Magnetic dipole moments of odd-odd $N=Z$ nuclei

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The experimental data of the magnetic dipole moments of low-lying states in odd-odd $N=Z$ nuclei, which has increased recently by a factor of 2, are revisited within the simple shell model, taking the wave functions to be of the form $\Psi_{\rm core}(J=0,T=0)\Psi_{np}(J,T=0)$. Good agreement with the updated experimental data is obtained within the jj-coupling scheme $\Psi_{np}[(nlj)^2J,(T=0)]$, using the Schmidt values for the g factor and also by using effective g factor deduced from nearby odd-A nuclei. We have also carried out a calculation of μ within the LS-coupling scheme $\Psi_{np}[(1/2)^2S(nl)^2L,J,(T=0)]$ and also obtained good agreement with data. The success of these simple models in reproducing the experimental data is discussed.
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I. INTRODUCTION

The magnetic dipole moment μ of nuclear states, determined as expectation values of a single particle operator, has played an important role in understanding the structure of nuclear states [1]. When compared with experimental data on μ, the calculated values may provide a sensitive test for nuclear models. In particular, the magnetic dipole moments of light nuclei has been studied within the spherical shell model by many authors, see for example Refs. [1–6]. In this work we revisit the magnetic dipole moments of low-lying states in odd-odd $N=Z$ nuclei with $A=N+Z\leqslant 46$. Since the early works [1–6], the number of experimental data on μ [7,8] has increased by a factor of 2 and now include those of the states ^{10}B(1^+), ^{18}F(3^+), ^{22}Na(1^+), ^{26}Al(5^+), ^{26}Al(3^+), ^{38}K(7^+), and ^{46}V(3^+). We carry out a study of μ within a simple shell model, assuming that the states are described by the wave functions $\Psi_{\rm core}(J=0, T=0)\Psi_{np}(J,T=0)$, where $\Psi_{\rm core}$ describes the $(N$-1, Z-1) even-even core and the odd-neutron and odd-proton occupy the same single particle orbit (nl) with a total angular momentum J and isospin $T=0$.

We carry out calculations of μ within the jj-coupling scheme $\Psi_{np}[(nlj)^2J,(T=0)]$, using the Schmidt values for the neutron (g_n) and proton (g_p) g factors and also by using effective g factors deduced from experimental values of μ in nearby odd-A (mainly mirror) nuclei. We point out that this model has been used earlier [3]. However, in the present work we also consider more recent data, taken from Refs. [7, 8], and also obtain good agreement between theory and experiment.

Nuclei exhibit phenomena indicative of few-body correlations in the many-body system, which are due to the short-range character of the nucleon-nucleon interaction [9–13]. This has led to many investigations of light nuclei in terms of various cluster models, i.e., using wave functions embodying cluster correlations (with a proper account of the Pauli principle). In particular, the magnetic moment of the ground state ^{6}Li(1^+) is well described within the LS-coupling model for the odd-neutron and odd-proton system and within the simple core-deuteron cluster wave function. We have therefore carried out calculations of μ of odd-odd $N=Z$ nuclei within the simple LS-coupling scheme $\Psi_{np}[(1/2)^2(nl)^2L,J,T=0]$. The values of L and S were then fixed by the corresponding experimental value of μ. Good agreement with the experimental data was also obtained. It is interesting to note that similar results for μ were obtained within a simplistic core-deuteron cluster model. In this model the states are described in terms of the single particle wave function of a deuteron with principle quantum number N, orbital angular momentum L, and intrinsic angular momentum $S=1$, coupled to a total angular momentum J, and with total isospin $T=0$. It should be pointed out that in this model L must be even and therefore, $S=1$ and $L=J\pm 1$. To obtain a physical insight into the success of the LS-coupling scheme and the core-deuteron model in reproducing the experimental data on μ, we have also carried out a decomposing of the shell model wave functions within the jj-coupling scheme in terms of the LS-coupling scheme wave functions and in terms of the cluster wave functions and determined the corresponding amplitudes of these wave functions.

