Variation of moments of inertia with angular momentum and systematics of bandhead moments of inertia of superdeformed bands

S. X. Liu and J. Y. Zeng
Department of Physics, Peking University, Beijing 100871, China

(Received 23 February 1998)

The variation of the kinematic and dynamic moments of inertia and the bandhead moments of inertia J_0 systematics of superdeformed (SD) bands in the $A\sim190$ region are investigated, which turns out to be helpful in the spin prediction of SD bands. The spins of about 70 SD bands in the $A\sim190$ region are predicted by these approaches in combination with the usually adopted best-fit method. The J_0 systematics seems to be very useful to the understanding of the properties of excited SD bands and the implication of identical SD bands. [S0556-2813(98)02711-3]

PACS number(s): 21.10.Re, 21.60.Ev, 27.70.+q, 27.80.+w

I. INTRODUCTION

Since the observation of the first high-spin superdeformed (SD) band in ^{152}Dy [1], SD bands have been intensively studied in several mass regions $A\sim190$, 150, 130, and 80. However, while the intraband energies are easy to detect with modern Ge arrays, it is difficult to observe the link between the SD band and normally deformed (ND) states with known spins. Therefore, the exact excitation energies, spins, and parities of SD bands remain unknown. In the past few years several approaches to predicting the spins of SD bands have been suggested [2–5]. Recently the discrete γ rays connecting states of the yrast SD band ^{194}Hg(1) to ND states with known spins were discovered [6], and the spins and excitation energies of all members of ^{194}Hg(1) were established experimentally. Immediately, the spins and excitation energies of the yrast SD band ^{194}Pb(1) [7–9] and ^{194}Hg(3) [10] were established. Therefore, the measured spins of these SD bands provide a significant test of the validity of these approaches. It is noted that all the available approaches profit from the comparison of the calculated transition energies or moments of inertia with the experimental results and usually are referred to as the best-fit method (BFM). It was found that the rms (root-mean-square) deviation of the calculated results with experiments, χ, depends on the number of transitions involved, and in some cases χ is insensitive to the suggested spin, i.e., the rms deviations may be close to each other for two or more spin propositions, in this case it is difficult to make a unique spin proposition. Particularly, if a significant bandmixing occurs in the transitions involved in the least squares fitting, the rms deviation may display some irregularities and make the suggestion of the exit spin values more difficult.

In Sec. II we suggest another approach (approach II) to the spin proposition of a rotational band. In this approach, on the one hand, one can extract the kinematic and dynamic moments of inertia by using the experimental intraband $E2$ transition energies as follow:

$$J^{(1)}(I-1)/\hbar^2=\frac{2I-1}{E_\gamma(I\to I-2)}, \qquad (1)$$

$$J^{(2)}(I)/\hbar^2=\frac{4}{E_\gamma(I+2\to I)-E_\gamma(I\to I-2)}. \qquad (2)$$

It is seen that, while the extracted $J^{(1)}$ depends on the spin proposition, $J^{(2)}$ does not. On the other hand, according to the now available expressions for rotational bands (Bohr-Mottelson's $I(I+1)$ expansion [11], Harris' ω^2 expansion [12], ab expression [13,14], etc.), some properties concerning the variation of $J^{(1)}$ and $J^{(2)}$ with angular momentum (or rotational frequency) can be found (for details see Sec. II). Thus, we may investigate whether the extracted $J^{(1)}$ and $J^{(2)}$ display these properties, which may be used as a very useful guideline for the spin proposition of a rotational band. It is found that for all well-established ND rotational bands and the fission isomeric bands (SD bands with low spins down to $I=0$ in the $A\sim240$ mass region), the spin propositions by this approach are in agreement with the experiments (see Figs. 1–4). It is encouraging to note that for the high-spin SD bands, ^{194}Hg(1), ^{194}Hg(3) and ^{194}Pb(1), the spin propositions by this approach are also in agreement with experiments (see Fig. 5). Therefore, we believe the proposed spins by this approach are reliable for both ND and SD bands.

By using the measured spins or suggested spins made by the BFM and approach II, it is found that the observed intraband transition energies of both ND and SD bands can be reproduced very well by the ab formula [see Eq. (10)], or its revised abc formula [15] [see Eq. (13)] with rms deviations $\chi\leqslant10^{-3}$. As illustrative examples, the comparison of the calculated E_γ's with experiments for the SD bands ^{194}Hg(1,2,3) and ^{192}Hg(1) are given in Table I (see Sec. III). Therefore, it is reasonable to expect that the variation of the kinematic and dynamic moments of inertia extracted by Eqs. (1) and (2) can be faithfully reproduced by the calculation using the abc formula [see Eqs. (14), (15) and Fig. 6]. Therefore, the bandhead moment of inertia calculated by Eq. (16) is meaningful, though an actual SD band may not extend to very low spins. In fact, the J_0 thus extracted may be considered as another equivalent parameter characterizing a rotational band, and depends on the intrinsic structure of a rotational band. It is interesting to note that the J_0 values thus extracted are usually more sensitive to the suggested spin for a rotational band than the rms deviation χ; i.e., while for some SD bands the χ values may be close to each other for two or more suggested spins, the extracted J_0 values are rather different for different suggested spins. It is found that in the $A\sim190$ region, with increasing (decreasing) the suggested spin by one, the J_0 values will increase (decrease) by about 10% (see Tables II and III). Analysis shows that the J_0 systematics of SD bands may provide another useful guideline for the spin propositions of SD bands in the $A\sim190$ region, which will be discussed in Sec. III.

In Sec. IV, the spin propositions of about seventy SD bands observed in the $A\sim190$ region are made by using the above three approaches. It is found that the spins of most SD bands in the $A\sim190$ region (except a very few) can be proposed consistently and reliably, which are given in Tables IV–VII. The J_0 systematics of SD bands in the $A\sim190$ region is discussed in detail, which turns out to be very useful not only for the spin proposition of a SD band, but also for the understanding of its intrinsic structure (excitation mechanism etc.). A summary is given in Sec. V.

II. VARIATION OF MOMENTS OF INERTIA WITH ANGULAR MOMENTUM

Based on very general symmetry arguments, Bohr and Mottelson pointed out [11] that, under the adiabatic approximation, the rotational energy of an axially symmetric nucleus may be expanded as

$$E(\xi)=A\xi^2+B\xi^4+C\xi^6+D\xi^8+\cdots, \tag{3}$$

$\xi^2=I(I+1)$ (for $K=0$ band). The expression for a $K\neq0$ band takes the form similar to Eq. (3), but includes a bandhead energy and ξ^2 is replaced by $I(I+1)-K^2$. It was well established that extensive ND bands can be described rather well by Eq. (3) except in the bandcrossing region. Systematic analyses of a large number of ND bands showed [11,16] that $|B/A|\sim10^{-3}$, $|C/A|\sim10^{-6}$, $|D/A|\sim10^{-9}$, etc.; i.e., the convergence of the $I(I+1)$ expansion is satisfactory. For SD bands, the convergence is even better [2,17], ($|B/A|\sim10^{-4}$, $|C/A|\sim10^{-8}$, etc.). The kinematic and dynamic moments of inertia are

$$J^{(1)}/\hbar^2=\left(\frac{1}{\xi}\frac{dE}{d\xi}\right)^{-1}=\frac{1}{2A}\left(1+\frac{2B}{A}\xi^2+\frac{3C}{A}\xi^4+\cdots\right)^{-1}, \tag{4}$$

$$J^{(2)}/\hbar^2=\left(\frac{d^2E}{d\xi^2}\right)^{-1}=\frac{1}{2A}\left(1+\frac{6B}{A}\xi^2+\frac{15C}{A}\xi^4+\cdots\right)^{-1}. \tag{5}$$

Another useful expression for nuclear rotational spectra is the Harris ω^2 expression [12] $[\omega=(1/\hbar)dE/d\xi]$

$$E(\omega)=\alpha\omega^2+\beta\omega^4+\gamma\omega^6+\delta\omega^8+\cdots, \tag{6}$$

whose convergence is believed [11] to be superior to the $I(I+1)$ expansion (3), and particularly the Harris two-parameter expansion

$$E(\omega)=\alpha\omega^2+\beta\omega^4 \tag{7}$$

was shown [18] to be equivalent to the variable moment of inertia model [19], and was widely used in the high-spin nuclear physics. The kinematic and dynamic moments of inertia are given by

$$J^{(1)}/\hbar^2=2\alpha+\frac{4}{3}\beta\omega^2+\frac{6}{5}\gamma\omega^4+\cdots, \tag{8}$$

$$J^{(2)}/\hbar^2=2\alpha+4\beta\omega^2+6\gamma\omega^4+\cdots. \tag{9}$$

In [14], the ab formula

$$E(I)=a[\sqrt{1+bI(I+1)}-1] \tag{10}$$

was derived from the Bohr Hamiltonian for a well-deformed nucleus with small axial asymmetry ($\sin^4 3\gamma\ll1$). This expression had been suggested empirically by Holmberg and Lipas [13]. It was found that extensive amount of ND bands can be described very well by this simple expression and the improved abc expression [see Eq. (13)] [15,17]. The kinematic and dynamic moments of inertia are given by

$$J^{(1)}/\hbar^2=J_0[1+bI(I+1)]^{1/2}, \tag{11}$$

$$J^{(2)}/\hbar^2=J_0[1+bI(I+1)]^{3/2}, \tag{12}$$

where $J_0=\hbar^2/ab$ is referred to as the bandhead moment of inertia.

According to the above-mentioned expressions (or similar expressions) for rotational bands, which turned out to be valid for ND rotational bands, the variation of kinematic and dynamic moments of inertia of a rotational band with angular momentum (or angular frequency) should have the following properties (except $K=1/2$ bands and the case of significant bandmixing):

(A) $\lim_{\xi\to0} J^{(1)}=\lim_{\xi\to0} J^{(2)}=J_0$ (the bandhead moment of inertia).

(B) $J^{(1)}$ and $J^{(2)}$ *monotonically increase* with I (for $B<0$ or $\beta,b>0$), or *decrease with* I (for $B>0$ or $\beta,b<0$), and $d\ln J^{(2)}/d\xi\approx3d\ln J^{(1)}/d\xi$.

(C) Within the parametrizations considered in this paper, the $J^{(1)}$ vs ξ and $J^{(2)}$ vs ξ plots *never cross with each other at nonzero spins*.

(D) Within the parametrizations considered in this paper, $\lim_{\xi\to0} dJ^{(1)}/d\xi=\lim_{\xi\to0} dJ^{(2)}/d\xi=0$, i.e., as $\xi\to0$, $J^{(1)}$ vs ξ and $J^{(2)}$ vs ξ plots become *horizontal*.

