Superdeformed identical bands ^{152}Dy(1) and ^{151}Tb(2) in supersymmetry with a many-body interaction

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(Received 12 February 1998)

With the supersymmetry model being extended to include many-body interactions and with the single-particle Routhian being taken into account, the identical superdeformed (SD) bands ^{152}Dy(1) and ^{151}Tb(2) are investigated. The calculated results agree with experimental data quantitatively well. It indicates that the supersymmetry with a many-body interaction is probably a source of the identical SD bands. Meanwhile to describe various aspects of the identical bands, the single-particle Routhian should also be taken into account. [S0556-2813(98)06708-9]

PACS number(s): 21.10.Re, 21.60.Fw, 23.20.Lv, 27.80.+w

The new generation of large γ-ray detector arrays has revealed many superdeformed (SD) bands in $A\sim190$, 150, 130, 80, and even 60 mass regions. The deexcitation γ-ray energies of many pairs of these bands are identical to each other within 1/1000 (isospectral) or, to the same precision, differ by 1/2 or $\pm1/4$ of the energy spacing of the consecutive γ rays. This fascinating phenomenon is referred to as identical bands (IB's). Since the observation in different nuclei of long sequences of γ rays that have equal energies had not been expected, but for more than 50 years it had been believed that each γ-ray spectrum uniquely belonged to a specific nucleus, the phenomenon of identical bands has solicited the greatest amount of effort in both experimental exploration and theoretical investigation (for a review see Ref. [1]). To explain this phenomenon many theoretical approaches have been put forward. To describe particular IB's in a limited region, the particle-plus-core coupling [2] is quite powerful [3,4]. To understand the phenomenon globally, both the nonrelativistic mean field (MF) theory (see, for example, Refs. [5,6]) and the relativistic mean field (RMF) theory [7,8] have been quite successful. Meanwhile, symmetry-based approaches, such as the pseudo-SU(3) models [9], the supersymmetric models [10–12], and the fermion dynamical symmetry model [13], have also been exploited. Especially, realistic supersymmetric partners seem to be observed practically by the appearance of identical bands in even-even and neighbor odd-A nuclei. However, theoretical predictions in terms of supersymmetry up to now have not yet reproduced the experimental data quantitatively well. In this paper, by extending the supersymmetry approach [10,11] to include many-body interactions, we describe the most typical identical SD bands: the yrast SD band of ^{152}Dy [^{152}Dy(1)] and the first excited SD band of ^{151}Tb [^{151}Tb(2)].

The experimentally observed identical bands show that the superdeformed nuclear energy spectra are degenerate. The degeneracy in quantum-mechanical spectra is usually a sign of the presence of a symmetry. The appearance of identical γ-ray energies in even-even and odd-A SD bands leads to the identical SD bands being explained in terms of certain dynamical supersymmetries [10–12], which are often discussed in the context of the interacting boson-fermion model (IBFM) [14,10,11]. The states in a supermultiplet can be classified with the group chain

$$\begin{array}{ccccccc} \mathrm{U}(m,n)\supset & \mathrm{U}_B(m)\otimes & \mathrm{U}_F(n)\supset\cdots\supset & \mathrm{SO}_{B+F}(3)\otimes & \mathrm{SU}_F(n')\supset & \mathrm{Spin}(3), \\ [N] & [N_B]_m & [N_F]_n & L & S & I \end{array} \tag{1}$$

where m is determined by the constituent of the bosons, n by the single-particle energy levels of the fermion, and n' by the total pseudospin of the fermion. $N=N_B+N_F$ is the total number of particles with N_B and N_F the boson and fermion numbers, respectively. For even-even and odd-A nuclei, it reads $(N_B,N_F)=(N,0)$, and $(N-1,1)$, respectively. The excitation spectrum belongs to this group chain and can be given as [1,10,11]

$$E=E_0(N_B,N_F)+C_L L(L+1)+C_I I(I+1), \tag{2}$$

where $E(N_B,N_F)$ is the contribution of the groups $\mathrm{U}_B(m)$, $\mathrm{U}_F(n)$, etc., with boson number N_B and fermion number N_F. L is the effective-core angular momentum which is composed of the angular momentum of the bosonic core R_B and the pseudo-orbital angular momentum of the fermion R_F. I is the total spin of the nucleus, $\vec{I}=\vec{L}+\vec{S}$. Taking $R_B\equiv0$, we get the pseudo-SU(3) limit with $\vec{I}=\vec{R}+\vec{S}$ in which R is the total pseudo-orbital angular momentum of the fermion and S is the total pseudospin. With an effective aligned angular momentum i being introduced, Eq. (2) can be given as