The plan of this paper is the following. In the next section we provide, for completeness, the basic expressions needed to calculate μ and to carry out the decomposing of the shell model wave functions within the jj-coupling scheme in terms of the LS-coupling scheme wave functions and in terms of the core-deuteron cluster model wave functions. In Sec. III we present the results of the calculations of μ within the shell model jj-coupling scheme and LS-coupling scheme wave functions and the values of the amplitude of the core-deuteron cluster wave function in the shell model jj-coupling scheme wave function. In Sec. IV we provide some conclusions.

II. FORMALISM

For the purpose of completeness we give here some basic expressions [1] needed to calculate μ in nuclei. The magnetic dipole moment operator is given by

$$\vec{\mu}=\sum_{i=1}^{A}\frac{1+\tau_{zi}}{2}(g_l^p\vec{l}_i+g_s^p\vec{s}_i)+\sum_{i=1}^{A}\frac{1-\tau_{zi}}{2}g_s^n\vec{s}_i, \qquad (1)$$

where τ_{zi} is the third component of the isospin operator with eigenvalues of 1 and -1 for proton and neutron, respectively, and

$$g_s^p=5.5856,\ g_s^n=-3.8263,\ g_l^p=1,\ \text{and}\ g_l^n=0, \qquad (2)$$

in units of nuclear magneton (μ_N) $\mu_0=e\hbar/2m_pc$. The magnetic dipole moment μ of a state with a definite total angular momentum J is defined by

$$\mu=\langle JM=J|\vec{\mu}|JM=J\rangle=gJ, \qquad (3)$$

where the g factor of the state is introduced in Eq. (3). A useful relation obtained from the Lande formula [1] for the calculation of g is

$$g=\frac{\langle JM|\vec{\mu}\cdot\vec{J}|JM\rangle}{J(J+1)}. \qquad (4)$$

A. Shell model description within the jj-coupling scheme

In the independent particle model, the magnetic dipole moment μ of a nucleon in an orbit (nlj) is given [1] by the Schmidt values $\mu=g_jj$ with

$$g_j=\frac{2g_ll+g_s}{2l+1}\quad\text{for}\quad j=l+\frac{1}{2},$$

$$g_j=\frac{2g_l(l+1)-g_s}{2l+1}\quad\text{for}\quad j=l-\frac{1}{2}. \qquad (5)$$

In the simple shell model, for odd-A nucleus the magnetic dipole model is due to the odd nucleon, since the core has $J=0$. Considering mirror nuclei, the sum of their magnetic dipole moments is given [3] by the isoscalar part of Eq. (1),

$$\mu(M_T=1/2)+\mu(M_T=-1/2)=\left[g_l^p\pm\frac{g_s^p+g_s^n-1}{2l+1}\right]\quad\text{for}\quad j=l\pm\frac{1}{2}. \qquad (6)$$

It is well known that deviations of the experimental values of μ from the corresponding Schmidt values exist and it is common to introduce the effective g factors g_j^n and g_j^p. In the the jj-coupling scheme shell model, the wave functions of the states in odd-odd $N=Z$ nuclei under consideration, are given by $\Psi_{\text{core}}(J=0,\ T=0)\times\Psi_{np}[(nlj)^2J,T=0]$. Since in these states the valence nucleons occupy the same orbit j we can write [1]

$$\vec{\mu}=\sum_{i=1}\frac{1+\tau_{zi}}{2}g_j^p\vec{j}_i+\sum_{i=1}\frac{1-\tau_{zi}}{2}g_j^n\vec{j}_i$$
$$=\frac{1}{2}(g_j^p+g_j^n)\vec{J}+\sum_{i=1}\frac{1}{2}\tau_{zi}(g_j^p-g_j^n)\vec{j}_i. \qquad (7)$$

The contribution of the isovector part of Eq. (7) to μ vanishes for these states (with $T=0$) and we have

$$\mu_{j^2J}=\frac{1}{2}(g_j^p+g_j^n)J. \qquad (8)$$

Using the Schmidt's values, Eq. (5), for g_j^n and g_j^p we obtain

$$\mu_{j^2J}=\left[0.5\pm\frac{0.38}{2l+1}\right]J\quad\text{for}\quad j=l\pm\frac{1}{2}. \qquad (9)$$

Note that the ratio between Eq. (6) and Eq. (9) is $2j/J$.

