Measurement of collectivity of collective flow in relativistic heavy-ion collisions using particle group correlations

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Based on a particle groups correlation function, a new type of inferring collectivity of collective flow is proposed in this paper. Using this method, the particle group correlations arising from collective flow are analyzed for collisions of 1.2*A* GeV Ar+BaI_2 and 2.1*A* GeV Ne+NaF in the Bevalac streamer chamber. The results have been compared with Monte Carlo simulation, which show that the collectivities are between 80 and 95 % for the experimental events in collisions of Ar+BaI_2, and between 75 and 95 % for the experimental events in collisions of Ne+NaF. [S0556-2813(98)01611-2]

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I. INTRODUCTION

The collective sideward flow produced in the relativistic heavy-ion collisions (''collective flow'') provided valuable information for the exploration of the dynamical mechanism of nucleus-nucleus collision [1]. In 1992, Jiang *et al.* proposed the concept of collectivity and inferred it for 0.4*A* GeV Ar+Pb collisions. Since then, the study of collective flow had some comparatively important developments. A number of experiments and models have indicated that the strength of collective flow reflects the quantitative property of collective flow and the collectivity of collective flow more deeply reflects the essential property of collective flow [2–7]. Strength and collectivity are two complementary aspects describing the collective flow and are closely related to anisotropic transverse motion [8,9]. Thus, the study on collectivity of collective flow is an important subject in the field of relativistic heavy-ion collisions.

Unlike other previous investigations of collectivity [9,10] using particle correlations, an analysis of collectivity is first discussed using particle group correlations in this paper. This new attempt is an uninvestigated research direction. The new method analyzes collectivity by means of a correlation function of N ($N>2$) particle groups, which is based on methods introduced by Jiang *et al.* [9], Danielewicz and Odyniec [11], but has significant developments added to them. On the one hand, this new method can fully utilize transverse momentum information of particles in the final state by using both the azimuthal angle (introduced by Jiang *et al.*) and the magnitude of transverse momentum. On the other hand, the experimental data analysis indicates that the correlation of two particle groups is not sensitive to the collectivity of collective flow, but the correlation of N ($N>2$) particle groups becomes more and more sensitive to the collectivity of collective flow as N increases in value. Thus, more difficulties arise when the inference of collectivity of collective flow is taken into account using the analysis introduced by Danielewicz and G. Odyniec [11] as well as by Beckmann *et al.* and others [12,13].

Much is known about particle correlations [1–11], but little is known about particle group correlations. Two questions must be raised. First, how can one study particle group correlations quantitatively? Secondly, how do these correlations provide the possibility of conducting any analysis of strength and collectivity of collective flow? Therefore, to answer these questions a more intuitive approach follows in which phenomenological simulations are used to aid analysis of the experimental results.

The goal of this paper is to study the properties of correlation among particle groups and to seek new analyses, which can improve the measurement accuracy of collectivity. In Sec. II, the information of experimental data is briefly introduced and the correlation function of N particle groups is defined. In Sec. III, N particle group correlations in the experimental events are analyzed. In Sec. IV, the collectivity is inferred by comparing experimental results with Monte Carlo simulations. Finally, the conclusion is given.

II. EXPERIMENT AND THE CORRELATION FUNCTION OF *N* PARTICLE GROUPS

The experimental data samples for this investigation come from the two Bevalac streamer chamber 4 π experiments, in which the systems 1.2*A* GeV Ar+BaI_2 and 2.1*A* GeV Ne+NaF were studied. A description in more detail can be found in [14,15]. These are 786 events with multiplicity $M \geqslant 30$ for Ar+BaI_2 and 2707 events with multiplicity $M \geqslant 13$ for Ne+NaF. Assuming a simple geometrical picture, the impact parameters are between 0 and 6.0 fm for Ar+BaI_2 and 0 and 5.0 fm for Ne+NaF. To avoid the effect of experimental factors such as particle misidentification, and absorption and energy loss in the target [15], the polar angles on the experimental data have been cut, $\theta_{\mathrm{lab}} \geqslant 8°$ [14] for Ar+BaI_2 and $\theta_{\mathrm{lab}} \geqslant 4°$ [15] for Ne+NaF. The corresponding ranges of multiplicity M' after cuts are between 16 and 53 for Ar+BaI_2 and 4 and 23 for Ne+NaF, and the average values $\langle M' \rangle$ of multiplicity are 24 for Ar+BaI_2 and 16 for Ne+NaF.