$$E=E_0'(N_B,N_F)+(C_L+C_I)(I-i)(I-i+1), \qquad (3)$$

where

$$i=\frac{C_L S}{C_L+C_I}.$$

It is apparent that if $C_L=0$, $i=0$, Eq. (3) gives the strong-coupling limit. If $C_I=0$, $i=S$, Eq. (3) gives the pseudospin-decoupling limit. Adjusting the ratio of C_L/C_I, we can get an arbitrary alignment i. Taking $I'=I-i$, Eq. (3) can be rewritten as

$$E=E_0'(N_B,N_F)+C_{I'}I'(I'+1). \qquad (4)$$

Calculating the γ-ray energies of the even-even and odd-A nuclei with Eq. (4), we can get the alignment.

It is definite that Eq. (4) can only generate the rotational band with constant dynamical moment of inertia, if the $C_{I'}$ is taken as a constant which has been commonly implemented [1,10,11]. However, experimental data (for a compilation see Ref. [15]) show that the dynamical moment of inertia $\mathcal{J}^{(2)}$ of most of the SD bands changes smoothly with variation of the rotational frequency $\hbar\omega$ (or the angular momentum). In particular, a turnover takes place in the curve of $\mathcal{J}^{(2)}$ versus $\hbar\omega$ [16]. In the light of variable moment of inertia models [17,18], it has been shown [19] that, with

$$C_{I'}=\frac{C_0}{1+f_1I'(I'+1)+f_2I'^2(I'+1)^2}, \qquad (5)$$

where C_0, f_1, and f_2 are parameters, the turnover and the smooth variance of the $\mathcal{J}^{(2)}$ with $\hbar\omega$ can be reproduced well. We can then rewrite Eq. (4) as

$$E=E_0'(N_B,N_F)+\frac{C_0}{1+f_1I'(I'+1)+f_2I'^2(I'+1)^2}\times I'(I'+1). \qquad (6)$$

With Eq. (6) under the commonly used assumption [1,10,11] that $E_0'(N_B,N_F)$ is a constant for a nucleus, we calculated the $E2$ γ-ray energies and the dynamical moment of inertia of SD band 1 of ^{152}Dy (with $N_F=0$, $N_B=N$; N is the total particle number) and the first excited SD band of ^{151}Tb (with $N_F=1$, $N_B=N-1$). After a nonlinear least squares fitting to the experimentally observed E_γ energies, we get the spin assignment and the $E2$ γ-ray energies. The best fitted parameters for the band ^{152}Dy(1) are $C_0=5.393$ keV, $f_1=-1.045\times10^{-5}$, and $f_2=5.858\times10^{-10}$ and those for the band ^{151}Tb(2) are $C_0=5.256$ keV, $f_1=-1.522\times10^{-5}$, and $f_2=1.203\times10^{-9}$. The calculated results are listed in the third and the Cal(A) column of Table I, respectively. With

$$\mathcal{J}^{(2)}=\frac{4\hbar^2}{E_\gamma(I+2)-E_\gamma(I)}, \qquad (7)$$

we get the dynamical moment of inertia of the two bands. The obtained results and the comparison with experimental data are shown in Figs. 1 and 2, respectively. From the table and the figures we know that SD band ^{152}Dy(1) is reproduced excellently, but the calculated result of SD band ^{151}Tb(2) does not agree with experimental data well, especially for the γ-ray energies and the dynamical moment of inertia, $\mathcal{J}^{(2)}$, at higher rotational frequency.

However, the above calculation is not consistent with the concept of supersymmetry. In the context of supersymmetry, the coefficients to evaluate the E_γ's of band ^{151}Tb(2) should be the same as those used for band ^{152}Dy(1). The calculation in this way gives the result of the E_γ spectrum of ^{151}Tb(2) as listed in the column Cal(B) of Table I. The deduced dynamical moment of inertia of the band is illustrated in Fig. 3. The table and the figure show that the $E2$ γ-ray spectrum has not yet been described well even though the dynamical moment of inertia, $\mathcal{J}^{(2)}$, is reproduced pretty well.

