Odd-even differences in moments of inertia

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(Received 2 July 1998)

Blocking effect and the effect of the Coriolis interaction are studied as the causes of the odd-even difference in the moments of inertia. The cranking formula is used to calculate the moments of inertia and the pairing Hamiltonian is diagonalized to obtain the state vectors and their energies that are necessary to calculate the moments of inertia. The moment of inertia is divided into two parts. One part $I_{\Lambda_{n=2}}$ comes from a part of the angular momentum operator that changes the seniority by 2. This part is the contribution from the even core in the odd-particle system and the blocking effect emerges in this part. The other part $I_{\Delta v=0}$ comes from that which does not change the seniority. This part corresponds to the contribution of the Coriolis interaction to the moment of inertia. Calculations are made for systems with $N=95-101$. Three typical examples are analyzed. The first example shows the importance of $I_{\Delta v=0}$, the second example shows that the moment of inertia becomes an increasing function of *G*, the strength of the pairing interaction, because of the predominance of $I_{\Delta v=0}$, and the third example shows the situation where $I_{\Delta v=0}$ is negligibly small. It is also shown that the blocking of a special level by the last odd particle brings about the smaller value for $I_{\Delta p=2}$ relative to the moment of inertia for the neighboring even-particle system. $[$ S0556-2813(98)01412-5 $]$

PACS number(s): 21.60 .Ev

I. INTRODUCTION

The recent discovery of identical bands has received a great deal of attention $[1]$. The identical bands were first found among two superdeformed bands $[2]$ and soon after the same phenomenon was found also in the normally deformed region $[3-5]$. The appearance of the identical bands is therefore not the special phenomenon for the superdeformed region but the phenomenon observed in a wide range of mass numbers and of excitation.

This phenomenon was particularly surprising because it has been believed that the moments of inertia for odd-*A* nuclei are systematically larger than those for the neighboring even-even nuclei. As summarized by Bohr and Mottelson [6], odd-even differences in moments of inertia are thought to result for two reasons. The first one arises from the pair correlations, i.e., the presence of the odd particle leads to a reduction of the pairing energy gap Δ and hence increases the moments of inertia. This gives the systematic increases of moments of inertia in odd-*A* nuclei. The additional increase in moments of inertia arises from the second-order effect of the Coriolis coupling between one-quasiparticle states.

The former systematic effect is the blocking effect of the last odd particle. Their discussion was made based on the BCS theory, where the moment of inertia was thought to be a function of Δ . Strictly speaking, however, in order to take the blocking effect into account, we have to introduce different quasiparticle bases for each blocked level and hence different Δ for each blocked level. Consequently, we encounter a crucial difficulty in the calculation of the matrix elements between different quasiparticle bases. Therefore, it is desirable to treat the pairing interaction in such a way that the blocking effect can be treated properly in the study of the odd-even difference in moments of inertia. Along this line, Zeng *et al.* [7] studied the odd-even differences in moments of inertia with their particle number conserving (PNC) treatment of the pairing interaction, have shown that the experimental large fluctuations in the odd-even differences in moments of inertia could be reproduced satisfactorily by their PNC treatment, and pointed out that the proper treatment of the blocking effect is essential in reproducing the odd-even differences in the moment of inertia. In their study, however, the two origins of the odd-even differences in moments of inertia mentioned above are mixed together and we cannot see if the difference really came from the blocking effect.

As concerns the latter systematic effect, Hamamoto and Udagawa $[8]$ studied the Coriolis force effect on the last odd particle and its contribution on the rotational parameters. They calculated the moments of inertia for odd-*A* nuclei based on the cranking model and showed that the Coriolis coupling played an essential role in the reproduction of the characteristic orbital dependence of the largeness of the moments of inertia for odd-*A* nuclei. However, their calculation was based on the BCS approximation and the blocking effect was not taken into account.