B. Shell model description within the LS-coupling scheme

Assuming that the odd-odd $N=Z$ nuclear states are well described in the LS-coupling scheme in the form of $\Psi_{\text{core}}(J=0,\ T=0)\times\Psi_{np}[(1/2)^2S(nl)^2L,J,T=0]$, we rewrite Eq. (1) in the form

$$\vec{\mu}=\sum_{i=1}^{A}\frac{1}{2}[\vec{l}_i+(g_s^p+g_s^n)\vec{s}_i]+\sum_{i=1}^{A}\frac{1}{2}\tau_{zi}[\vec{l}_i+(g_s^p-g_s^n)\vec{s}_i]. \qquad (10)$$

We have for the states with well defined isospin $(T=0)$ that the magnetic dipole moment operator is given by

$$\vec{\mu}=g_L\vec{L}+g_S\vec{S}, \qquad (11)$$

where L and S are the orbital angular momentum and the internal spin of neutron-proton system, respectively, and

$$g_L=0.5\ \mu_N\quad\text{and}\quad g_S=0.88\ \mu_N \qquad (12)$$

Using Eqs. (4) and (11) we have that μ, for a state with a well defined total angular momentum J, is given by

$$\mu_{LSJ}=g_L\left[\frac{J(J+1)+L(L+1)-S(S+1)}{2(J+1)}\right]+g_S\left[\frac{J(J+1)-L(L+1)+S(S+1)}{2(J+1)}\right]. \qquad (13)$$

Adopting the g factors of Eq. (12) we then have

$$\mu_{LSJ}=0.5J\quad\text{for}\quad L=J,$$

$$\mu_{LSJ}=\left[0.69+0.19\,\frac{2-L(L+1)}{J(J+1)}\right]J\quad\text{for}\quad L=J\pm1. \qquad (14)$$

It is important to point out that expression (13) is also obtained within the simple core-deuteron cluster model with the values of g_L and g_S given by 0.5 and 0.86 μ_N (the magnetic dipole moment of the deuteron), respectively.

TABLE I. Shell model calculation of magnetic dipole moments in odd-odd $N=Z$ nuclei within the jj-coupling scheme. The free-g factors are the Schmidt values and for the effective g factors see text.

State	E_x (MeV)	nlj	μ (μ_N) Exp.	Free g	Effective g
$d(1^+)$	0	$0s_{1/2}$	0.86	0.88	
^{6}Li(1^+)	0	$0p_{3/2}$	0.82	0.63	0.75
^{10}B(3^+)	0	$0p_{3/2}$	1.80	1.88	1.73
^{10}B(1^+)	0.72	$0p_{3/2}$	0.63 ± 0.12	0.63	0.58
^{14}N(1^+)	0	$0p_{1/2}$	0.40	0.37	0.37
^{18}F(3^+)	0.94	$0d_{5/2}$	1.68 ± 0.15	1.73	1.70
^{18}F(5^+)	1.12	$0d_{5/2}$	2.86 ± 0.03	2.88	2.83
^{22}Na(3^+)	0	$0d_{5/2}$	1.75	1.73	1.79
^{22}Na(1^+)	0.58	$0d_{5/2}$	0.53 ± 0.1	0.58	0.60
^{26}Al(5^+)	0	$0d_{5/2}$	2.80	2.88	2.79
^{26}Al(3^+)	0.42	$0d_{5/2}$	1.95 ± 0.45	1.73	1.67
^{38}K(3^+)	0	$0d_{3/2}$	1.37 ± 0.01	1.27	1.35
^{38}K(7^+)	3.46	$0f_{7/2}$	3.84 ± 0.01	3.88	3.92
^{46}V(3^+)	0.80	$0f_{7/2}$	1.64 ± 0.03	1.66	1.69

C. Overlap between the wave functions of the models

To obtain a physical insight on the success of the LS-coupling scheme shell model and of the core-deuteron cluster model in describing the structure of low-lying states in odd-odd $N=Z$ nuclei, we carry out a decomposition of the jj-coupling scheme shell model wave function and determine its overlap with the wave functions of these models. For this purpose, we first carry out the transformation from the jj-coupling scheme to the LS-coupling scheme [1]:

$$\psi[(nlj)^2J]=\sum \left\langle \frac{1}{2}l(j)\frac{1}{2}l(j)J\middle|\frac{1}{2}\frac{1}{2}(S)ll(L)J\right\rangle \times\psi\left[\left(\frac{1}{2}\right)^2(S)l^2(L)J\right]. \quad (15)$$

We have omitted the isospin wave function with $T=0$. The amplitude of the LS-scheme wave function $\langle \frac{1}{2}l(j)\frac{1}{2}l(j)J|\frac{1}{2}\frac{1}{2}(S)ll(L)J\rangle$ can be taken from Ref. [1].

