Lack of additivity in mass-190 superdeformed bands

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We investigate the independent quasiparticle picture for superdeformed nuclei of the mass-190 region, which is expected to lead to additivity in certain physical quantities. Obvious deviations from additivity are found from both experimental data and projected shell model calculations. The cause of the deviation can be decomposed and identified through this model. Our study suggests that the independent quasiparticle picture may not be fully appropriate for this mass region.
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The simple independent quasiparticle (qp) picture leads to additivity of single-particle motion and is often a useful initial model of nuclear structure. However, it has been known for some time that additivity of quasiparticles often fails for normally deformed (ND) nuclei (see [1] and references therein). Since additivity implies that residual interactions among qp orbitals are negligible, a study of additivity and its violation can provide important information concerning the nature of residual interactions among quasiparticles in realistic nuclei.

One might expect that additivity should hold better for superdeformed (SD) bands because of larger and more stable deformation and weaker pairing. There has been substantial recent progress in experimental measurements for superdeformed nuclei. For example, linking transitions between SD and ND states have been identified for some nuclei [2,3] that permit the first determination of absolute spin values for some SD bands, and more precise measurements of the relative quadrupole moments in superdeformed bands are becoming available [4,5]. This progress offers for the first time the possibility to thoroughly examine the additivity issue in SD nuclei. It is crucial to understand to what extent the independent particle picture works and, if a deviation from exact additivity rule is observed, what is the cause.

A quantitative check of additivity requires that an appropriate indicator be chosen. The *alignment* (more properly, *relative angular momentum* or *relative spin* [6,7]), i, defined by the difference in spin of a band relative to a reference band at a given transition energy, is a useful quantity for this purpose because the spin is quantized and can now be precisely determined from SD-ND linking transitions in favorable experimental circumstances. On the theoretical side, the projected shell model (PSM) has been shown to be remarkably successful in predicting the spin values of SD bands where they have been measured in the $A=130$ and 190 mass regions [8–10].

Thus, we are now in a position to compare spins for a multi-qp band to those obtained from a sum of the component qp bands and thereby examine the validity of the additivity hypothesis as a function of transition energy. In this paper we illustrate the simplest example of such an approach: a comparison of the relative spins for a given band of an odd-odd nucleus to the sum of the relative spins of corresponding bands for the neighboring odd-neutron and odd-proton nuclei before the first band crossing. This permits a direct test of whether measured alignment gains can be described simply as a sum of the alignments of a few individual orbitals in the absence of residual quasiparticle interactions, or whether the alignment gain is a more collective phenomenon involving many interacting orbitals.

The choice of a reference band is crucial in extracting reliable alignments. For situations where only two bands (say, a ground band and an s-band) are involved and they interact weakly, the ground band can serve as reference that is easily constructed (by fitting the lower-spin yrast states to obtain the Harris parameters [11]). The relative spin for the s-band is then ascertained by subtracting the reference band from the s-band. Such cases correspond to a sharp backbending in the spin versus rotational frequency plot. However, if the interaction is strong between two bands, or many bands interplay with each other, which leads to a strong effective interaction, one typically obtains a smoothly rising spin-frequency curve and it is difficult to construct a reference using the simple approach described above. A common recipe in this case is to take the neighboring even-even nucleus as the reference (the so-called core nucleus). In the mass-190 superdeformed nuclei that we examine here, the smoothly rising moments of inertia imply that the choice of reference is not a simple one.

We begin our analysis of additivity by using a core nucleus as reference for a typical group of SD bands with excellent data in the mass-190 region. Figure 1 shows the experimental relative spin (1) for the neutron $j_{15/2}$ band in ^{191}Hg [12], (2) for the proton $i_{13/2}$ band (with two signature partners) in ^{191}Tl [13], and (3) for their sum. This sum is to be compared with the relative spin for the corresponding $\nu j_{15/2}\pi i_{13/2}$ bands in ^{192}Tl [14], which we do by choosing as a reference the yrast SD-band of ^{190}Hg [15] (top) and the yrast SD-band of ^{192}Hg [16] (bottom). It is obvious from Fig. 1 that there are serious deviations from the additivity rule in either case. The deviation from additivity is seen not only in the absolute value but more important, also seen in the curvature and in the amount of signature splitting. Although the spin assignments for these bands are not yet con-