These two effects that cause the odd-even differences in moments of inertia have different origins and characters. The blocking effect emerges as the increase in moments of inertia of the even core in odd-particle system and is considered to give a systematic increase in moments of inertia by about 15%. The effect of the Coriolis coupling on the moments of inertia, on the other hand, comes from the coupling of two near-lying bands with $\Delta K = 1$ and is considered to give the large fluctuation in the odd-even differences according to the level that the last odd particle occupies. The main motive of the present study is to investigate the interweaving of these two origins of the odd-even difference in moments of inertia

separately and to see their contributions in each configuration.

II. FORMULATIONS

A. Cranking formula

In the present analysis we deal with a simple system that has an axially symmetric deformation. This system is supposed to rotate around the *x* axis, which is perpendicular to the symmetry axis of the nuclear deformation. The moment of inertia for the ground state of this system is given by the well known formula $[9]$

$$
I = 2\hbar^2 \sum_{\text{ex}} \frac{|\langle \text{ex} | J_x | \text{gr} \rangle|^2}{E_{\text{ex}} - E_{\text{gr}}},\tag{1a}
$$

where

$$
J_x = \sum_{\mu,\nu} (j_x)_{\mu\nu} (c_{\mu+}^{\dagger} c_{\nu+} - c_{\nu-}^{\dagger} c_{\mu-}). \tag{1b}
$$

In Eq. (1a), $|gr\rangle$ and $|ex\rangle$ are the ground and excited states with energies E_{gr} and E_{ex} , respectively. The symbol $(j_x)_{\mu\nu}$ in Eq. $(1b)$ stands for the matrix element of the *x* component of the angular momentum with respect to the single-particle states $\mu \sigma$ and $\nu \sigma$. ($\sigma = \pm$ represents the signature.)

For the convenience of later discussion, we rewrite the creation operator of a nucleon in terms of the pair operator $S_v^{\dagger} \equiv c_{v+}^{\dagger} c_{v-}^{\dagger}$ and the unpaired nucleon operator $a_{v\sigma}^{\dagger} \equiv c_{v\sigma}^{\dagger} (1$ $-c \frac{1}{\nu \sigma} c \frac{1}{\nu \sigma}$ as [10]

$$
c_{\nu\sigma}^{\dagger} = a_{\nu\sigma}^{\dagger} + \sigma S_{\nu}^{\dagger} a_{\nu\sigma}.
$$
 (2)

The substitution of Eq. (2) into Eq. $(1b)$ leads us to the expression

$$
J_x = J_x^S + J_x^A, \tag{3a}
$$

where

$$
J_x^S = \sum_{\mu,\nu} (j_x)_{\mu\nu} \sum_{\sigma} \sigma(a_{\mu\sigma}^{\dagger} a_{\nu\sigma} + S_{\mu}^{\dagger} a_{\nu\sigma}^{\dagger} a_{\mu\sigma} S_{\nu})
$$
 (3b)

and

$$
J_x^A = \sum_{\mu,\nu,\sigma} (j_x)_{\mu\nu} (a_{\mu\sigma}^\dagger a_{\nu\bar{\sigma}}^\dagger S_\nu + S_\mu^\dagger a_{\mu\bar{\sigma}} a_{\nu\sigma}). \tag{3c}
$$

Hereinafter we call the number of unpaired nucleons the seniority *v*. It is obvious from this expression that the operator J_x consists of two parts: one J_x^S that does not change the seniority and another J_x^{A} that changes the seniority by 2.

B. States and energies

We obtain the ground and excited states and their energies in Eq. $(1a)$ by diagonalizing the Hamiltonian

$$
H = \sum_{\mu,\sigma} \epsilon_{\mu} c^{\dagger}_{\mu\sigma} c_{\mu\sigma} - G \sum_{\mu,\nu} c^{\dagger}_{\mu} c^{\dagger}_{\mu} - c_{\nu} c_{\nu+}, \qquad (4)
$$

where ϵ_{μ} denotes Nilsson's single-particle energy and *G* is the strength of the pairing interaction. This Hamiltonian is rewritten in terms of S^{\dagger} , *S*, a^{\dagger} , and *a* as

$$
H = H_S + H_a, \t\t(5a)
$$

where

$$
H_S = \sum_{\mu} 2 \epsilon_{\mu} S_{\mu}^{\dagger} S_{\mu} - G \sum_{\mu, \nu} S_{\mu}^{\dagger} S_{\nu}
$$
 (5b)

and

$$
H_a = \sum_{\mu,\sigma} \epsilon_{\mu} a^{\dagger}_{\mu\sigma} a_{\mu\sigma}.
$$
 (5c)