Considering the amplitude of the cluster-deuteron wave function we have $S=1$ and therefore, L in Eq. (15) must be even. To carry out the transformation of $\psi[(nl)^2L]$ to the center of mass and relative coordinates of the n-p system, we now assume that the radial wave functions can be well approximated by those of a harmonic oscillator single particle potential. Using the Talmi transformation we can write

$$\psi(nl\ nlL)=\sum \langle nl\ nl\ L|N\lambda\ n'\ \Lambda\ L\rangle\psi(N\lambda n'\Lambda L), \quad (16)$$

where N and n' are the principle quantum numbers and λ and Λ are the orbital angular momenta associated with the center of mass and relative motions, respectively. Substituting Eq. (16) in Eq. (15) we write

$$\psi[(nlj)^2J]=\sum \left\langle \frac{1}{2}l(j)\frac{1}{2}l(j)J\middle|\frac{1}{2}\frac{1}{2}(S)l^2(L)J\right\rangle \times\langle nl\ nl\ L|N\lambda\ n'\ \Lambda\ L\rangle\langle S\ M_S\ L\ M_L|JM\rangle \times\langle\lambda\ M_\lambda\ \Lambda\ M_\Lambda|LM_L\rangle\psi_{N\lambda M_\lambda}(\vec{R})\chi^S_{M_S}\psi_{n'\Lambda M_\Lambda}(\vec{r}). \quad (17)$$

To extract the amplitude of a deuteronlike wave function from Eq. (17), we must have $\Lambda=0$, $M_\Lambda=0$, $n'=0$, and $S=1$ for the internal deuteron wave function $\psi_d^{SM_S}$. Therefore, $\lambda=L$, $M_\lambda=M_L$, and $\langle\lambda M_\lambda\Lambda\ M_\Lambda|LM_L\rangle=1$. The amplitude of the deuteronlike single particle wave function

$$\psi_{SLJM}=\sum \langle S\ M_S\ L\ M_L|JM\rangle\psi_{NLM_L}(\vec{R})\psi_d^{SM_S} \quad (18)$$

is then given by

$$\left\langle \frac{1}{2}l(j)\frac{1}{2}l(j)J\middle|\frac{1}{2}\frac{1}{2}(S)l^2(L)J\right\rangle\langle nl\ nl\ L|NL\ 00\ L\rangle. \quad (19)$$

with $T=0$, $S=1$, L even, and J odd. The coefficients of the $jj\rightarrow LS$ transformation are taken from Ref. [1] and those of the Talmi transformation are taken from Ref. [14].

III. RESULTS AND DISCUSSION

A. The *jj*-coupling scheme

In Table I we list the low-lying states in odd-odd $N=Z$ nuclei with mass $A\leqslant46$ and the corresponding experimental values [7,8] of the magnetic dipole moments μ. Also shown in Table I are the assigned (nlj) orbit of the simple shell model configuration $(nlj)^2(J)$ of the odd-neutron–odd-proton system and the predicted values of μ within this model. We present in Table I the calculated values of μ obtained from Eq. (9), i.e., using the free nucleon g factors [Schmidt values, Eq. (5)] and those obtained from Eq. (8)

using the effective g factors g_n and g_p deduced from experimental data of μ in nearby odd-A (mainly mirror) nuclei. The corresponding g_n and g_p factors were determined as follows.