Assuming the multiplicity of the collision event is M, and then this event is randomly and equally divided into N particle groups as far as possible, each particle group is a subset Ω_i in this collision event, where $i=1,2,...,N$. The vector $\mathbf{Q}_i$ of each subset Ω_i is formed by using the transverse momentum $\mathbf{p}_j^t$ for the jth particle of the ith particle group in the event.

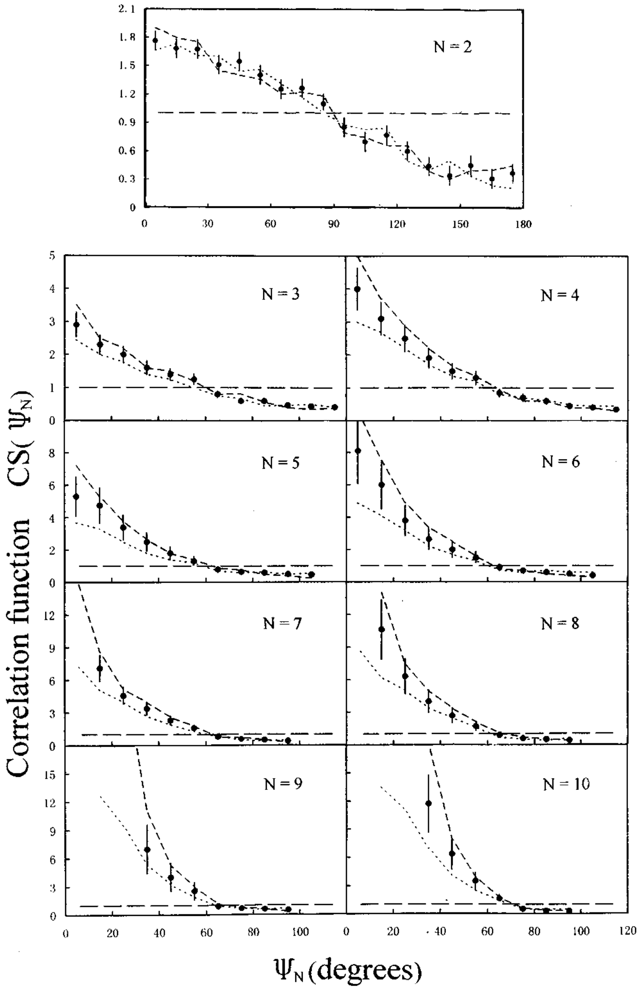

FIG. 1. The correlation function of particle groups $CS(\psi_N)$ while $N=2-10$ for Ar+BaI$_2$. The experimental data are indicated with error bars (points), the Monte Carlo data of simulated collective flow with a dotted line ($\alpha=80\%$) and a dashed line ($\alpha=95\%$).

$$\mathbf{Q}_i=\sum_{j=1}^{M/N} \omega_j \mathbf{p}_j^t, \quad (i=1,2...,N), \tag{1}$$

where $\omega_j=0$ for pions. For the baryons, $\omega_j=1$ for rapidity $y_{\rm lab}>y_{\rm c.m.}+\delta$, $\omega_j=-1$ for rapidity $y_{\rm lab}<y_{\rm c.m.}-\delta$, and $\omega_j=0$ otherwise. Here $y_{\rm c.m.}$ is the center-of-mass rapidity of the system. The number of M/N is made to be an integer that is as large as possible. The quantity δ is inserted to remove particles adjacent to the center-of-mass rapidity so as to avoid any effect on the result of collective flow analysis. The value of δ is 0.2 in this paper, which approximately excludes 20% of detected nuclear fragments for Ar+BaI$_2$ and 23% for Ne+NaF. The magnitude and direction of the vector $\mathbf{Q}_i$ reflect the extent to which the transverse momentum distribution of corresponding particle groups preferentially concentrate at a certain direction.