As the seniority of the ground state of our system is *v* $=1$ and the operator J_x changes the seniority up to 2, the states we have to take into account are those with $v=1$ and $v=3$. Suppose we are dealing with the $(2N+1)$ -particle system. The eigenstate of the Hamiltonian $(5a)$ with $v=1$ is written as

$$
|v=1[\alpha];m\rangle=a_{\alpha\sigma}^{\dagger}\sum_{i}V_{m}(\alpha;i)|N;i[\alpha]\rangle, \qquad (6a)
$$

where

$$
|N;i[\alpha]\rangle = S^{\dagger}_{\nu_1} S^{\dagger}_{\nu_2} \cdots S^{\dagger}_{\nu_N} |0\rangle
$$
 (6b)

and

$$
i = \{ \nu_1, \nu_2, \dots, \nu_N \}. \tag{6c}
$$

Here $V_m(\alpha;i)$ are the eigenvectors obtained by the diagonalization of *H* and the energy eigenvalue of this state is denoted as $E_{m,[\alpha]}$. In this expression, the unpaired nucleon is considered to occupy the level $\alpha\sigma$. As this level is occupied by this unpaired nucleon, we remove this level from the model space when we diagonalize the pairing Hamiltonian (5b). The symbol α in the argument of V_m and $[\alpha]$ in the ket vector in Eq. $(6a)$ are used to show this removed level. The symbol *i* stands for the possible combination of the occupied single-particle levels by the *N* nucleon pairs and *m* is the energetic order of the states obtained by the diagonalization of Eq. $(5a)$ and the lowest state is denoted by $m=0$. The ground state of our system is written as

$$
|v=1[\alpha_g];0\rangle = a_{\alpha_g\sigma_g}^{\dagger} \sum_i V_0(\alpha_g; i)|N; i[\alpha_g]\rangle, \qquad (7)
$$

where α_g is the level that is occupied by the unpaired nucleon in the ground state. In the same way, the eigenstate with $v=3$ is written as

$$
|v=3[\alpha_1\alpha_2\alpha_3];m\rangle
$$

= $a_{\alpha_1\sigma_1}^{\dagger}a_{\alpha_2\sigma_2}^{\dagger}a_{\alpha_3\sigma_3}^{\dagger}$

$$
\times \sum_{i} V_m(\alpha_1\alpha_2\alpha_3;i)|N-1;i[\alpha_1\alpha_2\alpha_3]\rangle, \qquad (8)
$$

and its energy eigenvalue is denoted as $E_{m,\lceil \alpha_1\alpha_2\alpha_3\rceil}$.

C. Contribution of J_x^S and J_x^A

The moment of inertia expressed in Eq. $(1a)$ is therefore written as

$$
I = I_{\Delta v = 0} + I_{\Delta v = 2},\tag{9a}
$$

where

$$
I_{\Delta v=0} = 2\hbar^2 \sum_{m,\alpha \neq \alpha_g} \frac{|\langle v=1[\alpha];m|J_x^S|v=1[\alpha_g];0\rangle|^2}{E_{m,[\alpha]}-E_{0,[\alpha_g]}} \tag{9b}
$$

and

 $I_{\Delta v=2}$

$$
=2\hbar^{2}\sum_{m,\{\alpha_{1}\alpha_{2}\alpha_{3}\}}\frac{|\langle v=3[\alpha_{1}\alpha_{2}\alpha_{3}];m|J_{x}^{A}|v=1[\alpha_{g}];0\rangle|^{2}}{E_{m,[\alpha_{1}\alpha_{2}\alpha_{3}]}-E_{0,[\alpha_{g}]}}.
$$
\n(9c)

