Energies and residues of the nucleon resonances $N(1535)$ and $N(1650)$

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We extract pole positions for the $N(1535)$ and $N(1650)$ resonances using two different models. The positions are determined from fits to different subsets of the existing $\pi N\rightarrow\pi N$, $\pi N\rightarrow\eta N$, and $\gamma p\rightarrow\eta p$ data and found to be $1510(10)-i85(15)$MeV and $1660(10)-i70(10)$MeV, when the data is described in terms of two poles. Sensitivity to the choice of fitted data is explored. The corresponding $\pi\pi$ and $\eta\eta$ residues of these poles are also extracted. [S0556-2813(98)07612-2]

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I. INTRODUCTION

Properties of the $N(1535)$ are difficult to extract from $\pi N\rightarrow\pi N$ and $\gamma N\rightarrow\pi N$ due to the nearby ηN threshold [1]. As a result, a number of recent analyses have been based on data from $\pi^-p\rightarrow\eta n$ and $\gamma p\rightarrow\eta p$. These studies and coupled-channel analyses including pion production data, generally find values for the $N(1535)$ pole position, mass, width, and photodecay amplitudes which differ from those obtained from pion production data alone [2–8]. While these more recent studies suggest that some $N(1535)$ properties should be revised, the modification of any single quantity is complicated due to correlations. An example is the ξ_p parameter used by Mukhopadhyay and Collaborators [9]. This combination of the photodecay amplitude ($A_{1/2}$), total (Γ_T) and ηN (Γ_η) widths is relatively stable, even though values of $A_{1/2}$ and Γ_T vary by factors of 2. Manley [10] has also noted that near-threshold $\pi^-p\rightarrow\eta n$ data provide little sensitivity to different parameter choices.

In Ref. [11] a two-channel K-matrix model was presented for S-wave πN and ηN scattering up to a center-of-mass energy of about 1700 MeV. There the main motivation was to extract the eta-nucleon scattering length (a) and effective range (r_0) and to determine their uncertainties allowed by the existing $\pi N\rightarrow\pi N$ amplitudes [12], $\pi N\rightarrow\eta N$ compilation [2], and $\gamma p\rightarrow\eta p$ [3] data. Below, this model will now be used to estimate the energies and residues of the S-wave nucleon resonances $N(1535)$ and $N(1650)$ as complex poles of the T matrix. Any problems with the $N(1535)$ may carry over to the nearby $N(1650)$ resonance, as the properties of these two resonances are extracted from the same (S_{11}) πN partial wave and the same photoproduction multipole.

In the model of Ref. [11] two poles corresponding to these resonances were included in the K matrix, and their energies were tuned along with other parameters to give a fit to the data. However, in principle, the positions of these T- and K-matrix poles can be quite different. Furthermore, it is the T-matrix poles that are of physical significance—hence their tabulation in Ref. [13]. A second reason for determining T-matrix pole positions is the greater variation of Breit-Wigner parameters within different parametrization schemes. For each pole, we have also extracted the corresponding residue.

The present study differs from most [8] of those carried out previously in that we have explored the effect of using different models and fitting different data sets. We have also considered, for πN elastic scattering, the effect of fitting the original experimental data rather than the amplitudes extracted from these data. In the next section, we compare the model used in Ref. [11] to that used in the VPI analyses. These two models have been utilized in our fits. In Sec. III, we show our results and consider the factors responsible for differences in the extracted resonance parameters.

II. FORMALISM

The model of Green and Wycech is fully described in Ref. [11]. Here we repeat only the main elements, in order to facilitate comparisons with the VPI analyses. Both models are based on a three-channel K-matrix formalism. In Ref. [11], a narrow energy range was chosen in order to justify the neglect of partial waves beyond $l=0$. In the VPI fits, higher partial waves were included in fits which spanned a much wider energy range. However, these fits, while employing a multichannel formalism, were not constrained by η-production data.

