Pion-three-nucleon problem with two-cluster connected-kernel equations

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A new set of integral equations for the coupled πNNN-NNN problem is obtained starting from the observation that this system breaks into fragments in a nontrivial way. Assuming the particles as distinguishable, there are indeed four modes of fragmentation into two clusters, while in the standard three-body problem there are three possible two-cluster partitions and conversely the four-body problem has seven different possibilities. The pion-three-nucleon collision problem is formulated through the integral-equation approach by taking into account the proper fragmentation of the system. The final result does not depend on the assumption of separability of the two-body *t* matrices. Then, the quasiparticle method *à la* Grassberger and Sandhas is applied and effective two-cluster connected-kernel equations are obtained. The corresponding bound-state problem is also formulated, and the resulting homogeneous equation provides an approach which generalizes the commonly used approaches via 3N Hamiltonians (where the meson degrees of freedom are usually suppressed) to describe the three-nucleon bound-state problem. [S0556-2813(98)03712-1]

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I. INTRODUCTION

In the past, there have been various attempts to generalize the integral-equation approach to the quantum few-body problem, and specifically the *N*-body formulation of Sandhas and collaborators [1,2], to obtain a formulation of the pion-three-nucleon problem with the aim to handle this different problem (where the number of particles is not fixed) with the same nonperturbative computational techniques which have been developed and widely tested in standard few-body applications.

In the standard *N*-body approach, as is well known, repeated applications of Faddeev's three-body treatment [3] and, in every step of the quasiparticle method [4], lead to effective two-body equations for the collision processes between composite particles. A few authors [5,6] some years ago proposed a treatment of the πNNN problem where quasiparticle equations were assumed from the very beginning as a starting ansatz. The treatment of Ref. [5] started from the coupled pion-three-nucleon (πNNN-NNN) dynamics and successfully arrived at a connected-kernel integral formulation of the problem, however, two-body equations describing binary collisions between composite particles of the *complete* system have not been obtained, since the amplitudes were represented in terms of cluster partitions of the four- and three-body spaces as if these were two completely disjoint sectors. In Ref. [6] the underlying three- and four-body dynamics has been approximated by phenomenological multicluster two- and three-body relativistic equations, including a 24-channel effective two-body equation which then was solved numerically and compared with pion production data; however in this case it was not possible to show that the approach is linked to or can be directly obtained from the underlying three- and four-body dynamics.

More recently, there has been another attempt to find a better formulation of the coupled πNNN-NNN problem [7]. The approach is more general than the previous ones since it does not assume from the beginning the quasiparticle (separable) ansatz but relies on the four-body chain-labeled formalism *à la* Yakubovskĭ [8] and extends this formalism to the πNNN situation where the pion can disappear through the πNN vertex interaction. In this case, by repeated use of the quasiparticle method (in close analogy with the standard four-body formulation [2]) effective two-body equations for the collision problem between composite fragments of the whole system have been found, but it has been shown in a subsequent analysis [9] that (i) the leading equation has a disconnected kernel and (ii) the amplitudes referring to the various rearrangement processes have intrinsic ambiguities and cannot be univocally identified with the physical collision processes. Both problems cannot be solved in that formalism unless one disregards certain diagrams referring to the 2+2 partitions, thereby making an approximation which at the least breaks unitarity.

Since all the above-mentioned approaches achieved only a limited success in the attempt to generalize the Grassberger-Sandhas transition operator formalism (or the equivalent Faddeev-Yakubovskĭ Green function formalism) to the pion-three-nucleon problem, one may raise the question whether these multiparticle approaches are well suited to treat the multinucleon dynamics in the presence of an absorbable pion. This paper is mainly focused on this important question and arrives at an affirmative (although not general) conclusion: It is indeed possible to generalize the Faddeev-Yakubovskĭ-Alt-Grassberger-Sandhas formalism, developed for the quantum-mechanical treatment of a fixed number of bodies, to the case of the pion-multinucleon dynamics, at least under the assumption that the proper Fock space with its infinite number of particles (unavoidable whenever production and/or absorption occurs) is truncated and the sole states with at most one dynamical pion are retained. The formalism illustrated in the next section is indeed an approximate, effective description of the three-nucleon collision problem below the production threshold of the second pion, and within the limits set by the truncation of the Hilbert space to three and four particles it is shown that it is possible to obtain the formal solution of the coupled πNNN-NNN

collision problem in terms of effective two-cluster connected-kernel equations.

The approach begins with a set of equations, originally developed for the coupled πNN-NN problem by Thomas and Rinat [10], and later extended by Afnan and Blankleider [11], and following a somewhat different method by Avishai and Mizutani [12]. Their final equations merge the three-body dynamics in the πNN sector together with the two-body dynamics of the NN sector and provide the additional couplings between the two sectors. The introduction of the quasiparticle (separable) ansatz for the two-body t matrices allows one to derive two-body effective equations coupling together the two-cluster partitions of the whole system, in close similarity with the Alt-Grassberger-Sandhas (AGS) [1] quasiparticle formalism for the pure three-body problem. An important aspect of this formalism is that it satisfies unitarity by construction at both two- and three-body level [11,12] provided that for the input two-body t matrices the off-shell unitarity relation is assumed, that the Green functions in the no-pion sector include the pion-loop self-energy diagrams, that the πNN vertex is properly dressed with the contribution coming from the nonpolar πN interaction, and that at least the one-pion exchange (OPE) contribution of the NN interaction is treated nonstatically. All these features have been carefully maintained [13] in the equations herein used as input for the pion-three-nucleon problem. Another aspect worth mentioning here is that the relativistic dynamics of the system can be incorporated in these sets of equations by modification of the Green functions, along the lines of the relativistic three-particle isobar approach in the Aaron-Amado-Young model, or by using the Blankenbecker-Sugar reduction method to eliminate the time component from the integration variables in the four-dimensional covariant equations. We refer to the books [14,15] and to the references contained therein for these possible relativistic reformulations of the problem.

It must be acknowledged, on the other hand, that in spite of all these attractive features the input equations we start with are not free from conceptual problems. The difficulties unavoidably arise when a truncation of the Hilbert space up to a limited number of particles is introduced in the context of a time-ordered perturbation theory. Such problems manifest themselves directly, e.g., in the dressing of the multinucleon propagator which turns out to be incomplete since in the one-pion truncation approximation two nucleons (not to speak of three) cannot be dressed at the same time. This problem is known as the nucleon renormalization problem [16,17], and has important practical consequences in that the effective πNN coupling constant in the multinucleon media becomes systematically smaller than the one used as input to describe the pion-nucleon subsystem dynamics, thereby producing an underestimation of the predictions for the pp-πd cross section. Also, it has been shown [18] that the inclusion of equal-time dressing contributions introduces significant modifications in the two-nucleon Green's function, at least for energies above the pion threshold while at lower energies the differences are smaller. Full dressing of the multinucleon propagator entering in the πNN-NN dynamical equations can be obtained by means of the representation in terms of convolution integrals [19]. Another associated difficulty is that in this dynamical approach, while certain time-ordered diagrams are included, others which are topologically equivalent but differ only for the choice of the time ordering remain excluded. A significant example, besides the equal-time two-nucleon self-energy diagram, is the Jennings mechanism [20] in πd scattering, which has to be included in order to correctly reproduce the deuteron tensor polarization T_{20}. This mechanism is not included in standard πNN-NN equations, while other diagrams differing merely for the time order are included. An approximate but effective solution to these problems consists in generalizing the πNN-NN formulation by including contributions from the two-pion Hilbert space, so that the important diagrams with two pions at the same time are not missing. The model developed and discussed in Ref. [21] represents an approach formulated along these lines. It does not solve, however, the general problem of arbitrarily excluding certain diagrams on the ground of the sole difference in the time order, but simply refers the problem to the truncation at a higher-order level. The difficulty can be circumvented if one starts with a four-dimensional covariant approach rather than from the ''old-fashioned'' time-ordered perturbation theory. In this way diagrams differing merely by the time order are systematically collected together in one single covariant diagram. Covariant four-dimensional equations which are free from double countings have been obtained only very recently for the πNN-NN system [22], and their extension to the three-nucleon case is not within the scope of this work.

With these limitations, we arrive in this paper at the identification of a new set of coupled integral equations for the πNNN-NNN dynamics, whose kernel is connected after iteration. The structure of the new coupled equation is chain labeled, as in the standard four-body (Yakubowskĭ) approach. However, the chain labeling of the equation is richer in structure than the standard four-body formulation, because of the coupling with the three-nucleon space. We identify this structure starting from the study of the fragmentation of the πNNN-NNN system into two clusters. We then adapt to this context the quasiparticle method, and obtain the reformulation of the πNNN-NNN dynamics in terms of a multiparticle two-cluster equation, where the effective two-cluster potential is given by particle-exchange diagrams. We also formulate the bound-state problem, and show that it can be obtained as a solution of the homogeneous problem associated to the new dynamical equation obtained in this paper. We finally provide also the rules to calculate the physically interesting scattering amplitudes starting from the solutions of these new dynamical equations.

The paper is organized as follows: In Section II the four partitions of the whole system into two clusters are introduced. This partition mode has no counterparts either in the four-body sector (where there are seven two-cluster partitions) or in the three-body sector (three two-cluster partitions) but allows the two sectors to dialogue. Then, to obtain the new integral-equation formulation, the following steps are taken. First, the input equations are reformulated in a matrix Lippmann-Schwinger-type (LS) form where the role of the t matrix (denoted $\mathbf{T}^{(3)}$ in matrix notation) is played by the multiparticle transition amplitudes referring to all possible three-cluster partitions of the system. Secondly, the dynamical equations (again in LS form) for the subsystems identified by two-cluster partitions are introduced. Then a

new sum rule is introduced with respect to the two-cluster partition index, for the ''generalized'' potential $\mathbf{V}^{(3)}$ (that is, the operator that plays the role of the potential in the input LS equation). Subsequently, from the set of three-cluster amplitudes $\mathbf{T}^{(3)}$ the two-cluster disconnected contributions are extracted. Finally, by means of the previous results, a new equation for the remaining connected part of $\mathbf{T}^{(3)}$ has been derived. The result by no means relies on the assumptions that the subsytem t matrices or amplitudes have a rank-one structure. It is to be noted that in the standard N-body problem it is possible to recover the whole Grassberger-Sandhas (GS) multiparticle formulation, and rederive their final connected-kernel equations by recursive application of the procedure made by the steps just mentioned above, since this recursive procedure allows us to extract from the N-body collision amplitude the whole set of disconnected contributions ranging from the highest level (corresponding to partitions of the system in $N-1$ clusters), down to the lowest level of disconnectedness where the system is partitioned into two clusters [7]. This fact emphasizes the close analogies between the GS formulation and the approach here adopted to solve the πNNN problem.

In Sec. III the quasiparticle formalism is introduced. The quasiparticle method is applied once in the four-body sector and a second time simultaneously in both three- and four-body sectors, to exhibit diagrammatically the connected-kernel structure of the theory, and to recast the result in terms of coupled multiparticle equations for the two-cluster dynamics, since this is physically more transparent and easier to communicate. The equations are discussed in terms of coalescence diagrams and particular attention is paid to the nonstandard role of the pion. All the driving terms of the final two-cluster coupled equations are exchange-type diagrams and are shown to connect the entire set of equations.

In Sec. IV the bound-state equation for the coupled πNNN-NNN system is derived. As is well known, in the two-nucleon system the bound-state wave function can be expressed as the negative-energy solution of the homogeneous equation whose kernel is transposed with respect to that of the two-body LS equation, and similarly the three-nucleon bound-state wave function can be expressed in terms of the negative-energy solution of the homogeneous equation whose kernel is transposed with respect to that of the AGS equation. The homogeneous solution of the coupled πNN-NN equations provides the natural way to include the pion dynamics in the two-nucleon bound-state wave function, and from this fact it is shown that it is possible to derive a three-nucleon bound-state wave function (explicitly including the pion dynamics) which can be given as a solution of a new homogeneous equation whose kernel is similarly related to that of the equation we have derived in Sec. II for the multiparticle collision problem. If we switch off the couplings due to the πNN vertices the homogeneous equation splits into two independent ones (with of course two independent spectra): one whose kernel is referable to the Faddeev-AGS one for the pure three-nucleon sector, and another homogeneous Yakubowskǐ-GS-type equation for the pure four-particle bound state. With the complete equation it is possible to merge the three- and four-particle aspects of the problem, thus providing, for the three-nucleon system, a bound-state equation of new structure which generalizes the ones investigated so far. The approach may also serve as guidance to develop a consistent formulation which divides the interaction of the three-nucleon system between a two- and a three-body force. In fact, the need of three-body forces naturally arises in theories where the meson degrees of freedom are suppressed and the three nucleons are depicted as pointlike quantum particles interacting via local two-body potentials. The common procedure relies on symmetry principles to evaluate certain three-nucleon irreducible diagrams, selected on physical grounds to give the dominant contribution to the three-body force (πN s-wave interaction at threshold [23,24], or p-wave Δ excitation at intermediate energies [25,26]). The approach here discussed performs the complete resummation of the whole multiple scattering series including all one-pion intermediate states, provides the source of all reducible and irreducible three-nucleon contributions of the one-pion type, and furthermore sets the proper framework for their nonperturbative handling.

In Sec. V attention is returned to the collision problem and in particular to the rules for calculating the scattering amplitudes for all possible combinations of multiparticle fragmentation involved in the collision. Finally, in Sec. VI a brief summary and the conclusions are given.

II. CLUSTER DECOMPOSITION OF THE PION-THREE-NUCLEON SYSTEM

We consider as starting point the result obtained in Ref. [13]. Here the dynamical equations coupling all the partitions of the πNNN system into three clusters have been derived following the diagrammatic approach and applying nontrivial properties of the four-body transition operators defined within the standard AGS theory. In this manner, it was possible to obtain an equation for new amplitudes where scattering processes, pion production, and absorption are coupled in a unitary treatment.