^{6}Li(1^+) $E_x=0.0$, $\mu=0.82\ \mu_N$. Using the free g factors in Eq. (9) we obtain for the assigned $(0p_{3/2})^2$ configuration the value of $\mu=0.63\ \mu_N$. The corresponding $A=5$ mirror nuclei are unstable and the values of μ are unknown. If we adopt the experimental values of $\mu=3.439$ and $-1.178\ \mu_N$ of the $J=3/2^-$ ground states of ^{9}Li and ^{9}Be, we have the values of 2.29 and $-0.785\ \mu_N$ for the effective g_n and g_p factors, respectively. Substituting these values of g in Eq. (8) we obtain the value of $\mu=0.753\ \mu_N$ for ^{6}Li(1^+), which is close to the experimental value.

^{10}B(3^+) $E_x=0.0$, $\mu=1.80\ \mu_N$. From Eq. (9) we find for the $(0p_{3/2})^2$ configuration the value of $\mu=1.88\ \mu_N$. Since the magnetic dipole moment of the $3/2^-$ state of ^{9}B is unknown, we use the experimental values of $\mu=2.689$ and $-0.964\ \mu_N$ of the $3/2^-$ ground states of ^{11}B and ^{11}C to extract the effective g_n and g_p, respectively. Substituting in Eq. (8) we obtain $\mu=1.725\ \mu_N$ for the ^{10}B(3^+) state.

^{10}B(1^+) $E_x=0.718$ MeV, $\mu=0.63\pm0.12\ \mu_N$. Using Eq. (9) for the $(0p_{3/2})^2$ configuration gives $\mu=0.63\ \mu_N$. Using the effective g factors deduced from the experimental values of μ of the $3/2^-$ ground states of ^{11}B and ^{11}C we obtain from Eq. (8) the value $\mu=0.575\ \mu_N$.

^{14}N(1^+) $E_x=0.0$, $\mu=0.40\ \mu_N$. From Eq. (9) we obtain for the $(0p_{1/2})^2$ configuration the value of $\mu=0.37\ \mu_N$. The given experimental value of μ for the $1/2^-$ ground state of ^{13}N is $|0.322|\ \mu_N$ and the corresponding one for ^{13}C is $0.7024\ \mu_N$. Taking μ of ^{13}N to be negative we find using Eq. (8), the value $\mu=0.37\ \mu_N$ for the ^{14}N(1^+) state.

^{18}F(3^+) $E_x=0.937$ MeV, $\mu=1.68\pm0.15\ \mu_N$. Using Eq. (9) with the $(0d_{5/2})^2$ configuration we obtain the value of $\mu=1.73\ \mu_N$. Extracting the effective values of g_n and g_p from the experimental values of $\mu=-1.894$ and $4.722\ \mu_N$ of the $5/2^+$ ground states of ^{17}O and ^{17}F, respectively, we find using Eq. (8) the value of $\mu=1.698\ \mu_N$ for the ^{18}F(3^+) state.

^{18}F(5^+) $E_x=1.12$ MeV, $\mu=2.86\pm0.03\ \mu_N$. Carrying out similar calculations as for the ^{18}F(3^+) state we find using Eq. (9) and Eq. (8) the values of $\mu=2.88$ and $2.83\ \mu_N$, respectively.

^{22}Na(3^+) $E_x=0.0$, $\mu=1.75\ \mu_N$. Using Eq. (9) with the $(0d_{5/2})^2$ configuration we obtain $\mu=1.73\ \mu_N$. The experimental value of μ for the $5/2^+$ state at 0.332 MeV in ^{21}Na is $3.70\pm0.25\ \mu_N$ and the average experimental value of μ for the $5/2^+$ state at 0.351 MeV in ^{21}Ne is $|0.715\pm0.24|\ \mu_N$. Taking μ of the $5/2^+$ state in ^{21}Ne to be negative, we find from Eq. (8) that $\mu=1.79$ for the ^{22}Na(3^+) state.

^{22}Na(1^+) $E_x=0.583$ MeV, $\mu=0.53\ \mu_N$. Carrying out similar analysis as for the ^{22}Na(3^+) state we obtain from Eq. (9) and Eq. (8) the values of $\mu=0.58$ and $0.60\ \mu_N$, respectively.