In the fits of Ref. [11], S-wave scattering was considered in a system consisting of the two channels πN and ηN—here denoted simply by the indices π and η. Then the K matrix and the corresponding T matrix, which are related by $T=K+iKqT$, can be written as

$$K=\begin{pmatrix} K_{\pi\pi} & K_{\eta\pi} \\ K_{\pi\eta} & K_{\eta\eta}\end{pmatrix} \quad \text{and}$$

$$T=\begin{pmatrix} \dfrac{A_{\pi\pi}}{1-iq_\pi A_{\pi\pi}} & \dfrac{A_{\eta\pi}}{1-iq_\eta A_{\eta\eta}} \\ \dfrac{A_{\pi\eta}}{1-iq_\eta A_{\eta\eta}} & \dfrac{A_{\eta\eta}}{1-iq_\eta A_{\eta\eta}}\end{pmatrix}, \tag{1}$$

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where $q_{\pi,\eta}$ are the center-of-mass momenta of the two mesons in the two channels π,η. The channel scattering lengths A_{ij} are expressed in terms of the K-matrix elements as

$$A_{\pi\pi}=K_{\pi\pi}+iK_{\pi\eta}^2 q_\eta/(1-iq_\eta K_{\eta\eta}),$$

$$A_{\eta\pi}=K_{\eta\pi}/(1-iq_\pi K_{\pi\pi}),$$

$$A_{\eta\eta}=K_{\eta\eta}+iK_{\eta\pi}^2 q_\pi/(1-iq_\pi K_{\pi\pi}). \tag{2}$$

As discussed in Ref. [11], these K matrices are designed to account directly for several observed features of the experimental data such as the presence of two S-wave πN resonances and allow both to have a coupling to the two-pion channel. The latter channel is not treated explicitly, but introduced by reducing a three channel K matrix for πN, ηN, and $\pi\pi N$ into the two channel form in Eq. (1). In addition, the second resonance is not coupled to the ηN channel—a feature indicated by experiment. The resultant K matrices in this two channel model are then as follows:

$$K_{\pi\pi}\to\frac{\gamma_\pi(0)}{E_0-E}+\frac{\gamma_\pi(1)}{E_1-E}+i\frac{K_{\pi 3}q_3K_{3\pi}}{1-iq_3K_{33}},$$

$$K_{\pi\eta}\to K_{\pi\eta}+\frac{\sqrt{\gamma_\pi(0)\gamma_\eta}}{E_0-E}+i\frac{K_{\pi 3}q_3K_{3\eta}}{1-iq_3K_{33}},$$

$$K_{\eta\eta}\to K_{\eta\eta}+\frac{\gamma_\eta}{E_0-E}+i\frac{K_{\eta 3}q_3K_{3\eta}}{1-iq_3K_{33}}, \tag{3}$$

where

$$K_{33}=\frac{\gamma_3(0)}{E_0-E}+\frac{\gamma_3(1)}{E_1-E},$$

$$K_{\pi 3}=\frac{\sqrt{\gamma_\pi(0)\gamma_3(0)}}{E_0-E}+\frac{\sqrt{\gamma_\pi(1)\gamma_3(1)}}{E_1-E},$$

$$K_{\eta 3}=\frac{\sqrt{\gamma_\eta\gamma_3(0)}}{E_0-E}, \tag{4}$$

and q_3 is a three-body $\pi\pi N$ phase space. In all there were nine parameters in the K matrices and one parameter for normalizing the photoproduction data.

In the second (VPI) approach, a Chew-Mandlestam K matrix has been used [12] to couple the elastic πN channel to two inelastic channels, ηN and $\pi\Delta$ (in an l=2 state). One starts with a 3×3 matrix:

$$K=\begin{pmatrix} K_{\pi\pi} & K_{\pi\eta} & K_{\pi\Delta}\\ K_{\pi\eta} & K_{\eta\eta} & 0\\ K_{\pi\Delta} & 0 & K_{\Delta\Delta}\end{pmatrix}. \tag{5}$$