The final coupled equations were formally identical to the Thomas-Rinat-Afnan-Blankleider-Avishai-Mizutani (TRABAM) equations, originally designed for the coupled πNN system:

$$U_{ab}=G_0^{-1}\overline{\delta}_{ab}+\sum_c \overline{\delta}_{ac}t_cG_0U_{cb}+F_ag_0U_b^\dagger, \quad (2.1a)$$

$$U_a^\dagger=F_a^\dagger+\mathcal{V}g_0U_a^\dagger+\sum_c F_c^\dagger G_0t_cG_0U_{ca}, \quad (2.1b)$$

$$U_a=F_a+\sum_c \overline{\delta}_{ac}t_cG_0U_c+F_ag_0U, \quad (2.1c)$$

$$U=\mathcal{V}+\mathcal{V}g_0U+\sum_c F_c^\dagger G_0t_cG_0U_c. \quad (2.1d)$$

We briefly recall the meaning of the symbols, referring to Ref. [13] and to the references therein contained for more detailed explanations. The transition matrices U_{ab} and U represent the scattering amplitudes for the three-fragment collision processes in the four-particle and three-nucleon sectors, respectively, while $U_a^\dagger$ and U_b are the corresponding absorption and production amplitudes.

The two-body t matrices acting between all the possible pairs (labeled ''a'') of the four-particle sector are denoted by t_a, while F_a ($F_a^\dagger$) are calculated from the elementary πNN production (absorption) vertices in a manner that is detailed below. As for the notation, it must be observed that the absorption amplitude $U_a^\dagger$ is not directly associated to the corresponding production amplitude via Hermitian conjugation, since the effect of complex conjugation on the boundary conditions must be taken into account. The same considerations apply for the πNN vertices, as these include the energy-dependent distortion effects due to the nonpolar πN interaction [11]. Moreover we omit for conciseness the dependence upon the total energy of the system, E, since its role can be easily recovered by resorting to the analogy with the standard few-body case.

The operator G_0 represents the free four-body Green's function and g_0 denotes the free three-nucleon Green's function (with the inclusion of the pion self-energy contributions). The boundary conditions are fixed by approaching the right-hand cut in the complex energy plane from above. Finally, $\mathcal{V}$ represents the total interaction acting among the three nucleons, and is given by the sum over the three pairwise nuclear interactions, which must include the nonstatic OPE diagrams. For the sake of simplicity, we will not assume the occurrence of a residual three-body force, although irreducible three-nucleon forces can be—and indeed have already been—accommodated in formalisms of this sort [5]; we will, however, add in Sec. VI a discussion on the subject under a general perspective. Equations (2.1) can be viewed (or reinterpreted) as a generalized Lippmann-Schwinger equation; in fact if we restrict the description to the zero-pion sector, which corresponds to freezing the pion degrees of freedom, the set of equations collapses to the well-known Lippmann-Schwinger equation describing the standard quantum-mechanical situation of nucleons interacting through the nuclear potential, i.e.,

$$U=\mathcal{V}+\mathcal{V}g_0U, \tag{2.2}$$

and for the simpler two-nucleon system, U corresponds to the well-known nucleon-nucleon t matrix. Equations (2.1) generalize the above equation by providing a direct link between the three-nucleon space and the three-cluster rearrangement processes in the four-particle space. As is obvious, the index a (or b, etc.) denotes the particle pair, either πN or NN, which forms the composite fragment in the four-body space. With $\overline{\delta}_{ab}(\equiv 1-\delta_{ab})$ it is 1 if the pairs a, b are different, 0 otherwise. The link between the two spaces is made possible by the operators F_a and $F_a^\dagger$, defined in terms of the elementary pion production or absorption vertices,

$$F_a=\sum_{i=1}^{3}\overline{\delta}_{ia}f_i, \quad F_a^\dagger=\sum_{i=1}^{3}\overline{\delta}_{ia}f_i^\dagger. \tag{2.3}$$

Here, ''i'' has a twofold meaning since it denotes the nucleon which emits (or absorbs) the pion and at the same time the corresponding pion-nucleon pair. As mentioned above, the employed elementary vertices have to be dressed by the distortion effects of the nonpolar contribution to the πN t matrix, $f_i=(1+t_iG_0)f_i^{(o)}$, and similar distortions hold for $f_i^\dagger$.

The analogy with the standard LS equation can be best exploited by formally rewriting the TRABAM equations as a matrix LS equation

$$\mathbf{T}^{(3)}=\mathbf{V}^{(3)}+\mathbf{V}^{(3)}\mathbf{G}_0^{(3)}\mathbf{T}^{(3)}, \tag{2.4}$$

where all operators are now 7×7 matrix operators with indices spanning all the three-cluster partitions of the πNNN-NNN system. This can be obtained by introducing the following definitions:

$$\mathbf{G}_0^{(3)}\equiv\begin{pmatrix} G_0t_aG_0\delta_{ab} & 0 \\ 0 & g_0 \end{pmatrix}, \tag{2.5}$$

$$\mathbf{V}^{(3)}\equiv\begin{pmatrix} G_0^{-1}\overline{\delta}_{ab} & F_a \\ F_b^\dagger & \mathcal{V} \end{pmatrix}, \tag{2.6}$$

$$\mathbf{T}^{(3)}\equiv\begin{pmatrix} U_{ab} & U_a \\ U_b^\dagger & U \end{pmatrix}. \tag{2.7}$$

While for the πNN problem the above equation is already connected and couples all the possible two-cluster partitions of the system (which include the two-nucleon state without pions), in the πNNN case the same equation couples only three-cluster partitions, thus leading to the nonconnectedness of the equation. This problem can be immediately understood by reasoning in terms of classes of ''disconnected'' diagrams. In Eqs. (2.1) all diagrams connecting only two of the four particles have been subtracted, via the t matrices. These same diagrams, if considered in the πNN case, group the system into two fragments, hence all the remaining diagrams contained in Eqs. (2.1) must connect the whole equation. However in the πNNN case such two-body diagrams arrange the system into three clusters; therefore Eqs. (2.1) contain either diagrams connecting the entire system, or diagrams arranging the system into two fragments. One has to isolate this last class of diagrams of higher connectivity but still ''disconnected'' before the correct equation can be found. This scenario is perfectly analogous to the situation for the standard few-body problem, where the Faddeev-AGS equation solves the three-body problem but leaves the four-body problem still out of reach. In the four-body problem one must introduce the partitions into two clusters and repeat the same logical scheme to obtain four-body connected-kernel equations of Yakubovskĭ-GS type.

From the above considerations it is clear that great attention must be paid first in finding the correct two-cluster partitions for the system and then one can proceed toward πNNN-NNN connected-kernel equations. Conversely, in the approach attempted previously [7] the two-cluster partitions are identified literally with the seven two-cluster partitions of the standard four-body problem, while in the three-nucleon space the homologous partitions were playing a secondary role. That fragmentation scheme, depicted in Table I, leads to the difficulties observed in Ref. [9], where it was found that the resulting two-cluster amplitudes had intrinsic ambiguities and the kernel of the resolving equation was not connected. Both aspects originate from the same problem; the

TABLE I. The seven two-cluster partitions of the πNNN-NNN system in previous approaches.

a'	πNNN sector	NNN sector
1	$N_1\ (N_2\ N_3\ \pi)$	$N_1(N_2\ N_3)$
2	$N_2\ (N_3\ N_1\ \pi)$	$N_2(N_3\ N_1)$
3	$N_3\ (N_1\ N_2\ \pi)$	$N_3(N_1\ N_2)$
4	$(\pi\ N_1)\ (N_2\ N_3)$	$N_1(N_2\ N_3)$
5	$(\pi\ N_2)\ (N_3\ N_1)$	$N_2(N_3\ N_1)$
6	$(\pi\ N_3)\ (N_1\ N_2)$	$N_3(N_1\ N_2)$
7	$\pi\ (N_1\ N_2\ N_3)$	

(non) proper identification of the physical partitions of the complete system into two clusters.

In the current approach, we identify only four two-cluster partitions, listed in Table II. We label these partitions with the index s, spanning from 0 to 3. The partition $s=0$ represents the only genuine four-body partition of the πNNN system and corresponds to the last partition reported in Table I. Here the pion is isolated from the rest of the system, hence there is no direct coupling with the zero-pion sector. The remaining partitions with $s=1$, 2, and 3 exhibit a new structure with no counterparts in the standard few-body theories. Each partition represents a physical cluster decomposition which can be detected as an asymptotic channel and where, according to Table II, one two-cluster no-pion state is coupled with two different two-cluster one-pion states.

We can now introduce the equations for the channel (or subsystem) dynamics. First we have to define the channel interaction v_s. (We will assume $s\neq 0$ since the $s=0$ case will be discussed separately with standard few-body techniques.) When $s\neq 0$ the subsystem interaction couples the zero-pion sector with the one-pion sector and one has to define the action of v_s in each sector. In the one-pion sector v_s is labeled by the chain-of-partition index, $\{a'a\}$, where a' represents one of the possible partitions (two, for a given $s\neq 0$) into two clusters of the four-body sector, while a represents one of the possible three-cluster partitions which can be obtained from the sequential breakup of the partition a'. Therefore the structure of v_s in the one-pion sector can be best represented as

$$v_s=(v_s)_{a'a,b'b}, \tag{2.8}$$

where the partition indexes fulfill the chain conditions $a\subset a'\subset s$ and $b\subset b'\subset s$. In the no-pion sector, the index s is sufficient to identify the two-cluster partition of the system, since for $s\neq 0$ there is a one to one correspondence between the index s and the spectator nucleon, as can be directly inferred from Table II. Thus, in the three-nucleon sector we denote the two-nucleon potential by

$$v_s=(v_s)_{-,-}. \tag{2.9}$$

Up to now we have identified the diagonal blocks of the channel interaction; however, it is obvious that the index structure of the diagonal block fixes unavoidably the structure of the off-diagonal couplings between the two sectors, e.g.,

$$v_s=(v_s)_{a'a,-}. \tag{2.10}$$

The way the channel interaction operates is rather remarkable and deserves further comments: note that if we drop all the explicit links to the one-pion sector the interaction operator collapses to the standard two-nucleon interaction. In this case, the one-pion sector affects the channel interaction only through the OPE diagram, this being explicitly included in the interaction. Thus the present approach implies a highly nontrivial generalization of what we identify as the NN potential in the three-nucleon system. For a given $s\neq 0$, the nucleon-nucleon potential becomes a matrix operator acting not only as a standard two-nucleon potential in the three-nucleon space, but acquires extra components and couplings to the chain-of-partition space of the four-body sector. For instance, for $s=1$, v_s not only represents the standard NN potential between nucleons 2 and 3, but has further couplings in the one-pion sector to all possible sequential breakups of the four-body system which are allowed by the given s. And there is more than this. In addition there is a fourth interaction term (for $s=0$) which has no direct action in the three-nucleon space since it operates only in the four-body sector and in particular in the chain of partitions obtained from the sequential breakup of the $\pi+(\mathrm{NNN})$ channel.

Up to now we have discussed the general structure of the channel interactions, but we have not yet given its explicit expressions. To accomplish this we write

$$(v_s)_{a'a,b'b}=G_0^{-1}\overline{\delta}_{ab}\delta_{a'b'}\delta_{a,b\subset a'}\delta_{a',b'\subset s} \tag{2.11}$$

for the interaction in the one-pion sector, while in the no-pion sector (only for $s\neq 0$)

$$(v_s)_{-,-}=\mathcal{V}_s \tag{2.12}$$

denotes the pair potential between the two interacting nucleons, representing the nonstatic OPE diagram (as well as other possible static contributions which phenomenologically take into account more complicated diagrams such as heavy-boson exchanges and/or multipion exchanges). Finally, the off-diagonal interactions connecting the three-nucleon and four-body sectors are defined by

$$(v_s)_{a'a,-}=\sum_{i=1}^{3} f_i\overline{\delta}_{ia}\delta_{i,a\subset a'}\delta_{a'\subset s}\equiv(f_s)_{a'a} \tag{2.13a}$$

and

$$(v_s)_{-,b'b}=\sum_{i=1}^{3} f_i^{\dagger}\overline{\delta}_{ib}\delta_{i,b\subset b'}\delta_{b'\subset s}\equiv(f_s^{\dagger})_{b'b}. \tag{2.13b}$$

TABLE II. The two-cluster partitions of the πNNN-NNN system defined in this approach.

s	πNNN sector	NNN sector
0	$\pi\ (N_1\ N_2\ N_3)$	
1	$N_1\ (N_2\ N_3\ \pi);\ (\pi\ N_1)\ (N_2\ N_3)$	$N_1(N_2\ N_3)$
2	$N_2\ (N_3\ N_1\ \pi);\ (\pi\ N_2)\ (N_3\ N_1)$	$N_2(N_3\ N_1)$
3	$N_3\ (N_1\ N_2\ \pi);\ (\pi\ N_3)\ (N_1\ N_2)$	$N_3(N_1\ N_2)$

It must be observed that Table II is crucial for discussing the structure of the subamplitudes. For each $s \neq 0$, there are two two-cluster partitions in the four-body sector and one two-cluster partition in the three-nucleon sector. Then, in the four-body sector, there are five possible sequential breakups for a given s (three when the partition is of type $3+1$, and two when it is of the form $2+2$), and in the three-nucleon sector there is an additional one associated with the breakup of the nucleonic pair. In conclusion we have a total number of six components for each channel interaction with $s \neq 0$. The case $s=0$ is obviously simpler, since the corresponding fragmentation mode passes through one single two-cluster partition (of type $3+1$) of the four-body sector with no couplings to the three-nucleon sector. As is well known, this standard four-body partition has three possible ulterior fragmentations into three clusters. The subsystem interaction v_s for $s=0$ couples together only these three components.