Now that N $\mathbf{Q}_i$ vectors correspond to N azimuthal $\phi(\mathbf{Q}_i)$, then we define the correlative angle variable ψ_N for N particle groups as

$$\psi_N=\left(\prod^{K} \Delta\phi_{ij}\right)^{1/K}, \quad K=N(N-1)/2, \tag{2}$$

where $\Delta\phi_{ij}$ is the smaller azimuthal separation between vectors Q_i and Q_j formed by arbitrary two-particle groups in the same collision event, and $\Delta\phi_{ij}=\arccos[(\mathbf{Q}_i\cdot\mathbf{Q}_j)/(|\mathbf{Q}_i|\cdot|\mathbf{Q}_j|)]$, $0\leqslant\psi_N\leqslant\pi$. The product runs over all K azimuthal separations $\Delta\phi_{ij}$ formed from the N particle groups.

The number of N particle group subevents constructed from each event of multiplicity M is $g=M!/(M—n)!n!$. Because the experimental sample contains a wide range of multiplicity, events with higher M completely swamp those with lower M at large N when all the subevents have equal weight. To compensate for this, using the method given by Jiang *et al.* [9,10], all the contributions for the subevents of N particle groups are weighted by M/g, thus ensuring that each event makes a contribution to the final result that is proportional to the multiplicity M of the event.

Adapting the approach of the azimuthal correlation function, we define the correlation function of N particle groups as

$$CS(\psi_N)=\frac{PS(\psi_N)}{PSM(\psi_N)}, \tag{3}$$

where $PS(\psi_N)$ and $PSM(\psi_N)$ are the distribution probabilities of N particle groups with ψ_N for the experimental and Monte Carlo events, respectively. Monte Carlo events are generated [10,11,14,15] by randomly mixing particles from different experimental events within the same multiplicity range such that there should not be particle group correlations arising from the collective flow in such events. Thus, we can extract information about N particle group correlations through studying $CS(\psi_N)$. The correlation function of N particle groups is the ratio of the weighted number of correlated two-particle groups to uncorrelated two-particle groups within the same bin of azimuthal difference ψ_N.

Known from the definition of the variable ψ_N, ψ_N contains more information, and it is related to the azimuthal angles and the magnitudes of the transverse momentum of N particles, and also considers the forward and backward emitted fragments in the center-of-mass system of the fireball. So, the correlation function $C(\psi_N)$ introduced by Jiang *et al.* [9] is generalized to the correlation function $CS(\psi_N)$. The values of $CS(\psi_N)$ reflect the correlative extent among N particle groups. Thus, through having studied the distribution of the correlation function $CS(\psi_N)$ in the range $0\leqslant\psi_N\leqslant\pi$ as a whole, the particle group correlations can be quantitatively analyzed.

III. ANALYSIS OF THE PARTICLE GROUP CORRELATIONS FOR THE EXPERIMENTAL EVENTS

Figures 1 and 2 show the variation of $CS(\psi_N)$ with ψ_N for the two experimental data sets, respectively, while $N=2-10$. Points with error bars represent the experimental data. Through having studied the distribution of the correlation function $CS(\psi_N)$ in the range $0\leqslant\psi_N\leqslant\pi$, not only the particle group correlations but also the strength and collectivity of collective flow can be quantitatively analyzed. Apparently, the fact that the value of the correlation function $CS(\psi_N)$ is bigger in the minor ψ_N range, whereas the value is smaller in the major ψ_N range, indicates the particle group correlations exist among the particles in the final state. Spe-