Following the methods outlined in Ref. [12], the T matrix is written in the form

$$T=\rho^{1/2}K(1-CK)^{-1}\rho^{1/2}, \tag{6}$$

and abbreviated as $T=\rho^{1/2}\overline{T}\rho^{1/2}$. In this notation, the elastic T matrix is given by

$$\overline{T}_{\pi\pi}=\frac{\overline{K}}{1-C_{\pi\pi}\overline{K}}, \tag{7}$$

where

$$\overline{K}=K_{\pi\pi}+\frac{C_{\eta N}K_{\pi\eta}^2}{1-C_{\eta N}K_{\eta\eta}}+\frac{C_{\pi\Delta}K_{\pi\Delta}^2}{1-C_{\pi\Delta}K_{\Delta\Delta}}, \tag{8}$$

C_i being a dispersion integral [12] of phase space factors over the appropriate unitarity cut, and $\rho=\mathrm{Im}\,C$. Inelastic channels are given by

$$\overline{T}_{\pi i}=\frac{(1+C_{\pi\pi}\overline{T}_{\pi\pi})}{(1-C_iK_{ii})}K_{\pi i}. \tag{9}$$

TABLE I. Dependence of the fit, using the form of Ref. [11], with variations of the branching to $\pi\pi N$. Notation is GW1X, where the parameter combination (see text) $q_3\gamma_3(0)$ takes on the value X/100. χ^2 values are given for the fitted (2771) π^-p, (452) charge-exchange (CEX), (53) η photoproduction [3], and (11) $\pi^-p\to\eta n$ total cross-section [2] data. The $I=3/2$ amplitudes have been fixed at the VPI values. The S11 column shows how well this fit to data reproduces the (60) VPI S11 single-energy points. The 2π column shows the corresponding two-pion branching ratio as a percentage.

Soln.	Total	π^-p	CEX	(γ,η)[3]	(π,η)[2]	S11	2π
GW10	7717	6153	1447	66	51	105	0
GW11	7671	6169	1410	62	30	98	2.6
GW12	7717	6245	1393	57	22	101	5.2
GW13	7783	6323	1377	67	16	100	7.8
GW14	7861	6401	1374	73	13	104	10.0

III. FITS TO DATA AND AMPLITUDES

In Ref. [11] the ten parameters were determined by fitting the η-production data of Refs. [2,3] and the energy dependent S11 $\pi N\to\pi N$ amplitudes of Ref. [12] over the center-of-mass energy range $1350\leqslant E_{\mathrm{c.m.}}\leqslant 1700$ MeV. However, a better approach is to fit the $\pi N\to\pi N$ experimental data directly, thus avoiding the intermediate step of extracting partial-wave amplitudes. Since the above K-matrix formalism is designed only for $T_{\pi\pi}(S11)$, the other partial waves are in the form advocated in Ref. [12]. The procedure is, therefore, to first fit with this latter form all of the πN

TABLE II. Notation as in Table I. Here the two-term form (see text) has been used to fit η photoproduction data. The VPI solution is a fit to the elastic πN scattering database from threshold to 2.1 GeV (with only forward dispersion-relation constraints), including the η-production data.

Soln.	Total	π^-p	CEX	(γ,η)[3]	(π,η)[2]	S11	2π
GW20	7687	6144	1442	51	50	126	0
GW21	7599	6102	1420	52	25	121	1.1
GW22	7627	6147	1410	53	17	122	2.3
GW23	7690	6227	1398	50	15	125	3.5
GW24	7774	6309	1395	52	18	128	4.8
VPI	7539	6040	1397	53	49	85	–

TABLE III. The optimized parameters defining the K matrices for GW11 and GW21 in Tables I and II.