^{26}Al(5^+) $E_x=0.0$, $\mu=2.80\ \mu_N$ [8]. Using Eq. (9) with the $(0d_{5/2})^2$ configuration we obtain the value of $\mu=2.88\ \mu_N$. Using Eq. (8) and the experimental values of $\mu=3.646$ and $-0.855\ \mu_N$ for the $5/2^+$ ground states of ^{25}Al and ^{25}Mg, respectively, we obtain the value of $\mu=2.79\ \mu_N$ for the ^{26}Al(5^+) state.

^{26}Al(3^+) $E_x=0.42$ MeV, $\mu=1.95\pm0.45\ \mu_N$. Carrying out similar analysis as for the ^{26}Al(5^+) states we obtain from Eq. (9) and Eq. (8) the values of $\mu=1.73$ and $1.674\ \mu_N$, respectively.

^{38}K(3^+) $E_x=0.0$, $\mu=1.37\pm0.01\ \mu_N$. Using Eq. (9) and the $(0d_{3/2})^2$ configuration we obtain the value of $\mu=1.27\ \mu_N$. Using Eq. (8) and the experimental values of $\mu=1.145$ and $0.203\ \mu_N$ for the $3/2^+$ ground states of ^{37}Ar and ^{37}K, respectively, we obtain $\mu=1.35\ \mu_N$ for the ^{38}K(3^+) state.

^{38}K(7^+) $E_x=3.46$ MeV, $\mu=3.84\pm0.01\ \mu_N$. We first note that we assign the shell model configuration of $(0f_{7/2})^2$ for this 7^+ state in ^{38}K. Using Eq. (9) we obtain the value $\mu=3.88\ \mu_N$. The $7/2^-$ state at 1.611 MeV of ^{37}Ar has a measured $\mu=-1.33\pm0.05\ \mu_N$ and the 1.379 MeV state in ^{37}K has a measured g factor of $g=1.5\pm0.1\ \mu_N$ with a possible spin assignment of $5/2^-$ or $7/2^-$. Assuming a spin assignment of $7/2^-$ for the 1.379 MeV state in ^{37}K we obtain, using Eq. (8) and the $(0f_{7/2})^2$ configuration, the value of $\mu=3.92\ \mu_N$ for the ^{38}K(7^+) state. We note that using Eq. (8) and the values of $\mu=5.535$ and $-1.595\ \mu_N$ for the $7/2^-$ ground states of ^{41}Sc and ^{41}Ca, respectively, we obtain the value of $\mu=3.94\ \mu_N$ for the ^{38}K(7^+) state.

^{46}V(3^+) $E_x=0.802$ MeV, $\mu=1.64\pm0.03\ \mu_N$. Using Eq. (9) and the assignment of $(0f_{7/2})^2$, we obtain the value of $\mu=1.66\ \mu_N$. The experimental value of μ of ^{45}V is not available. Using Eq. (8) and the experimental values of μ for the ground states of ^{41}Sc and ^{41}Ca we obtain the value $\mu=1.69\ \mu_N$.

The results presented in Table I demonstrate that a good agreement with the experimental values of μ in odd-odd $N=Z$ nuclei is obtained using the simple shell model assignments of the $(nlj)^2$ configurations and the Schmidt values for the g factors (free g factors). Moreover, in specific cases, such as the states of ^{6}Li(1^+) and ^{38}K(3^+), the adoption of effective g factors, deduced from the experimental values of μ in odd-A nuclei, leads to a better agreement with data. We note that our results are in agreement with similar analysis of the magnetic dipole moments of odd-odd nuclei, carried out in Refs. [1–6]. However, in this work we have considered all the experimental values of g and μ, known at present, which include the additional cases of ^{10}B(1^+), ^{18}F(3^+), ^{22}Na(1^+), ^{26}Al(5^+), ^{26}Al(3^+), ^{38}K(7^+), and ^{46}V(3^+).

B. The LS-coupling scheme

In Table II we present the values of μ obtained within the simple LS-coupling scheme shell model assuming $S=1$ and using the free g factors of Eq. (12). We note that the results of core-deuteron cluster model, using the free g factor of Eq. (12) coincide with those shown in Table II. We also indicate in Table II the quantum numbers NLJ of the deuteron single particle orbit for which μ is close to the experimental data. The orbital angular momentum L is determined by μ and N is then obtained from the energy condition [1]

$$2(2n+l)=2N+L. \tag{20}$$

Also shown in Table II, in parenthesis, are the values of μ obtained using the other possible value of L and those ob-

TABLE II. Shell model calculation of magnetic dipole moments in odd-odd $N=Z$ nuclei within the LS-coupling scheme assuming $S=1$. The principle quantum number N is that associated with the deuteron-like single particle orbit.