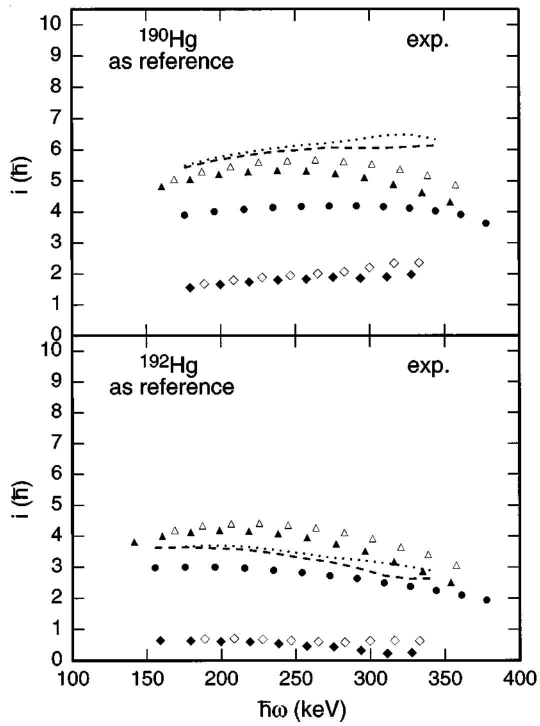

FIG. 1. Experimental relative spin for the $j_{15/2}$ band in ^{191}Hg [12] (dots), the $i_{13/2}$ band (with two signature partners) in ^{191}Tl [13] (diamonds), and their sum (dotted and dashed lines), plus the corresponding bands in ^{192}Tl [14] (triangles). The reference is the yrast SD-band of ^{190}Hg [15] (top), and the yrast SD-band of ^{192}Hg [16] (bottom).

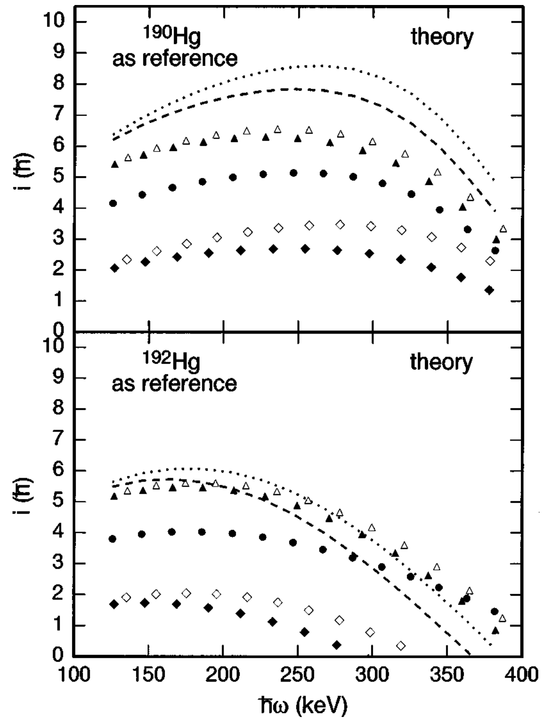

FIG. 2. As for Fig. 1, except all values are from projected shell model calculations.

firmed by measurements, they are based on very reasonable guesses. A shift of the spin assignment will move these curves up or down, which would affect the agreement in the absolute magnitude of i values, but the difference in curvature and in the amount of signature splitting would not be influenced by spin reassignment.

Let us now examine the same quantities from a pure theoretical perspective. The projected shell model calculations were done in the standard way described in [9,17]. The Nilsson plus BCS mean field solution is taken as the basis, in which average particle number conservation is guaranteed. The PSM many-body wave function is a superposition of angular momentum projected multi-qp states,

$$|\psi^I_M\rangle=\sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa}|\varphi_\kappa\rangle, \quad (1)$$

where $|\varphi_\kappa\rangle$ denotes basis states consisting of the qp-vacuum, two-quasi-neutron and -proton, and four-qp states for even-even nuclei; one-quasi-neutron (-proton) plus three-qp states for odd-neutron (-proton) nuclei; and one-quasi-proton plus one-quasi-neutron states for odd-odd nuclei, respectively. A quadrupole plus pairing Hamiltonian is used in the PSM with inclusion of both monopole and quadrupole pairing [17],

$$\hat{H}=\hat{H}_0-\frac{\chi}{2}\sum_\mu \hat{Q}^\dagger_\mu\hat{Q}_\mu-G_M\hat{P}^\dagger\hat{P}-G_Q\sum_\mu \hat{P}^\dagger_\mu\hat{P}_\mu. \quad (2)$$

The interaction strengths were set to values common for nuclei in this mass region [9,10], and the deformation of the bases was fixed at $\epsilon_2=0.45$ for all nuclei calculated in this paper.

