericson, Nucl. Phys. **A416**, 281 (1984).

[39] N. L. Rodning and L. D. Knutson, Phys. Rev. Lett. **57**, 2248 (1986).