For each of these four different modes of fragmentation into two clusters, we introduce the subamplitudes $\mathbf{t}_s$ having the same chain-labeled structure of the channel interactions, with six components

$$\mathbf{t}_s = \begin{pmatrix} (t_s)_{a'a,b'b} & (t_s)_{a'a,-} \\ (t_s)_{-,b'b} & (t_s)_{-,-} \end{pmatrix} \tag{2.14}$$

for $s \neq 0$. These subamplitudes represent the solutions of the equation for the subsystem dynamics which can be explicitly written as

$$(t_s)_{a'a,b'b} = G_0^{-1}\overline{\delta}_{ab}\delta_{a'b'} + \sum_{c'(\subset s)} \sum_{c(\subset c')} \overline{\delta}_{ac}\delta_{a'c'} t_c G_0 (t_s)_{c'c,b'b} + (f_s)_{a'a} g_0 (t_s)_{-,b'b}, \tag{2.15a}$$

$$(t_s)_{-,b'b} = (f_s^\dagger)_{b'b} + \mathcal{V}_s g_0 (t_s)_{-,b'b} + \sum_{c'(\subset s)} \sum_{c(\subset c')} (f_s^\dagger)_{c'c} G_0 t_c G_0 (t_s)_{c'c,b'b}, \tag{2.15b}$$

$$(t_s)_{a'a,-} = (f_s)_{a'a} + \sum_{c'(\subset s)} \sum_{c(\subset c')} \overline{\delta}_{ac}\delta_{a'c'} t_c G_0 (t_s)_{c'c,-} + (f_s)_{a'a} g_0 (t_s)_{-,-}, \tag{2.15c}$$

$$(t_s)_{-,-} = \mathcal{V}_s + \mathcal{V}_s g_0 (t_s)_{-,-} + \sum_{c'(\subset s)} \sum_{c(\subset c')} (f_s^\dagger)_{c'c} G_0 t_c G_0 (t_s)_{c'c,-}, \tag{2.15d}$$

with $a \subset a' \subset s$ and $b \subset b' \subset s$.

One can directly compare the structure of these equations with the previously discussed TRABAM equations, Eqs. (2.1). They are obviously similar, the former being the dynamical equation for the whole system, the latter carrying the information for the internal dynamics with respect to the partition s. In Eqs. (2.15) a careful disentanglement has been made of which components contribute within the same subsystem, according to the scheme illustrated in Table II. We observe that for each partition $s \neq 0$ the no-pion sector acts as a doorway state and couples two different two-cluster partitions a' of the four-body sector. The operators $(f_s)_{a'a}$ and $(f_s^\dagger)_{b'b}$ are fundamental in this sense, since without these the two-cluster partitions of the four-body sector would remain uncoupled (as happens in the standard four-body theory).

When $s=0$ the subamplitude is a genuine four-body subamplitude, identified by one single two-cluster partition of the four-body system. The corresponding channel equation has the standard three-component AGS structure (in the presence of a spectator particle)

$$(u_{a'})_{ab} = G_0^{-1}\overline{\delta}_{ab} + \sum_{c(\subset a')} \overline{\delta}_{ac} t_c G_0 (u_{a'})_{cb}, \tag{2.16}$$

with $a,b \subset a'$.

We prefer to rewrite such an equation for the $s=0$ subamplitude as follows:

$$(t_s)_{a'a,b'b} = G_0^{-1}\overline{\delta}_{ab}\delta_{a'b'} + \sum_{c'(\subset s)} \sum_{c(\subset c')} \overline{\delta}_{ac}\delta_{a'c'} t_c G_0 (t_s)_{c'c,b'b}, \tag{2.17}$$

(where $a \subset a' \subset s$ and $b \subset b' \subset s$) with the position

$$\mathbf{t}_s = (t_s)_{a'a,b'b} \equiv (u_{a'})_{a,b}\delta_{a'b'}. \tag{2.18}$$

Clearly, Eq. (2.17) is not the simplest way to write a standard AGS equation, however, it does correspond to the standard AGS equation, Eq. (2.16), since only the $[(NNN)\ \pi]$ partition is relevant for $s=0$ (hence $a'=b'=c' = [(NNN)\ \pi]$). The form given by Eq. (2.17) has the advantage that it treats the $s=0$ subamplitude with the same formalism which must be introduced to describe the much more complex $s \neq 0$ subamplitudes. In this way the index structure of all subamplitudes, including the $s=0$ one, is given by the same rules.

Up to now we have discussed the partition modes of the πNNN-NNN system into two clusters and have given the corresponding subsystem equations. We show now that the channel interaction v_s satisfies a sum-rule property. For convenience, we discuss separately the effect of the sum rule in the various sectors.

In the four-body sector, the driving term (total interaction) of the TRABAM equations is a matrix potential with components ranging within the six three-cluster partitions of the system $V_{ab}^{(3)} = G_0^{-1}\overline{\delta}_{ab}$. In the same sector the channel interaction has a structure which is conceptually more complicated, since for each partition s the potential in the four-body sector is a matrix potential ranging between all the possible chains of partitions corresponding to each s: $(v_s)_{a'a,b'b} = G_0^{-1}\overline{\delta}_{ab}\delta_{a'b'}\delta_{ab\subset a'}\delta_{a'\subset s}$. In particular, for each partition with $s \neq 0$ we have five chains while for $s=0$ there are three chains. The total corresponds to the 18 Yakubovskĭ components necessary for the complete dynamical description of four-body states. We observe that the following sum rule holds:

$$(V^{(3)})_{ab}=\sum_{s=0}^{3}\sum_{a',b'(\subset s)}(v_s)_{a'a,b'b}. \tag{2.19}$$

This can be easily demonstrated once it has been realized that the right-hand term can be rewritten as $\Sigma_{a'}G_0^{-1}\overline{\delta}_{ab}\delta_{ab\subset a'}$.

Similarly, for the interaction operators connecting the four-body and the three-nucleon sectors, we observe the following sum rules:

$$F_a=\sum_s\sum_{a'(\subset s)}(f_s)_{a'a}, \tag{2.20a}$$

$$F_a^{\dagger}=\sum_s\sum_{a'(\subset s)}(f_s^{\dagger})_{a'a}. \tag{2.20b}$$

They both come from the identity

$$\overline{\delta}_{ia}=\sum_s\sum_{a'}\overline{\delta}_{ia}\delta_{i,a\subset a'}\delta_{a'\subset s}, \tag{2.21}$$

which can be demonstrated by observing (from Table II) that a partition a' corresponds to one single subsystem s and a pair of *different* three-cluster partitions i,a corresponds to one single two-cluster partition a'. Furthermore, we observe that the $s=0$ contribution to the sum over s is identically null since there are no pion-nucleon pairs which can be identified from the sequential breakup of the π (NNN) partition.

Finally, in the no-pion sector, $\mathcal{V}$ represents the sum over all the pair interactions among the three nucleons,

$$\mathcal{V}=\sum_s \mathcal{V}_s, \tag{2.22}$$

having assumed that only two-body NN potentials are given as input. The sum over the three s components (from 1 to 3) saturates the total interaction in the three-nucleon sector (the $s=0$ case does not contribute here as well as in the vertex interactions).

We summarize the results obtained so far.

(I) Our starting point is given by the TRABAM equations which have been symbolically rewritten as a matrix LS equation connecting all the three-cluster partitions (in both sectors) of the system:

$$\mathbf{T}^{(3)}=\mathbf{V}^{(3)}+\mathbf{V}^{(3)}\mathbf{G}_0^{(3)}\mathbf{T}^{(3)}. \tag{2.23}$$

(II) We have introduced the dynamical equations for the subamplitudes. Since we have already expressed these equations in detail [in Eqs. (2.15)], we rewrite the same equations in a more compact matrix form, namely,

$$\mathbf{t}_s=\mathbf{v}_s+\mathbf{v}_s\mathbf{G}_0^{(3)}\mathbf{t}_s. \tag{2.24}$$

It has to be recalled that only when $s\neq 0$ is there a direct coupling to the three-nucleon sector. The operators involved in Eq. (2.24) act in a conceptually more complex space, if compared to the three-cluster partition space of $\mathbf{T}^{(3)}$, $\mathbf{V}^{(3)}$, and $\mathbf{G}_0^{(3)}$, since it involves the chain-of-partition labeling of the Yakubovskĭ approach. Therefore care must be taken in considering the operatorial product $\mathbf{v}_s\mathbf{G}_0^{(3)}\mathbf{t}_s$ since the operators are defined in different spaces, as can be directly seen by inspection of the detailed formulas (2.15) previously reported.

(III) Within this formalism, we can collect the three sum rules previously discussed in a more general and compact sum rule

$$\mathbf{V}^{(3)}\equiv\begin{pmatrix}(V^{(3)})_{ab} & (V^{(3)})_{a,-}\\ (V^{(3)})_{-,b} & (V^{(3)})_{-,-}\end{pmatrix}=\begin{pmatrix}\sum_s\sum_{a',b'(\subset s)}(v_s)_{a'a,b'b} & \sum_s\sum_{a'(\subset s)}(v_s)_{a'a,-}\\ \sum_s\sum_{b'(\subset s)}(v_s)_{-,b'b} & \sum_s(v_s)_{-,-}\end{pmatrix}. \tag{2.25}$$

(IV) We now can proceed in analogy with the methods developed in standard N-body theory, namely, we introduce the new unknowns U, with the following definition:

$$\begin{aligned}(T^{(3)})_{ab}=&\sum_s\sum_{a',b'(\subset s)}(t_s)_{a'a,b'b}+\sum_{s,s'}\sum_{a',c'(\subset s)}\sum_{d',b'(\subset s')}\sum_{c(\subset c')}\sum_{d(\subset d')}(t_s)_{a'a,c'c}G_0t_cG_0(U_{s,s'})_{c'c,d'd}G_0t_dG_0(t_{s'})_{d'd,b'b}\\ &+\sum_{s,s'}\sum_{a',c'(\subset s)}\sum_{b'(\subset s')}\sum_{c(\subset c')}(t_s)_{a'a,c'c}G_0t_cG_0(U_{s,s'})_{c'c,-}g_0(t_{s'})_{-,b'b}\\ &+\sum_{s,s'}\sum_{a'(\subset s)}\sum_{d',b'(\subset s')}\sum_{d(\subset d')}(t_s)_{a'a,-}g_0(U_{s,s'})_{-,d'd}G_0t_dG_0(t_{s'})_{d'd,b'b}\\ &+\sum_{s,s'}\sum_{a'(\subset s)}\sum_{b'(\subset s')}(t_s)_{a'a,-}g_0(U_{s,s'})_{-,-}g_0(t_{s'})_{-,b'b},\end{aligned} \tag{2.26a}$$

$$(T^{(3)})_{a-}=\sum_{s}\sum_{a'(\subset s)}(t_s)_{a'a,-}+\sum_{s,s'}\sum_{a',c'(\subset s)}\sum_{d'(\subset s')}\sum_{c(\subset c')}\sum_{d(\subset d')}(t_s)_{a'a,c'c}G_0t_cG_0(U_{s,s'})_{c'c,d'd}G_0t_dG_0(t_{s'})_{d'd,-}$$
$$+\sum_{s,s'}\sum_{a',c'(\subset s)}\sum_{c(\subset c')}(t_s)_{a'a,c'c}G_0t_cG_0(U_{s,s'})_{c'c,-}g_0(t_{s'})_{-,-}$$
$$+\sum_{s,s'}\sum_{a'(\subset s)}\sum_{d'(\subset s')}\sum_{d(\subset d')}(t_s)_{a'a,-}g_0(U_{s,s'})_{-,d'd}G_0t_dG_0(t_{s'})_{d'd,-}+\sum_{s,s'}\sum_{a'(\subset s)}(t_s)_{a'a,-}g_0(U_{s,s'})_{-,-}g_0(t_{s'})_{-,-},\tag{2.26b}$$

$$(T^{(3)})_{-b}=\sum_{s}\sum_{b'(\subset s)}(t_s)_{-,b'b}+\sum_{s,s'}\sum_{c'(\subset s)}\sum_{d',b'(\subset s')}\sum_{c(\subset c')}\sum_{d(\subset d')}(t_s)_{-,c'c}G_0t_cG_0(U_{s,s'})_{c'c,d'd}G_0t_dG_0(t_{s'})_{d'd,b'b}$$
$$+\sum_{s,s'}\sum_{c'(\subset s)}\sum_{b'(\subset s')}\sum_{c(\subset c')}(t_s)_{-,c'c}G_0t_cG_0(U_{s,s'})_{c'c,-}g_0(t_{s'})_{-,b'b}$$
$$+\sum_{s,s'}\sum_{d',b'(\subset s')}\sum_{d(\subset d')}(t_s)_{-,-}g_0(U_{s,s'})_{-,d'd}G_0t_dG_0(t_{s'})_{d'd,b'b}+\sum_{s,s'}\sum_{b'(\subset s')}(t_s)_{-,-}g_0(U_{s,s'})_{-,-}g_0(t_{s'})_{-,b'b},\tag{2.26c}$$

$$(T^{(3)})_{-,-}=\sum_{s}(t_s)_{-,-}+\sum_{s,s'}\sum_{c'(\subset s)}\sum_{b'(\subset s')}\sum_{c(\subset c')}\sum_{d(\subset d')}(t_s)_{-,c'c}G_0t_cG_0(U_{s,s'})_{c'c,d'd}G_0t_dG_0(t_{s'})_{d'd,-}$$
$$+\sum_{s,s'}\sum_{c'(\subset s)}\sum_{c(\subset c')}(t_s)_{-,c'c}G_0t_cG_0(U_{s,s'})_{c'c,-}g_0(t_{s'})_{-,-}$$
$$+\sum_{s,s'}\sum_{a'(\subset s)}\sum_{d'(\subset s')}\sum_{d(\subset d')}(t_s)_{a'a,-}g_0(U_{s,s'})_{-,d'd}G_0t_dG_0(t_{s'})_{d'd,-}+\sum_{s,s'}(t_s)_{-,-}g_0(U_{s,s'})_{-,-}g_0(t_{s'})_{-,-}.\tag{2.26d}$$