(E) Within the parametrizations considered in this paper, both $J^{(1)}$ vs ξ and $J^{(2)}$ vs ξ plots are *concave upwards* (for $B<0$ or $\beta,b>0$) or *downwards* (for $B>0$ or $\beta,b<0$).

Thus, for an energy band which is considered as a rotational band, it is expected that the energy spectra satisfy (at least, approximately) the relations (3), (6), (10), etc., thus the extracted $J^{(1)}$ and $J^{(2)}$ by using Eqs. (1) and (2) using the experimental intraband transition energies should have these properties. It is found that for all ND rotational bands whose spins have been measured experimentally, the extracted $J^{(1)}$ and $J^{(2)}$ do exhibit these properties (except $K=1/2$ bands and significant bandmixing cases). However, if the spins are artificially increased or decreased by one or two, some of these properties will obviously disappear. Thus, whether the extracted $J^{(1)}$ and $J^{(2)}$ exhibit these properties may be used as a very useful guideline for the spin proposition. Some illustrative examples are given below. Examples 1, 2, and 3 are for ND rotational bands (see Figs. 1–3). Example 4 (see Fig. 4) is for the fission isomeric band in ^{240}Pu. Therefore, it seems reasonable to expect that the extracted $J^{(1)}$ and $J^{(2)}$ of

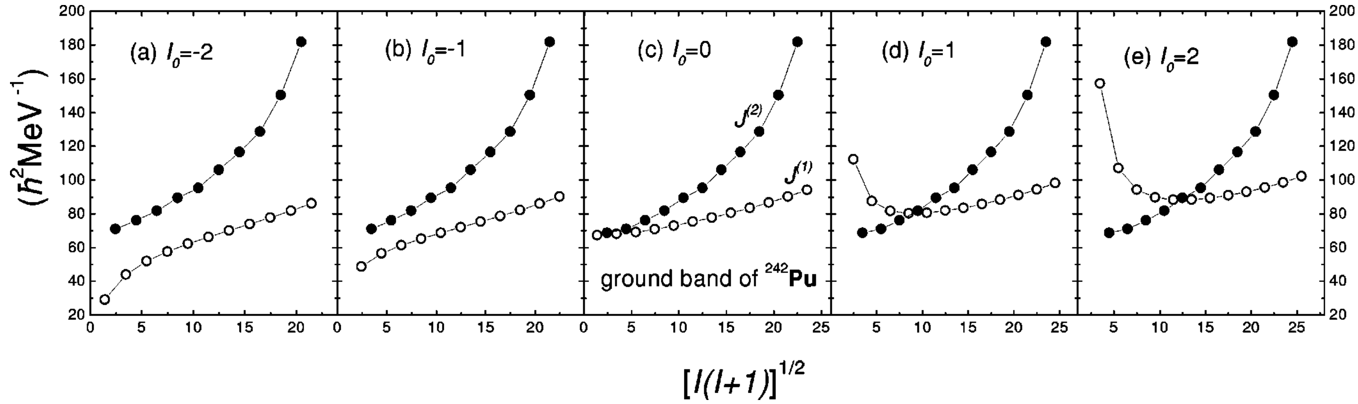

FIG. 1. $J^{(1)}$ and $J^{(2)}$ plots for the ground-state band of ^{242}Pu. The open and solid circles represent the $J^{(1)}$ and $J^{(2)}$ extracted by Eqs. (1) and (2) using the experimental intraband $E2$ transition energies. The shape of the $J^{(1)}$ plot depends on the spin proposition, but the shape of the $J^{(2)}$ plot does not. In (c) the experimental spin sequence $I=0,2,4,\ldots$ is adopted. In (d) and (e), the spin of each level is artificially increased by 1 and 2, respectively; i.e., the experimental spin sequence is replaced by $I=1,3,5,\ldots$ and $I=2,4,6,\ldots$, respectively. In (b) and (a), the spin of each level is artificially decreased by 1 and 2, respectively.

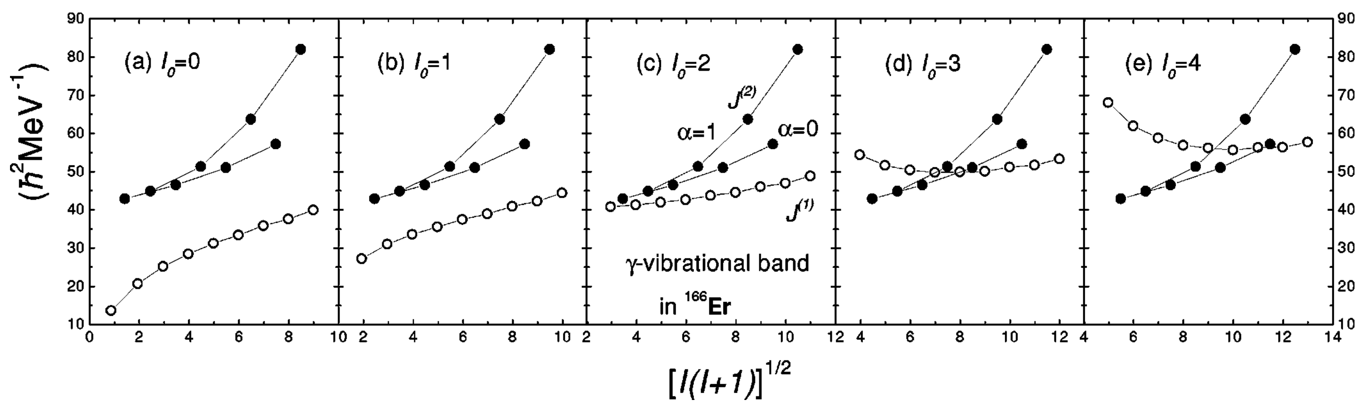

FIG. 2. The same as Fig. 1, but for the γ-vibrational band ($K^\pi=2^+$) of ^{166}Er. A significant signature splitting in $J^{(2)}$ is seen.

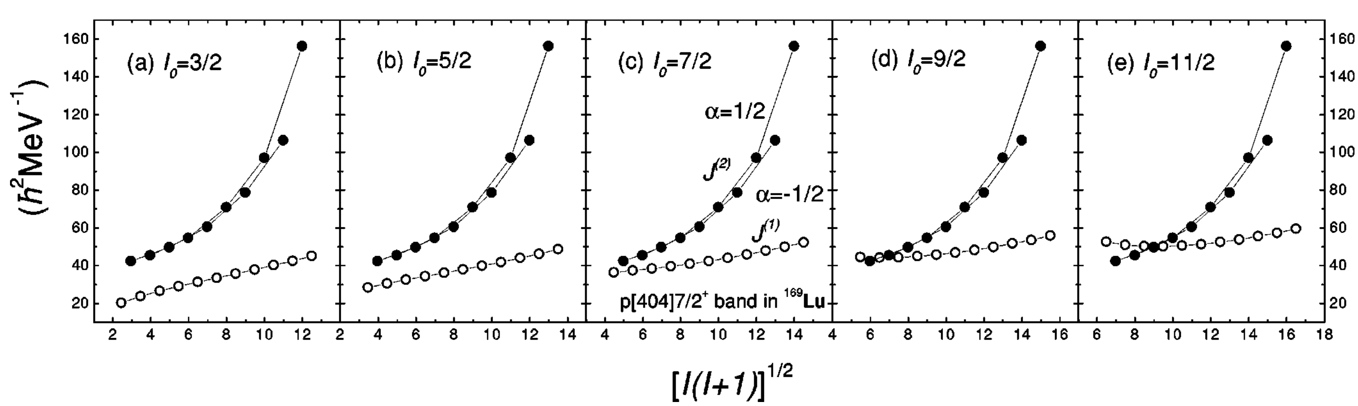

FIG. 3. The same as Fig. 1, but for the proton [404]7/2 band of ^{169}Lu. A very small signature splitting in $J^{(2)}$ is seen, which can be understood from the very weak Coriolis response of the low j and high Ω orbit [404]7/2.

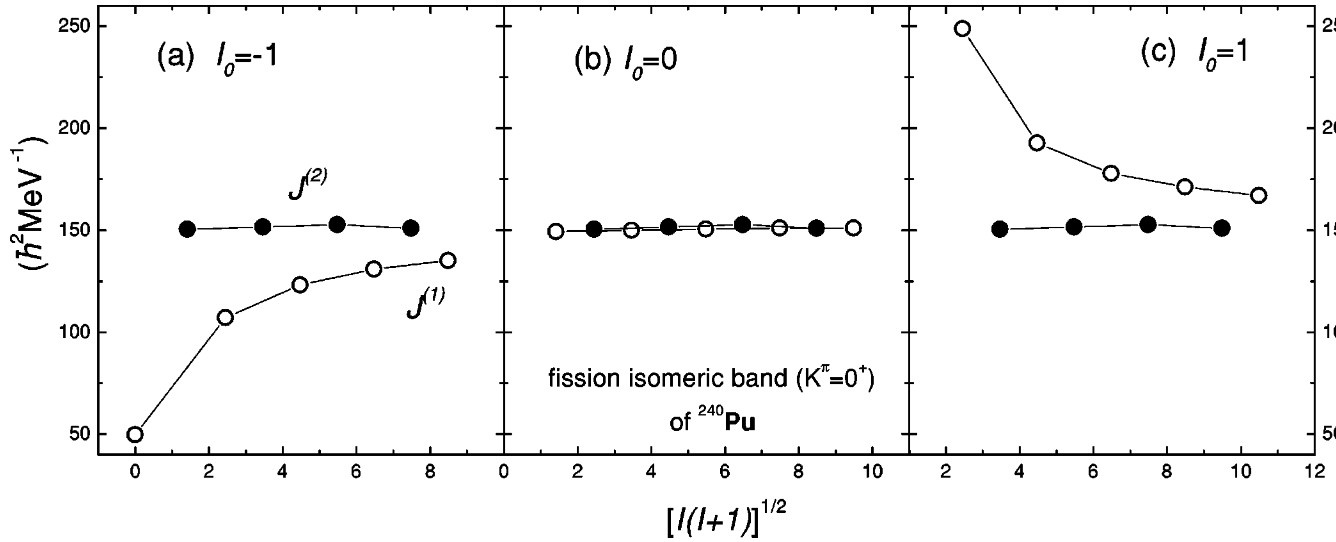

FIG. 4. $J^{(1)}$ and $J^{(2)}$ plots for the fission isomeric band in ^{240}Pu. In (b), the experimental spin sequence $I=0,2,4,6,8,10$ is adopted. In (c) [(a)], the spin of each level is artificially increased (decreased) by 1.

