Double-Λ hypernuclei in the Skyrme-Hartree-Fock approach and nuclear core polarization

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Extension of the Skyrme-Hartree-Fock approach to double-Λ hypernuclei is presented. Several Skyrme-like ΛΛ potentials are fitted to the binding energy of ${}^{13}_{\Lambda\Lambda}$B. So-called ΛΛ bond energy $\Delta B_{\Lambda\Lambda}$ appears generally from a complicated interplay between the ΛΛ potential, the nuclear core polarization, and Λ spatial distributions. The core polarization gives a positive contribution to the bond energy even in the absence of ΛΛ interaction. This contribution can be substantial even in heavy double-Λ hypernuclei. The greatest uncertainties in extraction of the ΛΛ potential from empirical data arise if hyperons contract the core, a ΛΛ potential is of a short range, and the nuclear incompressibility is small. [S0556-2813(98)02912-4]

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I. INTRODUCTION

After first observations of ΛΛ hypernuclei in the 1960s [1], great attention has been attracted to this subject up to now. The most striking feature of these systems is the unique possibility to study hyperon-hyperon interaction.

Only 3 species (${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B) have been identified [1,2]. Nevertheless, numerous theoretical studies have been made. The majority of them were addressed to the species measured experimentally. Three-body $\alpha+\Lambda+\Lambda$ variational schemes were applied most usually to ${}^{6}_{\Lambda\Lambda}$He (e.g., [3–5]) since the polarization (i.e., distortion by the hyperons) of the nuclear core ^{4}He is probably small [6]. Otherwise, the polarization of the loose core in ${}^{10}_{\Lambda\Lambda}$Be is evidently significant [7], and this hypernucleus can be treated in four-body $\alpha+\alpha+\Lambda+\Lambda$ models, while the α particles preserve their entities [7,5,8]. In some sense, these species exemplify two extreme cases: small (${}^{6}_{\Lambda\Lambda}$He) and large (${}^{10}_{\Lambda\Lambda}$Be) core polarizations. The third species (${}^{13}_{\Lambda\Lambda}$B), discovered much more recently [2], was treated in a three-body ^{11}B$+\Lambda+\Lambda$ model [9].

It was understood that extraction of the ΛΛ potential from the experimental binding energies is generally a rather complicated problem. Particularly, ΛΛ dynamics in a double-Λ hypernucleus is dictated strongly by Λ-core potential properties [10]. Even Λ-core potentials fitted equally to the binding energy of a ${}^{A+1}_{\Lambda}$Z hypernucleus can lead to clearly different results for the ${}^{A+2}_{\Lambda\Lambda}$Z hypernucleus. It is driven mainly by radii of Λ orbits, and, therefore, by shapes of Λ-core potentials. For instance, a Λ-core potential with a central repulsion generates spatially expanded Λ distributions and leads to a less ΛΛ attraction energy than that for a purely attractive Λ-core potential [10,11]. This ambiguity is retained also for heavy double-Λ hypernuclei [12].

In this paper, we present an extension of the well-known Skyrme-Hartree-Fock approach to double-Λ hypernuclei. Being a common tool in nuclear physics, this approach is widely used also for single-Λ hypernuclei. It is rather simple but feasible enough to incorporate a complicated structure of *NN*, Λ*N*, and ΛΛ effective interactions in hypernuclei. Whereas few-body models are obviously more appropriate for the lightest systems, the Skyrme-Hartree-Fock approach is suitable for studying of average properties of arbitrary (not too light) hypernuclei.

In the earlier stage, Hartree-Fock techniques were used only once for ${}^{6}_{\Lambda\Lambda}$He [13]. Recently, Hartree-Fock calculations for ${}^{13}_{\Lambda\Lambda}$B with finite-range density-dependent ΛΛ potentials were made [14]. The present approach has been applied to the study of binding energies of heavy ΛΛ hypernuclei [12].

Various phenomenological as well as meson-exchange motivated forms were employed for the ΛΛ interaction in earlier studies. Based on the data available, some selection between Nijmegen potential models was made [15,16]. It was shown also that coupling of the ΛΛ and ΞN channels is significant. Therefore, ΛΛ interaction in hypernuclei can differ from the free interaction and be generally density-dependent [15,16,14]. Moreover, hypernuclear ΛΛ interaction may be different in principle in different hypernuclei (e.g., an anomalously weak $\Lambda\Lambda-\Xi N$ coupling occurs in ${}^{6}_{\Lambda\Lambda}$He, so the ΛΛ interaction is damped) [17]. Otherwise, an especially strong $\Lambda\Lambda-\Xi N$ coupling is anticipated in ${}^{5}_{\Lambda\Lambda}$H [18].

However, we adopt here a purely phenomenological strategy. Since data on double-Λ hypernuclei are still scarce, it is questionable now to consider ΛΛ potential in detail. Instead, we examine several simplified Skyrme-like ΛΛ potentials, which are derived without any microscopic base, and combine them with various empirical Λ*N* and *NN* potentials. Our aim is not to deduce the true ΛΛ potential, but rather to study the interplay between average potential properties and hypernuclear properties. We try also to understand uncertainties in the ΛΛ potential extraction from data, which arise from Λ*N* and *NN* potential ambiguities. Particularly, we emphasize possible implications of the nuclear core polarization by the hyperons.

In Sec. II, we present the extension of the Skyrme-Hartree-Fock formalism to double-Λ (and also to multi-Λ) hypernuclei. A simplified treatment of implications of core polarization to binding energies of double-Λ hypernuclei is the subject of Sec. III. Parameters for the ΛΛ potential are derived in Sec. IV. In Sec. V, some illustrative calculations for ${}^{13}_{\Lambda\Lambda}$B as well as for heavier hypernuclei are discussed. A

brief conclusion is presented in Sec. VI. Preliminary, partial results of the presented study were reported in [19].