Now, we substitute Eqs. (2.25) and (2.26) into Eq. (2.23), and use repeatedly Eq. (2.24). We find that

$$(U_{s,s'})_{a'a,b'b}=(G_0t_aG_0)^{-1}\delta_{ab}(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'b'})+\sum_{s''}\sum_{c',d'(\subset s'')}\sum_{d(\subset d')}(\bar{\delta}_{ss''}+\delta_{ss''}\bar{\delta}_{a'c'})(t_{s''})_{c'a,d'd}G_0t_dG_0(U_{s'',s'})_{d'd,b'b}$$
$$+\sum_{s''}\sum_{c'(\subset s'')}(\bar{\delta}_{ss''}+\delta_{ss''}\bar{\delta}_{a'c'})(t_{s''})_{c'a,-}g_0(U_{s'',s'})_{-,b'b},\tag{2.27a}$$

$$(U_{s,s'})_{-,b'b}=\sum_{s''}\sum_{d'(\subset s'')}\sum_{d(\subset d')}(\bar{\delta}_{ss''})(t_{s''})_{-,d'd}G_0t_dG_0(U_{s'',s'})_{d'd,b'b}+\sum_{s''}(\bar{\delta}_{ss''})(t_{s''})_{-,-}g_0(U_{s'',s'})_{-,b'b},\tag{2.27b}$$

$$(U_{s,s'})_{a'a,-}=\sum_{s''}\sum_{c',d'(\subset s'')}\sum_{d(\subset d')}(\bar{\delta}_{ss''}+\delta_{ss''}\bar{\delta}_{a'c'})(t_{s''})_{c'a,d'd}G_0t_dG_0(U_{s'',s'})_{d'd,-}$$
$$+\sum_{s''}\sum_{c'(\subset s'')}(\bar{\delta}_{ss''}+\delta_{ss''}\bar{\delta}_{a'c'})(t_{s''})_{c'a,-}g_0(U_{s'',s'})_{-,-},\tag{2.27c}$$

$$(U_{s,s'})_{-,-}=(g_0)^{-1}(\bar{\delta}_{ss'})+\sum_{s''}(\bar{\delta}_{ss''})(t_{s''})_{-,-}g_0(U_{s'',s'})_{-,-}+\sum_{s''}\sum_{d'(\subset s'')}\sum_{d(\subset d')}(\bar{\delta}_{ss''})(t_{s''})_{-,d'd}G_0t_dG_0(U_{s'',s'})_{d'd,-},\tag{2.27d}$$

with $a \subset a' \subset s$ and $b \subset b' \subset s'$. We stress that there is always a relation between the chains of partitions of the four-body sector, $\{a'a\}$, and the two-cluster partitions s, since for a given s, the allowed partitions ($a' \subset s$) are listed in Table II. Keeping this in mind, it is obvious that $(\overline{\delta}_{ss'} + \delta_{ss'}\overline{\delta}_{a'b'}) = \overline{\delta}_{a'b'}$.

These four coupled equations represent the main theoretical result of the paper. The first two equations couple four-body scattering and pion absorption, while the last two couple three-nucleon scattering with pion production. The equations decouple into ordinary four- and three-body equations if we switch off the couplings between the three- and four-particle channels, however, this is much less obvious than the corresponding decoupling for the simpler πNN system. To show how this happens one must first observe that the four two-cluster partitions of the whole system decouple into the seven two-cluster partitions of the four-body sector, plus the three two-cluster partitions of the three-nucleon sector. Moreover all the production/absorption amplitudes vanish, and therefore Eq. (2.27d) changes into the standard three-component AGS equation, and Eq. (2.27a) becomes precisely the standard 18-component GS equation. With the pion-nucleon vertex interaction switched on, we have instead a new 21-component equation which is remarkably different in structure. In the Appendix it is shown that the kernel of this set of coupled equations leads exclusively to connected diagrams, after two iterations.

In the following we intend to discuss the properties of this set of equations in the light of the quasiparticle interpretation. Then we will derive the corresponding bound-state equation and finally give the rules for calculating the collision amplitudes for rearrangement and breakup processes.

III. THE QUASIPARTICLE FORMALISM

The introduction of the quasiparticle formalism is in principle not indispensable, since direct solutions of multivariable few-body-type integral equations are possible by resorting to the nowadays available computational tools. The historical reason for introducing the quasiparticle method is that it reduces by one unit the dimensionality of the multiparticle equation whenever the method is applied. By repeated applications of the method, one reduces the problem to the solution of a two-cluster multiparticle equation in one single variable, after angular momentum decomposition. However, the quasiparticle or separable method not only represents a converging approximation scheme but it also allows us to reinterpret the previously obtained equations in a physically more transparent way, and by translating the theory in terms of coalescence diagrams, it allows us to exhibit diagrammatically the connected-kernel properties of the final equations.

To introduce the quasiparticle formalism, we derive first the amplitude for the fully unclusterized reaction process. This corresponds to the four to four amplitude, denoted by $T(1|1)$, describing the process of a free collision of the four particles. The amplitude for this process is linked to the TRABAM amplitudes previously defined, $\mathbf{T}^{(3)}$ (we remind the reader that such amplitudes for the πNNN-NNN system refer to all three to three processes). To obtain this link, we resort to Ref. [13] where the TRABAM theory for the πNNN-NNN system has been discussed within the diagrammatic approach.

As shown in Ref. [13], if we apply the last-cut lemma to the $4 \leftarrow 4$ (i.e., $\pi\mathrm{NNN} \leftarrow \pi\mathrm{NNN}$) amplitude we obtain

$$T(1|1) = T(1|0) g_0 T(0|1)_1 + T(1|1)_1, \tag{3.1}$$

while applying the first-cut lemma to the $4 \leftarrow 3$ (i.e., $\pi\mathrm{NNN} \leftarrow \mathrm{NNN}$) amplitude yields

$$T(1|0) = T(1|0)_1 [1 + g_0 T(0|0)]. \tag{3.2}$$

The subscript ''1'' denotes that the given amplitude contains at least one pion in all the intermediate states.

Similar assumptions for $T(0|1)_1$ and $T(1|0)_1$ yield [to the lowest order, see Eqs. (2.6), (2.8), and (2.10) of Ref. [13]]

$$T(1|0)_1 = \left(\sum_i f_i^{(o)} \right) + T(1|1)_1 G_0 \left(\sum_i f_i^{(o)} \right), \tag{3.3}$$

$$T(0|1)_1 = \left(\sum_i f_i^{(o)\dagger} \right) + \left(\sum_i f_i^{(o)\dagger} \right) G_0 T(1|1)_1, \tag{3.4}$$

and from these last equations we obtain

$$T(1|1) = T(1|1)_1 + [1 + T(1|1)_1 G_0] \left(\sum_i f_i^{(o)} \right) \times [g_0 + g_0 T(0|0) g_0] \left(\sum_i f_i^{(o)\dagger} \right) [1 + G_0 T(1|1)_1]. \tag{3.5}$$

If we identify $T(1|1)_1$ with the standard four-body four to four amplitude, $T(1|1)_1 = U_{00}$, we can use the relations connecting the various AGS amplitudes,

$$U_{00} = U_{0i}(1 + G_0 t_i) - G_0^{-1}, \tag{3.6}$$

$$U_{00} = (1 + t_i G_0) U_{i0} - G_0^{-1}. \tag{3.7}$$

By substituting the two expressions in the previous formula we get

$$T(1|1) = U_{00} + \sum_{ij} U_{0i} G_0 f_i [g_0 + g_0 T(0|0) g_0] f_j^\dagger U_{j0} \tag{3.8}$$

and recalling that

$$U_{00} = \sum_a t_a + \sum_{a,b} t_a G_0 U_{ab} G_0 t_b, \tag{3.9}$$

$$U_{0i} = G_0^{-1} + \sum_{c=1,6} t_c G_0 U_{ci}, \tag{3.10}$$

$$U_{i0} = G_0^{-1} + \sum_{c=1,6} U_{ic} G_0 t_c, \tag{3.11}$$

we obtain

$$T(1|1)=\sum_a t_a+\sum_{a,b} t_a G_0 U_{ab} G_0 t_b + \sum_{aijb} t_a G_0 U_{ai} f_i [g_0+g_0 T(0|0) g_0] f_j^\dagger U_{jb} G_0 t_b + \sum_{aij} t_a G_0 U_{ai} f_i [g_0+g_0 T(0|0) g_0] f_j^\dagger + \sum_{ijb} f_i [g_0+g_0 T(0|0) g_0] f_j^\dagger U_{jb} G_0 t_b + \sum_{ij} f_i [g_0+g_0 T(0|0) g_0] f_j^\dagger . \quad (3.12)$$

By the use of the AGS equations (see Ref. [13], pp. 1238–1240), it is possible to directly express the above amplitude in terms of the TRABAM amplitudes for the three-cluster partitions of the system, thereby obtaining the final result

$$T(1|1)=\sum_a t_a+\sum_{ij} f_i g_0 f_j^\dagger+\sum_{a,b} t_a G_0 (T^{(3)})_{ab} G_0 t_b + \sum_{aj} t_a G_0 (T^{(3)})_{a-} g_0 f_j^\dagger+\sum_{ib} f_i g_0 (T^{(3)})_{-b} G_0 t_b + \sum_{ij} f_i g_0 (T^{(3)})_{-,-} g_0 f_j^\dagger . \quad (3.13)$$

It must be observed that in previous studies [7,9] the second, and the three last terms were missing in the reported expressions for the fully unclusterized amplitude $T(1|1)$. In particular, the simplest pole-type diagrams $\Sigma_{ij} f_i g_0 f_j^\dagger$ were not considered in that approach.

We now introduce the quasiparticle method. According to this method, the two-body t matrix is represented by means of the separable ansatz,

$$t_a(z)=|a^{(3)}(z)\rangle \tau_a^{(3)}(z) \langle a^{(3)}(z)|. \quad (3.14)$$

When calculating the matrix element of this operator in the four-body space, we obtain

$$\langle \mathbf{p q_1 q_2}|t_a|\mathbf{p' q_1' q_2'}\rangle = \delta(\mathbf{q_1'}-\mathbf{q_1})\,\delta(\mathbf{q_2'}-\mathbf{q_2}) \langle \mathbf{p}|a^{(3)}(z-\Delta)\rangle \times \tau_a^{(3)}(z-\Delta)\langle a^{(3)}(z-\Delta)|\mathbf{p'}\rangle, \quad (3.15)$$

where it is assumed that $\mathbf{p}$ is the relative momentum of the pair a, while $\mathbf{q_1}$, $\mathbf{q_2}$ are the Jacobi coordinates for the two spectators and the c.m. of the pair (considered in toto as a three-body system), and $z-\Delta(\mathbf{q_1},\mathbf{q_2})$ the kinetic energy of the pair a with respect to its c.m.

Here, for simplicity, we have assumed a rank-one structure, but the extension of the formalism to higher ranks is straightforward, although practical extensions might require a major computational work. Depending on the specific separable expansion method, the states may or may not depend on the parametric energy, z. Moreover, $\langle a^{(3)}(z)|$ does not necessarily have to be the adjoint of $|a^{(3)}(z)\rangle$; for instance, in the case of Weinberg states a possible choice is $\langle a^{(3)}(z)|=|a^{(3)}(z^*)\rangle^\dagger$, but depending on the normalization conventions other choices are also possible [27]. We have no reasons here for analyzing in detail the technical differences which characterize the variety of separable-expansion methods available in the literature (for this we refer to Ref. [15]); as long as they correctly reproduce the polar structure of the subsystem t matrices we generically denote all these methods as ''quasiparticle'' approaches, although the quasiparticle idea historically refers to the application in terms of Weinberg states [28].

We note that the separable assumption affects only the four-body space, given that the two-body t matrices t_a act within this space, and, by means of the form Eq. (3.14), the fully unclusterized amplitude becomes [omitting the superscript ''(3)'' in the states $|a\rangle$]

$$T(1|1)=\sum_a |a\rangle \tau_a^{(3)} \langle a|+\sum_{ij} f_i g_0 f_j^\dagger+\sum_{ab} |a\rangle \tau_a^{(3)} X_{ab}^{(3)} \tau_b^{(3)} \langle b| + \sum_{aj} |a\rangle \tau_a^{(3)} X_a^{(3)} g_0 f_j^\dagger+\sum_{ib} f_i g_0 X_b^{\dagger(3)} \tau_b^{(3)} \langle b| + \sum_{ij} f_i g_0 X^{(3)} g_0 f_j^\dagger , \quad (3.16)$$

where the folded amplitudes are given according to the equations

$$X_{ab}^{(3)}=\langle a|G_0 (T^{(3)})_{ab} G_0|b\rangle, \quad (3.17a)$$

$$X_a^{(3)}=\langle a|G_0 (T^{(3)})_{a-}, \quad (3.17b)$$

$$X_b^{\dagger(3)}=(T^{(3)})_{-b} G_0|b\rangle, \quad (3.17c)$$

$$X^{(3)}\equiv (T^{(3)})_{-,-} . \quad (3.17d)$$

In the $\mathbf{X}^{(3)}$ amplitudes the variable describing the internal structure of the pair has been integrated over, thereby reducing the dimensionality of the corresponding dynamical equation. Such a quasiparticle equation for the $\mathbf{X}^{(3)}$ amplitudes has been given in Eq. (2.6) of Ref. [5]. However, it is known that the equation is not connected for the pion-three-nucleon problem [5,13].