Recalling the calculation procedure more carefully we know that the problem may result from the term $E_0'(N_B,N_F)$ which has been taken as a constant for ^{151}Tb(2). Investigating the spectrum-generating procedure one can get

$$E_0'(N_B,N_F)=\varepsilon_BN_B+\varepsilon_FN_F+E_0''(N_B,N_F).$$

It is evident that the terms ε_BN_B and $E_0''(N_B,N_F)$ contribute really nothing to the γ-ray energies. However, the contribution of the term ε_FN_F to E_γ cannot be ignored, since the single fermion energy ε_F is usually rotational frequency dependent, which is usually referred to as a single-particle Routhian, i.e., $\varepsilon_F=\varepsilon_F(\hbar\omega)$. Considering the rotational frequency dependence of the ε_F we can rewrite Eq. (6) as

$$E=E_0'''(N_B,N_F)+\varepsilon_FN_F+\frac{C_0}{1+f_1I'(I'+1)+f_2I'^2(I'+1)^2}I'(I'+1), \qquad (8)$$

where $E_0'''(N_B,N_F)$ is a constant for a nucleus with boson number N_B and fermion number N_F.

To fix the $E2$ γ-ray energies with Eq. (8) one should determine the single-particle Routhian at first. Many calculations (for a review see Ref. [1]) and experimental data analysis (see for example Ref. [20]) have indicated that SD band ^{151}Tb(2) is based on the [301]1/2 Nillson orbital of the proton hole. Nevertheless, the single-particle Routhian of the proton hole [301]1/2 orbital has not yet been determined uniquely from different calculations [8,21,22]. Taking the average value of the results of Refs. [8,21], and [22], we can get the single-particle Routhain approximately as $\varepsilon_F(\hbar\omega)=-0.125(\hbar\omega)^2-0.425\hbar\omega-6.017$ (MeV). With $\varepsilon_F(\hbar\omega)$ being taken as a perturbation, we have reevaluated the $E2$ γ-ray energies of SD band ^{151}Tb(2) with the parameters C_0, f_1, and f_2 being the same as those for band ^{152}Dy(1). The obtained result is listed in the column Cal(C) of Table I. Based on this γ-ray spectrum we get the dynamical moment

TABLE I. Calculated $E2$ γ-ray energies of SD bands ^{152}Dy(1) and ^{151}Tb(2) and comparison with experiment (the experimental data are taken from Ref. [15]). The Cal(A) refers to the calculated result with C_0, f_1, and f_2 being fixed artificially by the fitting. Cal(B) denotes the result with C_0, f_1, and f_2 being taken as the same as those for band ^{152}Dy(1). Cal(C) represents the result with the parameters maintained the same as those for band ^{152}Dy(1) and with the single-particle energy of the proton hole being considered simultaneously.

	^{152}Dy(1)				^{151}Tb(2)		
Spin	Expt.	Cal.	Spin	Expt.	Cal(A)	Cal(B)	Cal(C)
28	602.4(1)	602.18	28.5	602.1(8)	601.64	613.38	602.34
30	647.5(1)	647.25	30.5	646.4(5)	646.46	658.57	646.90
32	692.7(1)	692.63	32.5	691.9(5)	691.60	704.02	692.71
34	738.1(1)	738.27	34.5	737.4(3)	737.05	749.72	737.72
36	784.0(1)	784.18	36.5	783.4(3)	782.82	795.70	783.63
38	829.9(1)	830.45	38.5	828.7(3)	828.88	841.93	829.00
40	876.4(1)	876.78	40.5	874.8(3)	875.22	888.43	875.10
42	923.2(1)	923.48	42.5	921.6(4)	921.81	935.19	921.73
44	970.2(1)	970.43	44.5	968.1(4)	968.63	982.20	968.55
46	1017.4(1)	1017.62	46.5	1015.7(4)	1015.64	1029.45	1016.15
48	1064.9(1)	1065.04	48.5	1063.0(5)	1062.80	1076.93	1063.06
50	1112.7(1)	1112.67	50.5	1110.5(6)	1110.04	1124.61	1111.04
52	1160.5(1)	1160.36	52.5	1158.6(6)	1157.32	1172.49	1158.76
54	1208.6(1)	1208.50	54.5	1206.9(8)	1204.55	1220.52	1206.80
56	1256.6(1)	1256.63	56.5	1254.8(6)	1251.66	1268.68	1254.82
58	1304.8(1)	1304.87	58.5	1303.2(8)	1298.56	1316.94	1302.94
60	1352.9(1)	1353.17	60.5	1352.0(8)	1345.15	1365.25	1351.30
62	1401.3(1)	1401.49	62.5	1339.5(9)	1391.32	1413.57	1399.27
64	1449.6(1)	1449.79	64.5	1448.3(9)	1436.96	1461.85	1447.55
66	1497.8(2)	1498.00	66.5	1495.0(11)	1481.92	1510.04	1495.44
68	1545.6(3)	1546.07					