II. FORMALISM

We employ the $\Lambda\Lambda$ potential in the usual Skyrme-type form:

$$V_{\Lambda\Lambda}=\lambda_0\delta(\mathbf{r}_1-\mathbf{r}_2)+\tfrac{1}{2}\lambda_1[\mathbf{k}'^2\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}^2]+\lambda_2\mathbf{k}'\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}+\lambda_3\delta(\mathbf{r}_1-\mathbf{r}_2)\rho_N^{\alpha}\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right), \quad (1)$$

where ρ_N is the nucleon density; other notations are standard [20,21].

The formal extension of the Skyrme-Hartree-Fock approach [20,21] to multi-Λ systems is more or less straightforward. We present it in reference to the related formalism for single-Λ hypernuclei of Rayet [21], describing only additional terms, which appear due to $\Lambda\Lambda$ interaction.

The Hamiltonian density for a triple-even ${}^{A+n}_{n\Lambda}$Z system is

$$H=H_{S\Lambda}+H_{\Lambda\Lambda}, \quad (2)$$

where $H_{S\Lambda}$ is formally the same as the Hamiltonian density of the ${}^{A+1}_{\Lambda}$Z hypernucleus [21], and

$$H_{\Lambda\Lambda}=\frac{1}{4}\lambda_0\rho_\Lambda^2+\frac{1}{8}(\lambda_1+3\lambda_2)\rho_\Lambda\tau_\Lambda+\frac{3}{32}(\lambda_2-\lambda_1)\rho_\Lambda\nabla^2\rho_\Lambda+\frac{1}{4}\lambda_3\rho_\Lambda^2\rho_N^{\alpha}. \quad (3)$$

Here ρ_Λ is the hyperon density and τ_Λ is the corresponding kinetic energy density. Following Ref. [20] and the latter common practice, we omit a term incorporating the Λ spin density in Eq. (3).

The Hartree-Fock equations for baryon single-particle radial wave functions $R_\gamma(r)$ in a spherical system are

$$\frac{\hbar^2}{2m_B^*}\left[-R_\gamma''+\frac{l(l+1)}{r^2}R_\gamma\right]-\left(\frac{\hbar^2}{2m_B^*}\right)'R_\gamma'+U_BR_\gamma=e_\gamma R_\gamma, \quad (4)$$

where primes mean differentiation with respect to the radial coordinate. State label γ denotes the nlj numbers and also the type of baryon B. Single-particle potentials U_B and Λ effective mass m_Λ^* acquire additional terms as follows:

$$U_\Lambda=(U_\Lambda)_{S\Lambda}+\frac{1}{2}\lambda_0\rho_\Lambda+\frac{1}{8}(\lambda_1+3\lambda_2)\left(\frac{\rho_\Lambda'}{r}+\tau_\Lambda\right)+\frac{3}{16}(\lambda_2-\lambda_1)\left(\rho_\Lambda''+\frac{2\rho_\Lambda'}{r}\right)+\frac{1}{2}\lambda_3\rho_\Lambda\rho_N^{\alpha}, \quad (5)$$

$$U_{p,n}=(U_{p,n})_{S\Lambda}+\frac{\alpha}{4}\lambda_3\rho_\Lambda^2\rho_N^{\alpha-1}, \quad (6)$$

$$\frac{\hbar^2}{2m_\Lambda^*}=\left(\frac{\hbar^2}{2m_\Lambda^*}\right)_{S\Lambda}+\frac{1}{8}(\lambda_1+3\lambda_2)\rho_\Lambda, \quad (7)$$

where the subscript $S\Lambda$ again refers to the same expressions as for a single-Λ hypernucleus. The nucleon effective masses remain formally unchanged.

Equations (4)–(7) are valid, strictly speaking, for triply closed shell hypernuclei. However, such an approach is commonly applied also for unclosed shells by means of the so-called spherical (or filling) approximation. In this approximation, an unclosed shell (nlj) with N baryons is replaced by a closed shell with the states $(nljm)$ occupied with occupation number $N/(2j+1)$ for each m. In the calculations, we restrict ourselves to the ground states of double-Λ hypernuclei, so the Λ shell is actually closed. However, the spherical approximation will be used for nucleonic shells.

For the ground states of double-Λ hypernuclei, the contribution of the p-wave interaction amplitude λ_2 vanishes exactly. However, the related terms are retained above for generality.

One can also introduce a three-body $\Lambda\Lambda N$ force:

$$V_{\Lambda\Lambda N}=\lambda_3\delta(\mathbf{r}_1-\mathbf{r}_N)\,\delta(\mathbf{r}_2-\mathbf{r}_N) \quad (8)$$

instead of the last (density-dependent) term in Eq. (1). The force (8) is equivalent to the density-dependent force from Eq. (1) at $\alpha=1$, i.e., it gives the same energy density and Hartree-Fock equations.

Density-dependent terms at $\alpha\leqslant 1$ are usual in nuclear Skyrme-Hartree-Fock calculations. The possible physical meaning of the density dependence in the $\Lambda\Lambda$ case may originate from the $\Lambda\Lambda-\Xi N$ coupling [15,16]. However, it should be noted that some pathological features may arise in general at $\alpha<1$, in contrast to similar NN and ΛN density-dependent interactions. It is seen from Eq. (6) that the additional term in the single-nucleon potential, proportional to $\rho_N^{\alpha-1}$, may diverge outside a nucleus at $\alpha<1$ if ρ_Λ^2 falls too slowly, so a confining nucleon potential may appear. However, physically it is clear that at low nucleon densities, the $\Lambda\Lambda$ potential must be linear in the density. Forms with $\alpha<1$ may be applied to the ground states as well as to majority of the excited states since ρ_Λ^2 falls more rapidly than $\rho_N^{\alpha-1}$ grows. The pathology can be actual for states with both Λ's on near-threshold levels. This case requires a special treatment of the density-dependent $\Lambda\Lambda$ force.