We solve the problem by introducing the representation given in Eq. (2.26) which allows us to express the three-cluster partition amplitudes in terms of the new quantities t_s and $U_{ss'}$,

$$
\begin{aligned}
T(1|1)=&\sum_a t_a+\sum_{ij} f_i g_0 f_j^\dagger+\sum_s \sum_{a',b'(\subset s)} \sum_{a(\subset a')} \sum_{b(\subset b')} t_a G_0(t_s)_{a'ab'b} G_0 t_b \\
&+\sum_{ss'} \sum_{a',c'(\subset s)} \sum_{b',d'(\subset s')} \sum_{a(\subset a')} \sum_{c(\subset c')} \sum_{b(\subset b')} \sum_{d(\subset d')} t_a G_0(t_s)_{a'a,c'c} G_0 t_c G_0(U_{ss'})_{c'c,d'd} G_0 t_d G_0(t_{s'})_{d'd,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{a'(\subset s)} \sum_{b',d'(\subset s')} \sum_{a(\subset a')} \sum_{b(\subset b')} \sum_{d(\subset d')} t_a G_0(t_s)_{a'a,-} g_0(U_{ss'})_{-,d'd} G_0 t_d G_0(t_{s'})_{d'd,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{a',c'(\subset s)} \sum_{b'(\subset s')} \sum_{a(\subset a')} \sum_{c(\subset c')} \sum_{b(\subset b')} t_a G_0(t_s)_{a'a,c'c} G_0 t_c G_0(U_{ss'})_{c'c,-} g_0(t_{s'})_{-,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{a'(\subset s)} \sum_{b'(\subset s')} \sum_{a(\subset a')} \sum_{b(\subset b')} t_a G_0(t_s)_{a'a,-} g_0(U_{ss'})_{-,-} g_0(t_{s'})_{-,b'b} G_0 t_b \\
&+\sum_s \sum_{a'(\subset s)} \sum_{a(\subset a')} t_a G_0(t_s)_{a'a,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{a',c'(\subset s)} \sum_{a(\subset a')} \sum_{c(\subset c')} \sum_{b'(\subset s')} \sum_{b(\subset b')} t_a G_0(t_s)_{a'a,c'c} G_0 t_c G_0(U_{ss'})_{c'c,b'b} G_0 t_b G_0(t_{s'})_{b'b,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{a'(\subset s)} \sum_{a(\subset a')} \sum_{b'(\subset s')} \sum_{b(\subset b')} t_a G_0(t_s)_{a'a,-} g_0(U_{ss'})_{-,b'b} G_0 t_b G_0(t_{s'})_{b'b,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{a',c'(\subset s)} \sum_{a(\subset a')} \sum_{c(\subset c')} t_a G_0(t_s)_{a'a,c'c} G_0 t_c G_0(U_{ss'})_{c'c,-} g_0(t_{s'})_{-,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{a'(\subset s)} \sum_{a(\subset a')} t_a G_0(t_s)_{a'a,-} g_0(U_{ss'})_{-,-} g_0(t_{s'})_{-,-} g_0\Big(\sum_j f_j^\dagger\Big)+\sum_s \sum_{b'(\subset s)} \sum_{b(\subset b')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{b',d'(\subset s')} \sum_{b(\subset b')} \sum_{d(\subset d')} \sum_{c'(\subset s)} \sum_{c(\subset c')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,c'c} G_0 t_c G_0(U_{ss'})_{c'c,d'd} G_0 t_d G_0(t_{s'})_{d'd,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{b',d'(\subset s')} \sum_{b(\subset b')} \sum_{d(\subset d')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,-} g_0(U_{ss'})_{-,d'd} G_0 t_d G_0(t_{s'})_{d'd,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{b'(\subset s')} \sum_{b(\subset b')} \sum_{c'(\subset s)} \sum_{c(\subset c')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,c'c} G_0 t_c G_0(U_{ss'})_{c'c,-} g_0(t_{s'})_{-,b'b} G_0 t_b \\
&+\sum_{ss'} \sum_{b'(\subset s')} \sum_{b(\subset b')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,-} g_0(U_{ss'})_{-,-} g_0(t_{s'})_{-,b'b} G_0 t_b+\sum_s \Big(\sum_i f_i\Big) g_0(t_s)_{-,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{c'(\subset s)} \sum_{c(\subset c')} \sum_{d'(\subset s')} \sum_{d(\subset d')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,c'c} G_0 t_c G_0(U_{ss'})_{c'c,d'd} G_0 t_d G_0(t_{s'})_{d'd,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{d'(\subset s')} \sum_{d(\subset d')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,-} g_0(U_{ss'})_{-,d'd} G_0 t_d G_0(t_{s'})_{d'd,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \sum_{c'(\subset s)} \sum_{c(\subset c')} \Big(\sum_i f_i\Big) g_0(t_s)_{-,c'c} G_0 t_c G_0(U_{ss'})_{c'c,-} g_0(t_{s'})_{-,-} g_0\Big(\sum_j f_j^\dagger\Big) \\
&+\sum_{ss'} \Big(\sum_i f_i\Big) g_0(t_s)_{-,-} g_0(U_{ss'})_{-,-} g_0(t_{s'})_{-,-} g_0\Big(\sum_j f_j^\dagger\Big). \qquad (3.18)
\end{aligned}
$$

If we introduce at this point the quasiparticle expansion Eq. (3.14) we obtain $T(1|1)$ expressed in terms of new folded amplitudes referring to the subsystem (or channel) dynamics

$$(x_s)_{a'a,b'b}=\langle a|G_0(t_s)_{a'a,b'b}G_0|b\rangle, \tag{3.19a}$$

$$(x_s)_{a'a,-}=\langle a|G_0(t_s)_{a'a,-}, \tag{3.19b}$$

$$(x_s^\dagger)_{-,b'b}=(t_s)_{-,b'b}G_0|b\rangle, \tag{3.19c}$$

$$(x_s)_{-,-}\equiv(t_s)_{-,-}, \tag{3.19d}$$

and to the total system

$$(X_{ss'})_{a'a,b'b}=\langle a|G_0(U_{ss'})_{a'a,b'b}G_0|b\rangle, \tag{3.20a}$$

$$(X_{ss'})_{a'a,-}=\langle a|G_0(U_{ss'})_{a'a,-}, \tag{3.20b}$$

$$(X_{ss'}^\dagger)_{-,b'b}=(U_{ss'})_{-,b'b}G_0|b\rangle, \tag{3.20c}$$

$$(X_{ss'})_{-,-}\equiv(U_{ss'})_{-,-}. \tag{3.20d}$$

The corresponding expression of $T(1|1)$ in terms of $\mathbf{x}_s$ and $\mathbf{X}_{ss'}$ will be omitted for brevity but the derivation is quite obvious starting from Eq. (3.18): the quantities $U_{ss'}$, t_s, endowed where appropriate with the Green function G_0, are replaced by $\mathbf{X}_{ss'}$ and $\mathbf{x}_s$, respectively, while the two-body t matrix t_a is substituted with $\tau_a^{(3)}$. Finally $\tau_a^{(3)}$ is further dressed with the state vector $|a^{(3)}\rangle$ ($\langle a^{(3)}|$) if the left (right) state refers to the asymptotic state rather than to an intermediate state.

The quasiparticle equation for the subsystem amplitudes can be immediately obtained by folding the equation (2.15) between the states $\langle a|G_0$ and $G_0|b\rangle$. The result is

$$\mathbf{x}_s=\mathbf{z}_s+\mathbf{z}_s\mathcal{G}^{(3)}\mathbf{x}_s, \tag{3.21}$$

where

$$\mathbf{z}_s=\begin{pmatrix}(z_s)_{a'a,b'b} & (z_s)_{a'a,-}\\ (z_s^\dagger)_{-,b'b} & (z_s)_{-,-}\end{pmatrix}$$

$$=\begin{pmatrix}\langle a|G_0|b\rangle\delta_{a'b'}\overline{\delta}_{ab}\delta_{ab\subset a'}\delta_{a'\subset s} & \langle a|G_0(f_s)_{a'a}\\ (f_s^\dagger)_{b'b}G_0|b\rangle & \mathcal{V}_s\end{pmatrix} \tag{3.22}$$

and with the three-cluster (quasiparticle) propagator given by

$$\mathcal{G}^{(3)}=\begin{pmatrix}\tau_a^{(3)}\delta_{ab} & 0\\ 0 & g_0\end{pmatrix}. \tag{3.23}$$

Within the same matrix formalism, the solution of the equation for the subsystems is represented as

$$\mathbf{x}_s=\begin{pmatrix}(x_s)_{a'a,b'b} & (x_s)_{a'a,-}\\ (x_s^\dagger)_{-,b'b} & (x_s)_{-,-}\end{pmatrix}, \tag{3.24}$$

where the elements (for each value of s) are spanned by chains of partitions in the one-pion sector, completed with the additional component in the no-pion zone (in case $s\neq 0$), in close analogy with the quantities $\mathbf{t}_s$. This leads to 6×6 matrices for $s\neq 0$, while we have the standard 3×3 matrix for the $s=0$ partition. Obviously, the same considerations previously observed for $\mathbf{v}_s\mathbf{G}_0^{(3)}\mathbf{t}_s$ apply also for $\mathbf{z}_s\mathcal{G}^{(3)}\mathbf{x}_s$.

It might be useful to illustrate diagrammatically what the six components represent, e.g., for $s=1$, as has been done in Fig. 1. Here the diagrams representing the z interaction, i.e., the driving term of Eq. (3.21), have been drawn. The figure represents the diagrams in a square grid denoting the 6×6 interaction matrix. Both columns and rows are ordered so that the first three elements represent the $(\pi,\ N_2)$, $(\pi,\ N_3)$, and $(N_2,\ N_3)$ pairs originating from the $[(\pi,\ N_2,\ N_3),\ N_1]$ two-cluster partition, the fourth and fifth elements represent the $(N_2,\ N_3)$ and $(\pi,\ N_1)$ pairs obtained from the breakup of the second two-cluster partition, $[(N_2,\ N_3),\ (\pi,\ N_1)]$, and finally the last element denotes the no-pion state with the three nucleons all disentangled.

In the bottom-right corner, one easily recognizes the two-nucleon OPE diagram, which is therefore extended in the present formulation to embrace the entire set of diagrams shown by the figure. As a matter of fact, for obvious reasons of simplicity two diagrams have been omitted. One is a second OPE diagram, similar to that already shown but with the opposite time ordering, and then (in the third row and last column) there should be another diagram where the red and green lines (nucleons ''2'' and ''3'') are interchanged. It is clear that the same situation occurs in the symmetric case (third column and last row). In passing we observe that if one considers the iterations of this driving term, i.e., $\mathbf{z}_s\mathcal{G}^{(3)}\mathbf{z}_s$ and so on, it is possible to generate in one single step all the disconnected diagrams which have been illustrated in Figs. 6 and 8 of Ref. [5]. The last diagram of Fig. 6, in particular, represents an off-energy-shell effect where the two-nucleon scattering amplitude (the one defined in the four-body sector) appears while the spectator nucleon undergoes an intermediate pion emission-reabsorption process. This self-energy contribution has to be explicitly taken into account at the present stage of the theory, and plays a role which is analogous to a two-body scattering process in the presence of a virtual dissociation of a composite spectator. The relevance of that process is well known in standard N-body theory.

In the same way as done for the subsystem dynamics, from Eq. (2.27) it is possible to obtain the following equation for the folded amplitudes referring to the entire system, which we write as

$$\mathbf{X}_{ss'}=\mathcal{G}^{(3)-1}\overline{\Delta}_{ss'}+\sum_{s''}\overline{\Delta}_{ss''}\mathbf{x}_{s''}\mathcal{G}^{(3)}\mathbf{X}_{s''s'}. \tag{3.25}$$

Here, we have introduced a new matrix operator, $\overline{\Delta}$, defined as follows:

$$(\overline{\Delta}_{ss'})_{a'a,b'b}\equiv\delta_{ab}\overline{\delta}_{a'b'}=\delta_{ab}(\overline{\delta}_{ss'}+\delta_{ss'}\overline{\delta}_{a'b'}), \tag{3.26a}$$

$$(\overline{\Delta}_{ss'})_{a'a,-}\equiv 0, \tag{3.26b}$$

$$(\overline{\Delta}_{ss'})_{-,b'b}\equiv 0, \tag{3.26c}$$

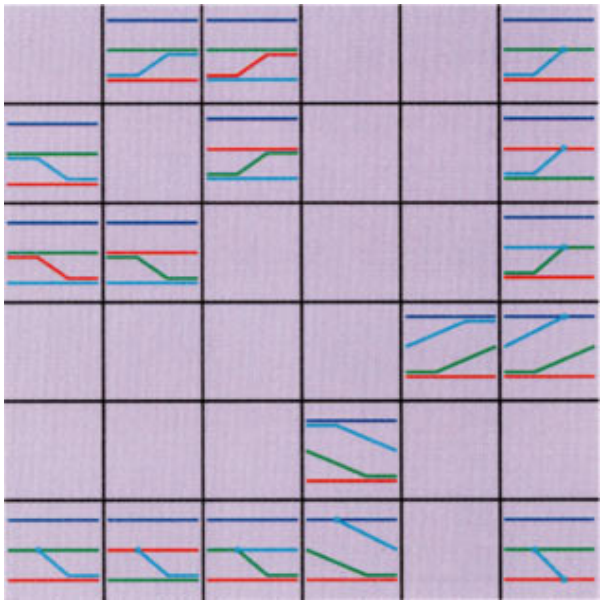

FIG. 1. (Color) Disconnected three-cluster exchange diagrams $\mathbf{z}_s$, for $s=1$. These diagrams contribute to the interaction between nucleons ''2'' and ''3'' (green and red lines, respectively). The blue line (nucleon ''1'') is always disconnected from the green and red ones, for any iteration of the diagrams belonging to this set. The pale blue line represents the pion.

$$(\overline{\Delta}_{ss'})_{-,-} \equiv \overline{\delta}_{ss'}. \tag{3.26d}$$

At this point, we can proceed with the iteration of the quasiparticle expansion, and introduce the separable structure for the four subamplitudes of the system,

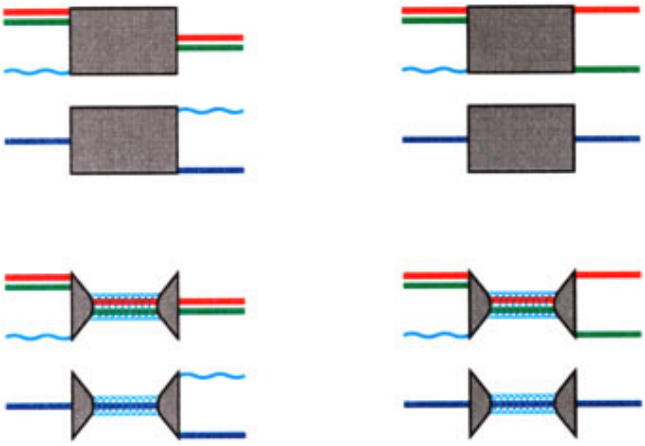

FIG. 2. (Color) Examples of disconnected three-cluster amplitudes, for $s=1$. The two diagrams on top of the figure represent the subamplitude $\mathbf{x}_s$, with $s=1$. In particular, the case $(x_s)_{a'a,b'b}$ with $a=b$ and $a'\neq b'$ has been chosen for the top-left diagram, while the top-right diagram represents the production subamplitude $(x_s)_{a'a,-}$. The corresponding diagrams on the bottom side denote the very same amplitudes in the quasiparticle formalism. Here, the intermediate propagation of the multiparticle two-fragment partition is exhibited by drawing the nucleonic lines surrounded by a pionic concentric line. For $s=1$ the three possible intermediate two-cluster components are $[(\pi N_2 N_3)N_1]$, $[(N_2 N_3)(N_1\pi)]$, and $[(N_2 N_3)N_1]$.

