Direct capture astrophysical *S* factors at low energy

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(Received 11 February 1998)

We investigate the energy dependence of the astrophysical S factors for the reactions $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$, the primary source of high-energy solar neutrinos in the solar pp chain, and $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$, an important reaction in the CNO cycle. Both of these reactions have predicted S factors which rise at low energies; we find the source of this behavior to be a pole in the S factor at a center-of-mass energy $E=-E_{\mathrm{B}}$, the point where the energy of the emitted photon vanishes. The pole arises from a divergence of the radial integrals.
[S0556-2813(98)03907-7]

PACS number(s): 26.65.+t, 25.40.Lw, 25.60.Pj, 25.10.+s

The $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$ reaction, at center-of-mass energies E near 20 keV, plays an important role in the production of solar neutrinos [1]. The neutrinos from the subsequent decay of $^{8}\mathrm{B}$ provide the high-energy neutrinos to which many solar neutrino detectors are sensitive. The cross section for this reaction is conventionally expressed in terms of the S_{17} factor, where the S factor is defined in terms of the cross section σ by

$$S(E)=\sigma(E)E\ \exp[2\pi\eta(E)], \tag{1}$$

where $\eta(E)=Z_1Z_2\alpha\sqrt{\mu c^2/2E}$ is the Sommerfeld parameter for nuclei of charges Z_1 and Z_2 and reduced mass μ. The exponential factor in the definition of S removes the rapid energy dependence of the cross section due to Coulomb repulsion between the two nuclei. In the stellar core the probability of capture of protons by $^{7}\mathrm{Be}$, obtained by folding the thermal distribution of nuclei with the cross section, peaks at ~ 20 keV. Because the cross section diminishes exponentially at low energies, the only method of obtaining information about S_{17} at those energies is to extrapolate data taken at experimentally accessible energies ($E>100$ keV).

The $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}^*$ ($\frac{1}{2}^+;\frac{1}{2}$, 0.498 MeV [2]) reaction, which occurs in the CNO cycle, is of little importance for energy production in the Sun but of greater importance for hotter stars. As is the case for $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$, extrapolation of data taken at high energies is necessary to obtain the S factor at energies applicable in the stellar core, $E\sim 25$ keV.

Direct capture calculations [3] of these two reactions [4–6] predict an upturn in the S factor at threshold. As the capture in both reactions is primarily external, the S factors at astrophysical energies are determined by the product of the spectroscopic factor, the asymptotic normalization of the final (bound) state wave functions, and a purely Coulombic term. As the spectroscopic factor is independent of energy, the energy dependence of the S factor, away from resonances, may be studied without detailed knowledge of the nuclear structure.

In each reaction the weakly bound final state ($E_{\mathrm{B}}=137.5$ keV [7] for $^{8}\mathrm{B}$ and $E_{\mathrm{B}}=105.2$ keV for the first excited state of $^{17}\mathrm{F}$ [2]) causes the S factor to rise as the center-of-mass energy approaches 0. In the case of $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}^*$ both the data and direct-capture calculations [4,8] show clear evidence of this low-energy rise. The $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$ capture to the ground state shows no such rise because the final state is more deeply bound and has higher angular momentum. For $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$ the upturn occurs below the lowest experimental point, and so it is only observed in the calculations.

Williams and Koonin [5] do an explicit expansion about zero energy for S_{17} and give the first two coefficients in a Taylor series for the logarithmic derivative of S_{17} as $-2.350\ \mathrm{MeV}^{-1}$ and $28.3\ \mathrm{MeV}^{-2}$. Using these in a (1,1) Padé approximant gives

$$S_{17}=\frac{(1+4.85E)}{(1+7.20E)}, \tag{2}$$

where E is in MeV. This Padé approximant has a pole at -139 keV which is very close to their bound state energy of 136 keV. Thus we see that in the region of the threshold the bound state is important and induces a pole. In fact, a Taylor series expansion would converge only with a radius of the binding energy — barely to the region that is experimentally accessible. Hence any functional form for the extrapolation to zero energy should contain the contribution from the pole.

Following [5], we write the astrophysical S factor as

$$S=C(I_0^2+2I_2^2)E_\gamma^3(J_{11}\beta_{11}^2+J_{12}\beta_{12}^2)\frac{1}{1-e^{-2\pi\eta}}, \tag{3}$$

where

$$I_L=\int_0^\infty dr r^2\psi_{iL}(r)\psi_f(r)/k, \tag{4}$$

$$C=\frac{5\pi}{9}\frac{1}{(\hbar c)^3}(2\pi\eta k)e^2\mu^2\left(\frac{Z_1}{M_1}-\frac{Z_2}{M_2}\right)^2. \tag{5}$$

In Eq. (3), J_{LS} is the spectroscopic factor for a given angular momentum L and channel spin S, β_{LS} is the asymptotic normalization of the bound state wave function, E_γ is the photon energy, and k is the momentum of the incident proton. The final bound state wave function $\psi_f(r)$ is normalized asymptotically to $\psi_f(r)=W_{\alpha,l}(\kappa r)/r$ while the initial wave function reduces to the regular Coulomb wave function divided by $\sqrt{2\pi\eta/(e^{2\pi\eta}-1)}$. The unusual choice of normal-