The c.m. energy is eliminated in the standard approximate way [20,21]. The diagonal part of the c.m. kinetic energy

$$T_{cm}=\sum_{i=1}^{A+n}\frac{\mathbf{p}_i^2}{2(Am_N+nm_\Lambda)} \quad (9)$$

is subtracted from the total energy, whereas the nondiagonal part, including scalar products of single-particle momenta $\mathbf{p}_i$, is neglected. Notice that the additional nondiagonal contributions of Λ's vanish exactly in the ground states of double-Λ hypernuclei.

The main quantity in $\Lambda\Lambda$ hypernuclei is the $\Lambda\Lambda$ bond energy

$$\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2B_\Lambda, \quad (10)$$

where $B_{\Lambda\Lambda}$ is the separation energy of the hyperon pair from a ${}^{A+2}_{\Lambda\Lambda}$Z hypernucleus and B_Λ is the hyperon separation energy from the ${}^{A+1}_{\Lambda}$Z one. Since the bond energy is expressed

in terms of total binding energies B as $\Delta B_{\Lambda\Lambda}=B(^{A+2}_{\Lambda\Lambda}\mathrm{Z})+B(^{A}\mathrm{Z})-2B(^{A+1}_{\Lambda}\mathrm{Z})$, possible uncertainties in the c.m. treatment are mainly canceled in the bond energy. A special attention to the c.m. correction is needed, however, when the core polarization energies are considered (Sec. III).

III. CORE POLARIZATION AND THE ΛΛ BOND ENERGY

Let us first consider the interplay between the ΛΛ bond energy and the nuclear core polarization in the framework of an oversimplified model similar to those considered by many authors (e.g., [22–25] for single-Λ hypernuclei and [26] for double-Λ ones).

We start from a single-Λ $^{A+1}_{\Lambda}\mathrm{Z}$ hypernucleus with the total energy $E_1=E_N+E_\Lambda$, where E_N is determined by the Hamiltonian of the nucleonic fraction only, and E_Λ includes Λ kinetic energy and Λ-nucleon interaction. We suppose that both E_N and E_Λ depend on the single parameter R, which can be attributed to the core radius. Then the condition $dE_N/dR(R_0)=0$ gives the radius R_0 of the $^{A}\mathrm{Z}$ nucleus. Treating the core distortion by Λ perturbatively, we have

$$\frac{dE_1}{dR}(R_1)\simeq\frac{d^2E_N}{dR^2}(R_0)\cdot(R_1-R_0)+\frac{dE_\Lambda}{dR}(R_0)=0. \quad (11)$$

Equation (11) gives the core radius R_1 and then a measure of the core distorsion δR_1:

$$\delta R_1\equiv\frac{R_1-R_0}{R_0}=-\frac{K_\Lambda}{\kappa_A}, \quad (12)$$

$$K_\Lambda=R_0\frac{dE_\Lambda}{dR}(R_0), \quad (13)$$

$$\kappa_A=R_0^2\frac{d^2E_N}{dR^2}(R_0). \quad (14)$$

The hypernuclear energy now is

$$\begin{aligned}E_1(R_1)&=E_N(R_0)+\frac{1}{2}\kappa_A\delta R_1^2+E_\Lambda(R_0)+K_\Lambda\delta R_1\\&=E_N(R_0)+\frac{1}{2}\frac{K_\Lambda^2}{\kappa_A}+E_\Lambda(R_0)-\frac{K_\Lambda^2}{\kappa_A}\\&=E_1(R_0)-\frac{1}{2}\frac{K_\Lambda^2}{\kappa_A}.\end{aligned} \quad (15)$$

Evidently, the core polarization energy $E_1^*=K_\Lambda^2/2\kappa_A$, while $B_\Lambda=B_\Lambda^0+E_1^*$, where $B_\Lambda^0=-E_\Lambda(R_0)$ is the Λ binding energy with respect to the rigid core.

Moving to the energy E_2 of the $^{A+2}_{\Lambda\Lambda}\mathrm{Z}$ hypernucleus in the ground state, one has $E_2(R)=E_N(R)+2E_\Lambda(R)+E_{\Lambda\Lambda}(R)$, where the last term represents the contribution of ΛΛ interaction. Assuming the dependence of $E_{\Lambda\Lambda}$ on R as too weak to change R significantly and deriving the core radius R_2, we obtain

$$\delta R_2\equiv\frac{R_2-R_0}{R_0}=-\frac{2K_\Lambda}{\kappa_A}=2\,\delta R_1, \quad (16)$$

$$E_2(R_2)=E_N(R_0)+2E_\Lambda(R_0)-\frac{2K_\Lambda^2}{\kappa_A}+E_{\Lambda\Lambda}(R_2), \quad (17)$$

$$E_2^*=4E_1^*, \quad (18)$$

where R_2 is the core radius and E_2^* is the core polarization energy in the double-Λ hypernucleus.

From Eq. (17),

$$B_{\Lambda\Lambda}=-2E_\Lambda(R_0)-E_{\Lambda\Lambda}(R_2)+\frac{2K_\Lambda^2}{\kappa_A}, \quad (19)$$

and

$$\begin{aligned}\Delta B_{\Lambda\Lambda}&=-E_{\Lambda\Lambda}(R_2)+\frac{K_\Lambda^2}{\kappa_A}\\&=-E_{\Lambda\Lambda}(R_2)+\frac{E_2^*}{2}\\&=-E_{\Lambda\Lambda}(R_2)+2E_1^*.\end{aligned} \quad (20)$$

Here $E_{\Lambda\Lambda}(R_2)=E_{\Lambda\Lambda}(R_0)+\delta E_{\Lambda\Lambda}$ includes the rigid-core contribution $E_{\Lambda\Lambda}(R_0)$ and a correction $\delta E_{\Lambda\Lambda}$ vanishing either in a rigid-core approximation or when ΛΛ interaction is zero. A similar decomposition of the bond energy has been suggested by Bodmer and Ali [7]. We relate here the polarization energies in the single-Λ and double-Λ hypernuclei and emphasize that the second terms in Eqs. (20) give a positive contribution to $\Delta B_{\Lambda\Lambda}$ independently of the ΛΛ interaction. Even in the absence of ΛΛ interaction, $\Delta B_{\Lambda\Lambda}>0$. However, this contribution may be small, if the polarization is weak.