	GW11	GW21		GW11	GW21
$K_{\eta\eta}$	0.1078	−0.8336	$\gamma_\pi(0)$	0.0640	0.1220
$K_{\pi\eta}$	0.0157	−0.1051	$\gamma_\pi(1)$	0.1071	0.0913
E_0(MeV)	1538.5	1582.5	γ_η	0.2283	0.6027
E_1(MeV)	1681.6	1678.8	$\gamma_3(0)q_3(1535)$(MeV)	1.97	1.97
			$\gamma_3(1)q_3(1650)$(MeV)	18.1	16.5

$\rightarrow\pi N$ data over the full energy range (2.1 GeV) utilized by the VPI analyses. This fit is referred to as solution VPI. Data are then refitted, using the form of Ref. [11], over the energy range $1350\leqslant E_{\mathrm{c.m.}}\leqslant 1700$ MeV, along with the η-production data, with the non-S11 amplitudes kept fixed. In this case only the parameters of the above K-matrix model are adjusted. These fits are referred to as solutions GW1X, where $X/100$ denotes the parameter combination $q_3\gamma_3(0)$ related to the $\pi\pi N$ branching for the $N(1535)$. The value of $X/100$ was varied from 0.00 to 0.04 [corresponding to $\pi\pi N$ branching fractions ranging from 0 to 10% for $N(1535)$], thus generating solutions GW10 to GW14. The results of these fits are given in Table I.

We also considered the effect of modifying the form used in fitting the η photoproduction data. As a first step, an additional energy dependence was added. This amounted to replacing A(phot) in Ref. [11] by $A(\mathrm{phot})+B(\mathrm{phot})[E_{\mathrm{c.m.}}-1485]/100$. However, this had little overall effect with B(phot) being an order of magnitude smaller than A(phot). A second two-parameter form

$$A\propto\alpha(1+iT_{\eta N})+\frac{\beta}{q_\eta}T_{\eta N}, \qquad (10)$$

analogous to that used in pion photoproduction [14], was also used. In the above, α and β were taken simply as constants. This form was labeled GW2X, with X retaining its earlier meaning, and was used to generate the results presented in Table II. In comparing Tables I and II, one should note that, while the $\pi\pi N$ branching varies essentially linearly with X in either GW1X or GW2X, the same X in GW1X and GW2X does not give the same branching, as the other parameters are very different in the two models. The actual values for the nine parameters are given in Table III for GW11 and GW21—the solutions with the smallest χ^2. In Table IV these parameters are converted into the more conventional form of Ref. [11]. Here it is seen that for GW21

TABLE IV. The parameters in Table III expressed in terms of widths and branching ratios as in Ref. [11]. Throughout η(1650,B) is fixed at zero.

	GW11	GW21
Γ(1535,total)(MeV)	151.6	354.4
η(1535,B)	0.576	0.663
π(1535,B)	0.398	0.326
Γ(1650,total)(MeV)	150.4	133.3
π(1650,B)	0.769	0.758

TABLE V. The real and imaginary parts of poles ($E_P-i\Gamma_P/2$) in the complex energy plane compared with those quoted in Ref. [13]. The numbers in parentheses correspond to the third S11 pole, for which some evidence was found in Ref. [16].

Reference	E_P(1535) (MeV)	Γ_P(1535)/2 (MeV)	E_P(1650) (MeV)	Γ_P(1650)/2 (MeV)
Arndt [16]	1501	62	1673(1689)	41(96)
Höhler [17]	1487	–	1670	82
Cutkosky [15]	1510±50	130±40	1640±20	75±15
This paper				
VPI	1510±3	73±3	1666(1668)	41(147)
VPI90 [19]	1499	55	1657	80
GW10	1510±8	87±5	1662±3	70±5
GW11	1514±9	90±6	1658±4	69±5
GW20	1502±3	80±3	1667±2	60±4
GW21	1509±3	82±4	1663±2	60±4

some of these parameters are very different from their on-energy-shell counterparts, whereas those for GW11 are very similar to the on-energy-shell parameters in Table I of Ref. [11]. The errors quoted in this table from Ref. [11] will be used later, when error estimates on the pole positions and residues are made.