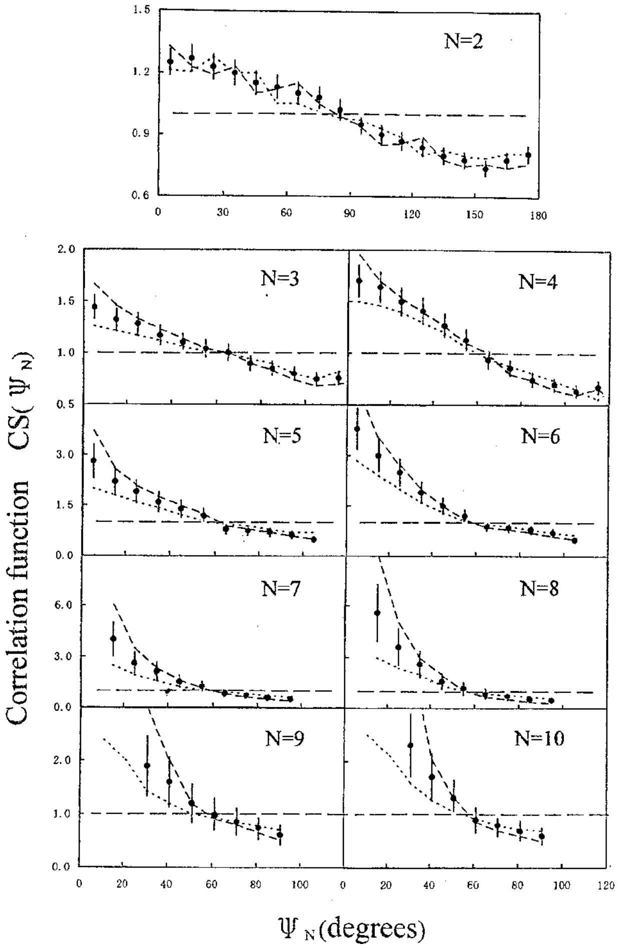

FIG. 2. The correlation function of particle groups $CS(\psi_N)$ while $N=2-10$ for Ne+NaF. The experimental data are indicated with error bars (points), the Monte Carlo data of simulated collective flow with a dotted line ($\alpha=75\%$) and a dashed line ($\alpha=95\%$).

cifically, the distribution of the correlation function $CS(\psi_N)$ in the range $0\leqslant\psi_N\leqslant\pi/2$ reflects more clearly the correlation of the particle groups than that in the whole range. With the increase of the N value, the differential value between $CS(\psi_N)$ and 1 increase rapidly. Therefore, the correlation strength among particle groups can be measured quantitatively by means of the formula $|CS(\psi_N)-1|$. The distribution of $CS(\psi_N)$ includes all cumulative effects of correlations formed from the N particle groups. Affected by the statistics of experimental events, the $CS(\psi_N)$ values err by 80%, while $N=7-8$ in the range $0\leqslant\psi_N\leqslant\pi/12$ and $N=9-10$ in the range $0\leqslant\psi_N\leqslant\pi/6$ for the two experimental data sets. $N=10$ is the maximum value permissible in the two experimental data analyses. Therefore, in Figs. 1 and 2, we removed the experimental data points of these corresponding ranges for $N=7-8$ in the range $0\leqslant\psi_N\leqslant\pi/12$ and $N=9-10$ in the range $0\leqslant\psi_N\leqslant\pi/6$.

When $N=2$, the analysis above is similar to the transverse momentum analysis introduced by Danielewicz and Odyniec [11]. Similarly, Beckmann *et al.* [12,13] divided the particles of each event into two-particle groups—one with forward emitted fragments and the other with backward emitted fragments—in the center-of-mass system, and studied the correlation among particle groups using vector $\mathbf{W}_s$ (see, for detail, [12,13]). The analysis mentioned above [11–13] could be used to analyze the strength of collective flow in the experimental events, but could not be used to infer the collectivity of collective flow yet. The reason is that the variables selected include the summation of the transverse momentum of all particles, and only the correlation between two-particle groups in the event is considered, so, results in the information of events could not be fully utilized. Besides, experimental data analyses indicate that the analysis introduced by Liu *et al.* (their analysis considered only the forward emitted fragments [10]) as well as by Jiang *et al.* (their analysis did not consider the magnitude of particle transverse momentum [9]) is not more sensitive to the collectivity of collective flow than the new method.

Next, we will infer the collectivity in the collective flow for experimental events by comparison with Monte Carlo simulation.

IV. INFERENCE OF THE COLLECTIVITY OF COLLECTIVE FLOW FOR EXPERIMENTAL EVENTS

In order to reveal the number of particles participating in the collective directed motion, the collectivity parameter α can be defined as

$$\alpha=\left(\frac{M_1}{M}\right)\times 100\%, \tag{4}$$

where M is the multiplicity in the event and M_1 is the number of particles participating in the collective directed motion. In this paper, the collectivity of the experimental sample is phenomenologically inferred through comparison of the correlation function $CS(\psi_N)$ of N particle groups for the Monte Carlo simulation with that for experimental data. The collective order N is between 2 and 10. The following explains exactly what to do. The first step is to generate a cascade sample of noncollective flow.