SD bands by Eqs. (1) and (2) using a correct spin proposition should exhibit these properties. It is encouraging to note that for the SD bands ^{194}Hg(1) and ^{194}Hg(3), the extracted $J^{(1)}$ and $J^{(2)}$ by (1) and (2) using the experimental E_γ's and spins do exhibit these properties (see Fig. 5).

1. The ground-state band of ^{242}Pu

In Fig. 1(c), the $J^{(1)}$ and $J^{(2)}$ of the ground-state band in ^{242}Pu are extracted by Eqs. (1) and (2) using the experimental transition energies and the measured spin sequence I

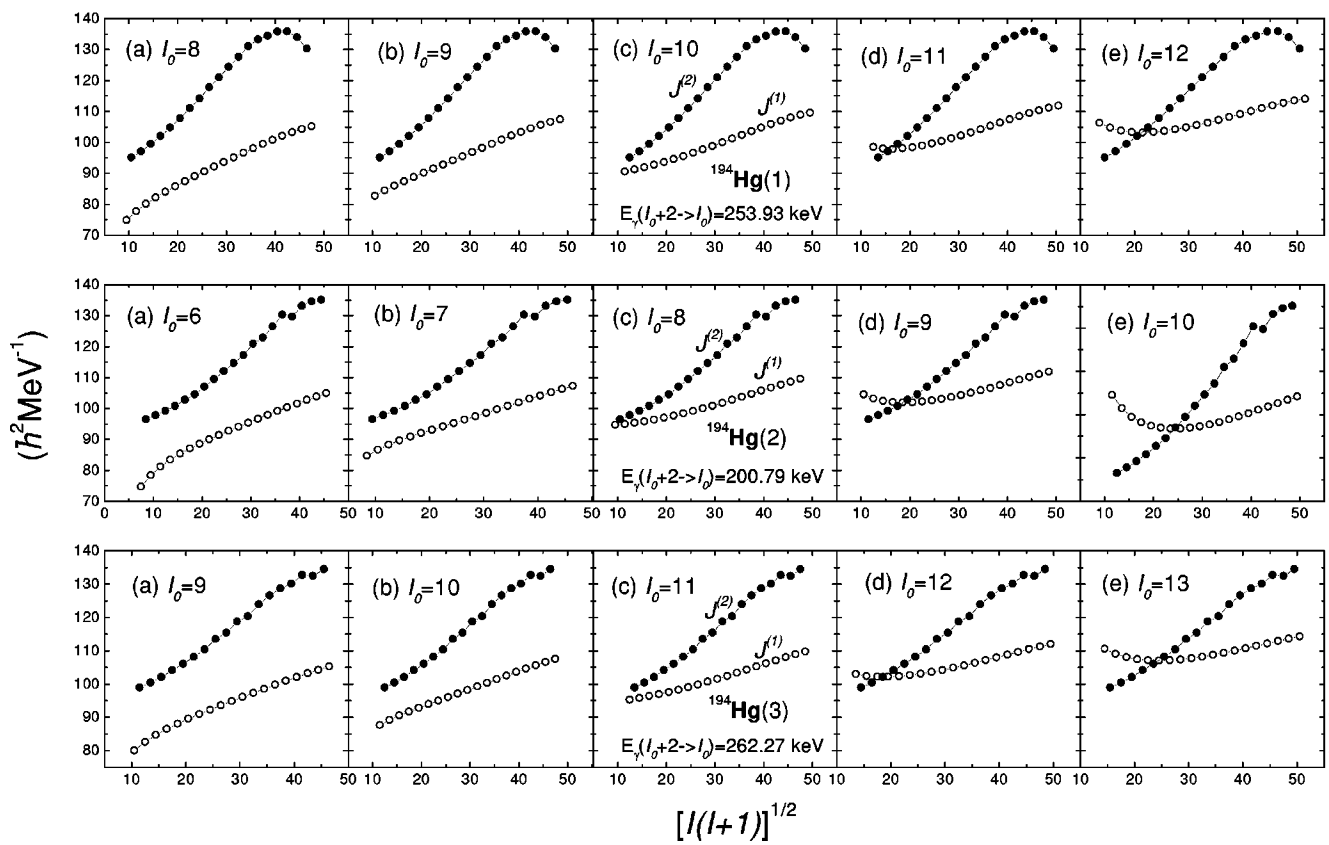

FIG. 5. $J^{(1)}$ and $J^{(2)}$ plots for the yrast SD band ^{194}Hg(1) and excited signature partner SD bands ^{194}Hg(2,3). The open and solid circles denote the $J^{(1)}$ and $J^{(2)}$ extracted by Eqs. (1) and (2) using the experimental intraband $E2$ transition energies. For ^{194}Hg(1) and ^{194}Hg(3), the spin sequences have been established experimentally [6,10], which are adopted in (c). It is seen that the variations of $J^{(1)}$ and $J^{(2)}$ with angular momentum do exhibit the five properties (a)–(e). In (d) and (e), the spin of each level is artificially increased by 1 and 2, respectively. It is seen that $J^{(1)}$ and $J^{(2)}$ plots cross with each other at nonzero spin. In (b) and (a), the spin of each level is artificially decreased by 1 and 2, respectively. It is seen that $J^{(1)}$ and $J^{(2)}$ do not tend to the same limit with decreasing I.

$=0,2,4,\ldots$. It is seen that all the five properties (a)–(e) display obviously. However, if the spin of each level is artificially increased by 1 [Fig. 1(d)] or 2 [Fig. 1(e)], i.e., the measured spin sequence $I=0,2,4,\ldots$ is replaced by $I=1,3,5,\ldots$, or $I=2,4,6,\ldots$, it is seen that, while the shape of the $J^{(2)}$ vs ξ plot remains unchanged, the shape and location of the $J^{(1)}$ vs ξ plot change significantly, and some of the five properties disappear obviously. Particularly, the $J^{(1)}$ vs ξ and $J^{(2)}$ vs ξ plots cross with each other and the extracted $J^{(1)}$ increases dramatically as $I\to 0$ rather than becomes horizontal. On the other hand, if the spin of each level is artificially decreased by 1 [Fig. 1(b)] or 2 [Fig. 1(a)], also some of the five properties disappear. Particularly, as $I\to 0$, $J^{(1)}$ and $J^{(2)}$ do not tend to the same limit. Moreover, while the $J^{(2)}$ vs ξ plot is always concave upwards, the $J^{(1)}$ vs ξ plot becomes concave downwards.

2. The γ-vibrational band ($K^\pi=2^+$) of ^{166}Er

In Fig. 2 is shown the analysis for the γ-vibrational band ($K^\pi=2^+$) of ^{166}Er. There exists an obvious signature splitting for the $J^{(2)}$ plot. It is clearly seen that when the measured spins are used [Fig. 2(c)], the extracted $J^{(1)}$ and $J^{(2)}$ plots do have the five properties (a)–(e). On the contrary, if the spins are artificially increased [Figs. 2(d) and 2(e)] or decreased [Figs. 2(b) and 2(a)], some of the five properties no longer exist.

3. The $K^\pi=7/2^+$ (proton [404]7/2) band of ^{169}Lu

In Fig. 3 is displayed the $J^{(1)}$ and $J^{(2)}$ plots for the ND band $K^\pi=7/2^+$ (proton [404]7/2) of odd-A nucleus ^{169}Lu. A slight signature splitting ($\alpha=\pm 1/2$) in the $J^{(2)}$ vs ξ plot is observed, which is easily understood because [404]7/2 is a low j and high Ω orbit [20]. When the measured spins are adopted [Fig. 3(c)], the angular momentum variation of $J^{(1)}$ and $J^{(2)}$ do have the five properties. In contrast, if incorrect spins [Figs. 3(a), 3(b), 3(d), and 3(e)] are assumed, some of these properties disappear obviously.

4. The fission isomeric band in ^{240}Pu

It is interesting to note that even for the fission isomeric bands (SD bands with low spins down to $I=0$ in the $A\sim 240$ region) the extracted $J^{(1)}$ and $J^{(2)}$ by Eqs. (1) and (2) using the measured spins [Fig. 4(b)] also exhibit the five properties (a)–(e).

5. The yrast SD band ^{194}Hg(1) and excited SD bands ^{194}Hg(2,3)

The spins of ^{194}Hg(1) and ^{194}Hg(3) have been measured experimentally. It is interesting to note that for the correct spin proposition [Fig. 5(c)] which is in agreement with experiment, the extracted $J^{(1)}$ and $J^{(2)}$ do exhibit the five properties (a)–(e) for $I\leqslant 40$, which is in agreement with the fact that the intraband transition energies (for $I\leqslant 40$) can be reproduced nicely by the ab expression (10) or the abc expression (13) (see Table I). The observed downturn of $J^{(2)}$ for $I>40$ implies that a significant change in the intrinsic structure of the SD band may have happened, then the E_γ's (for $I>40$) no longer nicely follow the same relation as that for the lower spins ($I\leqslant 40$).

The analyses for most SD bands in the $A\sim 190$ region are similar. The proposed spins for these SD bands are summarized in Tables IV–VII.

III. SYSTEMATICS OF BANDHEAD MOMENTS OF INERTIA

In [15,17] it was found that if the measured spins or correct spin propositions are adopted, the experimental intraband transition energies can be reproduced very well by the ab formula (10) or its improved version, the abc formula [15]

$$E(I)=a[\sqrt{1+bI(I+1)}-1]+cI(I+1), \tag{13}$$

in which the influence of the anharmonic (β^4) term of potential energy in the Bohr Hamiltonian has been taken into account, thus the abc formula is especially suitable for the description of SD bands with small axial asymmetry ($\sin^4 3\gamma \ll 1$). The corresponding kinematic and dynamic moments of inertia are

$$\hbar^2/J^{(1)}=ab[1+bI(I+1)]^{-1/2}+2c, \tag{14}$$

$$\hbar^2/J^{(2)}=ab[1+bI(I+1)]^{-3/2}+2c. \tag{15}$$

The bandhead moment of inertia is

$$J_0=\hbar^2/(ab+2c). \tag{16}$$

The examples given in Table I show that using the measured spins of the SD bands ^{194}Hg(1) and ^{194}Hg(3), the large number of E_γ's ($I\leqslant 40$) can be reproduced very nicely by the simple abc expression (13) with rms deviation $\chi\approx 0.30\times 10^{-3}$ and $\chi\approx 0.15\times 10^{-3}$, respectively, which are less than the usual experimental errors in transition energies. Similar situations are found for the SD bands ^{192}Hg(1) and ^{194}Hg(2) (see Table I) and most other SD bands in the $A\sim 190$ region, provided correct spin propositions are adopted. Therefore, it is reasonable to expect that the variation of $J^{(1)}$ and $J^{(2)}$ with angular momentum can be faithfully reproduced by Eqs. (14) and (15). The comparison of the $J^{(1)}$ and $J^{(2)}$ calculated by Eqs. (14) and (15) with those extracted by Eqs. (1) and (2) for the SD bands ^{192}Hg(1), ^{194}Hg(1,2,3) is displayed in Fig. 6, and a very satisfactory agreement is obtained. Therefore, it is expected that the extracted bandhead moment of inertia J_0 by Eq. (16) is meaningful, though an actual SD band may stop at low spins due to the change in its intrinsic structure (e.g., the SD barrier disappears). In fact, J_0 depends on the intrinsic structure of a rotational band and may be considered as another parametrization of observed transition energies [$J_0=\hbar^2/2A$ for Bohr-Mottelson's $I(I+1)$ expansion, $J_0=2\alpha$ for Harris' ω^2 expansion, $J_0=\hbar^2/(ab+2c)$ for the abc expression, etc.]. It is interesting to note that while the extracted J_0 values (16) for the yrast SD bands ^{194}Hg(1) and ^{194}Pb(1) are close to each other, $J_0[^{194}\mathrm{Hg}(1)]=88.6\hbar^2\ \mathrm{MeV}^{-1}$ and $J_0[^{194}\mathrm{Pb}(1)]=87.6\hbar^2\ \mathrm{MeV}^{-1}$, the extracted J_0 values of the excited SD

TABLE I. Comparison of the calculated and experimental E_γ's for the SD bands ^{192}Hg(1) and ^{194}Hg(1,2,3). The spins of the SD bands ^{194}Hg(1) and ^{194}Hg(3) have been measured experimentally by Khoo *et al.* [6] and Hackman *et al.* [10], respectively.