State	E_x (MeV)	NL	μ (μ_N) Exp.	Free g	Fit
$d(1^+)$	0	$0S$	0.86	0.88	0.82
^{6}Li(1^+)	0	$1S$	0.82	0.88 ($0D$, 0.31)	0.82
^{10}B(3^+)	0	$0D$	1.80	1.88	1.86
^{10}B(1^+)	0.72	$1S$	0.63 ± 0.12	0.88 ($0D$, 0.31)	0.82
^{14}N(1^+)	0	$0D$	0.40	0.31 ($1S$, 0.88)	0.37
^{18}F(3^+)	0.94	$1D$	1.68 ± 0.15	1.88 ($0G$, 1.22)	1.86
^{18}F(5^+)	1.12	$0G$	2.86 ± 0.03	2.88	2.91
^{22}Na(3^+)	0	$1D$	1.75	1.88 ($0G$, 1.22)	1.86
^{22}Na(1^+)	0.58	$2S$	0.53 ± 0.1	0.88 ($1D$, 0.31)	0.82
^{26}Al(5^+)	0	$0G$	2.80	2.88	2.91
^{26}Al(3^+)	0.42	$1D$	1.95 ± 0.45	1.88 ($0G$, 1.22)	1.86
^{38}K(3^+)	0	$0G$	1.37 ± 0.01	1.22 ($1D$, 1.88)	1.34
^{38}K(7^+)	3.46	$0I$	3.84 ± 0.01	3.88	3.46
^{46}V(3^+)	0.80	$2D$	1.64 ± 0.03	1.88 ($0G$, 1.22)	1.86

tained from Eq. (13) by a least square fit with $g_L=0.5218\ \mu_N$ and $g_S=0.8208\ \mu_N$.

It is seen from Table II that quite good agreement is obtained between the experimental values of μ and those calculated within the LS-coupling scheme, with $S=1$ and an appropriate selection of the values of the orbital angular momentum L, except for the states ^{10}B(1^+), ^{22}Na(1^+), and ^{46}V(3^+) where the $L=J$, $S=0$ assignment leads to the values of $\mu=0.5$, 0.5, and 1.5 μ_N, respectively, in a better agreement with data and with the prediction of the jj-coupling scheme. Considering the values of L in Table II, which provide better fit to the data on μ, one observes the following interesting result. In the LS-coupling scheme shell model (or core-deuteron model) one should adopt the value of $L=J-1$ when the corresponding shell model jj-coupling scheme configuration is $(nlj)^2$ with $j=l+1/2$ and the value of $L=J+1$ when $j=l-1/2$. This result is consistent with the intuitive classical picture of addition of angular momenta.

C. The overlap between the wave functions

To obtain a better physical insight and understand the success of the LS-coupling scheme shell model and core-deuteron cluster model in reproducing the experimental values of μ considered in Table II, we have calculated using Eq. (19), the overlap $\langle j^2|d\rangle$ between the jj-coupling scheme shell model wave function and the core-deuteron model wave function and present the results in Table III. Also shown in Table III are the shell model g factors in the jj-coupling scheme and the core-deuteron g factors for the cases of $L=J\pm1$. It is seen from Table III that for a shell model configuration $(nlj)^2$ the $\langle j^2|d\rangle$ overlap associated with the $L=J-1$ is larger (smaller) than that associated with the $L=J+1$ state for the case of $j=l+1/2$ ($j=l-1/2$) by a factor of about 10. This result justifies the determination of L carried out in the calculation of μ in Table II and explains the success of the core-deuteron model in reproducing the experimental data on μ. (Note the large differences between the value of g associated with $L=J-1$ and $L=J+1$.) However, since the values of μ calculated within the core-deuteron cluster model coincide with those obtained within the LS-coupling shell model, the success of the core-deuteron cluster model in reproducing the experimental data on μ cannot be taken as evidence for strong deuteronlike correlations and one should consider other physical quantities.