(1) A cascade sample is generated with the impact parameter $b=0$ fm and then selected by use of the cascade model [5]. In these events, there are no explicit effects of collective flow because this model does not explicitly incorporate the effects of nuclear equation of state.

(2) The polar angles cut mentioned in the above experimental events are applied to this cascade sample. It has been verified that the distribution of multiplicity M^*, of inclusive transverse momentum and of the rapidity after the cut conforms to the experimental data.

(3) Corresponding to experimental events, the impact parameter b of each cascade sample can be randomly redefined in the range 0–6 fm for Ar+BaI_2 and of 0–5 fm for Ne+NaF, respectively. A cascade sample with statistics five times that of the experimental sample is generated.

The second step is to confirm the flow parameter f_0. For every event of multiplicity M^*, a component of collective directed motion $f_0 p_i^{\mathrm{flow}}$ is added to the projection of the transverse momentum on the reaction plane [9,10] for each fragment, where f_0 is a free parameter that controls the strength of the flow, i is the order sign of the fragment and $i\in[16,53]$ for Ar+BaI_2 and $i\in[4,23]$ for Ne+NaF, respectively. p_i^{flow} is expressed as [16]

$$p_i^{\mathrm{flow}}=A_i B_i u_i |\cos(\phi_i)|, \tag{5}$$

where A_i is the mass number of the ith fragment. B_i is defined as

$$B_i = \sin^{2/3}\left(\pi \frac{b}{b_{\max}}\right), \tag{6}$$

where b is the impact parameter, $b_{\max}=R_P+R_T$, R_P and R_T are the radii of the projectile and the target, respectively, u_i is the ratio of the ith fragment rapidity and the projectile rapidity in the center-of-mass, ϕ_i is the angle of the transverse momentum vector relative to the reaction plane. Equation (5) phenomenologically describes the dependence of transverse collective flow on fragment mass, rapidity, and azimuthal angle relative to the reaction plane, and the impact parameter. This prescription is certainly not uniquely constrained by them, but it maintains momentum and energy conservation for each event, and it is approximately consistent with the available experimental data [9,10,15,16].

Adjusting the parameter f_0 to reproduce the experimental events for the distribution of the function correlation of two-particle groups $CS(\psi_2)$ or the average value of transverse momentum in the reaction plane as a function of the rapidity $\langle p_\perp^x \rangle(y)$ [11], f_0 is found to be equal to 350 MeV for Ar+BaI$_2$ and 200 MeV for Ne+NaF, respectively, in this paper.

The third step is to produce a Monte Carlo sample of the collective flow and inference the collectivity α.

(1) A Monte Carlo sample with collectivity α and the flow parameter f_0 is obtained as follows. M_1^* fragments are randomly selected from the cascade sample of multiplicity M^* obtained in the first step (3), and then a component of collective directed motion $f_0 p_i^{\text{flow}}/\alpha$ is added to the projection of the transverse momentum on the reaction plane [9,10] for each fragment [following Eq. (4), $\alpha = M_1^*/M^*$]. Adding the collective directed motion component in this way can ensure that the distributions of $CS(\psi_2)$ and $\langle p_\perp^x \rangle(y)$ in the Monte Carlo sample are consistent with that of experimental data sample as α is changed.

(2) The parameter f_0 is kept unchanged and the parameter α is adjusted. If the correlation function of particle groups $CS(\psi_N)$ for the Monte Carlo sample is consistent with the experimental data sample in the range of the present experimental precision, then the value of α will be the collectivity of particles in the final state for these experiments. The quantitative measurements of collectivity are specified by the range of fractions α which results in an acceptable overall agreement with experimental sample for $N=3-10$ in Figs. 1 and 2.

In Fig. 1, the dashed line and the dotted line represent the Monte Carlo data for $\alpha=80\%$ and $\alpha=95\%$, respectively. Similarly, in Fig. 2, the dashed line and the dotted line represent the Monte Carlo data for $\alpha=75\%$ and $\alpha=95\%$, respectively. The errors of the dashed line and dotted line are approximately half of that of the experimental points. Analysis shows that α value ranges for experimental events are between 80 and 95 % for Ar+BaI$_2$ and between 75 and 95 % for Ne+NaF, respectively.