I	^{192}Hg(1), $\alpha=0$ $E_\gamma(I+2\to I)$ (keV)		^{194}Hg(1), $\alpha=0$ $E_\gamma(I+2\to I)$ (keV)		^{194}Hg(2), $\alpha=0$ $E_\gamma(I+2\to I)$ (keV)		^{194}Hg(3), $\alpha=1$ $E_\gamma(I+3\to I+1)$ (keV)	
	Expt. [33]	Calc.[a]	Expt. [22]	Calc.[b]	Expt. [22]	Calc.[c]	Expt. [22]	Calc.[d]
40	792.7	792.6	783.7	783.1	777.7	777.6	793.5	793.2
38	762.3	762.3	753.9	754.0	746.9	747.4	762.8	762.9
36	731.5	731.5	723.9	724.2	716.2	716.5	731.7	731.9
34	700.1	700.0	693.4	693.6	684.6	684.8	700.1	700.2
32	668.1	668.0	662.1	662.3	652.0	652.3	667.8	667.8
30	634.9	635.2	629.9	630.1	619.0	619.0	634.6	634.7
28	601.7	601.7	596.9	596.9	584.8	584.8	600.9	600.9
26	567.4	567.4	562.9	562.9	549.9	549.9	566.3	566.2
24	532.1	532.1	527.9	527.8	514.2	514.1	531.0	530.9
22	496.0	496.0	491.9	491.7	477.7	477.5	494.8	494.7
20	458.8	458.9	454.8	454.7	440.3	440.1	457.8	457.8
18	421.1	420.8	416.6	416.5	402.0	401.9	420.1	420.1
16	381.6	381.6	377.4	377.4	363.1	363.0	381.7	381.7
14	341.4	341.4	337.2	337.2	323.4	323.4	342.5	342.5
12	300.1	300.1	296.0	296.0	283.1	283.1	302.7	302.7
10	257.8	257.6	253.9	254.0	242.2	242.3	262.3	262.3
8	214.4	214.5			200.8	200.9		
	[a] $a=6474.55$ keV		[b] $a=11607.5$ keV		[c] $a=24126.9$ keV		[d] $a=16222.1$ keV	
	$b=8.31535\times10^{-4}$		$b=5.54489\times10^{-4}$		$b=2.91891\times10^{-4}$		$b=3.64544\times10^{-4}$	
	$c=3.04890$ keV		$c=2.41591$ keV		$c=1.81208$ keV		$c=2.37013$ keV	
	$\chi(\text{rms})=0.26\times10^{-3}$		$\chi(\text{rms})=0.30\times10^{-3}$		$\chi(\text{rms})=0.42\times10^{-3}$		$\chi(\text{rms})=0.15\times10^{-3}$	
	$J_0=87.1\hbar^2$ MeV^{-1}		$J_0=88.6\hbar^2$ MeV^{-1}		$J_0=93.8\hbar^2$ MeV^{-1}		$J_0=93.9\hbar^2$ MeV^{-1}	

band ^{194}Hg(3) is much larger, $J_0[^{194}\text{Hg}(3)]=93.9\hbar^2$ MeV^{-1}. This is quite like the situation in ND nuclei, i.e., the bandhead moments of inertia of ground-state (quasiparticle vacuum) bands of neighboring even-even nuclei are usually close to each other, but the J_0 values are systematically larger for excited bands than for ground-state bands. For example, the bandhead moments of inertia of the ground-state band of ^{168}Er is very close to that of ^{166}Er, $J_0(^{166}\text{Er})\approx37.0\hbar^2$ MeV^{-1}, $J_0(^{168}\text{Er})\approx37.5\hbar^2$ MeV^{-1}, but for the γ-vibrational ($K^\pi=2^+$) band of ^{168}Er, $J_0=40.0\hbar^2$ MeV^{-1}. It is important to note that if the spin proposition is increased (decreased) by 1, the extracted J_0 value will increase (decrease) by about 10% for SD bands in the $A\sim190$ region. Some illustrative examples are given in Table II (yrast SD bands in eight even-even nuclei) and Table III (three pairs of signature partner SD bands). In column 3 of Tables II and III, three suggested spins for each SD band are presented and the corresponding bandhead moments of inertia are given in column 6. The χ in column 4 is the relative rms deviation of the calculated transition ener-

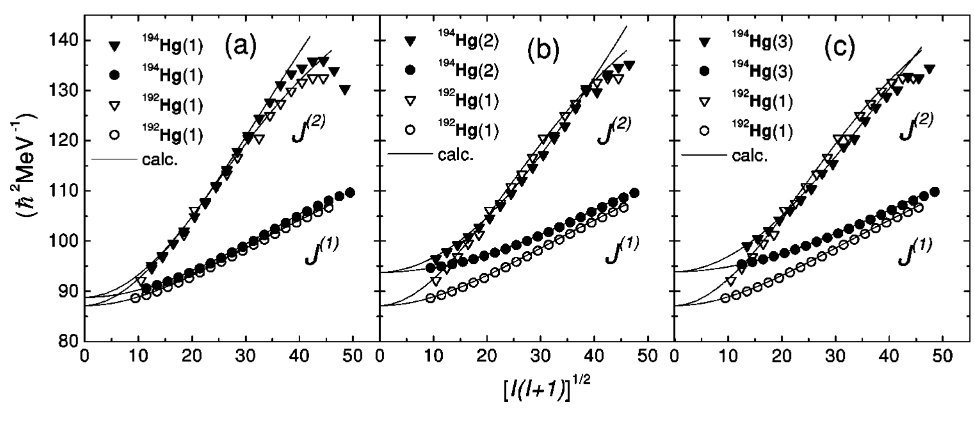

FIG. 6. Comparison between the experimental $J^{(1)}$ and $J^{(2)}$ extracted by Eqs. (1) and (2) and the calculated $J^{(1)}$ and $J^{(2)}$ by Eqs. (14) and (15). The experimental $J^{(1)}$ and $J^{(2)}$ of ^{192}Hg(1) are denoted by open circle and down triangle, respectively. The experimental $J^{(1)}$ and $J^{(2)}$ of ^{194}Hg(1,2,3) are denoted by solid circle and down triangle, respectively. The solid lines represent the calculated results using the parameters a, b, and c determined by the least squares fitting (see Table I).

TABLE II. Comparison of the spin propositions by the BFM, approach II, and the J_0 systematics for the yrast SD bands in even-even nuclei.

Yrast SD band	$E_\gamma(I_0+2\rightarrow I_0)$ (keV)	I_0	χ (10^{-3})	Approach II	J_0 ($\hbar^2$ MeV^{-1})
		5	3.42	no	68.9
^{198}Po(1)	175.91	**6**	**0.88**	**yes**	**84.2**
		7	9.06	no	94.5
		11	1.09	no	77.5
^{198}Pb(1)	305.1	**12**	**0.39**	**yes**	**86.8**
		13	2.84	no	93.8
		5	7.71	no	74.6
^{196}Pb(1)	171.5	**6**	**1.20**	**yes**	**87.2**
		7	13.75	no	95.4
		3	2.09	no	75.2
^{194}Pb(1)	124.9	**4**	**0.68**	**yes**	**87.6**
		5	6.11	no	96.6
		9	2.48	no	80.0
^{194}Hg(1)	253.93	**10**	**0.67**	**yes**	**88.6**
		11	4.36	no	96.2
		7	3.56	no	76.8
^{192}Hg(1)	214.4	**8**	**0.26**	**yes**	**87.1**
		9	5.98	no	95.4
		9	1.31	no	73.2
^{192}Pb(1)	262.5	**10**	**0.65**	**yes**	**84.5**
		11	1.64	no	93.8
		11	0.56	no	75.3
^{190}Hg(1)	316.9	**12**	**0.23**	**yes**	**82.5**
		13	0.33	no	89.1

gies by Eq. (13) with the experimental results. In column 5 is presented the analysis using approach II, where ‘‘yes’’ means the variations of the extracted $J^{(1)}$ and $J^{(2)}$ by Eqs. (1) and (2) with angular momentum do have the five properties (a)–(e), and ‘‘no’’ means some of the five properties no longer exist. It is seen, for these SD bands in Tables II and III, that the spin propositions by the three approaches are consistent with each other, and also in agreement with the

TABLE III. The same as Table II, but for three pairs of signature partner SD bands.