Note that Eq. (18) contradicts to a naive guess $E_2^*=2E_1^*$ suggested in Ref. [27]. This suggestion implies in a nearly zero core polarization contribution to $\Delta B_{\Lambda\Lambda}$, contrary to Eq. (20).

Considering the bond energy in the rigid-core approximation, it is possible to introduce two quantities alternatively. First, it is

$$\Delta B^0_{\Lambda\Lambda}=B^0_{\Lambda\Lambda}-2B_\Lambda, \quad (21)$$

where $B^0_{\Lambda\Lambda}=-2E_\Lambda(R_0)-E_{\Lambda\Lambda}(R_0)$. Secondly,

$$\Delta B^{00}_{\Lambda\Lambda}=B^0_{\Lambda\Lambda}-2B^0_\Lambda. \quad (22)$$

The latter quantity corresponds to the bond energy calculated consistently in the rigid-core model. The former one simulates the value extracted from $B_{\Lambda\Lambda}$ with the use of actual (e.g., empirical) B_Λ. Obviously, $\Delta B^0_{\Lambda\Lambda}=\Delta B_{\Lambda\Lambda}+\delta E_{\Lambda\Lambda}-4E_1^*$ and $B^{00}_{\Lambda\Lambda}=\Delta B_{\Lambda\Lambda}+\delta E_{\Lambda\Lambda}-2E_1^*$. Assuming $\delta E_{\Lambda\Lambda}=E_{\Lambda\Lambda}(R_2)-E_{\Lambda\Lambda}(R_0)$ to be small since implicit $E_{\Lambda\Lambda}$ dependence on the core radius is weak, we have

$$\Delta B^0_{\Lambda\Lambda}=\Delta B_{\Lambda\Lambda}-E_2^*, \quad (23)$$

$$\Delta B^{00}_{\Lambda\Lambda}=\Delta B_{\Lambda\Lambda}-\frac{E_2^*}{2}. \quad (24)$$

Evidently, κ_A is the usual finite nucleus incompressibility times A, while K_Λ measures the Λ ability to polarize the core. Relations between this ability and features of the ΛN interaction have been studied by many authors (see references collected in [24]). Some estimations for K_Λ in a nuclear matter model were performed in [24]. It is worthwhile that the core polarization is not necessarily contraction. Whereas a simple local purely attractive two-body ΛN force leads to the core contraction, density-dependent forces and nonlocality act competitively and can result in core dilatation. In this case, $K_\Lambda<0$. On the other hand, the polarization contribution to $\Delta B_{\Lambda\Lambda}$ is always positive.

Recently, Bodmer *et al.* [25] suggested that short-range ΛN correlations can influence significantly the polarization picture. Considering ${}^{5}_{\Lambda}$He with a local density-independent ΛN interaction with a short-range repulsion, they argued that the core polarization without the correlations is contraction, and the correlations have a dilutive effect. As we address to heavier systems and use more comprehensive phenomenological ΛN interactions, incorporating effectively some short-range repulsion due to density-dependent forces, we believe that neglect of dynamical two-body correlations is not crucial. However, this point deserves further study.

Defining the core polarization energy from Hartree-Fock calculations, one should take into account an ambiguity arising from the c.m. treatment [28]. Namely, the main differences between calculated core energies in ^{A}Z, ${}^{A+1}_{\Lambda}$Z, and ${}^{A+2}_{\Lambda\Lambda}$Z come from differences in masses of these species, and not from dynamical rearrangement of the core. The energy differences (about 1 MeV for single-Λ hypernuclei and 2 MeV for double-Λ hypernuclei with respect to corresponding nuclei in *p*-shell systems) survive even if action of Λ on the core is directly switched off. In this view, Rayet [28] evaluated a related correction and subtracted it from the core polarization energy. We employ another way. To eliminate the c.m. contribution, we define the dynamical core polarization energy as $E_1^*=E_N({}^{A+1}_{\Lambda}\mathrm{Z})-E'_N({}^{A+1}_{\Lambda}\mathrm{Z})$, where E_N is the total core energy from the full calculation, and E'_N is the core energy obtained with switched-off Λ action on the core. For double-Λ systems, the definition $E_2^*=E_N({}^{A+2}_{\Lambda\Lambda}\mathrm{Z})-E'_N({}^{A+2}_{\Lambda\Lambda}\mathrm{Z})$ is the same. It is seen below that just this dynamical core polarization energy is relevant for the bond energy and consistent with the simplified treatment above.

IV. FIT OF PARAMETERS

Available data on $\Lambda\Lambda$ hypernuclei are too scarce to give a sufficient footing for the $\Lambda\Lambda$ potential determination. Here, we try to use the above approach with a simplified version of the potential (1), examining several sets of parameters.

We drop the density-dependent term in Eq. (1) and constrain the other parameters by the bond energy $\Delta B_{\Lambda\Lambda}=4.8\pm0.7$ MeV [2,9,29] of the ${}^{13}_{\Lambda\Lambda}$B ground state only, since the Skyrme-Hartree-Fock approach is clearly unsuitable for ${}^{6}_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be. We recall that *p*-wave interaction amplitude λ_2 is irrelevant for the ground states. So we consider several pairs of the λ_0 and λ_1 values.

It is known that the Skyrme potential simulates effects of finite-range interaction via the momentum-dependent terms. Namely, while the λ_0 value represents the volume integral, the λ_1/λ_0 ratio is quadratic in the potential range. Alternatively, a positive λ_1 value may be treated as a repulsion at high relative momenta (short distances).