The part $I_{\Delta v=0}$ corresponds to the first term of Eq. (5-47) in Ref. $[6]$, which is proportional to

$$
\sum_{\alpha \neq \alpha_g} \frac{[(j_x)_{\alpha \alpha_g}]^2}{\varepsilon_\alpha - \varepsilon_{\alpha_g}} (u_\alpha u_{\alpha_g} + v_\alpha v_{\alpha_g})^2, \tag{10}
$$

where the ε 's are the quasiparticle energies and u and v are the usual *uv* factors that appear in the BCS theory. As stated in Ref. $[6]$, this term represents the effect of the Coriolis coupling between the one-quasiparticle states with $v=1, \alpha_g$ and $v=1,\alpha$. The denominator of this expression is not the sum of the quasiparticle energies but their subtraction. Therefore, this term is supposed to give the large fluctuations in the odd-even differences in moments of inertia according to the situation of the single-particle levels.

In Eq. $(5-47)$ in Ref. [6], there is another term proportional to

$$
-\sum_{\alpha \neq \alpha_g} \frac{\left[(j_x)_{\alpha \alpha_g} \right]^2}{\varepsilon_\alpha + \varepsilon_{\alpha_g}} (u_{\alpha_g} v_\alpha - v_{\alpha_g} u_\alpha)^2, \tag{11}
$$

which is explained to represent the effect of the odd particle in preventing some of the excitations that contribute to the rotational energy (hence to the moment of inertia) of the even-particle system. This term comes from the contribution of the excited state with three quasiparticles such as $\alpha_{\alpha}^{\dagger} \alpha_{\alpha}^{\dagger} \alpha_{\alpha}^{\dagger} |0\rangle$, where α^{\dagger} is the creation operator of a quasiparticle and $|0\rangle$ is the BCS vacuum. As our single-particle levels are doubly degenerate levels, we cannot create two quasiparticles in a single-particle level because two particles in a doubly degenerate level must be a pair. Therefore, this term should not be taken into account.

In the even-particle systems, we have only the term that comes from J_x^A ,

$$
I_{\text{even}} = 2\hbar^2 \sum_{m,\{\alpha_1\alpha_2\}} \frac{|\langle v=2[\alpha_1\alpha_2];m|J_x^A|gr\rangle|^2}{E_{m,[\alpha_1\alpha_2]}-E_{gr}}.\tag{12}
$$

This term corresponds to $I_{\Delta v=2}$ in the odd-particle system and the difference between $I_{\Delta v=2}$ and I_{even} comes from the presence of an additional particle in the former. Therefore, the blocking effect of the last odd particle on the moment of inertia is seen as the difference between $I_{\Delta v=2}$ and I_{even} .

III. NUMERICAL CALCULATIONS

The purpose of the present study is to see the mechanism of the odd-even differences in moments of inertia. We studied the moments of inertia for odd-particle systems whose neutron numbers range from 95 to 101. Although our aim is not to reproduce the precise experimental values of moments of inertia but to discuss qualitatively the odd-even differences in moments of inertia, we need to use the realistic single-particle levels and the wave functions because the moment of inertia is quite sensitive to the distribution of the single-particle levels and to the values of matrix elements $(j_x)_{\mu\nu}$. We used the same parameters as those in the previous paper $[11]$ in determining the single-particle states $[12]$. The values of the parameters are $\epsilon = 0.245$, $\kappa = 0.0637$, μ =0.420, and ϵ_4 = -0.015. The energy levels of the singleneutron states are shown in Fig. 1.