Signature partner SD bands	$E_\gamma(I_0+2\rightarrow I_0)$ (keV)	I_0	χ (10^{-3})	Approach II	J_0 ($\hbar^2$ MeV^{-1})
		7	4.50	no	83.3
^{194}Hg(2)	200.79	**8**	**0.42**	**yes**	**93.8**
		9	9.70	no	101.2
		10	2.21	no	84.9
^{194}Hg(3)	262.27	**11**	**0.15**	**yes**	**93.9**
		12	4.84	no	100.8
		8.5	1.92	no	85.2
^{193}Tl(1)	227.3	**9.5**	**0.81**	**yes**	**95.8**
		10.5	7.39	no	103.2
		7.5	3.15	no	85.3
^{193}Tl(2)	206.6	**8.5**	**1.13**	**yes**	**95.8**
		9.5	9.42	no	103.3
		9	1.45	no	86.6
^{192}Tl(c)	233.4	**10**	**0.86**	**yes**	**97.5**
		11	6.62	no	105.1
		8	2.06	no	86.0
^{192}Tl(d)	213.4	**9**	**0.61**	**yes**	**97.5**
		10	8.03	no	105.4

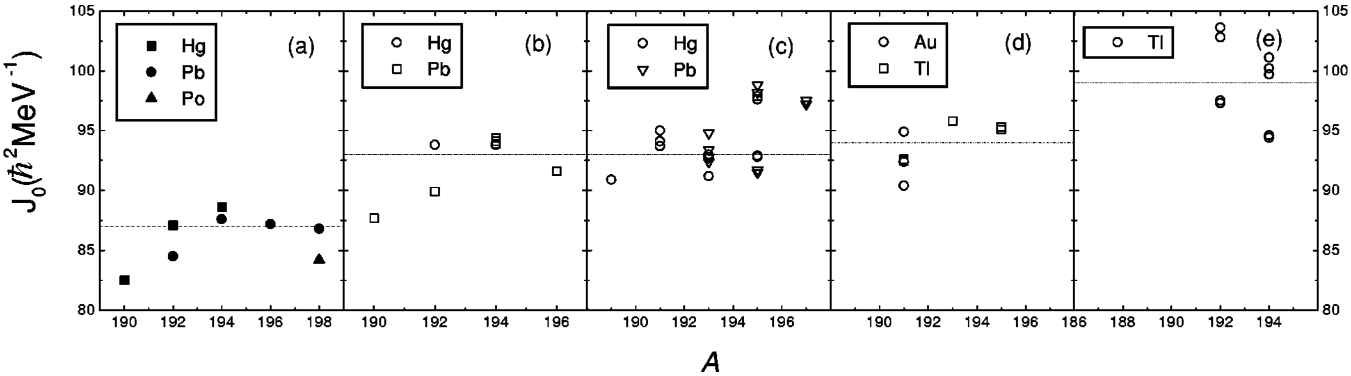

FIG. 7. The systematics of bandhead moments of inertia of SD bands in the $A \sim 190$ region. (a) yrast SD bands of even-even nuclei. (b) excited SD bands of even-even nuclei. (c) SD bands of odd-N nuclei. (d) SD bands of odd-Z nuclei. (e) SD bands of odd-odd nuclei.

experimentally measured spins for the SD bands ^{194}Hg(1), ^{194}Hg(3), and ^{194}Pb(1). The situations for most SD bands in the $A \sim 190$ region are similar. However, it is found that while it is hard to make a unique spin proposition by approach I (BFM) for some SD bands, one still may use the J_0 systematics in combination with approach II to make reasonable spin propositions (see discussions in Sec. IV). The J_0 systematics for SD bands in the $A \sim 190$ region is displayed in Fig. 7.

IV. RESULTS AND DISCUSSIONS

A. Proposed spins for SD bands in the $A \sim 190$ region

So far three approaches to predicting the spin of a rotational band by using the observed intraband γ transition energies are developed: (a) Approach I—the BFM (best-fit method), e.g., by Becker *et al.* [2], Zeng *et al.* [3], etc. (b) Approach II—investigating the variation of $J^{(1)}$ and $J^{(2)}$ extracted by Eqs. (1) and (2) with angular momentum. (c) Approach III—the J_0 (bandhead moments of inertia) systematics. All of them are based on the assumption that the energy spectra follow the usual available expressions for a rotational band [Bohr-Mottelson's $I(I+1)$ expansion (3), Harris ω^2 expansion (5), ab and abc expressions (10) and (13), the variable moment of inertia model [19], etc.] which had turned out to be suitable for describing ND bands.

The advantage of approach II to approach I is as follows. (1) The extracted $J^{(1)}(I\pm 1)$ and $J^{(2)}(I)$ depend only on the experimental values of $E_\gamma(I+2\rightarrow I)$ and $E_\gamma(I\rightarrow I-2)$, but are irrelevant to other transition energies and the number of transitions involved in the BFM. (2) Approach II can be managed very simply and no tedious least-squares calculation is needed. (3) While the magnitude of the rms deviation χ depends on which expression for the energy spectra and moments of inertia are used, the above-mentioned properties of the variation of $J^{(1)}$ and $J^{(2)}$ are common for all now available expressions for rotational spectra.

The advantage of approach III to approach I is as follows. On the one hand, for some SD bands, the rms deviation is not sensible to the spin proposition, i.e., the χ values are close to each other for two or more proposed spins, thus it is hard to make a unique spin proposition. On the other hand, the extracted J_0 values for SD bands in the $A \sim 190$ region are, in general, more sensitive than χ to the proposed spin, and from the J_0 systematics, unique spin propositions can be made for most SD bands in the $A \sim 190$ region.

The spins of most SD bands which are proposed consistently by the above mentioned three approaches are summarized in Tables IV (even-even nuclei), V (odd-N nuclei), VI (odd-Z nuclei) and VII (odd-odd nuclei). Some discussions are given below.

TABLE IV. Spin propositions for the SD bands in even-even nuclei and systematics of the bandhead moments of inertia. The I_0 values inside a bracket means the spin proposition has not been made very reliably. The spins of the SD bands ^{194}Hg(1) [6], ^{194}Hg(3) [10] and ^{194}Pb(1) [7] have been measured experimentally. The experimental data of transition energies are taken from [40,22,23,29–33,24,21].

Yrast SD band	$E_\gamma(I_0+2\rightarrow I_0)$ (keV)	I_0	J_0 ($\hbar^2$ MeV^{-1})	References
^{198}Po(1)	175.91	6	84.2	[29]
^{198}Pb(1)	305.1	12	86.8	[30]
^{196}Pb(1)	171.4	6	87.2	[31]
^{194}Pb(1)	124.9	4	87.6	[32]
^{194}Hg(1)	253.93	10	88.6	[22]
^{192}Hg(1)	214.4	8	87.1	[33]
^{192}Pb(1)	262.5	10	84.5	[24]
^{190}Hg(1)	316.9	12	82.5	[21]
Excited SD band				
^{196}Pb(2)	204.5	8	91.6	[31]
^{196}Pb(3)	226.7	9	91.6	[31]
^{194}Pb(2a)	241.2	10	94.4	[23]
^{194}Pb(2b)	260.9	11	94.1	[23]
^{194}Hg(2)	200.79	8	93.8	[40,22]
^{194}Hg(3)	262.27	11	93.9	[40,22]
^{192}Hg(2)	282.4	12	93.8	[33]
^{192}Hg(3)	333.1	14	89.9	[33]
^{190}Hg(2)	481.1	(23)	88.6	[21]
^{190}Hg(3)	318.0	13	87.7	[21]
^{190}Hg(4)	446.3	(20)	92.6	[21]

TABLE V. The same as Table IV, but for the SD bands in odd-N nuclei. The experimental data of transition energies are taken from [30,36,28,38,27,26,41].

SD band	$E_\gamma(I_0+2\rightarrow I_0)$ (keV)	I_0	J_0 ($\hbar^2$ MeV^{-1})	References
^{197}Pb(1)	184.4	7.5	97.5	[30]
^{197}Pb(2)	205.5	8.5	97.2	[30]
^{195}Pb(1)	182.13	7.5	98.8	[36]
^{195}Pb(2)	162.58	6.5	98.2	[36]
^{195}Pb(3)	236.19	9.5	91.5	[36]
^{195}Pb(4)	213.58	8.5	91.7	[36]
^{193}Pb(3)	250.6	10.5	94.8	[28]
^{193}Pb(4)	273.0	11.5	93.4	[28]
^{193}Pb(5)	212.9	8.5	92.8	[28]
^{193}Pb(6)	234.1	9.5	92.4	[28]
^{195}Hg(a)	333.9	14.5	92.8	[38]
^{195}Hg(b)	273.9	11.5	92.9	[38]
^{195}Hg(c)	284.5	12.5	97.6	[38]
^{195}Hg(d)	341.9	15.5	97.9	[38]
^{193}Hg(1)	233.2	9.5	92.7	[27]
^{193}Hg(2a)	254.0	10.5	93.0	[27]
^{193}Hg(2b)	254.0	10.5	93.0	[27]
^{193}Hg(3)	233.5	9.5	92.8	[27]
^{191}Hg(2)	252.4	10.5	94.1	[26]
^{191}Hg(3)	272.0	11.5	93.7	[26]
^{193}Pb(1)	277.2	11.5	92.0	[28]
^{193}Pb(2)	190.5	(7.5)	94.1	[28]
^{193}Hg(4)	291.0	(12.5)	92.3	[27]
^{193}Hg(5)	240.5	9.5	91.2	[27]
^{191}Hg(1)	310.9	13.5	95.0	[26]
^{189}Hg(1)	366.4	15.5	90.9	[41]

(1) ^{190}**Hg(2)**

According to the linking transitions between the SD bands ^{190}Hg(2) and ^{190}Hg(1), the spin of the lowest level observed in ^{190}Hg(2) was assigned to be $I_0=23$ $[E_\gamma(I_0+2\rightarrow I_0)=481.1$ keV] by Wilson *et al.* [21]. Analysis shows that it is hard to make a unique spin proposition by approaches I and II. From approach III, $I_0=23$ is the most reasonable choice, because for $I_0=23$, $J_0=88.6\hbar^2$ MeV^{-1}, which is consistent with the J_0 systematics (see Table II), but for $I_0=22$, $J_0=62.4\hbar^2$ MeV^{-1} which is much smaller than that in neighboring nuclei, and for $I_0=24$, $J_0=99.8\hbar^2$ MeV^{-1}, which is much larger than that in neighboring nuclei.

(2) ^{190}**Hg(3)**

For the SD band ^{190}Hg(3), $I_0=14$ $[E_\gamma(I_0+2\rightarrow I_0)=318.0$ keV] was suggested in [21] using the BFM by Becker *et al.* Analysis shows that, though it is hard to make a unique spin proposition by approach I, according to approach II and the J_0 systematics, $I_0=13$ is the most plausible choice:

I_0	$\chi(10^{-3})$	approach II	$J_0(\hbar^2$ MeV$^{-1})$
12	1.03	no	78.3
13	**0.65**	**yes**	**87.7**
14	0.74	no	95.8

TABLE VI. The same as Table IV, but for the SD bands in odd-Z nuclei. The experimental data of transition energies are taken from [39,44].