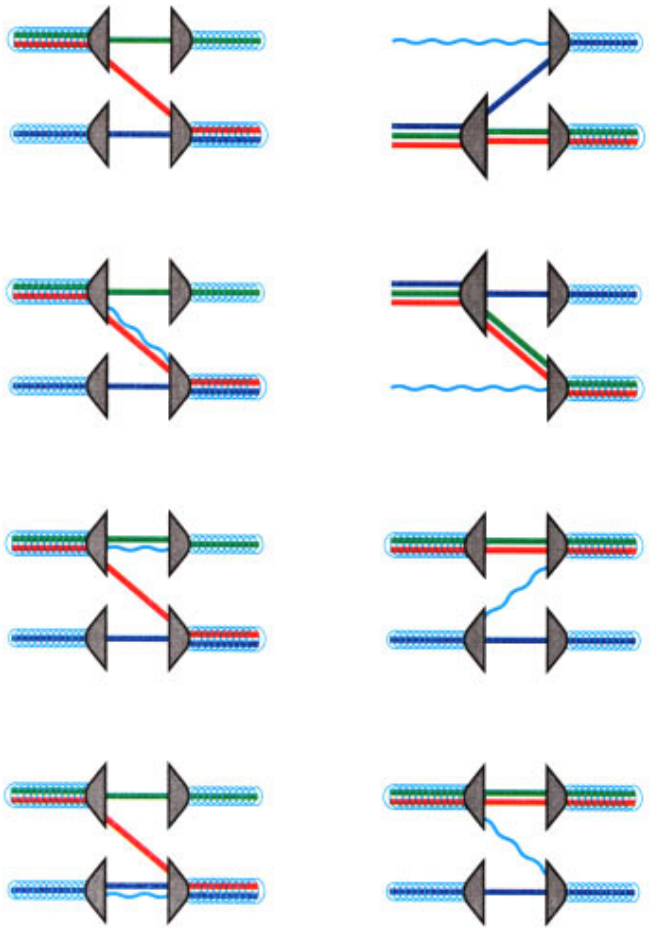

FIG. 3. (Color) Two-cluster exchange diagrams. The figure shows the exchange diagrams contributing to the two-cluster potential $Z^{(2)}_{ss'}$ of Eq. (3.28). The four diagrams on the left side contribute to $Z^{(2)}_{ss'}$ for $0\neq s\neq s'\neq 0$, while the two top diagrams on the right side contribute for $0=s\neq s'$, and finally the remaining two bottom diagrams contribute for $s=s'\neq 0$. There are no other diagrams to lowest order (aside from those obtained from permutation of the three colors) and they are all connecting-type diagrams.

$$(x_s)_{a'a,b'b} = |(s^{(2)})_{a'a}\rangle \tau^{(2)}_s \langle (s^{(2)})_{b'b}|, \tag{3.27a}$$

$$(x_s)_{a'a,-} = |(s^{(2)})_{a'a}\rangle \tau^{(2)}_s \langle (s^{(2)})_{-}|, \tag{3.27b}$$

$$(x^{\dagger}_s)_{-,b'b} = |(s^{(2)})_{-}\rangle \tau^{(2)}_s \langle (s^{(2)})_{b'b}|, \tag{3.27c}$$

$$(x_s)_{-,-} = |(s^{(2)})_{-}\rangle \tau^{(2)}_s \langle (s^{(2)})_{-}|. \tag{3.27d}$$

(As usual at this point, we must note that in case $s=0$ the states $|s^{(2)}\rangle$ have no components in the no-pion sector.)

In the upper side of Fig. 2 we represent two examples of disconnected amplitude $\mathbf{x}_s$, both referring to the partition $s=1$, where the blue line is not connected with the red and green ones. The boxlike diagram on the left represents a process connecting two states of the four-body sector. We have chosen the special case where the ''in'' and ''out'' three-cluster states coincide. In spite of this fact, the diagram does not represent a *diagonal* matrix element, because the three-cluster partition on the right coalesces into a 2+2 two-cluster partition, while the same three-cluster partition on the left has been originated from the breakup of the 3+1 parti-

tion. The boxlike diagram on the right represents a disconnected production amplitude, where there is a collision between nucleons ''2'' and ''3'' in the presence of the nucleon ''1,'' with the pion in the final three-cluster state. The selected production amplitude shows that the final three-cluster partition derives from the breakup of the 3+1 two-cluster partition, however, it must be kept in mind that the final three-cluster state can be obtained also from the 2+2 partition. This indicates that the role of the spectator nucleon (the blue line in the diagram) is not passive at all, since it can still interact with the pion. This contrasts with the standard three-particle case where the spectator merely plays a passive role. In the lower part of the figure, the same amplitudes are represented in the form of quasiparticle diagrams, thus reproducing Eqs. (3.27). The diagrams represent the processes passing through the intermediate propagation of a multiparticle two-cluster state, where the nucleon ''1'' is always disconnected from the other two. The pion, however, is shared between both parts without being physically exchanged from one to the other.

Introducing the new separable expansion of the subamplitudes in Eq. (3.25), and folding the equation with the new states $\mathcal{G}^{(3)}|s^{(2)}\rangle$ referring to the two-cluster partitions one obtains the final quasiparticle equation

$$X^{(2)}_{ss'}=Z^{(2)}_{ss'}+\sum_{s''} Z^{(2)}_{ss''}\mathcal{G}^{(2)}_{s''}X^{(2)}_{s''s'}, \tag{3.28}$$

where

$$Z^{(2)}_{ss'}=\langle s^{(2)}|\mathcal{G}^{(3)}\overline{\Delta}_{ss'}|s'^{(2)}\rangle\equiv\langle(s^{(2)})_-|g_0|(s'^{(2)})_-\rangle\overline{\delta}_{ss'}$$
$$+\sum_{a'(\subset s)}\sum_{b'(\subset s')}\sum_{a(\subset a',b')}\langle(s^{(2)})_{a'a}|\tau^{(3)}_a|(s'^{(2)})_{b'a}\rangle(\overline{\delta}_{ss'}+\delta_{ss'}\overline{\delta}_{a'b'}), \tag{3.29}$$

$$\mathcal{G}^{(2)}_s=\tau^{(2)}_s, \tag{3.30}$$

$$X^{(2)}_{ss'}=\langle s^{(2)}|\mathcal{G}^{(3)}\mathbf{X}_{ss'}\mathcal{G}^{(3)}|s'^{(2)}\rangle$$
$$\equiv\langle(s^{(2)})_-|g_0[(X_{ss'})_{-,-}]g_0|(s'^{(2)})_-\rangle+\sum_{a'(\subset s)}\sum_{b'(\subset s')}\sum_{a(\subset a')}\sum_{b(\subset b')}\langle(s^{(2)})_{a'a}|\tau^{(3)}_a[(X_{ss'})_{a'a,b'b}]\tau^{(3)}_b|(s'^{(2)})_{b'b}\rangle$$
$$+\sum_{a'(\subset s)}\sum_{a(\subset a')}\langle(s^{(2)})_{a'a}|\tau^{(3)}_a[(X_{ss'})_{a'a,-}]g_0|(s'^{(2)})_-\rangle+\sum_{b'(\subset s')}\sum_{b(\subset b')}\langle(s^{(2)})_-|g_0[(X_{ss'})_{-,b'b}]\tau^{(3)}_b|(s'^{(2)})_{b'b}\rangle. \tag{3.31}$$

The expression Eq. (3.28) represents the two-cluster connected-kernel equation which solves the πNNN-NNN problem. It represents the translation within the quasiparticle formalism of the general result represented by Eq. (2.27). In spite of the fact that Eq. (3.28) must be considered an approximated result holding only when the *t*-matrix separability is assumed, nevertheless the result should be considered under a very general perspective because the representation of the *t* matrix as a sum of separable terms is a mathematically converging procedure [28] and approaches of this kind have been demonstrated to work numerically [29] in few-body applications involving realistic nuclear interactions.

In Eq. (3.28) the complete dynamics of Eq. (2.27) is represented in terms of two-body multiparticle correlated states (bound states, or resonances, etc., for the subsystems). They give a physically clear description of the meaning of the general equations, otherwise difficult to interpret in terms of processes or diagrams. For instance, Eq. (3.28) can be easily compared with the AGS quasiparticle equation for the standard three-particle problem: Here, the equation is endowed with a fourth component (the $s=0$ component) which does not appear in the AGS equations, and the number of diagrams contributing to the *Z*-exchange terms are considerably larger with some of them giving rise to totally new mechanisms. This can be seen in Fig. 3 where the diagrams contributing to the *Z* terms [as expressed by Eq. (3.29)] are illustrated.

IV. THE BOUND-STATE EQUATION

This section is devoted to the discussion of the bound-state equation for the πNNN-NNN system. The equation we derive is in fact a bound-state equation for the three-nucleon system, but has the special feature that it incorporates explicitly the pion dynamics (limited to the degree of freedom of one pion), while in the standard approach this aspect is usually restricted in the limits of the OPE tail of the NN interaction.

In the AGS approach, the three-nucleon bound state is associated to the homogeneous solution of the AGS equation. In close similarity, here we seek the homogeneous solution of the TRABAM equation for the πNNN system.

According to the matrix notation previously introduced, we denote the homogeneous equation as

$$|\mathbf{\Gamma}^{(3)}\rangle=\mathbf{V}^{(3)}\mathbf{G}^{(3)}_0|\mathbf{\Gamma}^{(3)}\rangle, \tag{4.1}$$

where $|\mathbf{\Gamma}^{(3)}\rangle$ represents the state eigenvector of the operator

$\mathbf{V}^{(3)}\mathbf{G}_0^{(3)}$. Obviously $|\Phi^{(3)}\rangle = \mathbf{G}_0^{(3)}|\mathbf{\Gamma}^{(3)}\rangle$ represents the analogous eigenvector for the transposed kernel

$$|\Phi^{(3)}\rangle = \mathbf{G}_0^{(3)}\mathbf{V}^{(3)}|\Phi^{(3)}\rangle. \tag{4.2}$$

If we neglect all the couplings with the pion sector, this last equation represents precisely the Schrödinger equation for the three-nucleon system, with the constituents interacting through pairwise potentials and in such a case $|\Phi^{(3)}\rangle$ denotes simply the complete three-body Schrödinger wave function. Once the one-pion degrees of freedom are explicitly included in the theory, the equation acquires the typical TRABAM-like structure and couples the three-nucleon Schrödinger wave function with the six Faddeev-like components referring to the partition of the πNNN system into three clusters. Obviously, being the kernel of the homogeneous equation the same as discussed in the previous sections, we have an equation whose kernel is not connected. We proceed as follows.

We introduce the partitions of the system into two clusters and recall the interaction sum rule Eq. (2.25). Then, we define the new two-cluster-partition components for the wave function:

$$|(\Phi_s^{(2)})_{a'a}\rangle = G_0^{(3)} \sum_{b'(\subset s)} \sum_{b(\subset b')} (v_s)_{a'a,b'b} |(\Phi^{(3)})_b\rangle + G_0^{(3)}(v_s)_{a'a,-}|(\Phi^{(3)})_-\rangle \tag{4.3a}$$

and

$$|(\Phi_s^{(2)})_-\rangle = G_0^{(3)} \sum_{b'(\subset s)} \sum_{b(\subset b')} (v_s)_{-,b'b} |(\Phi^{(3)})_b\rangle + G_0^{(3)}(v_s)_{-,-}|(\Phi^{(3)})_-\rangle, \tag{4.3b}$$

where the first expression refers to components associated to the four-body sector while the second one to the components in the three-nucleon space.