TABLE I. Parameters of $\Lambda\Lambda$ potential (1): λ_0 and λ_1, and ranges μ of the ''equivalent'' single-Gaussian potentials.

Set	λ_0 (MeV fm^3)	λ_1 (MeV fm^5)	μ (fm)
S$\Lambda\Lambda$1	−312.6	57.5	0.61
S$\Lambda\Lambda$2	−437.7	240.7	1.05
S$\Lambda\Lambda$3	−831.8	922.9	1.49

For the main fit, we employ parameter set SkM* [30] for the *NN* potential and the 5th set from [31], denoted hereafter as YBZ5, for the ΛN potential. The former is used extensively in nuclear Hartree-Fock calculations. The latter was fitted [31] to the spectra of single-Λ hypernuclei measured at BNL [32], and it is also consistent [14] with the more recent KEK data [33].

To have some initial idea for the parameters, we take set $\Lambda1$ from [34], which is a simple Skyrme-like approximation to a single-Gaussian $\Lambda\Lambda$ potential with the two-pion-exchange range. This approximation is not very accurate, and set $\Lambda1$ underestimates the ${}^{13}_{\Lambda\Lambda}$B bond energy. Then we vary the λ_1 value, responsible to the potential range, to obtain $\Delta B_{\Lambda\Lambda}=4.8$ MeV (fit of the potential range) and derive set S$\Lambda\Lambda1$. Alternatively, we vary simultaneously the λ_0 and λ_1 values, keeping constant the λ_1/λ_0 ratio (fit of the potential depth at the fixed range) and find set S$\Lambda\Lambda2$. The second fit is repeated also with set $\Lambda3$ [34] of a greater range, and set S$\Lambda\Lambda3$ is obtained. Sets S$\Lambda\Lambda1$, S$\Lambda\Lambda2$, and S$\Lambda\Lambda3$ are presented in Table I. Ranges μ of the ''equivalent'' single-Gaussian potentials, which reflect qualitatively the corresponding property of the Skyrme potentials, are also shown.

It should be noted that set YBZ5 underestimates the Λ binding energy in ${}^{12}_{\Lambda}$B ($B_\Lambda=10.5$ MeV versus the experimental value $B_\Lambda^{\mathrm{exp}}=11.37\pm0.06$ MeV). The majority of the ΛN potentials are fitted to ${}^{12}_{\Lambda}$C ($B_\Lambda^{\mathrm{exp}}=10.76\pm0.19$ MeV) and/or ${}^{13}_{\Lambda}$C ($B_\Lambda^{\mathrm{exp}}=11.69\pm0.12$ MeV) binding energies. Since Hartree-Fock calculations with charge-symmetrical ΛN potentials give for B_Λ's in ${}^{12}_{\Lambda}$C and ${}^{12}_{\Lambda}$B just close values, this drawback is inherent for such potentials. However, the bond energy is rather stable to such uncertainty, since the ΛN interaction gives nearly equal contributions to $B_{\Lambda\Lambda}$ and $2B_\Lambda$, and they mostly cancel. We checked this point, strengthening artificially the ΛN potential to provide true B_Λ in ${}^{12}_{\Lambda}$B. The bond energy increases by only several hundredths of MeV for set S$\Lambda\Lambda3$ and by 0.25 MeV for set S$\Lambda\Lambda1$, which is evidently within the error bar. The strongest sensitivity in the last case results from the smallest range of the S$\Lambda\Lambda1$ potential. As a consequence, the energy of the $\Lambda\Lambda$ interaction grows significantly when hyperons become more bound and, therefore, more closely-spaced.

It is worthwhile also that, strictly speaking, the definition (10) of the bond energy in ${}^{13}_{\Lambda\Lambda}$B with a nonzero-spin core incorporates not B_Λ in ${}^{12}_{\Lambda}$B in the ground state, but rather a less value averaged over the spin-doublet states. The spin-doublet splitting in ${}^{12}_{\Lambda}$B is possibly small [16,35], but the related uncertainty cannot be excluded now.

TABLE II. Calculated properties of the ${}^{12}_{\Lambda}$B ground state: Λ binding energy B_Λ, core polarization energy E_1^*, rms radius of the Λ orbit r_Λ, and δR_1 [defined in Eq. (12)].

Potentials		B_Λ	r_Λ	E_1^*	δR_1
NN	ΛN	(MeV)	(fm)	(MeV)	(%)
	YBZ5	10.5	2.06	0.1	−0.6
SkM*	SKSH1	12.1	1.75	1.3	−3.5
	I	10.8	2.11	0.2	−0.8
	YBZ6	10.0	2.24	0.0	−0.1
	YBZ5	10.6	2.06	0.0	−0.5
Sk3	SKSH1	11.5	1.83	0.7	−2.1
	I	10.9	2.11	0.1	−0.6
	YBZ6	10.2	2.23	0.0	−0.1

Though our approach is not suitable for the lightest hypernuclei, we calculated the ΛΛ bond energies in ${}^{10}_{\Lambda\Lambda}$Be and ${}^{6}_{\Lambda\Lambda}$He and obtained for sets SΛΛ1, SΛΛ2, and SΛΛ3, respectively, 5.4, 5.1, and 4.8 MeV (${}^{10}_{\Lambda\Lambda}$Be, $\Delta B^{\rm exp}_{\Lambda\Lambda}=4.3\pm0.4$ MeV) and 6.1, 5.3, and 4.3 MeV (${}^{6}_{\Lambda\Lambda}$ He, $\Delta B^{\rm exp}_{\Lambda\Lambda}=4.7\pm0.5$ MeV). This comparison is not instructive quantitatively, because the approach is adequate neither for ^{4}He and ^{8}Be nor for ${}^{5}_{\Lambda}$He and ${}^{9}_{\Lambda}$Be. Nevertheless, it shows that the potentials obtained are of reasonable magnitudes.