SD band	$E_\gamma(I_0+2\rightarrow I_0)$ (keV)	I_0	J_0 ($\hbar^2$ MeV^{-1})	References
^{197}Bi(1)	166.2	6.5	95.8	[37,39]
^{197}Bi(2)	186.7	7.5	95.7	[37,39]
^{195}Tl(1)	146.2	5.5	95.3	[44]
^{195}Tl(2)	167.5	6.5	95.1	[44]
^{193}Tl(1)	227.3	9.5	95.8	[45]
^{193}Tl(2)	206.6	8.5	95.8	[45]
^{191}Tl(1)	358.9	13.5	92.6	[46]
^{191}Tl(2)	417.2	16.5	92.5	[46]
^{191}Au(2)	397.8	17.5	92.4	[47]
^{191}Au(3)	382.7	16.5	90.4	[47]
^{191}Au(1)	186.8	7.5	94.9	[47]

(3) ^{190}**Hg(4)**

For the SD band ^{190}Hg(4), a bandcrossing was observed in the lower spin states and no spin was proposed in [21]. We find that though it is hard to make a unique spin proposition by approaches I and II, from the J_0 systematics, $I_0=20$ $[E_\gamma(I_0+2\rightarrow I_0)=446.3$ keV] is the most reasonable choice:

I_0	$\chi(10^{-3})$	$J_0(\hbar^2$ MeV$^{-1})$
19	16.2	79.0
20	**16.2**	**92.6**
21	16.2	124.4

(4) ^{192}**Pb(1)**

Using the BFM by Becker *et al.*, in [24] the spin of the lowest level of ^{192}Pb(1) was suggested to be $I_0=10$ or 11 $[E_\gamma(I_0+2\rightarrow I_0)=262.5$ keV]. According to our analysis using the three approaches, the most reliable choice is $I_0=10$ rather than $I_0=11$, i.e., the signature is $\alpha=0$. Recently, the spin $I_0=10$ was assigned by McNabb *et al.* [25] according to the observed 2058 keV linking transition ($E1$) between the

TABLE VII. The same as Table IV, but for the SD bands in odd-odd nuclei. The experimental data of transition energies are taken from [48,35].

SD band	$E_\gamma(I_0+2\rightarrow I_0)$ (keV)	I_0	J_0 ($\hbar^2$ MeV^{-1})	References
^{194}Tl(1a)	268.0	12	99.7	[48]
^{194}Tl(1b)	209.3	9	99.7	[48]
^{194}Tl(2a)	240.5	10	94.6	[48]
^{194}Tl(2b)	220.3	9	94.4	[48]
^{194}Tl(3a)	187.9	8	100.2	[48]
^{194}Tl(3b)	207.0	9	101.1	[48]
^{192}Tl(a)	283.0	13	102.8	[35]
^{192}Tl(b)	337.5	16	103.6	[35]
^{192}Tl(c)	233.4	10	97.3	[35]
^{192}Tl(d)	213.4	9	97.5	[35]

SD band ^{192}Pb(1) and the yrast 9^- state. It was pointed out [25] there is every expectation that the yrast SD band ^{192}Pb(1) should have positive parity and even spin, because the yrast SD band in neighboring even-even nucleus ^{194}Pb(1) have been measured experimentally [7] to have positive parity and even spin. Moreover, no signature partner of ^{192}Pb(1) is observed, which argues that the band is built on a $K=0$ bandhead:

I_0	$\chi(10^{-3})$	approach II	$J_0(\hbar^2\ \mathrm{MeV}^{-1})$
9	1.31	no	73.2
10	**0.65**	**yes**	**84.5**
11	1.60	no	93.8

(5) ^{191}**Hg(4)**

The SD band ^{191}Hg(4) was considered as the signature partner of the yrast SD band ^{191}Hg(1) in [26], and the spin of its lowest level observed was proposed to be $I_0=12.5$ $[E_\gamma(I_0+2\rightarrow I_0)=280.9\ \mathrm{keV}]$. However, our analysis shows that the most plausible choice is $I_0=10.5$ rather than $I_0=12.5$. In fact, according to approach II, all $I_0\geqslant 11.5$ are not reasonable because $J^{(1)}$ vs ξ plot and $J^{(2)}$ vs ξ plot cross with each other. However, for $I_0=10.5$, the bandhead moment of inertia is $J_0=85.0\hbar^2\ \mathrm{MeV}^{-1}$, which is quite different from that of ^{191}Hg(1), $J_0=95.0$. Thus, the spin of ^{191}Hg(4) needs further investigation:

I	$\chi(10^{-3})$	approach II	$J_0(\hbar^2\ \mathrm{MeV}^{-1})$
9.5	1.73	no	76.7
10.5	**0.75**	**yes**	**85.0**
11.5	3.44	no	91.1
12.5	7.18	no	97.2

(6) ^{193}**Hg(4)**

In [27] the SD band ^{193}Hg(4) was considered as the signature partner of ^{193}Hg(5), and $I_0=13.5$ $[E_\gamma(I_0+2\rightarrow I_0)=291.0\ \mathrm{keV}]$ was suggested. No reliable spin proposition can be made by approach II, because an obvious bandcrossing is seen at the lower spin states of ^{193}Hg(4) in the $J^{(2)}$ vs ξ plot. Using approach I, also no unique spin proposition can be made. However, from the J_0 systematics, $I_0=12.5$ seems most reasonable and the corresponding bandhead moment of inertia $J_0=92.3\hbar^2\ \mathrm{MeV}^{-1}$ is close to that of ^{193}Hg(5), $J_0=91.2$:

^{193}Hg(4)			^{193}Hg(5)		
I_0	χ (10^{-3})	J_0 $(\hbar^2\ \mathrm{MeV}^{-1})$	I_0	χ (10^{-3})	J_0 $(\hbar^2\ \mathrm{MeV}^{-1})$
11.5	0.58	87.4	8.5	4.39	81.0
12.5	**0.63**	**92.3**	**9.5**	**1.68**	**91.2**
13.5	0.70	97.0	10.5	6.91	98.0

(7) ^{193}**Pb(1,2)**

In [28] ^{193}Pb(1,2) were considered as a pair of signature partner SD bands, and $I_0=13.5$ $[E_\gamma(I_0+2\rightarrow I_0)=277.2\ \mathrm{keV}]$ for ^{193}Pb(1) and $I_0=8.5$ $[E_\gamma(I_0+2\rightarrow I_0)=190.5\ \mathrm{keV}]$ for ^{193}Pb(2) were suggested. Our analysis shows that $I_0=13.5$ is in contradiction to approach II and the J_0 systematics. Recently, according to the linking transitions between ^{193}Pb(1) and several yrast ND states with known spins, the spin and parity assignment $I_0^\pi=11.5^-$ was made by Perris *et al.* [34]. This assignment is consistent with the J_0 systematics. If ^{193}Pb(2) is considered as the signature partner of ^{193}Pb(1), I_0 should be 8.5 or 6.5 for ^{193}Pb(2), and the corresponding $J_0=102.5$ or $79.6\hbar^2\ \mathrm{MeV}^{-1}$, which are quite different from that of ^{193}Pb(1) $(J_0=92.0\hbar^2\ \mathrm{MeV}^{-1})$. Thus, it is hard to consider them as a pair of signature partners. On the other hand, from the J_0 systematics, $I_0=7.5$ for ^{193}Pb(2) is reasonable. If so, ^{193}Pb(2) is no longer a signature partner of ^{193}Pb(1):

^{193}Pb(1)			^{193}Pb(2)		
I_0	$\chi(10^{-3})$	$J_0(\hbar^2\ \mathrm{MeV}^{-1})$	I_0	$\chi(10^{-3})$	$J_0(\hbar^2\ \mathrm{MeV}^{-1})$
10.5	1.54	79.0	6.5	3.22	79.6
11.5	**1.21**	**92.0**	**7.5**	**3.03**	**94.1**
12.5	2.38	100.5	8.5	13.2	102.5

(8) ^{192}**Tl(a,b)**

In [35] $I_0=15$ $[E_\gamma(I_0+2\rightarrow I_0)=283.0\ \mathrm{keV}]$ for ^{192}Tl(a) and $I_0=18$ $[E_\gamma(I_0+2\rightarrow I_0)=337.5\ \mathrm{keV}]$ for ^{192}Tl(b) were suggested by Fischer *et al.* However, from the J_0 systematics and approach I, $I_0=13$ for ^{192}Tl(a) and $I_0=16$ for ^{192}Tl(b) are reasonable. It is noted that for such spin propositions, both $J^{(1)}$ and $J^{(2)}$ keep almost constant with increasing spin, which can be understood from the double-blocking effect of the unpaired proton and neutron:

^{192}Tl(a)			^{192}Tl(b)		
I_0	$\chi(10^{-3})$	$J_0(\hbar^2\ \mathrm{MeV}^{-1})$	I_0	$\chi(10^{-3})$	$J_0(\hbar^2\ \mathrm{MeV}^{-1})$
12	3.37	94.0	15	1.35	94.9
13	**1.45**	**102.8**	**16**	**0.70**	**103.6**
14	3.23	109.9	17	6.20	108.4
15	6.56	117.2	18	4.44	116.5

(9) SD bands in Bi isotopes

In [37] by Clark *et al.*, two SD bands were established in Bi, but their isotopic assignment to ^{197}Bi is tentative. In [39] the

SD band 1 [$E_\gamma(I_0+2\rightarrow I_0)=166.2$ keV] was tentatively suggested to belong to ^{196}Bi and the SD band 2 [$E_\gamma(I_0+2\rightarrow I_0)=186.7$ keV] was suggested to belong to ^{197}Bi. According to approach II and III, $I_0\leqslant 5.5$ and $I_0\geqslant 7.0$ for SD band 1 are definitely unreasonable. Similarly, for SD band 2, $I_0\leqslant 6.5$ and $I_0\geqslant 8.0$ are also unreasonable. For SD band 1, $I_0=6.5$ seems more reasonable than $I_0=6.0$ both from approach II and the J_0 systematics of odd-Z nuclei (Table VI) and odd-odd nuclei (Table VII). Similarly, for SD band 2, $I_0=7.5$ seems more reasonable than $I_0=7.0$. Therefore, our analysis supports the isotopic assignment given in [37] and both SD bands may be reasonably considered as a pair of signature partner, of which the extracted bandhead moments of inertia are nearly the same, $J_0=95.8\hbar^2$ MeV^{-1}:

SD band 1				SD band 2			
$E_\gamma(I_0+2\rightarrow I_0)=166.2$ keV				$E_\gamma(I_0+2\rightarrow I_0)=186.7$ keV			
I_0	$\chi(10^{-3})$	approach II	$J_0(\hbar^2$ MeV$^{-1})$	I_0	$\chi(10^{-3})$	approach II	$J_0(\hbar^2$ MeV$^{-1})$
5.0	2.20	no	67.3	6.0	1.69	no	71.2
5.5	1.20	no	78.6	6.5	1.59	no	80.6
6.0	1.52	no	88.8	7.0	1.42	yes	89.2
6.5	**1.36**	**yes**	**95.8**	**7.5**	**1.75**	**yes**	**95.7**
7.0	8.01	no	100.4	8.0	5.93	no	100.3
7.5	14.1	no	105.1	8.5	14.7	no	103.5

Another SD band [$E_\gamma(I_0+2\rightarrow I_0)=261.5$ keV] reported in [39] was suggested to belong to ^{195}Bi. According to approach II and the J_0 systematics, $I_0=11$ seems the most reasonable proposition. Thus, this band seems to belong to ^{196}Bi. However, its spin proposition and isotope assignment need further investigation:

SD band [$E_\gamma(I_0+2\rightarrow I_0)=261.5$ keV]			
I_0	$\chi(10^{-3})$	approach II	$J_0(\hbar^2$ MeV$^{-1})$
10.0	1.37	no	82.2
10.5	1.24	yes	88.8
11.0	1.01	yes	94.5
11.5	1.83	no	98.6
12.0	3.31	no	102.8
12.5	7.67	no	105.4

B. Discussions about the J_0 systematics

From Tables IV–VII and Fig. 7, some useful information about the J_0 systematics for SD bands in the $A\sim190$ region can be obtained as follows.