Let us take the case of $N=95$, for example, to see our model space. As the last odd neutron is supposed to occupy the level [642 5/2] in the ground state, we set $\alpha_g = 9$ in Eq. (7) . We took 13 particles as the active particles including this last odd particle and took levels $3-15$ (from [530 1/2] to [624 9/2]) as the active levels. Therefore, we set $N=6$ from Eqs. $(6a)$ – (8) . We have studied the isotopes whose numbers of neutrons are from 95 to 101. For the case of $N=101$, for example, we set $\alpha_g=12$ and took levels 6–18 as the active levels.

The calculated values of moments of inertia strongly de-

FIG. 1. Energy levels used in the present calculations. These levels are obtained in a similar way to those of Nilsson *et al.* [12]. The parameters used are shown in the text.

FIG. 2. Pairing energy gap Δ (MeV) and the moments of inertia *I* (\hbar^2 MeV⁻¹) for *N*=96 (dotted lines), *N*=97 (solid lines), and $N=98$ (dashed lines). Thin dash-dotted lines show the contributions of the two terms in Eqs. (9) .

pend on the size of the model space and the value of *G*. Therefore, it is important to use appropriate values of *G* corresponding to the size of the model space we use. As we did in a previous paper $[11]$, we studied the behavior of the moments of inertia around the value of *G* that gives the appropriate value of the pairing energy gap $\Delta = G \sqrt{\langle S^{\dagger}S \rangle}$ [13].

IV. RESULTS AND DISCUSSION

In Fig. 2, the pairing energy gap Δ and the moment of inertia for the [523 5/2] band of $N=97$ are compared with those for the ground states of neighboring even-particle systems with $N=96$ and 98. As the value of Δ deduced from the odd-even mass difference in this region is about $\Delta=0.9$ MeV, we drew the horizontal line to show the position of Δ =0.9 MeV in the upper part of this figure and the vertical line to show the value of *G* for which we make the comparisons.

As expected, the value of Δ for *N*=97 is smaller than the values for $N=96$ and 98. This shows that the last odd particle blocked the pairing correlations and brought about the decrease in Δ . The moment of inertia for *N*=97, which is shown by the solid line in the lower part of this figure, is much larger than those for even-particle systems, which are shown by dotted and dashed lines for $N=96$ and 98, respectively.

It is a widely accepted scenario $\lceil 6 \rceil$ that the presence of the odd particle leads to a reduction of the pair correlation parameter Δ and hence of the rotational parameter A $=$ $\hbar^2/2I$. As a quantitative example, Bohr and Mottelson showed that a reduction of Δ^2 by a factor of 2 leads to an increase in the moment of inertia of 15%. In our calculation, values of $\left[\Delta(N=97)/\Delta(N=96)\right]^2$ and $\left[\Delta(N=97)/\Delta(N=97)\right]$ $(5-98)$ ² are 0.71 and 0.79, respectively, at $G=0.21$ MeV. Therefore, the blocking effect of the last odd particle on Δ is

FIG. 3. Pairing energy gap Δ (MeV) and the moments of inertia *I* (\hbar^2 MeV⁻¹) for *N*=94 (dotted lines), *N*=95 (solid lines), and $N=96$ (dashed lines). Thin dash-dotted lines show the contributions of the two terms in Eqs. (9) .

smaller than that expected by the above scenario. The increase in the moment of inertia is, however, much larger than expected. At *G*=0.21 MeV, *I*(*N*=95) is 29.1 \hbar^2 MeV⁻¹, whereas $I_{\text{even}}(N=96)$ is $16.5\hbar^2$ MeV⁻¹ and $I_{\text{even}}(N=98)$ is 19.5 \hbar^2 MeV⁻¹. The origin of this increase is also seen from this figure. The contributions from two terms in Eqs. (9) are shown by thin dash-dotted lines. As stated in Sec. II C, $I_{\Delta v=2}$ is considered to correspond to the contribution from the even core in the odd-particle system and the blocking effect emerges as the difference between $I_{\Delta v=2}$ and I_{even} . Our calculation shows that $[I_{\Delta v=2} - I_{\text{even}}(N=96)]/I_{\text{even}}(N)$ $=96$) is 0.15. Therefore, one can say that the contribution of the blocking effect of the last odd particle to the increase of moment of inertia is around 15%. (However, this seems to be an accidental hit and the decrease in Δ is much smaller in our calculation than expected.) This figure also shows that the main part of the odd-even difference in moments of inertia comes from $I_{\Delta v=0}$. This term is considered to correspond to the second order contribution of the Coriolis coupling. [See the first term of Eq. $(5-47)$ in $[6]$ and Eq. $(9b)$ of this paper.] Therefore, the main part of the odd-even difference in moments of inertia in this case comes from this second-order contribution of the Coriolis coupling.