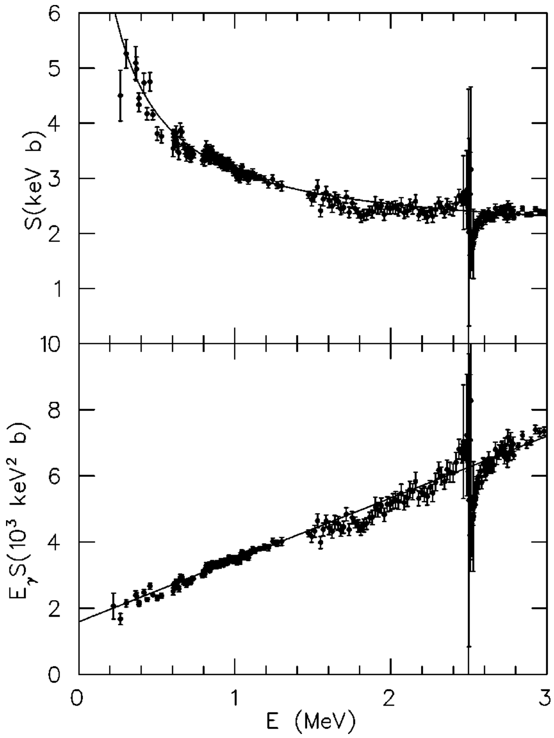

FIG. 1. Astrophysical S factor (top) and $E_\gamma S$ (bottom) for $^{16}O(p,\gamma)^{17}F^*$. The data of Morlock *et al.* [8] are compared to the fit as described in the text (solid line).

izations is just to eliminate uninteresting factors from the quantities of interest. Most of those factors have been collected in the coefficient C.

To investigate the behavior of the integrals in Eq. (3), we first consider $\psi_f(r)=W_{\alpha,1}(\kappa r)/r$ for all radii and take $\psi_{i0}(r)=F_0(kr)/[\sqrt{2\pi\eta}/(e^{2\pi\eta}-1)]$. The integral for the s wave then becomes

$$I_0=\int_0^\infty dr r W_{\alpha,1}(\kappa r)F_0(kr)/[k\sqrt{2\pi\eta}/(e^{2\pi\eta}-1)]. \quad (6)$$

At threshold, the integrands are peaked at large r: 40 fm for $^7Be(p,\gamma)^8B$, and 65 fm for $^{16}O(p,\gamma)^{17}F^*$. The tails of both integrands extend well beyond 100 fm and are, in each case, indicative of halo states. The integral is smooth as k passes through zero and diverges as $k\rightarrow i\kappa$ ($E\rightarrow -E_B$). The nature of the divergence is determined by the asymptotic forms of the Coulomb wave function and Whittaker function for large r. For large r the Whittaker function is proportional to $r^{-|\eta k|/\kappa}e^{-\kappa r}$ [3] (ηk is independent of k). While above threshold the Coulomb wave function oscillates at large radii, below threshold it is exponentially growing and is proportional to $r^{|\eta|}e^{|k|r}$. Thus the integrand approaches

$$r^{1-|\eta k|(1/\kappa-1/|k|)}\exp[-(\kappa-|k|)r] \quad (7)$$

for large r and the integral diverges as

$$I_0\propto 1/(\kappa-|k|)^2\propto 1/(E_B+E)^2=1/E_\gamma^2. \quad (8)$$

Since the integrand diverges as $1/E_\gamma^2$, the leading term and first correction term are both determined purely by the asymptotic behavior of the wave functions. The first correction term is not simply $1/E_\gamma$ but also involves logarithmic terms coming from the $r^{-|\eta k|(1/\kappa-1/|k|)}$ factor. The second correction term, of order E_γ^0, is not determined purely by the asymptotic value of the wave function alone but also depends on the wave function at finite r.

TABLE I. The numerical constants c_i and E_B used to determine the energy dependences, with the normalizations n used when displaying the curves with the discussed data sets.

Reaction	c_1 (MeV^{-1})	c_2 (MeV^{-2})	E_B (MeV)	n (keV^{2}b)
$^{16}O(p,\gamma)^{17}F^*$	1.18	0	0.1052	1.59×10^3
$^7Be(p,\gamma)^8B$	5.36	1.80	0.1375	3.74[a]
				2.99[b]

[a]Kavanagh *et al.* [10].
[b]Filippone *et al.* [9].

From Eq. (3), we see that the quadratic divergence of I_0 gives rise to a simple pole in S at $E_\gamma=0$. This suggests writing the S factor as a Laurent series:

$$S=d_{-1}E_\gamma^{-1}+d_0+d_1E_\gamma+\cdots. \quad (9)$$

As before the coefficients of the first two terms d_{-1} and d_0 are determined purely by the asymptotic forms of the wave functions while the third coefficient d_1 is also dependent on the short range properties of the wave functions.