1. J_0 systematics of yrast SD bands in even-even nuclei

For the yrast SD bands of even-even nuclei in the $A\sim190$ mass region, the extracted bandhead moments of inertia are close to each other, $J_0\sim(85\pm3)\hbar^2$ MeV^{-1}. If the suggested spin I_0 of each band is artificially increased by 1, J_0 will increase by about 10% ($J_0\sim95\hbar^2$ MeV^{-1}), which is much larger than the J_0 values of the yrast SD bands ^{194}Hg(1) and ^{194}Pb(1) ($J_0\sim88\hbar^2$ MeV^{-1}, see Table II), but is close to the J_0 values of excited SD bands [see Table IV and Fig. 7(b)]. On the contrary, if the suggested spin I_0 of each band is artificially decreased by 1, J_0 will decrease by about 10% ($J_0\sim74\hbar^2$ MeV^{-1}), which seems very unreasonable from the J_0 systematics. Therefore, we believe the spin propositions for the yrast SD bands of even-even nuclei given in Table II are reliable. It is interesting to note that, like ground-state bands observed in all ND even-even nuclei, *the signature of all yrast SD bands of even-even nuclei in the $A\sim190$ region*, according to the present analysis, *is* $\alpha=0$ rather than $\alpha=1$ (see Table II).

2. J_0 systematics of SD bands in odd-A nuclei

The bandhead moments of inertia of one-quasiparticle SD bands in odd-A nuclei, $J_0\sim(94\pm4)\hbar^2$ MeV^{-1}, are *systematically larger than* that of the yrast (quasiparticle vacuum) SD bands in neighboring even-even nuclei. The odd-even differences in bandhead moments of inertia in ND nuclei have been investigated in detail [20]. The odd-even differences in bandhead moments of inertia δJ_0 are mainly attributed to the blocking effect, which can be properly treated by the particle-number-conserving (PNC) treatment [20] of the cranked shell model Hamiltonian. Experimental results of ND nuclei show [11] that there exists very large fluctuation in δJ_0. According to the PNC calculation, the odd-even difference δJ_0 depends on the following. (a) The energy position of the single-particle level occupied by the odd particle. (b) The Coriolis response of the blocked single particle level. In particular, for an intruder high $j(N)$ and low Ω orbit, $\delta J_0/J_0$ may be very large. On the contrary, for a low j and high Ω orbit (e.g., proton [402]5/2, [404]7/2), $\delta J_0/J_0$ is very small, which may result in almost identical bands observed in ND neighboring odd-A and even-even nuclei. It is seen from Figs. 7(c) and 7(d) that in SD nuclei there also exists significant fluctuation in the differences in bandhead moments of inertia. From the observed $\delta J_0/J_0$, valuable information about the properties of the single-particle orbit occupied by an odd nucleon can be obtained.

3. J_0 systematics of excited SD bands in even-even nuclei

The J_0 values of excited SD bands in even-even nuclei (Hg, Pb), $J_0\sim93\hbar^2$ MeV^{-1}, are *systematically larger than* that of the corresponding yrast SD band by about 7%, and $\delta J_0=J_0$ (excited SD band)$-J_0$(yrast SD band) $\sim6\hbar^2$ MeV^{-1}. From Fig. 7 it is seen that while there exist large fluctuations in the bandhead moments of inertia of the SD bands in odd-N nuclei [Fig. 7(c)] and odd-odd nuclei [Fig. 7(e)], the J_0 systematics of the excited SD bands in even-even Hg and Pb nuclei display *rather regular behavior* [see Fig. 7(b) and Table IV], which seems to support the assumption [10] that some of the excited SD bands in even-even Hg and Pb nuclei may be vibrational excited bands.

If the excited SD bands in even-even nuclei are considered as two-quasiparticle SD bands [e.g., ^{194}Hg(2,3) were considered as ^{192}Hg(core)$\otimes[(\nu 624)9/2]\otimes[(\nu 512)5/2]$ in [40]], the difference δJ_0 may attribute to the blocking effects of two unpaired particles. For ND even-even nuclei in the rare-earth and actinide regions, it has been well established that the bandhead moments of inertia of excited two-quasiparticle bands are in general larger than that of the ground (quasiparticle vacuum) bands due to the blocking effects of two unpaired particles, and the difference in J_0 depends mainly on the Coriolis response of the single particle states occupied by two unpaired particles. Calculation [42] showed that, to account for the difference in the bandhead moments of inertia, $\delta J_0 = J_0(^{194}\text{Hg}(3)) - J_0(^{192}\text{Hg}(1)) \simeq 6.8\hbar^2\ \text{MeV}^{-1}$, the pairing interaction strength is much weaker in SD nuclei than in ND nuclei. According to Hackman *et al.* [10], the excited SD band ^{194}Hg(3) is an octupole vibrational band with $K^\pi = 2^-$, rather than a two-quasiparticle band. Because a vibrational state may be microscopically considered as a coherent superposition of a lot of two-quasiparticle states with the same spin and parity, the rather regular behavior of δJ_0 may be understandable from the average of blocking effects over a lot of single-particle orbits. It was well established [11] that the bandhead moments of inertia of γ-vibrational bands in even-even ND nuclei in the rare-earth and actinide regions are systematically larger than those of the ground bands and display rather regular behavior. For example, for the 821.1 keV $K^\pi = 2^+$ γ-vibrational band in ^{168}Er, $J_0 = 40.0\hbar^2\ \text{MeV}^{-1}$, but for the ground band, $J_0 = 37.5\hbar^2\ \text{MeV}^{-1}$, which is about 7% smaller than for the γ-vibrational band. The situation is similar for ND octupole vibrational bands.

4. J_0 systematics of SD bands in odd-odd nuclei

The bandhead moments of inertia of SD bands observed in odd-odd nuclei are *much larger than* ($>10\%$) that of the yrast SD bands in neighboring even-even nuclei. However, like the situation observed in ND odd-odd nuclei, there exist large fluctuations in J_0 values of these SD bands. Thus, it seems reasonable to consider these SD bands as two-quasiparticle bands, whose bandhead moments of inertia depend sensitively on the Coriolis response of the single-particle orbits occupied by the odd neutron and odd proton.

5. J_0 systematics of signature partner SD bands

It is seen that overwhelming majority of SD bands observed in odd-A nuclei (see Tables V and VI) and odd-odd nuclei (Table VII) and excited SD bands in even-even nuclei (Table IV) are signature partner SD bands. It is interesting to note that *the bandhead moments of inertia of each signature partner SD bands are almost identical*. In fact, in most cases, $\delta J_0/J_0 \approx 10^{-3}$. For examples, $J_0(^{194}\text{Hg}(2)) = J_0(^{194}\text{Hg}(3)) = 93.9\hbar^2\ \text{MeV}^{-1}$, $J_0(^{195}\text{Hg(a)}) = J_0(^{195}\text{Hg(b)}) = 92.9\hbar^2\ \text{MeV}^{-1}$, $J_0(^{196}\text{Pb}(2)) = J_0(^{196}\text{Pb}(3)) = 91.6\hbar^2\ \text{MeV}^{-1}$, $J_0(^{195}\text{Pb}(3)) = J_0(^{195}\text{Pb}(4)) = 91.6\hbar^2\ \text{MeV}^{-1}$, $J_0(^{193}\text{Tl}(1)) = J_0(^{193}\text{Tl}(2)) = 95.8\hbar^2\ \text{MeV}^{-1}$, $J_0(^{194}\text{Tl(1a)}) = J_0(^{194}\text{Tl(1b)}) = 99.7\hbar^2\ \text{MeV}^{-1}$, etc. Therefore, if there exists a significant difference in J_0 values of two SD bands, it is very hard to consider them as a pair of signature partner.

6. J_0 systematics of ''identical'' SD bands

The above analysis is conducive to understanding the implication for identical bands. The yrast SD band ^{192}Hg(1) was considered by Stephens *et al.* [43] as identical to the excited SD band ^{194}Hg(3), because the observed sequence of $E2$ transition energies are almost identical ($\delta E_\gamma/E_\gamma \sim 10^{-3}$) for the two SD bands (see columns 2 and 8 of Table I) within the frequency range $\hbar\omega \sim 0.2-0.4$ MeV (or $I \sim 20-40$), which implies their dynamical moments of inertia $J^{(2)}$ are almost identical in this frequency range [see Fig. 7(c)]. However, it is seen that at lower frequencies ($\hbar\omega \leqslant 0.2$ MeV, or $I \leqslant 20$), there exists obvious difference in $J^{(2)}$ of ^{194}Hg(3) and ^{192}Hg(1). Moreover, according to the experimental spin of ^{194}Hg(3) and the reliable spin proposition of ^{192}Hg(1), there exists significant difference in $J^{(1)}$ of ^{194}Hg(3) and ^{192}Hg(1) in the whole frequency range [see Fig. 7(c)]. Particularly, the bandhead moments of inertia (which depend intimately on the intrinsic structures of rotational bands) of the SD bands ^{194}Hg(3) and ^{192}Hg(1) are quite different, $J_0(^{194}\text{Hg}(3)) = 93.9\hbar^2\ \text{MeV}^{-1} \gg J_0(^{192}\text{Hg}(1)) = 87.1\hbar^2\ \text{MeV}^{-1}$. For two truly identical bands, it seems necessary that both bands have the same bandhead moments of inertia. Thus, it seems difficult to understand the implication of ''identity'' for the SD bands ^{192}Hg(1) and ^{194}Hg(3). The larger $J^{(2)}$ for ^{194}Hg(3) than for ^{192}Hg(1) at lower frequency ($\hbar\omega \leqslant 0.2$ MeV) and the larger $J^{(1)}$ for ^{194}Hg(3) than for ^{192}Hg(1) within the *whole* frequency range were explained microscopically [40,42,49–51] by pairing reduction due to blocking. Phenomenologically, the ω variation of the difference in angular momentum alignments

$$i = \Delta I_x = I_x(^{194}\text{Hg}(3)) - I_x(^{192}\text{Hg}(1)) = \omega[J^{(1)}(^{194}\text{Hg}(3)) - J^{(1)}(^{192}\text{Hg}(1))] \quad (17)$$

is shown in Fig. 8, which is taken from [49]. It is noted that a similar plot was given in Fig. 4(b) of [10]. It is seen that $i \approx 0$ at $\omega = 0$, but i increases *gradually* with ω, and $i \approx 1$ for $\hbar\omega \approx 0.2-0.4$ MeV.