As pointed out by many authors, the value of $I_{\Delta v=0}$ depends strongly on the single-particle orbit that the last odd particle occupies. It is pointed out that the Coriolis effect becomes very large when the last odd particle occupies the single-particle level whose total oscillator quantum number *N* is 6 [6,7]. As an example that shows the large effect of the Coriolis coupling, we compare the moments of inertia for [642 5/2] band in $N=95$ and the neighboring even-particle systems in Fig. 3. Before going into a detailed discussion, we examine if our result is reasonable in comparison to the experimental value. The experimental value of the moment of inertia for $[642 \frac{5}{2}]$ band of 161 Dy deduced from the two

m	$(j_x)_{\alpha\alpha_\rho}$	$F_{\alpha,m}$	$\Delta E_{\alpha,m}$	${C}_{\alpha,m}$	
	3.22	0.954	0.720	13.10	
	3.08	0.907	0.875	8.91	
	3.08	0.390	2.004	0.72	
	3.22	0.271	1.895	0.40	
	3.22	0.106	2.423	0.05	

TABLE I. Contributions from each excited state with $v=1$ on $I_{\Delta v=0}$ of Eq. (9b) at $G=0.205$ MeV. The sum of only two terms amounts to 95% of the total value. The total value of $\Sigma_{\alpha,m}C_{\alpha,m}$ is 23.27.

lowest observed energy levels is $79.70\hbar^2$ MeV⁻¹. Zeng *et al.* [7] calculated the moment of inertia for $[642 5/2]$ band of 161Dy by applying their PNC treatment to the cranking Hamiltonian and showed that they could reproduce this experimental value of moment of inertia. They obtained the values $58.96\hbar^2$ MeV⁻¹ and $14.33\hbar^2$ MeV⁻¹ for neutrons and protons, respectively, and the total value was 73.29 \hbar^2 MeV⁻¹. As seen from this figure, we obtained the value 57.6 \hbar^2 MeV⁻¹ at *G* = 0.205 MeV as the contribution from neutrons, which is very close to the value of Zeng *et al*. Therefore, our result seems to reproduce the experimental value of the moment of inertia to some extent.

The first thing we notice in this figure is the fact that $I(N=95)$ is an increasing function of *G*. This fact was pointed out by Hamamoto and Udagawa [8]. They discussed the derivative of the moment of inertia with respect to Δ and showed that the derivative becomes positive in some cases where the effect of the Coriolis force is large. In their study, the BCS approximation is used and the blocking effect is neglected in estimating the moment of inertia of the even core. Here we see the same fact as a result of more accurate calculation. As can be seen from this figure, $\partial(I_{\Delta v=0})/\partial G$ is positive and its absolute value is larger than that of $\partial(I_{\Delta v=2})/\partial G$. Therefore, the sum $\partial(I_{\Delta v=2})/\partial G$ $+\partial(I_{\Delta v=0})/\partial G$ becomes positive.