of inertia, $\mathcal{J}^{(2)}$, as illustrated in Fig. 4. The table and the figure manifest that both the $E2$ γ-ray energies and the dynamical moment of inertia can be simultaneously described excellently with the single-particle effect being considered. Since the relative alignment is fixed as 1/2 in the calculation inspired by supersymmetry and the contribution of the single-particle Routhian to the E_γ's is taken as almost a constant (in the range 11.01−14.59 keV), the final relative alignment of SD bands ^{151}Tb(2) to ^{152}Dy(1) remains 1/2. And the incremental alignment remains approximately zero. This agrees with experimental data very well too.

Along the line suggested in Ref. [18] we can identify that, when the parameters are taken as $f_1>0$, $f_2>0$ (or f_1

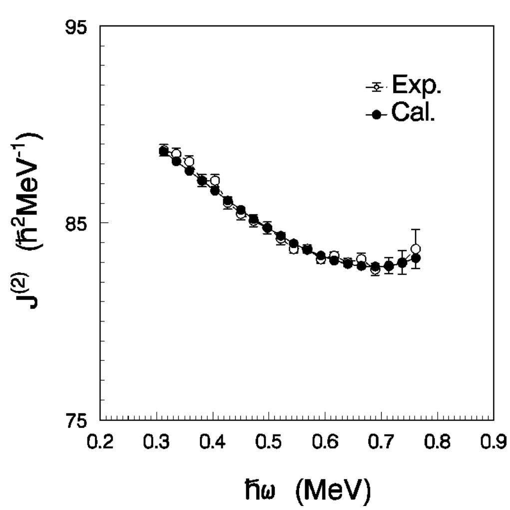

FIG. 1. Calculated result of the dynamical moments of inertia as a function of the rotational frequency of SD band 1 of ^{152}Dy and comparison with experiment. The experimental data are taken from Ref. [15].

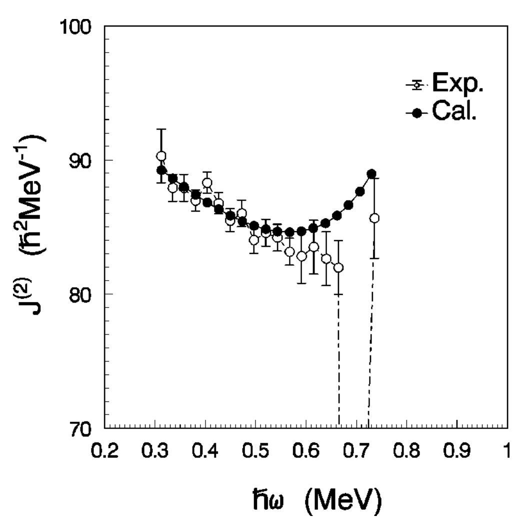

FIG. 2. The same as Fig. 1 but for the SD band ^{151}Tb(2) with the $E2$ γ-ray energies being fitted independent of those for ^{152}Dy(1).

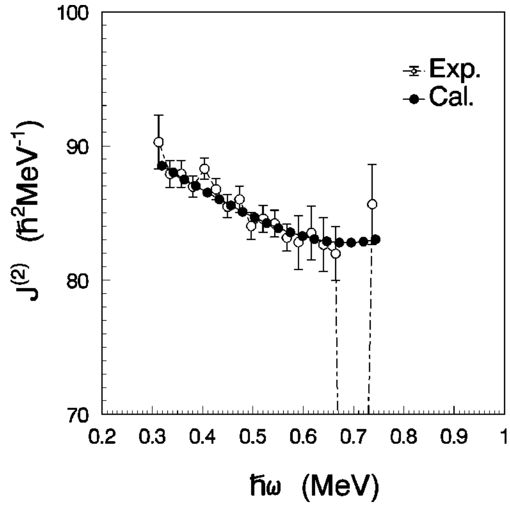

FIG. 3. The same as Fig. 2 but with parameters C_0, f_1, and f_2 being taken as the same as those for ^{152}Dy(1).