V. SUMMARY

In addition to the usually adopted approach (BFM) to the spin proposition of SD bands by using the experimental intraband transition energies, other two approaches to the spin prediction of a rotational band are developed. All three approaches are based on the assumption that the considered energy band can be described by the available expressions for rotational spectrum [e.g., Bohr-Mottelson's $I(I+1)$ expansion, Harris' ω^2 expansion, variable moment of inertia model, ab and abc expression, etc.]. The advantages of the

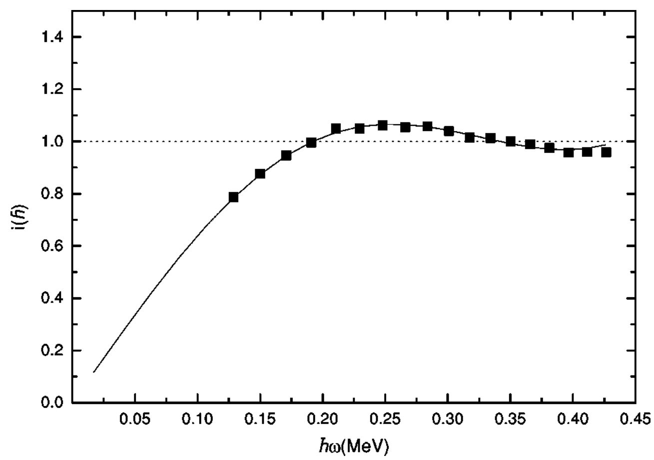

FIG. 8. The ω variation of the difference in angular momentum alignments of ^{194}Hg(3) and ^{192}Hg(1), $i=\Delta I_x=\omega[J^{(1)}(^{194}\mathrm{Hg}(3))-J^{(1)}(^{192}\mathrm{Hg}(1))]$, $\omega(I-1)=E_\gamma(I+2\rightarrow I)/2$. The solid square represents the experimental result extracted by Eq. (1). The solid line is the result calculated by Eq. (14) using the a, b, and c values given in Table I.

two approaches to the BFM are discussed. It is found that the spin propositions of most SD bands in the $A\sim190$ region can be made consistently by the three approaches and the results are given in Tables IV–VII. For the SD bands, ^{194}Hg(1), ^{194}Hg(3), and ^{194}Pb(1), the spin propositions are in agreement with experimental results. The variation of the kinematic and dynamic moments of inertia, particularly the bandhead moments of inertia J_0 systematics, are investigated in detail, which turn out to be very useful for the understanding of the properties of SD bands, e.g. the properties of excited SD bands, the implication of identical bands, etc.

Some remarks should be pointed out.

(1) For a few SD bands, the present spin propositions are different from that made in some previous papers. For example, the spin of the lowest level observed in ^{193}Pb(1) was proposed [28] by Ducroux *et al.* to be $I_0=13.5$ [$E_\gamma(I_0+2\rightarrow I_0)=277.2$ keV]. The present analysis shows I_0 should be 11.5. It is encouraging to note that this spin proposition is in agreement with the recent experimental result by Perris *et al.* [34]. Another example is the pair of signature partner SD bands ^{192}Tl(a,b). $I_0=15$ for ^{192}Tl(a) and $I_0=18$ for ^{192}Tl(b) were suggested by Fischer *et al.* [35]. However, according to the J_0 systematics and BFM, the most reasonable choice should be $I_0=13$ for ^{192}Tl(a) and $I_0=16$ for ^{192}Tl(b).

(2) For a few SD bands, the spin assignment was not made uniquely using the BFM by Becker *et al.* For example, $I_0=10$ or 11 for ^{192}Pb(1) was made in [24] using the BFM by Becker. However, our analysis shows that $I_0=10$ is the most reasonable choice, which is in agreement with the recent spin assignment by McNabb *et al.* [25].

(3) For the three SD bands observed in Bi isotopes [37,39], our analysis supports the isotopic assignment given in [37] by Clark *et al.*, i.e., two of them are a pair of signature partner SD bands in ^{197}Bi, $I_0=6.5$ [$E_\gamma(I_0+2\rightarrow I_0)=166.2$ keV] for ^{197}Bi(1), and $I_0=7.5$ [$E_\gamma(I_0+2\rightarrow I_0)=186.7$ keV] for ^{197}Bi(2).

(4) There are still a few SD bands whose spin propositions cannot be made uniquely, e.g., ^{191}Hg(4), ^{193}Hg(4), ^{192}Pb(2), etc., which need further investigation.

ACKNOWLEDGMENTS

This work was supported by the Natural Science Foundation of China and the Doctor Program Foundation of Institution of Higher Education of China.

[1] P. J. Twin *et al.*, Phys. Rev. Lett. **57**, 811 (1986).
[2] J. A. Becker *et al.*, Phys. Rev. C **46**, 889 (1992); J. E. Draper *et al.*, *ibid.* **42**, R1791 (1990).
[3] J. Y. Zeng *et al.*, Phys. Rev. C **44**, R1745 (1991); C. S. Wu *et al.*, *ibid.* **45**, 261 (1992).
[4] R. Piepenbring and K. V. Protasov, Z. Phys. A **345**, 7 (1993).
[5] F. R. Xu and J. M. Hu, Phys. Rev. C **49**, 1449 (1994).
[6] T. L. Khoo *et al.*, Phys. Rev. Lett. **76**, 1583 (1996).
[7] K. Hauschild *et al.*, Phys. Rev. C **55**, 2819 (1997).
[8] A. Lopez-Martens *et al.*, Phys. Lett. B **380**, 18 (1996).
[9] M. J. Brinkman *et al.*, Phys. Rev. C **53**, R1461 (1996).
[10] G. Hackman *et al.*, Phys. Rev. Lett. **79**, 4100 (1997).
[11] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2, Chap. 4.
[12] S. M. Harris, Phys. Rev. Lett. **13**, 663 (1964); Phys. Rev. **138**, A509 (1965).
[13] P. Holmberg and P. O. Lipas, Nucl. Phys. **A117**, 552 (1968).
[14] C. S. Wu and J. Y. Zeng, Commun. Theor. Phys. **8**, 51 (1987).
[15] H. X. Huang, C. S. Wu, and J. Y. Zeng, Phys. Rev. C **39**, 1617 (1989).
[16] F. X. Xu, C. S. Wu, and J. Y. Zeng, Phys. Rev. C **40**, 2337 (1989).
[17] Z. X. Hu and J. Y. Zeng, Phys. Rev. C **56**, 2523 (1997).
[18] A. Klein, R. M. Dreizler, and T. K. Das, Phys. Lett. **31B**, 333 (1970).
[19] G. Scharff-Goldhaker, C. Dove, and A. L. Goodman, Annu. Rev. Nucl. Sci. **26**, 239 (1996).
[20] J. Y. Zeng, Y. A. Lei, T. H. Jin, and Z. J. Zhao, Phys. Rev. C **50**, 746 (1994).
[21] A. N. Wilson *et al.*, Phys. Rev. C **54**, 559 (1996).
[22] R. Krücken *et al.*, Phys. Rev. C **54**, R2109 (1996).
[23] J. R. Hughes *et al.*, Phys. Rev. C **50**, R1265 (1994).
[24] L. Ducroux *et al.*, Z. Phys. A **352**, 13 (1995).
[25] D. P. McNabb *et al.*, Phys. Rev. C **56**, 2474 (1997).
[26] M. P. Carpenter *et al.*, Phys. Rev. C **51**, 2400 (1995).
[27] M. J. Joyce *et al.*, Phys. Lett. B **340**, 150 (1994).
[28] L. Ducroux *et al.*, Phys. Rev. C **53**, 2701 (1996).
[29] D. P. McNabb *et al.*, Phys. Rev. C **53**, R541 (1996).
[30] I. M. Hibbert *et al.*, Phys. Rev. C **54**, 2253 (1996).
[31] U. J. Van Severen *et al.*, Z. Phys. A **353**, 15 (1995).
[32] B. J. P. Gall *et al.*, Phys. Lett. B **345**, 124 (1995).
[33] P. Fallon *et al.*, Phys. Rev. C **51**, R1609 (1995).
[34] S. Perries *et al.*, Z. Phys. A **356**, 1 (1996).
[35] S. M. Fischer *et al.*, Phys. Rev. C **53**, 2126 (1996).
[36] L. P. Farris *et al.*, Phys. Rev. C **51**, R2288 (1995).
[37] R. M. Clark *et al.*, Phys. Rev. C **51**, R1052 (1995).

[38] G. Hackman *et al.*, Phys. Rev. C **55**, 148 (1997).
[39] R. M. Clark *et al.*, Phys. Rev. C **53**, 117 (1996).
[40] M. A. Riley *et al.*, Nucl. Phys. **A512**, 178 (1990).
[41] I. G. Bearden *et al.*, Z. Phys. A **341**, 491 (1992).
[42] Y. A. Lei, C. Z. Lin, and J. Y. Zeng, Sci. China, Ser. A **40**, 652 (1997).
[43] F. S. Stephens *et al.*, Phys. Rev. Lett. **64**, 2623 (1990); **65**, 301 (1990).
[44] J. Duprat *et al.*, Phys. Lett. B **341**, 6 (1994).
[45] S. Bouneau *et al.*, Phys. Rev. C **53**, R9 (1996).
[46] S. Pilotte *et al.*, Phys. Rev. C **49**, 718 (1994).
[47] C. Schück *et al.*, Phys. Rev. C **56**, R1667 (1997).
[48] F. Azaiez *et al.*, Phys. Rev. Lett. **66**, 1030 (1991).
[49] J. Y. Zeng and Y. A. Lei, Sci. China, Ser. A **36**, 1444 (1993).
[50] P. Fallon *et al.*, Phys. Lett. B **276**, 427 (1992).
[51] R. Wyss and W. Satula, Phys. Lett. B **351**, 393 (1995).