In Table I contributions from each excited state on $I_{\Delta v=0}$ of Eq. (9b) at $G = 0.205$ MeV are listed. Only five terms that have large contributions are listed. The matrix element in the numerator of each term in Eq. $(9b)$ is written as

$$
\langle v=1[\alpha];m|J_x^S|v=1[\alpha_g];0\rangle=(j_x)_{\alpha\alpha_g}\sigma_{\alpha_g}\delta_{\sigma_g\sigma_\alpha}F_{\alpha,m},\tag{13}
$$

where

$$
F_{\alpha,m} = \sum_{i',i} V_m(\alpha;i') V_0(\alpha_g;i)
$$

$$
\times \langle N;i'[\alpha]] \, 1 + S_{\alpha_g}^{\dagger} S_{\alpha} | N;i[\alpha_g] \rangle. \tag{14}
$$

Values of $(j_x)_{\alpha\alpha_g}$, $F_{\alpha,m}$ and the energy denominator $\Delta E_{\alpha,m}$ $\equiv E_{m,[\alpha]}-E_{0,[\alpha]}$ are listed together with the values of $C_{\alpha,m} = |\langle v=1[\dot{\alpha}], m | J_x^S | v=1[\alpha_g];0 \rangle|^2 / \Delta E_{\alpha,m}$. As seen from this table, the sum of only two terms amounts to 95% of the total value of $\Sigma_{\alpha,m}C_{\alpha,m}$ and these two terms have large values of $(j_x)_{\alpha\alpha}$ and $F_{\alpha,m}$ and small values of $\Delta E_{\alpha,m}$. In Table II the dependence of the largest two terms of $C_{\alpha,m}$ on *G* is shown. The factor $F_{\alpha,m}$ depends only weakly on *G*, whereas $\Delta E_{\alpha,m}$ decreases considerably as *G* increases and hence $I_{\Delta v=0}$ becomes an increasing function of *G*. This may correspond to the situation in BCS theory where the factor $uu + vv$ depends weakly on *G* and quasiparticle energies tend to degenerate as *G* increases.

The next thing we notice in this figure is the fact that the odd-even difference in moments of inertia is very large. As seen from the figure, the moments of inertia for neighboring systems are $17.3\hbar^2$ MeV⁻¹, whereas that for *N*=95 is 57.6 h^2 MeV⁻¹. It is apparent that this difference comes from $I_{\Delta v=0}$. The contribution of $I_{\Delta v=2}$ is, in this case, very small; in fact, $I_{\Delta v=2}$ is smaller than I_{even} . The fact that $I_{\Delta v=2}$ is smaller than I_{even} seems to contradict the traditional scenario for the odd-even difference in moments of inertia, i.e., moments of inertia for odd-particle systems become larger than those for even-particle systems because of the blocking effect of the last odd particle. In order to see the reason why we get smaller value for $I_{\Delta v=2}$ than I_{even} , the values of seven terms that have large contributions to I_{even} in Eq. (12) for the ground state band in $N=94$ are listed in Table III. Except for the first and third terms marked by an

TABLE II. Dependence of the largest two terms of *C*a,*m*[z^*v*51@a#;*m*u*Jx* S u*v*51@^a*g*#;0&z 2/D*E*a,*^m* on *^G*. The factors *^F*^a ¹ ,1 and *^F*^a ² ,1 depend only weakly on *G* whereas $\Delta E_{\alpha_1,1}$ and $\Delta E_{\alpha_2,1}$ decrease considerably as *G* increases and hence $I_{\Delta v=0}$ becomes an increasing function of *G*. α_1 stands for [651 3/2] and α_2 stands for [633 7/2].

G	$F_{\alpha_1,1}$	$\Delta E_{\alpha_1,1}$	$C_{\alpha_1,1}$	$F_{\alpha_2,\iota}$	$\Delta E_{\alpha_2,1}$	$C_{\alpha_{2},1}$	$\sum_{\alpha,m} C_{\alpha,m}$
0.15	0.969	0.841	11.574	0.926	1.052	7.715	20.385
0.17	0.961	0.804	11.931	0.914	0.992	7.981	21.154
0.19	0.956	0.758	12.503	0.908	0.926	8.449	22.238
0.21	0.953	0.708	13.304	0.907	0.856	9.115	23.669
0.23	0.953	0.657	14.329	0.910	0.784	9.997	25.480
0.25	0.955	0.608	15.553	0.915	0.718	11.035	27.610

TABLE III. The largest seven terms in Eq. (12) for the ground state band in $N=94$. Values marked with an asterisk do not contribute in the $[642 \t5/2]$ band in $N=95$ because this level is blocked by the last odd particle.