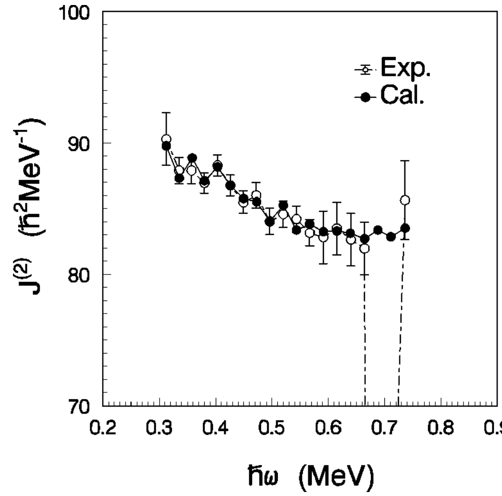

FIG. 4. The same as Fig. 3 but with the contribution of the single-particle energy of the proton hole being taken into account too.

<0, $f_2<0$), the antipairing (or pairing favorite) effect is considered. When they are taken as $f_1>0$, $f_2<0$ (or $f_1<0$, $f_2>0$), both antipairing and pairing effects are taken into account. Their magnitudes represent the influence of the effects on the rotational property. On the other hand, looking over the calculation process one can realize that the absolute values of the parameters f_1 and f_2 are very small. Equation (5) can then be expanded as

$$C_{I'}=C_0\{1-[f_1I'(I'+1)+f_2I'^2(I'+1)^2]+[f_1I'(I'+1)+f_2I'^2(I'+1)^2]^2-[f_1I'(I'+1)+f_2I'^2(I'+1)^2]^3+\cdots+(-1)^n[f_1I'(I'+1)+f_2I'^2(I'+1)^2]^n+\cdots\}.$$

Since $I'(I'+1)$ is the eigenvalue of the interaction with symmetries $SO_{B+F}(3)$ and Spin(3), the interaction generating the energy shown as Eqs. (6) and (8) is in fact a power series of the interaction with $SO_{B+F}(3)$ and Spin(3) symmetries. Therefore the interaction includes all the possible many-body interactions among the particles. The calculation indicates that only the two-body interaction [E is given as Eq. (4) with $C_{I'}$ being a constant] cannot reproduce the rotational frequency dependence of the dynamical moment of inertia. Only if the many-body interactions are taken into account can the variation of $\mathcal{J}^{(2)}$ against $\hbar\omega$ be generated well. This means that the many-body interactions play a much more crucial role in SD states than in normal deformation states. Nevertheless, the many-body interactions determine only the rotational property of the SD states globally. The fact that to describe the detailed γ-ray spectra well the effect of the single particle must be taken into account demonstrates that the interplay of the collective motion resulting from the many-body interaction and the single-particle motion dominates the superdeformed nuclear states, in particular the identical bands of the SD nuclei.

In summary, with the many-body interaction being included in the supersymmetry approach and the rotational frequency dependence of the single-particle energy being considered simultaneously, the identical SD bands in ^{152}Dy(1) and ^{151}Tb(2) have been described quantitatively well. It manifests that the supersymmetry with a many-body interaction is a possible source of the identical SD bands, where the many-body interaction plays a crucial role in governing the rotational property globally. Meanwhile the single-particle motion contributes a great deal to the detailed energy spectra of the states. The identical bands are dominated by not only the supersymmetry but also the complicated interplay of the many-body interaction and the single-particle property. Then, to describe all the aspects of the identical SD bands detailed information about the single-particle Routhian is required as well as the global property of the rotation.

This work was supported by the National Natural Science Foundation of China and a Grant from Peking University. Helpful discussions with Professor Hong-zhou Sun, Professor Qi-zhi Han, Professor Y.S. Chen, Professor En-guang Zhao, Professor C.S. Wu are acknowledged with thanks.

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