Further, using the methods introduced in Refs. [9, 10], the N-particle azimuthal correlation function $C(\psi_N)$ [9] and the N-particle transverse correlation function $F(V_N)$ [10] of the Monte Carlo simulation are compared with those of the experimental data, which shows the collectivity α is not less than 80% for Ar+BaI$_2$ and not less than 75% for Ne+NaF within the range of estimated error, respectively.

V. CONCLUSIONS

The correlation exists not only among particles but also among particle groups in the relativistic heavy-ion collisions. A new measurement of particle group correlations is proposed in this paper. This method is very sensitive to the collectivity in heavy-ion collisions. Using the new method, the particle group correlations arising from collective flow are studied with 4π data for 1.2A GeV Ar+BaI$_2$ and 2.1A GeV Ne+NaF collisions at the Bevalac streamer chamber. Comparing with Monte Carlo results, the collectivities of collective flow have been inferred to be between 80 and 95 % for the experimental events of Ar+BaI$_2$ collisions, and between 75 and 95 % for the experimental events of Ne+NaF collisions. The analysis indicates that the correlation function $CS(\psi_N)$ in this paper is sensitive to the collectivity and the strength of the collective flow. The new method provides an efficient approach to measure quantitatively the collectivity of collective flow with more constraint in the same data statistics and to further investigate the essentials of collective flow. Because the collectivity reflects the essential property of collective flow, it has become an important subject with which to seek new methods that are sensitive to collectivity [15] in the field of relativistic heavy-ion collisions.

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[1] P. A. DeYoung *et al.*, Phys. Rev. C **56**, 244 (1997).
[2] Henning Heiselberg and Axel P. Vischer, Phys. Rev. C **55**, 874 (1997).
[3] J. Barrette *et al.*, Phys. Rev. C **55**, 1420 (1997).
[4] S. Wang *et al.*, Phys. Rev. Lett. **76**, 3911 (1996).
[5] David E. Kahana, Declan Keane, Yang Pang, Tom Schlagel, and Shan Wang, Phys. Rev. Lett. **74**, 4404 (1995).
[6] J. P. Omtvedt, H. Mach, B. Fogelberg, D. Jerrestam, M. Hellström, L. Spanier, K. I. Erokhina, and V. I. Isakov, Phys. Rev. Lett. **75**, 3090 (1995).
[7] A. A. Chishti, P. Chowdhury, D. J. Blumenthal, P. J. Ennis, C. J. Lister, Ch. Winter, D. Vretenar, G. Bonsignori, and M. Savoia, Phys. Rev. C **48**, 2607 (1993).
[8] Liu Yiming, Zhang Weigang, Huo Lei, Wang Shan, D. Keane, S. Y. Chu, and S. Y. Fung, High Energy Phys. Nucl. Phys. **18**, 167 (1994).
[9] J. Jiang *et al.*, Phys. Rev. Lett. **68**, 2739 (1992).
[10] Liu Qingjun, Jiang Yuzhen, Wang Shan, Liu Yiming, J. Jiang, D. Keane, Y. Shao, S. Y. Fung, and S. Y. Chu, High Energy Phys. Nucl. Phys. **17**, 12 (1993).

[11] P. Danielewicz and G. Odyniec, Phys. Lett. **157B**, 146 (1985).
[12] P. Beckmann, H.-Å. Gustafsson, H. H. Gutbrod, K. H. Kampert, B. K. Olb, H. Lohner, A. M. Poskanzer, H. G. Ritter, H. R. Schmidt, and T. Siemiarzuk (Darstadt, GSI), 1987, PERTR. GSI87–09 (87, REC. FEB.) 10p.
[13] S. M. Kiselev, Phys. Lett. B **216**, 262 (1989).
[14] S. Wang, Y. Z. Jiang, Y. M. Liu, D. Keane, D. Beavis, S. Y. Chu, S. Y. Fung, M. Vient, C. Hartnack, and H. Stöcker, Phys. Rev. C **44**, 1091 (1991).
[15] Zhang Weigang, High Energy Phys. Nucl. Phys. **21**, 65 (1997).
[16] A. F. Barghouty, G. Fai, and D. Keane, Nucl. Phys. **A535**, 715 (1991).