α_1	α_{2}	\boldsymbol{m}	Value
$\begin{bmatrix} 651 & 3/2 \end{bmatrix}$	$\begin{bmatrix} 642 & 5/2 \end{bmatrix}$		$3.16*$
$[532 \ 3/2]$	$[523 \t5/2]$		1.72
$\begin{bmatrix} 642 & 5/2 \end{bmatrix}$	$\begin{bmatrix} 633 & 7/2 \end{bmatrix}$		$1.10*$
$[521 \ 3/2]$	$\begin{bmatrix} 512 & 5/2 \end{bmatrix}$		1.02
$[530 \ 1/2]$	$\begin{bmatrix} 521 & 3/2 \end{bmatrix}$		0.53
$[532 \ 3/2]$	$\begin{bmatrix} 521 & 1/2 \end{bmatrix}$		0.48
$[530 \ 1/2]$	$\begin{bmatrix} 521 & 1/2 \end{bmatrix}$		0.47

asterisk, we can find the corresponding terms in $I_{\Delta v=2}$ for the $[642 \t5/2]$ band in $N=95$ giving large contributions. However, as the last odd particle occupies the level $[642 5/2]$ and is blocking this level, a neutron of the broken pair is not allowed to jump into this level. Therefore, these two important terms in the even-particle system cannot contribute in the odd-particle system and hence we get a smaller value for $I_{\Delta v=2}$. This situation is not general because we saw in Fig. 2 (and will see in Fig. 4) that $I_{\Delta v=2}$ is larger than I_{even} . It seems to depend on the precise position of the blocked single-particle levels. Sun *et al.* [14] studied the blocking effect by projected shell model and argued that the blocking of [514 9/2] orbit produced a 30% reduction in Δ , but the moment of inertia decreased instead of the commonly expected increase. Although further investigation of their result has not yet been published, we think this can happen.

As the third example, a comparison is made in Fig. 4 for the $[521 \ 1/2]$ band in $N=101$. In this case, the contribution of $I_{\Delta v=0}$ is very small. Two configurations with $\alpha = [510]$ 1/2] and α = [521 3/2] have non-negligible matrix elements of $(j_x)_{\alpha\alpha}$ in this case. However, as these configurations have large values of $\Delta E_{\alpha,m}$, contributions of these terms are quite small. Consequently, the behavior of $I(N=101)$ with the change of *G* is quite similar to that of $I(N=100)$ and $I(N=102)$.

V. CONCLUSION

In summary, we have shown that $I_{\Delta v=0}$ in Eqs. (9) is decisive in odd-even differences in moments of inertia. This

FIG. 4. Pairing energy gap Δ (MeV) and the moments of inertia *I* (\hbar^2 MeV⁻¹) for *N*=100 (dotted lines), *N*=101 (solid lines), and $N=102$ (dashed lines). Thin dash-dotted lines show the contributions of the two terms in Eqs. (9) .

term strongly depends on the single-particle level that the last odd particle occupies. When the last odd particle occupying the single-particle level with the total oscillator quantum number is 6, $I_{\Delta v=0}$ dominates the behavior of the moment of inertia and the moment of inertia becomes an increasing function of the pairing interaction *G*. As to the blocking effect of the last odd particle, the presence of the last odd particle does not necessarily increase the contribution of the even core to the total moment of inertia of the odd-particle system. The blocking effect has been discussed in such a way that the moment of inertia is considered to be a function of Δ and the decrease in Δ brings about the increase of the moment of inertia. As we saw in this study, this traditional scenario is not always correct and the odd-even difference in moments of inertia should be examined individually according to the situation of the single-particle levels and the configurations.

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