V. RESULTS AND DISCUSSION

We perform calculations of the ${}^{13}_{\Lambda\Lambda}$B binding energy with the ΛΛ potential parameter sets from Table I and various Λ*N* sets. In addition to set YBZ5, we use also the sixth set from [31] (hereafter YBZ6) and set I from [14]. All these sets incorporate three-body Λ*NN* or density dependent Λ*N* forces of moderate strengths and, therefore, polarize cores slightly. They fit spectra of single-Λ hypernuclei well. Putting emphasis to implications of the nuclear core polarization, we examine also set SKSH1 from [36]. It is almost local and does not incorporate three-body or density-dependent forces at all, so it provides an extreme case of a strong core contraction. It should be noted that set SKSH1 is inadequate for heavy hypernuclei and also overestimates somewhat the level spacing in light ones. Overall fit of the spectra is poorer than those with the other interactions employed [14]. So set SKSH1 is considered rather as an extreme example. To study a role of nuclear incompressibility, we use set Sk3 [37] with a high nuclear matter incompressibility (355 MeV) besides set SkM* with a low one (217 MeV) for the *NN* potential.

The Λ binding energies and other quantities for ${}^{12}_{\Lambda}$B ground state are listed in Table II for various Λ*N* and *NN* interactions. It is seen that set SKSH1 induces an extremely strong contraction and also represents an extreme case of the smallest rms radii of hyperonic orbits r_Λ. Such a connection between the polarizing properties and the Λ rms radii is inherent for this type of the potentials. A repulsive Λ*NN* force pushes hyperon(s) as well as nucleons out of the dense central region. Otherwise, a purely attractive Λ*N* interaction, contracting the core, puts hyperon(s) closer to the center. Differences in r_Λ between the slightly polarizing sets originate from their different nonlocalities. Potential YBZ5 is local and gives, therefore, relatively small radii. Otherwise, set YBZ6 provides the greatest radii due to its greatest nonlocality. Realistic r_Λ's lie probably within the range confined to the YBZ5 and YBZ6 cases [14].

In Table III, results for ${}^{13}_{\Lambda\Lambda}$B with various combinations of

TABLE III. Calculated properties of the ${}^{13}_{\Lambda\Lambda}$B ground state for various potentials. Quantities $\Delta B_{\Lambda\Lambda}$, $\Delta B^{0}_{\Lambda\Lambda}$, $\Delta B^{00}_{\Lambda\Lambda}$, and δR_2 are defined in Eqs. (10), (21), (22), and (16), respectively. $\langle V_{\Lambda\Lambda}\rangle$ and $\langle V_{\Lambda\Lambda}\rangle^0$ are the expectation values of the ΛΛ interaction energy from the full and rigid-core calculations, r_Λ is the rms radius of the Λ orbit, and E_2^* is the core polarization energy. For the meaning of β, see the text.

	Potentials		β	$\Delta B_{\Lambda\Lambda}$	$\lvert\langle V_{\Lambda\Lambda}\rangle\rvert$	r_Λ	$\Delta B^{0}_{\Lambda\Lambda}$	$\Delta B^{00}_{\Lambda\Lambda}$	$\lvert\langle V_{\Lambda\Lambda}\rangle^0\rvert$	E_2^*	δR_2
ΛΛ	*NN*	Λ*N*		(MeV)	(MeV)	(fm)	(MeV)	(MeV)	(MeV)	(MeV)	(%)
		YBZ5	1	4.8	5.5	1.83	4.6	4.7	5.5	0.2	−1.3
		SKSH1	0.24	4.8	2.2	1.54	−1.3	1.2	1.4	7.4	−7.4
SΛΛ1	SkM*	I	1.08	4.8	5.5	1.88	4.4	4.6	5.3	0.4	−1.9
		YBZ6	1.34	4.8	5.4	2.02	4.7	4.7	5.3	0.1	−0.2
		YBZ5	1.00	4.8	5.6	1.83	4.6	4.7	5.5	0.2	−1.1
		SKSH1	0.49	4.8	3.8	1.63	1.2	2.6	2.9	3.9	−4.4
SΛΛ1	Sk3	I	1.08	4.8	5.4	1.89	4.5	4.6	5.3	0.3	−1.3
		YBZ6	1.32	4.8	5.3	2.02	4.7	4.7	5.3	0.1	−0.2
		YBZ5	1	4.8	5.0	1.90	4.6	4.7	5.0	0.2	−1.3
		SKSH1	0.33	4.8	2.1	1.59	−0.8	1.6	1.8	6.5	−7.2
SΛΛ2	SkM*	I	1.04	4.8	5.0	1.94	4.4	4.6	4.9	0.3	−1.8
		YBZ6	1.22	4.8	5.0	2.07	4.7	4.7	5.0	0.1	−0.3
		YBZ5	1	4.8	4.7	2.02	4.6	4.7	4.7	0.2	−1.2
		SKSH1	0.83	4.8	3.2	1.79	0.9	3.3	3.7	4.0	−6.2
SΛΛ3	SkM*	I	0.99	4.8	4.7	2.05	4.5	4.6	4.7	0.3	−1.7
		YBZ6	1.02	4.8	4.7	2.15	4.7	4.7	4.7	0.1	−0.3

the NN, ΛN, and $\Lambda\Lambda$ potentials are shown. For each combination, we readjust the $\Lambda\Lambda$ potential to reproduce $\Delta B_{\Lambda\Lambda} = 4.8$ MeV by multiplying both λ_0 and λ_1 by the same factor β (i.e., fixing the potential range). Deviations of β from unity provide a measure of ambiguity of the $\Lambda\Lambda$ potential extracted from data with the use of different NN and ΛN potentials.