In Fig. 2 we plot the same quantities as for Fig. 1, but with the results all taken from the PSM calculations. The theoretical curves reproduce the experimental trend shown in Fig. 1 very well; we note in particular that there is a similar deviation from the additivity rule in both the experimental and theoretical plots. As illustrated in Refs. [9,10], PSM calculations can reproduce nicely the experimental spin-frequency curves for SD bands of this mass region. Here, the agreement between Figs. 1 and 2 is less good because we are comparing the *relative spins*, which involves six bands from four different nuclei. However, the essential point of these two figures is that an obvious and similar deviation from additivity exists in both the data and the calculations. We find similar experimental and theoretical results for other groups of SD bands in this mass region as well.

The breakdown of additivity illustrated in Figs. 1 and 2 indicates that there is substantial residual quasiparticle interaction in realistic mass-190 superdeformed nuclei. For purposes of discussion, let us (approximately) separate the residual interactions responsible for deviations from the additivity rule for the last few quasiparticles into two categories: (1) the residual interaction between the quasiparticles and the core (the polarization effect), and (2) the additional residual interaction among the last few quasiparticles. It is obvious that the ansatz of using a neighboring even-even nucleus as the common core fails to minimize the polarization effect equally well for all nuclei compared because polarization is orbital-dependent. In addition, the residual interaction among the last few quasiparticles remains even after subtracting the common core. The additivity might in prin-

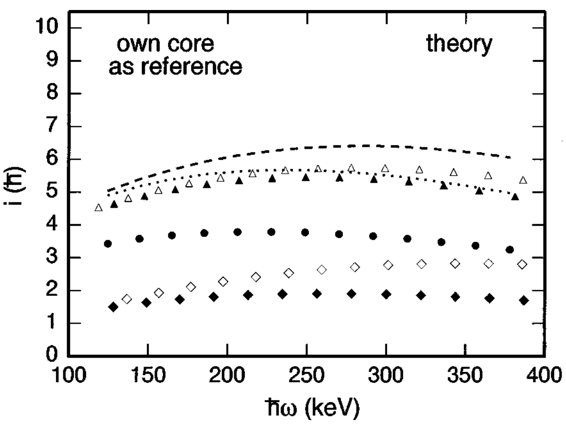

FIG. 3. As for Fig. 2, except the cores are calculated for each given band in odd-*A* and odd-odd nuclei and the configuration space is limited to the lowest quasiparticle number—see further explanation in the text.

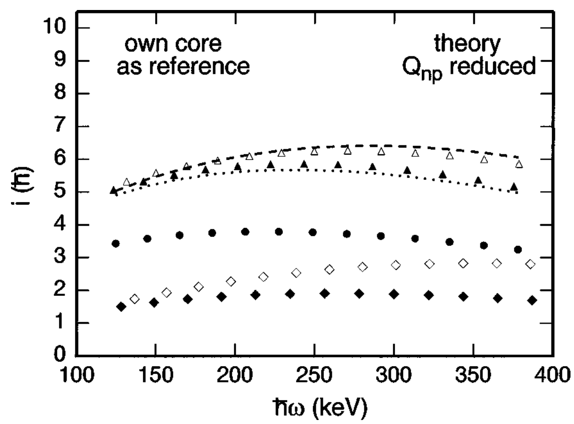

FIG. 4. As for Fig. 3, but in the diagonalization the Q_pQ_n coupling has been reduced to 95% of the full strength for the odd-odd nucleus.

ciple be restored if one could minimize these two types of residual interactions. The PSM offers such a possibility, because the quantitative agreement between Figs. 1 and 2 suggests that the PSM incorporates the same physics that produces deviations from additivity in the data.