In Fig. 1, we present the data of Morlock *et al.* [8] for the $^{16}O(p,\gamma)^{17}F^*$ S factor (top) and for the product $E_\gamma S$ (bottom). In the top panel, the energy dependence of the S factor is well approximated by the form

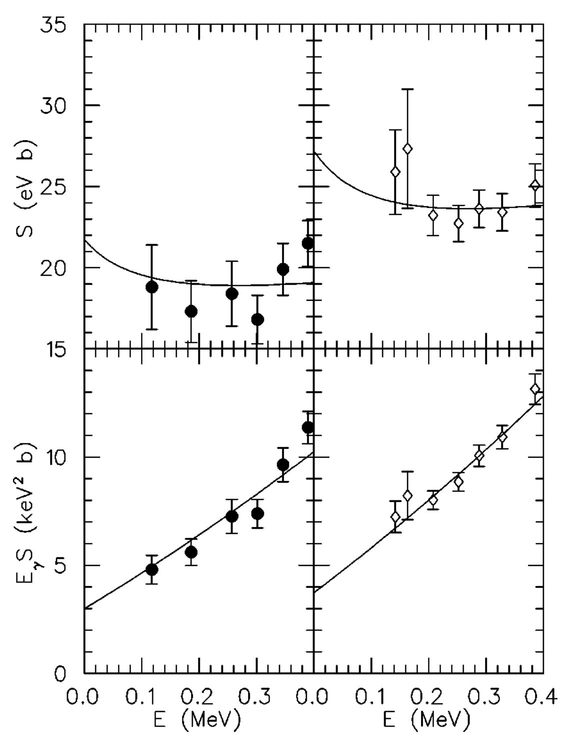

FIG. 2. The low-energy part of the astrophysical S factor and $E_\gamma S$ for $^7Be(p,\gamma)^8B$. The data of Filippone *et al.* [9] (circles) and Kavanagh *et al.* [10] (diamonds) are compared to the results of the calculations as described in the text (solid line).

$$S=n\frac{1+c_1E}{E_\gamma}=n\frac{1+c_1E}{E+E_{\rm B}}, \qquad (10)$$

where the constants c_1 and n are determined by the straight line fit to $E_\gamma S$ shown in the bottom panel. The numerical values are given in Table I. There is remarkable agreement with the data except near the resonance at 2.504 MeV. Equation (10) is a convenient form for fitting experimental data and is motivated by both the Padé approximant and Eq. (9).

In Fig. 2, the data of Filippone *et al.* [9] (circles) and Kavanagh *et al.* [10] (diamonds) for the S factor (top panels) and the product $E_\gamma S$ (bottom panels) for the ^{7}Be$(p,\gamma)^8$B reaction are presented for energies well below the $E=633$ keV $M1$ resonance. We take the data as normalized by Johnson *et al.* [11] to $\sigma_{dp}=157$ mb. The curves are similar in form to Eq. (10), but with a quadratic term added,

$$S=n\frac{1+c_1E+c_2E^2}{E_\gamma}=n\frac{1+c_1E+c_2E^2}{E+E_{\rm B}}. \qquad (11)$$

The values of n and c_i for this reaction are also listed in Table I. Different values of the normalization n are required to reproduce the data of Filippone *et al.* and Kavanagh *et al.*, but the c_i are determined from the threshold energy dependence of a direct-capture calculation following Ref. [5]. A cutoff radius of $r_0=2.3$ fm was chosen to be consistent with the phase shift and energy dependence found by Barker [12]. The upturn at threshold is clearly observed in the results of the calculation. The data are insufficient to determine this behavior or, equivalently, c_i. It will be very difficult to experimentally confirm this upturn since it is only pronounced below 100 keV. Fortunately it is theoretically well understood and both n and c_1 depend primarily on the asymptotic normalization, spectroscopic factor, and properties of the Coulomb force. Note that the curves presented in Fig. 2 should not be mistaken for a serious attempt at determining the S_{17} factor at zero energy; rather, they are illustrative of the energy dependence.

The straight line approximation for $E_\gamma S$ is valid for the ^{16}O$(p,\gamma)^{17}$F* reaction up to ≈ 3 MeV. However, the quadratic approximation for the ^{7}Be$(p,\gamma)^8$B reaction is not valid for energies above 0.4 MeV. Initially, the breakdown is caused by the resonance at 0.633 MeV. Above the resonance higher-order terms in E, arising predominantly from d-wave direct capture, become significant.

In conclusion we see that the threshold peak in the S factor is associated with weakly bound states and arises from a pole at $E_\gamma=0$. Those bound states in ^{8}B and ^{17}F are halo in nature and so the associated radial integrals are, by necessity, long range.

The authors would like to thank F. C. Barker, L. Buchmann, H. Fearing, T. Hemmert, K. Langanke, and E. Vogt for useful discussions and R. Morlock for making tables of his data available. Financial assistance from the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

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