IV. SUMMARY AND CONCLUSIONS

The magnetic dipole moments μ of low lying states in odd-odd $N=Z$ nuclei were studied in this work within the shell model and within a simple core-deuteron cluster model. The shell model wave functions were assumed to be of the form $\Psi_{\rm core}(J=0,\ T=0)\Psi_{np}\ (J,T=0)$, adopting the jj-coupling scheme as was done in earlier works [3] and also by adopting the LS-coupling scheme. However, since the analysis of μ carried out earlier in Refs. [2–6], the number of nuclear states with measured values of μ has increased by a factor of 2, with the addition of the experimental values of ^{10}B(1^+), ^{18}F(3^+), ^{22}Na(1^+), ^{26}Al(5^+), ^{26}Al(3^+), ^{38}K(7^+), and ^{46}V(3^+). As in Refs. [2–6], a good agreement with experimental values of μ was obtained within the jj-coupling shell model by using the free g factors, i.e., the Schmidt values of g_j^n and g_j^p. Using the effective g factors, deduced from experimental values of μ of states in nearby odd-A nuclei, a significantly better agreement with data is obtained for some cases, such as ^{6}Li(1^+) and ^{38}K(3^+).

An interesting result of the present work is that a similarly good agreement with the experimental values of μ was obtained within the LS-coupling scheme shell model assuming $S=1$, except for the states ^{10}B(1^+), ^{22}Na(1^+), and ^{46}V(3^+) were the $L=J$, $S=0$ assignment leads to a better agreement with data and with the prediction of the jj-coupling scheme. In this model the value of μ is sensitive to the value of the orbital angular momentum L. This enabled us to determine L

TABLE III. The overlap $\langle (nlj)^2|d\rangle$ between the jj-coupling scheme shell model and the core-deuteron wave function and the g factors.

jj-scheme configuration		$\langle j^2\|d\rangle$ $L=J-1$	$L=J+1$	g $(nlj)^2$ model	Deuteron model $L=J-1$,	$L=J+1$
$0p_{3/2}^2$	$J=1$	0.185	0.037	0.627	0.857	0.321
	3	0.500		0.627	0.619	
$0p_{1/2}^2$	$J=1$	0.019	0.370	0.373	0.857	0.321
$0d_{5/2}^2$	$J=1$	0.047	0.013	0.576	0.857	0.321
	3	0.051	0.009	0.576	0.619	0.411
	5	0.375		0.576	0.571	
$1s_{1/2}^2$	$J=1$	0.208		0.880	0.857	
$0d_{3/2}^2$	$J=1$	0.013	0.047	0.424	0.857	0.321
	3	0.003	0.308	0.424	0.619	0.411
$0f_{7/2}^2$	$J=1$	0.012	0.004	0.554	0.857	0.321
	3	0.009	0.003	0.554	0.619	0.411
	5	0.027	0.003	0.554	0.571	0.440
	7	0.313		0.554	0.551	
$1p_{3/2}^2$	$J=1$	0.097	0.012	0.627	0.857	0.321
	3	0.158		0.627	0.619	
$1p_{1/2}^2$	$J=1$	0.010	0.117	0.373	0.857	0.321
$0f_{5/2}^2$	$J=1$	0.005	0.010	0.446	0.857	0.321
	3	0.001	0.037	0.446	0.619	0.411
	5	0.0003	0.271	0.446	0.571	0.440

for the states with known experimental values of μ. It was found that $L=J-1$ ($L=J+1$) if the corresponding jj-coupling scheme shell model configuration $(nlj)^2$ has $j=l+1/2$ ($j=l-1/2$), as might be expected from the classical picture of addition of angular momenta. The calculated values of the amplitudes of the core-deuteron wave functions in the corresponding shell model wave functions support this assignment of the values of L and explain the success of the core-deuteron cluster model in reproducing the experimental values of μ. However, since the predictions of the core-deuteron model are identical with those of a simple shell model with pure configurations in the LS-coupling scheme, the success of the core-deuteron model in reproducing the experimental values of μ cannot be taken as clear evidence for strong deuteronlike correlations.

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