With this definition from the homogeneous equation for $|\Phi^{(3)}\rangle$, Eq. (4.2), it is possible to express the three-cluster components as sum over all the two-cluster partitions

$$|(\Phi^{(3)})_a\rangle = \sum_s \sum_{a'(\subset s)} |(\Phi_s^{(2)})_{a'a}\rangle, \tag{4.4a}$$

$$|(\Phi^{(3)})_-\rangle = \sum_s |(\Phi_s^{(2)})_-\rangle. \tag{4.4b}$$

From the last two equations it is possible to write a new homogeneous coupled equation whose solution directly yields the components $|\Phi_s^{(2)}\rangle$. We obtain

$$|(\Phi_s^{(2)})_{a'a}\rangle = \sum_{b'(\subset s)} \sum_{b(\subset b')} G_0^{(3)}(v_s)_{a'a,b'b} \times \sum_{s'} \sum_{c'(\subset s')} |(\Phi_{s'}^{(2)})_{c'b}\rangle + G_0^{(3)}(v_s)_{a'a,-} \sum_{s'} |(\Phi_{s'}^{(2)})_-\rangle \tag{4.5a}$$

and

$$|(\Phi_s^{(2)})_-\rangle = \sum_{b'(\subset s)} \sum_{b(\subset b')} G_0^{(3)}(v_s)_{-,b'b} \times \sum_{s'} \sum_{c'(\subset s')} |(\Phi_{s'}^{(2)})_{c'b}\rangle + G_0^{(3)}(v_s)_{-,-} \sum_{s'} |(\Phi_{s'}^{(2)})_-\rangle \tag{4.5b}$$

for the components in the four-body and three-nucleon sectors, respectively.

With simple algebraic manipulations we obtain

$$\begin{aligned} |(\Phi_s^{(2)})_{a'a}\rangle = & \sum_{b'(\subset s)} \sum_{c'(\subset s)} \sum_{b(\subset b',c')} \\ & \times G_0^{(3)}(v_s)_{a'a,b'b} |(\Phi_s^{(2)})_{c'b}\rangle \\ & + G_0^{(3)}(v_s)_{a'a,-}|(\Phi_s^{(2)})_-\rangle \\ & + \sum_{s'} \bar{\delta}_{ss'} \sum_{b'(\subset s)} \sum_{c'(\subset s')} \sum_{b(\subset b',c')} \\ & \times G_0^{(3)}(v_s)_{a'a,b'b} |(\Phi_{s'}^{(2)})_{c'b}\rangle \\ & + G_0^{(3)}(v_s)_{a'a,-} \sum_{s'} \bar{\delta}_{ss'} |(\Phi_{s'}^{(2)})_-\rangle \end{aligned} \tag{4.6a}$$

and

$$\begin{aligned} |(\Phi_s^{(2)})_-\rangle = & \sum_{b'(\subset s)} \sum_{c'(\subset s)} \sum_{b(\subset b',c')} G_0^{(3)}(v_s)_{-,b'b} |(\Phi_s^{(2)})_{c'b}\rangle \\ & + G_0^{(3)}(v_s)_{-,-}|(\Phi_s^{(2)})_-\rangle \\ & + \sum_{s'} \bar{\delta}_{ss'} \sum_{b'(\subset s)} \sum_{c'(\subset s)} \sum_{b(\subset b',c')} \\ & \times G_0^{(3)}(v_s)_{-,b'b} |(\Phi_{s'}^{(2)})_{c'b}\rangle \\ & + G_0^{(3)}(v_s)_{-,-} \sum_{s'} \bar{\delta}_{ss'} |(\Phi_{s'}^{(2)})_-\rangle . \end{aligned} \tag{4.6b}$$

The last two equations can be rewritten as

$$\begin{aligned} & |(\Phi_s^{(2)})_{a'a}\rangle - \sum_{b'(\subset s)} \sum_{b(\subset b')} G_0^{(3)}(v_s)_{a'a,b'b} |(\Phi_s^{(2)})_{b'b}\rangle \\ & \quad - G_0^{(3)}(v_s)_{a'a,-}|(\Phi_s^{(2)})_-\rangle \\ & = \sum_{s'} \sum_{b'(\subset s)} \sum_{c'(\subset s')} \sum_{b(\subset b',c')} \\ & \quad \times G_0^{(3)}(v_s)_{a'a,b'b} (\bar{\delta}_{ss'} + \delta_{ss'}\bar{\delta}_{b'c'}) |(\Phi_{s'}^{(2)})_{c'b}\rangle \\ & \quad + \sum_{s'} G_0^{(3)}(v_s)_{a'a,-} \bar{\delta}_{ss'} |(\Phi_{s'}^{(2)})_-\rangle \end{aligned} \tag{4.7a}$$

and

$$|(\Phi_s^{(2)})_-\rangle - \sum_{b'b} G_0^{(3)}(v_s)_{-,b'b}|(\Phi_s^{(2)})_{b'b}\rangle - G_0^{(3)}(v_s)_{-,-}|(\Phi_s^{(2)})_-\rangle$$
$$= \sum_{s'} \sum_{b'(\subset s)} \sum_{c'(\subset s')} \sum_{b(\subset b',c')} \times G_0^{(3)}(v_s)_{-,b'b}(\overline{\delta}_{ss'} + \delta_{ss'}\overline{\delta}_{b'c'})|(\Phi_{s'}^{(2)})_{c'b}\rangle + \sum_{s'} G_0^{(3)}(v_s)_{-,-}\overline{\delta}_{ss'}|(\Phi_{s'}^{(2)})_-\rangle. \quad (4.7b)$$

From these, employing the equations for the subsystem amplitudes, Eqs. (2.15), it is possible to obtain the final bound-state equation,

$$|(\Phi_s^{(2)})_{a'a}\rangle = \sum_{s'} \sum_{b'(\subset s)} \sum_{c'(\subset s')} \sum_{b(\subset b',c')} \times G_0 t_a G_0(t_s)_{a'a,b'b} \times (\overline{\delta}_{ss'} + \delta_{ss'}\overline{\delta}_{b'c'})|(\Phi_{s'}^{(2)})_{c'b}\rangle + \sum_{s'} G_0 t_a G_0(t_s)_{a'a,-}\overline{\delta}_{ss'}|(\Phi_{s'}^{(2)})_-\rangle \quad (4.8a)$$

and

$$|(\Phi_s^{(2)})_-\rangle = \sum_{s'} \sum_{b'(\subset s)} \sum_{c'(\subset s')} \sum_{b(\subset b',c')} g_0(t_s)_{-,b'b} \times (\overline{\delta}_{ss'} + \delta_{ss'}\overline{\delta}_{b'c'})|(\Phi_{s'}^{(2)})_{c'b}\rangle + \sum_{s'} g_0(t_s)_{-,-}\overline{\delta}_{ss'}|(\Phi_{s'}^{(2)})_-\rangle. \quad (4.8b)$$

Equations (4.8) represent the generalization of the bound-state three-nucleon equation and include in the three-nucleon dynamics also the pion dynamics. The bound-state wave function corresponds to the solution of the homogeneous equation whose kernel is transposed with respect to that of Eq. (2.27) for the scattering amplitudes.

In the no-pion sector the complete three-nucleon wave function is given simply by the sum over the three components $s=1,2,3$ (the $s=0$ case has no direct component in the no-pion sector):

$$|(\Phi^{(3)})_-\rangle = \sum_s |(\Phi_s^{(2)})_-\rangle. \quad (4.9)$$

This result is similar to that obtained in standard Faddeev theory, where the three-nucleon bound state is given by the sum over the three Faddeev components.

One may consider at this point the other component of the wave function, the one acting in the four-body sector (obviously, in the standard three-nucleon theory these components are set identically to zero). The wave function $|\Phi^{(3)}\rangle$ in the four-body sector spans the six three-cluster partitions of the πNNN system. In a standard four-body theory the complete wave function is given by the sum over these six Faddeev components. In the present theory we have to take into account the fact that a contribution to the wave function may arise by pion emission from the pure three-nucleon component, therefore the four-body component to the three-nucleon bound-state wave function is given by

$$|\Phi^{(4)}\rangle = \sum_{a=1}^{6} |\Phi_a^{(3)}\rangle + G_0\left(\sum_{i=1}^{3} f_i\right)|\Phi_-^{(3)}\rangle = \sum_s \sum_{a'(\subset s)} \sum_{a(\subset a')} |(\Phi_s^{(2)})_{a'a}\rangle + G_0\left(\sum_{i=1}^{3} f_i\right)\sum_s |(\Phi_s^{(2)})_-\rangle. \quad (4.10)$$

Equation (4.9) represents the residue of this term at the nucleon pole for a zero-energy pion.

V. REARRANGEMENT AND BREAKUP AMPLITUDES

In Sec. II we have restricted the discussion to the fully unclusterized amplitudes (four-to-four) or at most to the three-to-three amplitudes. Then in Sec. III we have given the rules to calculate $T(1|1)$ with the quasiparticle formalism. It is clear that from the phenomenological point of view the most interesting amplitudes are between channels involving the two-cluster partitions, or amplitudes where at least the incoming state refers to an asymptotic configuration where the system is partitioned into two clusters. To obtain such amplitudes, we start from the three-to-three amplitudes and apply the residue method. To this end we introduce the homogeneous equations associated with the two-cluster partition:

$$|\gamma_s^{(2)}(E_s)\rangle = \mathbf{v}_s(E_s)\mathbf{G}_0^{(3)}(E_s)|\gamma_s^{(2)}(E_s)\rangle, \quad (5.1)$$

where for each $s\neq 0$ the state $|\gamma_s^{(2)}\rangle$ represents a channel vector with one component in the no-pion sector and five components in the one-pion sector (corresponding to all possible chains of partitions starting from the $2+2$ and $3+1$ partitions compatible with s). For the special case $s=0$, the same equation couples only the three chains of partitions which start from the $\pi+(\mathrm{NNN})$ separation in two clusters, and has no components in the 3N sector. Similarly, one can introduce also the corresponding homogeneous equation for the bra states

$$\langle\gamma_s^{(2)}(E_s)| = \langle\gamma_s^{(2)}(E_s)|\mathbf{G}_0^{(3)}(E_s)\mathbf{v}_s(E_s). \quad (5.2)$$

Obviously for each s, with the transforming relations

$$|\gamma_s^{(2)}\rangle = \mathbf{v}_s|\phi_s^{(2)}\rangle, \quad (5.3)$$

$$|\phi_s^{(2)}\rangle = \mathbf{G}_0^{(3)}|\gamma_s^{(2)}\rangle, \quad (5.4)$$

it is possible to associate an asymptotic channel state satisfying a bound-state-type equation (the energy dependence has been omitted) for the two noninteracting fragments

$$|\phi_s^{(2)}\rangle = \mathbf{G}_0^{(3)}\mathbf{v}_s|\phi_s^{(2)}\rangle. \quad (5.5)$$

We can view explicitly how in case $s\neq 0$ the new equation couples the chain space in the four-body sector with the 3N space by writing in detail the homogeneous equation

$$|(\phi_s^{(2)})_{a'a}\rangle=\sum_{b(\subset a')} G_0 t_a \overline{\delta}_{ab}|(\phi_s^{(2)})_{a'b}\rangle + G_0 t_a G_0 (f_s)_{a'a}|(\phi_s^{(2)})_-\rangle, \quad (5.6a)$$

$$|(\phi_s^{(2)})_-\rangle=\sum_{b'(\subset s)}\sum_{b(\subset b')} g_0 (f_s^\dagger)_{b'b}|(\phi_s^{(2)})_{b'b}\rangle + g_0 \mathcal{V}_s|(\phi_s^{(2)})_-\rangle, \quad (5.6b)$$

while for $s=0$ we have a standard three-component (Faddeev-like) 3N bound-state equation, with the pion acting as a spectator. In case the couplings between the two spaces are switched off, each coupled six-component equation for $s\neq 0$ decouples into the three different equations, one single-component homogeneous equation for the NN pair in the presence of a spectator nucleon plus one three-component Faddeev equation for the $3+1$ partition, and one analogous two-component coupled equation for the corresponding $2+2$ partition. With the meson-nucleon vertex interaction turned on, these three different equations merge in one single coupled equation.

At energies E_s corresponding to nontrivial solutions of the homogeneous equations it follows that the solution of the inhomogeneous equation $\mathbf{t}_s$ has a pole, and around such values the t matrix for the subsystem can be represented in polar form

$$\mathbf{t}_s(z)\simeq|\gamma_s^{(2)}\rangle\frac{1}{z-E_s}\langle\gamma_s^{(2)}|+\cdots, \quad (5.7)$$

where the omitted contributions are nonsingular background remainders.

According to the residue method, the clusterized transition amplitudes can be obtained from the general expression for $\mathbf{T}^{(3)}$, Eqs. (2.26), by extracting the residues once the poles of the subamplitudes are exhibited. For instance, if we assume $s=0$ and $s'\neq 0$ and assuming that for E_s and $E_{s'}$ the associated homogeneous equations have a nontrivial (bound-state or narrow resonance) solution, then the corresponding two-cluster transition amplitude emerges as the residue of the double singularity in $\mathbf{T}^{(3)}$,

$$\mathbf{T}^{(3)}=|\overline{\gamma}_{s'}^{(2)}\rangle\frac{\mathcal{T}_{s's}}{(z-E_{s'})(z-E_s)}\langle\overline{\gamma}_s^{(2)}|+\cdots, \quad (5.8)$$

where the state vectors $|\gamma_s^{(2)}\rangle$ have been contracted by summing over the the two-body partitions of the four-body sector, $|(\overline{\gamma}_s^{(2)})_a\rangle=\Sigma_{a'(\subset s)}|(\gamma_s^{(2)})_{a'a}\rangle$.