Deviations of β's from unity are induced by two main effects: the core polarization and differences in r_Λ's generated by various ΛN potentials. The polarization, contributing positively to $\Delta B_{\Lambda\Lambda}$, tends to reduce a $\Lambda\Lambda$ potential fitted to a fixed bond energy. On the other hand, the greater is r_Λ the farther are the hyperons from each other, and, therefore, actual attraction between hyperons at the same $\Lambda\Lambda$ potential decreases. Thus, ΛN potentials giving greater r_Λ's require generally stronger $\Lambda\Lambda$ potentials. The last effect is most important for such short-range $\Lambda\Lambda$ potentials as S$\Lambda\Lambda$1, for which spacing of Λ's apart reduces their attraction crucially. Short-range $\Lambda\Lambda$ potentials, pulling the hyperons close together, also perturb significantly the Λ orbit with respect to the single-Λ hypernucleus, so that Λ-core attraction decreases.

For a strong core polarization and small r_Λ's (set SKSH1), we have very small β's with sets S$\Lambda\Lambda$1 and S$\Lambda\Lambda$2 for the above-mentioned reasons. For sets SkM* and S$\Lambda\Lambda$1, only a fourth of the potential, fitted with set YBZ5, is sufficient for the same bond energy. Obviously, a greater incompressibility (set Sk3) implies in a less polarization and, thus, a less deviation of β. On the other hand, β is much closer to unity with set S$\Lambda\Lambda$3 of a long range. We recall that calculations with set SKSH1 provide upper limits for the corresponding effects rather than their quantitative estimations.

The core polarization for sets I and YBZ6 is small as well as for incident set YBZ5. Due to greater r_Λ's, stronger $\Lambda\Lambda$ potentials ($\beta > 1$) are needed, if their ranges are short, independently from the incompressibility. For set I, β is rather close to unity, but the effect of the greater radii becomes meaningful for the YBZ6 set.

Some illustrative calculations [19] with a core-diluting potential, incorporating a huge ΛNN force and providing extremely high r_Λ's, gave β's typically not higher than those for set YBZ6. The reason is that the core polarization tends to diminish β and, therefore, acts competitively with the large radii.

It is seen also that the bond energies $\Delta B_{\Lambda\Lambda}$ differ from the expectation values $|\langle V_{\Lambda\Lambda}\rangle|$ of $\Lambda\Lambda$ potential energy. It means that routine assignment of $\Delta B_{\Lambda\Lambda}$ directly to $^1S_0$ $\Lambda\Lambda$ matrix element can lead to errors in evaluation of $\Lambda\Lambda$ interaction.

According to Sec. III, calculations with the frozen core ($\Delta B^0_{\Lambda\Lambda}$, $\Delta B^{00}_{\Lambda\Lambda}$, and $\langle V_{\Lambda\Lambda}\rangle^0$ in Table III) always underestimate the bond energy, though this underestimation is significant only for strongly polarizing ΛN potentials. It is worthwhile that $\Delta B^{00}_{\Lambda\Lambda}$ and $|\langle V_{\Lambda\Lambda}\rangle^0|$ are usually closer to each other than the corresponding quantities from the full calculation. Relations (16) and (18) between single-Λ and double-Λ hypernuclear core polarizations and Eqs. (23) and (24) between $\Delta B_{\Lambda\Lambda}$ from calculations with and without core polarization, obtained in Sec. III in a simple picture, remain valid qualitatively. Note that strong contraction (set SKSH1) can lead even to a negative $\Delta B^0_{\Lambda\Lambda}$. It is due to inconsistency of $B_{\Lambda\Lambda}$ calculated in the rigid core model with B_Λ from the full calculation.

TABLE IV. Bond energies: $\Delta B_{\Lambda\Lambda}$ from the full calculation and $\Delta B^{00}_{\Lambda\Lambda}$ from the rigid core calculation, and core polarization energy E^*_2. All quantities are in MeV. Parameter sets SkM*, SKSH1, and S$\Lambda\Lambda$2 (the last is renormalized according to Table III) are used for the NN, ΛN, and $\Lambda\Lambda$ potentials, respectively.

Hypernucleus	$\Delta B_{\Lambda\Lambda}$	$\Delta B^{00}_{\Lambda\Lambda}$	E^*_2
$^{42}_{\Lambda\Lambda}$Ca	2.3	1.0	2.5
$^{92}_{\Lambda\Lambda}$Zr	1.2	0.5	1.2
$^{140}_{\Lambda\Lambda}$La	0.9	0.4	0.8
$^{210}_{\Lambda\Lambda}$Pb	0.5	0.2	0.3

Summing up, uncertainties in the ΛN potential can lead to significant ambiguities in the $\Lambda\Lambda$ potential extraction for short and medium ranges of the $\Lambda\Lambda$ potentials. At the same time, these ambiguities are not high for the long-range potentials.

The A dependence of the bond energies was studied in various models in Ref. [12]. Here we discuss only implications of the core polarization.

It seems natural that core polarization effects become insignificant in heavier double-Λ systems. However, our calculations show that it is not necessarily the case. In Table IV, the bond energies for several $\Lambda\Lambda$ hypernuclei from the full Hartree-Fock calculation as well as from the rigid core approximation are shown for the strongly polarizing SKSH1 ΛN potential. As the $\Lambda\Lambda$ potential, set S$\Lambda\Lambda$2 (renormalized as described above) is used. It is seen that $\Delta B_{\Lambda\Lambda}$ is more than twice $\Delta B^{00}_{\Lambda\Lambda}$ up to $^{210}_{\Lambda\Lambda}$Pb. Of course, core polarization energy E^*_2 is very small with respect to the total binding energy, but not with respect to the bond energy. The relation (24) is fulfilled well again. At the same time, the bond energies (not presented here), calculated with nonpolarizing ΛN potentials, agree well with the rigid-core results at all A's.

The spatial distribution of the hyperons is described above in terms of the rms radius of the Λ orbit r_Λ. In few-body calculations, Jacobi variables $r_{\Lambda\Lambda}$ and $r_{\Lambda\Lambda A}$ are often employed. Here $r_{\Lambda\Lambda}$ is the rms distance between the hyperons, and $r_{\Lambda\Lambda A}$ is the rms distance between the $\Lambda\Lambda$ pair c.m. and the core c.m. Obviously, $r_{\Lambda\Lambda}$ relates to $\Lambda\Lambda$ interaction dynamics more directly than r_Λ.