In order to find the poles $E_P-i\Gamma_P/2$ of the T matrix in Eq. (1), the energy E appearing in Eqs. (3) and (4) and in the momenta q_π, q_η, and q_3 was everywhere converted into $E-i\Gamma/2$. It is a built-in feature of the present K-matrix formalism that the poles are at the same positions in all three matrix elements $T_{\pi\pi},T_{\eta\eta},T_{\pi\eta}$. This has been checked and found to be so within 10 keV.

The pole positions are given in Table V and compared with the current values in Ref. [13]. There it is seen that our results are consistent with previous values—especially those of Ref. [15]. As could be expected, our error bars are smaller due to the improvement in, and quantity of, the experimental

TABLE VI. The moduli($|r|$) and phases(θ) of the residues of the two poles in both $T_{\pi\pi}$ and $T_{\eta\eta}$ compared with those quoted in Ref. [13]. Residues for the VPI 1650 MeV resonance are not included, as the VPI fit has an added pole in this region.

T_{ii}	Reference	$\lvert r\rvert$(1535) (MeV)	θ(1535) (deg.)	$\lvert r\rvert$(1650) (MeV)	θ(1650) (deg.)
$T_{\pi\pi}$	Arndt [16]	31	−12	22(72)	29(−85)
	Höhler [17]			39	−37
	Cutkosky [15]	120±40	15±45	60±10	−77±25
	This paper				
$T_{\pi\pi}$	VPI	40	7		
	GW10	53±10	−1±10	54±5	−43±5
	GW11	57	1	54	−48
	GW20	43±5	−10±5	42±5	−32±5
	GW21	45	−5	42	−37
$T_{\eta\eta}$	VPI	41	−85		
	GW10	91±20	−53±10	8±5	122±10
	GW11	98	−48	11	127
	GW20	43±5	−120±5	6±10	14±15
	GW21	41	−121	8	15

data now being analyzed. From Table V it is also seen that, although there is a dependence of the pole positions on the two-pion branching, the differences—for the range of branchings considered—are essentially covered by the statistical errors on the positions.

In Table VI a corresponding comparison has been made for the moduli and phases of the residues of the $T_{\pi\pi}$ poles. This table also shows the moduli and phases for the two $T_{\eta\eta}$ poles. Again as a consistency check we confirm that the residues at the $T_{\pi\eta}$ poles are simply the square root of the $T_{\pi\pi}$ and $T_{\eta\eta}$ residues.

In addition to the above poles there is the possibility of having poles on other Riemann sheets—far from the physical region—that can be probed by systematically reversing the signs of q_π and q_η. These additional poles are quite symmetric—a point that can be understood in the limit where each K_{ij} is a single pole $\sqrt{\gamma_i\gamma_j}/(E_0-E)$. In this case the T matrix reduces to $T\propto[E_0-E-i\gamma_\pi q_\pi-i\gamma_\eta q_\eta]^{-1}$.

IV. CONCLUSIONS

In this paper we have extracted pole positions for the $N(1535)$ and $N(1650)$ resonances using two different models—the results being given in Table V. It is seen that the $N(1535)$ pole positions predicted by these two models agree within about 15 MeV, whereas some of the predictions of the earlier models [15–17] are considerably different. The $N(1650)$ pole values cannot be directly compared, as the most recent VPI fits have further poles and zeroes. However, if one compares to the 2 S11 resonance fit of Ref. [19], agreement with our $N(1650)$ values is much improved. The reasons for differences can be manifold: (a) the models used in the analysis are different, (b) different subsets of partial-wave amplitudes are fitted, (c) data versus amplitudes are fitted, (d) only certain data sets are fitted, e.g., only $\pi N\to\pi N$ or only $\pi N\to\pi N$ plus $\pi N\to\eta N$ etc., and (e) the energy ranges over which the data are fitted can differ. We explicitly considered one such possibility in our analysis, by including either the S11 πN partial-wave amplitude or the πN data. The $N(1535)$ pole position was found to be quite sensitive to this choice, shifting about 50 MeV higher if the partial-wave amplitudes were fitted. This sensitivity was also seen in the associated residues.