Our goal is to examine the above understandings on the cause of the deviation, but not to modify the model so that the data can be better reproduced. Therefore, from now on, what we will deal with is all from theoretical calculations, simply because, in experimental data, all kinds of interactions, including residual ones, are embodied and there is no direct and exact way to separate them. First, let us try to minimize the residual interaction between the last few quasiparticles and the core by constructing ''pure'' one-qp and two-qp states for an odd-*A* and an odd-odd nucleus, respectively, and limiting our configuration space in Eq. (1) by allowing only those lowest numbers of quasiparticles for each kind of system. That is, we use one-qp for odd-*A*, two-qp for odd-odd, and 0-qp for the core. This procedure eliminates the admixture of higher multi-qp states, thus effectively reducing the polarization effect. Furthermore, we shift the Fermi levels of the core in such a way that they lie in the appropriate positions for the neighboring odd-*A* and odd-odd nuclei. Consequently, the new core contains the vacuum state only and reflects the qp occupations of the corresponding odd-*A* or odd-odd nuclei. We term this new core the *own core* of an odd-*A* or odd-odd nucleus.

In Fig. 3, we present calculated results for the same bands studied in Fig. 2, but using this new procedure. Note that the reference now is neither ^{190}Hg nor ^{192}Hg: each odd-*A* or odd-odd nucleus has its own reference, and all curves and points come from PSM calculations. The agreement between the summed curves of the odd-neutron and odd-proton bands and those for the odd-odd nucleus is now much better. For the bands compared, this indicates that minimizing the polarization effect and forcing the residual interactions (the important one is the pairing correlation) to be similar in bands and their own cores leads to substantial improvement in quasiparticle additivity. Conversely, these results indicate clearly the reason that additivity fails for normal quasiparticles in the mass-190 superdeformed nuclei.

Close inspection of Fig. 3 reveals that there remain some small deviations from additivity. Thus, there are still residual interactions that influence odd-*A* and odd-odd systems differently, even though in the Hamiltonian (2) the same interaction strengths are used for all nuclei considered. The Q_pQ_n term in Eq. (2) is the only explicit proton-neutron interaction in the model [17]. Simple estimates based on counting valence particles [18] suggest that, the pn interaction between the last proton and last neutron is around a few percent (either counted from the spherical close shell or the SD ''closed shell'') of the total in ^{192}Tl. This percentage should approximately be the pn interaction left after subtraction of its own core for this odd-odd nucleus. Thus, we reduce the strength of the Q_pQ_n term by 5% for the odd-odd ^{192}Tl only and repeat the calculation of Fig. 3, with the corresponding results displayed in Fig. 4. Now the agreement in the absolute value, the curvature, and the amount of signature splitting is almost perfect, indicating that additivity among these corresponding theoretical bands has been restored almost exactly. These results may be viewed either as illustrating clearly the likely theoretical reasons for the failure of additivity in the mass-190 superdeformed nuclei, or as a suggestion for ways to produce improved quasiparticles that are approximately additive.

It is obvious that more SD-ND linking transition measurements and theoretical calculations are crucial for further understanding of the additivity rule and the nature of the residual interactions for superdeformed nuclei. However, the present results give considerable insight into the reason for failed quasiparticle additivity in the mass-190 superdeformed nuclei and may suggest paths to the development of more sophisticated quasiparticles that are additive. Although the particular decomposition of the residual interaction that we have employed here is model dependent, we may expect that the general conclusions that we have reached concerning additivity in this mass region will hold independent of this choice. Recently, an analysis of superdeformed bands in the mass-150 region concluded that the additivity rule is fulfilled well in the nuclei examined [19]. We intend now to check the additivity hypothesis for SD bands in the $A=150$ and 130 regions as well using these same methods, with the results to be reported elsewhere.

In summary, a substantial deviation from additivity for superdeformed bands in mass-190 nuclei is documented, indicating the presence of strong residual interactions among the quasiparticles and a failure of the simple independent quasiparticle picture. For the cases discussed, these residual interactions may be approximately decomposed into the quasiparticle-core interaction and an additional *pn* interaction between the last proton and neutron in odd-odd nuclei. Such an understanding is examined and proved by the PSM calculation for the group of bands chosen. We provide theoretical insight into the reason for this additivity failure by illustrating that once a proper core is constructed and subtracted from each odd-*A* and odd-odd nucleus, ''pure'' one-qp and two-qp states are obtained and additivity then holds much better. Finally, results from a small rescaling of the *pn* quadrupole interaction suggest that when the extra *pn* interaction between the last proton and last neutron in odd-odd nucleus is removed as well, additivity is almost totally restored.

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