In the Hartree-Fock approximation, these quantities are interrelated in a simple way. Namely, for the ground states of $\Lambda\Lambda$ hypernuclei

$$r_{\Lambda\Lambda} = \sqrt{2}\, r_\Lambda, \quad r_{\Lambda\Lambda A} = \frac{A + 2\xi}{\sqrt{2}A} r_\Lambda, \tag{25}$$

where $\xi = M_\Lambda / M_N$. It is seen that their ratios do not depend on any dynamical factors.

We calculated the ratio $r_{\Lambda\Lambda}/r_{\Lambda\Lambda A}$ from Eq. (25) for various A and compared them with results of a three-body ^{A}Z $+\Lambda+\Lambda$ calculation by Yamamoto *et al.* [9] with a finite-range $\Lambda\Lambda$ potential with a repulsive core. The ratios are shown in Table V.

TABLE V. Ratios of the rms distances $r_{\Lambda\Lambda}/r_{\Lambda\Lambda A}$ from the three-body calculation [9] and from Eq. (25).

Hypernucleus	Ref. [9]	Eq. (25)
${}^{6}_{\Lambda\Lambda}$He	1.53	1.26
${}^{8}_{\Lambda\Lambda}$Li	1.65	1.43
${}^{10}_{\Lambda\Lambda}$Be	1.67	1.54
${}^{12}_{\Lambda\Lambda}$B	1.70	1.62
${}^{13}_{\Lambda\Lambda}$B	1.71	1.65
${}^{14}_{\Lambda\Lambda}$C	1.71	1.69
${}^{16}_{\Lambda\Lambda}$N	1.72	1.71

It is seen that the Hartree-Fock relations (25) are inadequate for the lightest hypernuclei, while the agreement improves rapidly, when A increases, and becomes excellent at the end of the p shell. It is some indication that $\Lambda\Lambda$ correlations become of little importance at $A \gtrsim 10$, and $\Lambda\Lambda$ dynamics is described essentially by independent Λ orbitals. It is seen also that ${}^{13}_{\Lambda\Lambda}$B ($A=11$) is just near the limit of applicability of the approach. So we may hope that the fit of the $\Lambda\Lambda$ interaction to the bond energy in ${}^{13}_{\Lambda\Lambda}$B is reasonable at least qualitatively.

The applicability of the Hartree-Fock description depends, however, on properties of potentials used. Recently, Marcos *et al.* [27] have analyzed the role of the $\Lambda\Lambda$ correlations in a relativistic mean-field inspired model. They showed the contribution of the $\Lambda\Lambda$ correlations to the bond energy to depend strongly on the height of the repulsive core of the $\Lambda\Lambda$ potential. This contribution appears to be substantial at strong $\Lambda\Lambda\omega$ couplings (2/3 of the $NN\omega$ coupling) in terms of the relativistic mean-field theory. Obviously, Hartree-Fock approaches are inadequate for potentials with a very strong central repulsion. However, the $\Lambda\Lambda$ potentials used routinely in nonrelativistic calculations are usually less repulsive than those in $\sigma\omega$ models, so such repulsive cores (and their Skyrme-type simulations) are probably within the limits of applicability.

VI. CONCLUSION

We present an extension of the well-known Skyrme-Hartree-Fock approach to double-Λ hypernuclei. This method is quite simple and, on the other hand, flexible enough to incorporate a rather complicated structure of relevant baryonic effective interactions. It enables one to use great experience achieved in studies of nuclear and single-Λ hypernuclear systems.

As empirical knowledge of $\Lambda\Lambda$ hypernuclei is quite scarce, it is impossible to establish realistic $\Lambda\Lambda$ potentials now. Instead, we try to simulate extraction of this potential from data considering possible uncertainties encountered in this problem.

The bond energy of the $\Lambda\Lambda$ pair in a double-Λ hypernucleus is determined by not only the strength of $\Lambda\Lambda$ potential. It is driven by a complicated interplay of several factors, among which are also the spatial distribution of hyperons and the core polarization. Formally, a very detailed knowledge of not only $\Lambda\Lambda$, but also ΛN and NN interactions, is needed. However, various factors are of different significance in different cases, so it is important to deduce what conditions are really unfavorable for extracting of $\Lambda\Lambda$ potential from data. It is shown that the most ''dangerous'' combination is a ΛN interaction, contracting the core and generating small rms radii of Λ orbits, together with a short-range $\Lambda\Lambda$ potential and a low nuclear incompressibility.

Possibly, the simple $\Lambda\Lambda$ potentials used here are too schematic. However, calculations [14], made with more realistic $\Lambda\Lambda$ potentials along similar lines, do not contradict our conclusions. Nevertheless, it is interesting to study implications of realistic $\Lambda\Lambda$ potential shapes in their interplay with ΛN and NN potential properties more systematically.

We use different effective ΛN potentials, providing different Λ orbit radii and core polarizations. There is no direct knowledge of radii of Λ orbits as well as core polarization even in single-Λ hypernuclei up to now. Some attempts to deduce the radii indirectly from consistency of model calculations with experimental spectra of single-Λ hypernuclei [38,39,14] brought different results. Very strong sensitivity of the radii to model parameters were found [40] in the relativistic mean-field theory. Skyrme-Hartree-Fock calculations [21,31,41,14] indicate that the core polarization is probably a slight contraction. Another conclusion was drawn, however, from more comprehensive variational calculations [25,42], where a core dilatation was predicted. Recently, a strong core polarization of a more complex type (contraction of the neutron fraction and dilatation of the proton one) was inferred [43] from the quark-meson coupling model. Therefore, less model dependent ways to determine radii of the Λ orbits and core polarization properties are needed. We argue here that this problem is of importance particularly for studying $\Lambda\Lambda$ dynamics.

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