These various alternatives question the reliability of attempting to extrapolate into the complex energy plane the T matrix from a model that only fits a limited selection of data over a limited range of energies on the real energy axis. In view of this, it would be desirable to have quantitative estimates of the errors expected on these pole positions. Unfortunately, for those fits involving directly all of the $\pi N\to\pi N$, it is difficult to get a meaningful estimate of such errors. However, in the less ambitious approach of Ref. [11], only the $\pi N\to\pi N$ S11 amplitudes and their error bars were fitted, using the Minuit minimization procedure. This then gave error bars on the ten parameters defining the model, i.e., the E_i and γ_i in Eqs. (3),(4). Therefore, the errors δE_P and $\delta\Gamma_P/2$ on the pole positions $E_P-i\Gamma_P/2$ could be obtained by repeating the calculation a large number of times for a random selection of the nine parameters defining the K matrices of Eq. (1)—as discussed in Ref. [11]. This resulted in $\delta E_P(1535)\approx 10$ MeV, $\delta\Gamma_P/2(1535)\approx 10$ MeV, $\delta E_P(1650)\approx 5$ MeV, $\delta\Gamma_P/2(1650)\approx 5$ MeV—values that were not very dependent on the actual pole positions. Such estimates for δE_P, $\delta\Gamma_P/2$ are consistent with the spread of $E_P-i\Gamma_P/2$ values from the various fits GW1X and GW2X. They are also very close to the correlated error estimates listed in Table V. Furthermore, it is seen that the position of the first pole, as given in the VPI and GW models, is consistent within these errors. Therefore, if we were to quote a single ''best'' number for the pole positions involving only two poles, Tables V and VI suggest $E_P-i\Gamma_P/2(1535)=1510(10)-i85(15)$ and $E_P-i\Gamma_P/2(1650)=1660(10)-i70(10)$ and the corresponding residues $[|r|,\theta]$ for $T_{\pi\pi}$ being [50(10), 0(10)] and [45(10), –40(10)]. However, the residues for $T_{\eta\eta}$ depend strongly on the fit with the components for GW11 and GW21 differing by about a factor of 2 and with the VPI estimate being somewhat closer to that of GW21. Given these differences, we do not feel that an improved value for the $N(1535)$ photodecay amplitude can be determined from our fits.

In the above analysis the question of uniqueness arises. In the first model, the forms of Eqs. (3) and (4) are chosen with the physical idea in mind that there should be two basic resonances, which are compact in space (as in a quark model) and so may be expected to be well represented by a pole in the *K matrix* with a constant residue. Less compact objects would then need a form factor in place of the constant residue. This inclusion of explicit poles in the K matrix essentially guarantees poles in the T matrix in the vicinity of those in the K matrix. In the second of our models, poles in the K matrix can arise as a dynamical effect through coupling to high-lying closed channels as in Eq. (8). This alternative has also been discussed in Ref. [18], where the $N(1535)$ is treated as a $K\Lambda$ bound state. This type of ambiguity has a long history and has been discussed in most detail for the interpretation of the $\Lambda(1405)$—see Ref. [20]. However, as emphasized in Ref. [21], the truth is probably somewhere in between the two above possibilities, with both mechanisms playing a role. This seems to be supported in Ref. [22], where the authors conclude that the $N(1535)$ is not only generated by coupling to higher channels but ''appears to require a genuine three-quark component.'' In principle, with perfect data in all the relevant channels, the T matrix should be highly constrained, so that only one prescription would succeed. However, in practice, the data have error bars and only cover a limited range, so that both approaches could give a fit to some of the data, but yield different pole positions. As a next step in resolving this uncertainty, all of the available data in $\pi N\to\pi N$, $\pi N\to\eta N$, and $\eta N\to\eta N$ (from final-state interaction data in, for example, $\gamma p\to\eta p$) should be treated simultaneously and not simply the selections used above. Finally, we should note that our fits imply a value for the inelasticity due to $\pi\pi N$ channels. In comparing our solutions listed in Table V with the results of Ref. [23], we have found no serious disagreements over the fitted energy range.

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