The two-cluster transition matrix element is given by

$$\mathcal{T}_{s's}=\langle\phi_{s'}^{(2)}|U_{s's}|\phi_s^{(2)}\rangle = \sum_{a'(\subset s')}\sum_{a(\subset a')}\sum_{b'(\subset s)}\sum_{b(\subset b')} \times\langle(\phi_{s'}^{(2)})_{a'a}|(U_{s's})_{a'a,b'b}|(\phi_s^{(2)})_{b'b}\rangle + \sum_{b'(\subset s)}\sum_{b(\subset b')}\langle(\phi_{s'}^{(2)})_-|(U_{s's})_{-,b'b}|(\phi_s^{(2)})_{b'b}\rangle, \quad (5.9)$$

where in the last expression on the right the components for s' acting in each sector of the theory have been explicitly given. In this approach, such an amplitude represents the process $\pi+(\mathrm{NNN})\to \mathrm{N}+(\mathrm{NN})$ where the contributions of the type $(\pi\mathrm{N})+(\mathrm{NN})$, and $\mathrm{N}+(\mathrm{NN}\pi)$ are both dynamically included together with the $\mathrm{N}+(\mathrm{NN})$ partition.

We may at this point report on breakup reaction amplitudes, such as $[\pi\ (\mathrm{NNN})\to \mathrm{N\,N\,N}]$, $[\pi\ (\mathrm{NNN})\to \pi\ \mathrm{N}\ (\mathrm{NN})]$ and finally $[\pi\ (\mathrm{NNN})\to \mathrm{N\,N\,N}\ \pi]$. The first two can be obtained from $\mathbf{T}^{(3)}$ by extraction of the residue of a single two-cluster partition (bound-state) singularity, while for the last case one has to consider the single residue from Eq. (3.18).

We have (with $s=0$)

$$\mathcal{T}_{0s}[\mathrm{NNN}\leftarrow\pi(\mathrm{NNN})] = \sum_{s'}\sum_{a'(\subset s')}\sum_{a(\subset a')}\sum_{b'(\subset s)}\sum_{b(\subset b')} \times\langle(\chi_0^{(3)})|(t_{s'})_{-,a'a}G_0 t_a G_0 (U_{s's})_{a'a,b'b}|(\phi_s^{(2)})_{b'b}\rangle + \sum_{s'}\sum_{b'(\subset s)}\sum_{b(\subset b')} \times\langle(\chi_0^{(3)})|(t_{s'})_{-,-}g_0(U_{s's})_{-,b'b}|(\phi_s^{(2)})_{b'b}\rangle, \quad (5.10)$$

and

$$\mathcal{T}_{as}[(\mathrm{NN})\pi\mathrm{N}\leftarrow\pi(\mathrm{NNN})]=\sum_{s'}\sum_{a'(\subset s')}\sum_{c'(\subset s')}\sum_{c(\subset c')}\sum_{b'(\subset s)}\sum_{b(\subset b')}\langle\phi_a^{(3)}|(t_{s'})_{a'a,c'c}G_0 t_c G_0 (U_{s's})_{c'c,b'b}|(\phi_s^{(2)})_{b'b}\rangle + \sum_{s'}\sum_{a'(\subset s')}\sum_{b'(\subset s)}\sum_{b(\subset b')}\langle\phi_a^{(3)}|(t_{s'})_{a'a,-}g_0(U_{s's})_{-,b'b}|(\phi_s^{(2)})_{b'b}\rangle. \quad (5.11)$$

In this last case a represents the selected NN pair, in the presence of two remaining spectator particles, and $\langle \phi_a^{(3)}(E_a)| = \langle \phi_a^{(3)}(E_a)| v_a G_0(E_a)$ is the asymptotic three-cluster channel with two bound nucleons in the presence of two spectator particles.

The amplitude referring to the process with four outgoing fragments is

$$\begin{aligned}\mathcal{T}_{0s}[\,\pi\mathrm{NNN}\leftarrow\pi(\mathrm{NNN})\,] = &\sum_{s'}\sum_{a'(\subset s')}\sum_{a(\subset a')}\sum_{c'(\subset s')}\sum_{c(\subset c')}\sum_{b'(\subset s)}\sum_{b(\subset b')}\langle\chi_0^{(4)}|t_a G_0(t_{s'})_{a'a,c'c}G_0 t_c G_0(U_{s's})_{c'c,b'b}|(\phi_s^{(2)})_{b'b}\rangle \\ &+\sum_{s'}\sum_{a'(\subset s')}\sum_{a(\subset a')}\sum_{b'(\subset s)}\sum_{b(\subset b')}\langle\chi_0^{(4)}|t_a G_0(t_{s'})_{a'a,-}g_0(U_{s's})_{-,b'b}|(\phi_s^{(2)})_{b'b}\rangle \\ &+\sum_{s'}\sum_{c'(\subset s')}\sum_{c(\subset c')}\sum_{b'(\subset s)}\sum_{b(\subset b')}\langle\chi_0^{(4)}|\Big(\sum_i f_i\Big)g_0(t_{s'})_{-,c'c}G_0 t_c G_0(U_{s's})_{c'c,b'b}|(\phi_s^{(2)})_{b'b}\rangle \\ &+\sum_{s'}\sum_{b'(\subset s)}\sum_{b(\subset b')}\langle\chi_0^{(4)}|\Big(\sum_i f_i\Big)g_0(t_{s'})_{-,-}g_0(U_{s's})_{-,b'b}|(\phi_s^{(2)})_{b'b}\rangle. \end{aligned} \tag{5.12}$$

The states $\langle\chi_0^{(3,4)}|$ represent, respectively, the free three-nucleon and the four-particle asymptotic waves.

It is worthwhile to comment on one aspect common to all these amplitudes; namely, in all the physical reaction processes one has to sum over the possible (for a given s) two-cluster partitions of the four-body sector (herein denoted with a', b', and c'). The $s=0$ partition of the system is an exception only because it contains just one of these partitions (Table II). In other words, while the indices a' etc. are fundamental in the determination of the dynamical equation, they do not appear in the physical amplitudes, since these last quantities have to refer only to partitions of the whole system. This specific aspect of the coupled πNNN theory emerges from the structure of the dynamical equations, which are labeled in the four-body sector by chains of partitions, e.g., the pair $a'a$ with $a\subset a'$, while the physical amplitudes refer only to physical partitions of the complete system into clusters.

In the remaining part of this section we show how the quasiparticle method can be extended to the present situation to calculate the clusterized amplitudes. Previously, starting from the separable expansion $t_a^{(3)} = |a^{(3)}\rangle \tau_a^{(3)} \langle a^{(3)}|$ we arrived at the equation $\mathbf{x}_s = \mathbf{z}_s + \mathbf{z}_s \mathcal{G}^{(3)} \mathbf{x}_s$ where we observed that the operators $\mathbf{z}_s$ and $\mathbf{x}_s$ act in the chain-of-partition space of the four-body sector, and (for $s\neq 0$) in the space of two-cluster partitions of the three-nucleon sector. A second iteration of the quasiparticle method consisted in the exhibition of the separable structure for $\mathbf{x}_s$, $\mathbf{x}_s = |s^{(2)}\rangle \tau_s^{(2)} \langle s^{(2)}|$ which allowed us to derive the equation

$$X_{ss'}^{(2)} = Z_{ss'}^{(2)} + \sum_{s''} Z_{ss''}^{(2)} \mathcal{G}_{s''}^{(2)} X_{s''s'}^{(2)}. \tag{5.13}$$

In the general case, we have seen in this section that the two-cluster rearrangement amplitudes can be written as

$$\mathcal{T}_{ss'} = \langle \boldsymbol{\phi}_s^{(2)}| U_{ss'} |\boldsymbol{\phi}_s^{(2)}\rangle = \langle \boldsymbol{\gamma}_s^{(2)}| G_0^{(3)} U_{ss'} G_0^{(3)} |\boldsymbol{\gamma}_{s'}^{(2)}\rangle. \tag{5.14}$$

At this point, taking advantage of the separable expression for the t matrix in $(G^{(3)})_a = G_0 t_a G_0$ we obtain

$$\mathcal{T}_{ss'} = \langle \int_s^{(2)}| \mathcal{G}^{(3)} \mathbf{X}_{ss'} \mathcal{G}^{(3)} |\int_{s'}^{(2)}\rangle, \tag{5.15}$$

where

$$|\int_s^{(2)}\rangle \equiv \begin{pmatrix} |(\int_s^{(2)})_{a'a}\rangle \\ |(\int_s^{(2)})_-\rangle \end{pmatrix} = \begin{pmatrix} \langle a|G_0|(\gamma_s^{(2)})_{a'a}\rangle \\ |(\gamma_s^{(2)})_-\rangle \end{pmatrix}. \tag{5.16}$$

Stated in this form, this implies that to calculate the reaction amplitude $\mathcal{T}_{ss'}$ we have to solve the equation for $\mathbf{X}_{ss'}$ and the homogeneous equations for $|\boldsymbol{\gamma}_s^{(2)}\rangle$ to produce the states $|\int_s^{(2)}\rangle$. It is, however, possible to exploit the quasiparticle/separable structure for the t matrix in the homogeneous equations for the $|\boldsymbol{\gamma}_s^{(2)}\rangle$ and transform it into a homogeneous equation for the states $|\int_s^{(2)}\rangle$. This can be done by writing explicitly Eq. (5.1), folding its components in the four-body space with $\langle a|G_0$ to the left, and using Eq. (3.14). One obtains the homogeneous equation for the subsystem dynamics in quasiparticle form

$$|\int_s^{(2)}(E_s)\rangle = \mathbf{z}_s(E_s)\mathcal{G}^{(3)}(E_s)|\int_s^{(2)}(E_s)\rangle. \tag{5.17}$$

The corresponding inhomogeneous version of this equation has already been given in Eq. (3.21), where the block matrices $\mathbf{z}_s$ and $\mathcal{G}^{(3)}$ have been explicitly given. The fact that the states $|\int_s^{(2)}\rangle$ are eigensolution of the kernel for $\mathbf{x}_s$ implies that a particularly convenient expression arises when these states are used as a basis for the quasiparticle expansion of $\mathbf{x}_s$, i.e., $|s^{(2)}\rangle \equiv |\int_s^{(2)}(E_s)\rangle$. In that case the pole structure of $\mathbf{x}_s$ for $z\sim E_s$ naturally emerges in the quasiparticle expansion,

$$\mathbf{x}_s \simeq |s^{(2)}\rangle \frac{1}{z-E_s} \langle s^{(2)}|. \tag{5.18}$$

Treatments of the like, based upon the idea of pole dominance of the three-body subsystem operators in the kernel of

the standard four-body equations, have been suggested in various forms [31,15] (for a short review on recent applications, see also Ref. [32]).

With the idea of pole dominance, the solution of the final Eq. (3.28), i.e., the amplitudes $X^{(2)}_{ss'}$, when calculated on shell, directly yield the two-cluster reaction amplitudes $\mathcal{T}_{ss'}$,

$$X^{(2)}_{ss'} = \langle s^{(2)}|\mathcal{G}^{(3)}\mathbf{X}_{ss'}\mathcal{G}^{(3)}|s'^{(2)}\rangle = \langle \boldsymbol{\gamma}^{(2)}_s|G^{(3)}_0 U_{ss'} G^{(3)}_0|\boldsymbol{\gamma}^{(2)}_{s'}\rangle = \mathcal{T}_{ss'}\,. \tag{5.19}$$

VI. SUMMARY, CONCLUSIONS, AND OUTLOOK

This paper deals with the formulation of the three-nucleon problem with inclusion of an additional pionic degree of freedom. The subject implies confrontation of the rather difficult question of developing a few-body integral-equation approach with particle-nonconserving interactions. The problem is solved in the truncated Hilbert space defined by states with at most one pion, e.g., the coupled πNNN-NNN space. Attempts in this direction have been made before, but the solution here developed is original and more complete.

The first, crucial step has been the clarification of a rather delicate question of fragmentation of the system into two clusters (Table II). The meson-nucleon vertex interaction radically changes the cluster properties of the system with respect to the standard case. For instance, in the standard four-body case, the $3+1$ and $2+2$ partitions are not coupled, while in the πNNN system three over four $3+1$ partitions are coupled to the corresponding $2+2$ partitions, with the $2+1$ partition of the three-nucleon space acting as a doorway state. Only the remaining fourth $3+1$ partition (the one with the spectator pion) keeps its standard four-body role and conserves the number of particles. In other words, the subdivision of the coupled πNNN-NNN system into two-cluster partitions is transversal with respect to the separation of the system in terms of the number of particles, since three partitions do not conserve the number of particles while the fourth (denoted $s=0$) does so.

Then, the solution of the problem has been obtained by rewording the GS collision theory in terms of tools: (i) A disconnected dynamical equation of LS type for multicluster processes of the whole system ($T=V+VG_0T$); (ii) a similar integral-equation approach for the subsystem dynamics, which allows the systematic classification of the disconnected diagrams. ($t_s=v_s+v_sG_0t_s$); (iii) a sum-rule equation for the multicluster interaction, which prevents the overcounting or undercounting ($V=\Sigma_s v_s$); (iv) the systematic extraction of the disconnected contributions from the initial multicluster collision amplitude of the whole system ($T=\Sigma_s t_s+\Sigma_{ss'}t_sG_0U_{ss'}G_0t_{s'}$). In N-body scattering theory, from (i)–(iv), it is possible to obtain a new dynamical equation (of AGS type) for the amplitudes $U_{ss'}$, which can be formally recast into the LS form. Hence, by repeated applications of the method, it is possible to extract gradually all the disconnected subamplitudes, thereby obtaining at the end a connected-kernel formulation of the quantum N-body problem.

In the present paper we have shown that these multiparticle tools work also in the presence of particle-nonconserving interactions, at least if we choose as a starting point the integral-equation approach of Thomas, Rinat, Afnan, Blankleider, Avishai, and Mizutani. Obviously, the multiparticle method here developed cannot heal the typical limitations of such input formalisms based on truncation of the Hilbert spaces.

The final formulation is represented by the set of Eqs. (2.27), which generalizes the AGS three-nucleon approach. We have discussed this result in the light of the quasiparticle formalism, which allows a physically more transparent interpretation in terms of coalescence diagrams. Within this formalism the OPE diagram between two nucleons is treated at the same level of the particle-exchange diagrams between multiparticle clusters (Fig. 1). The final equation, Eq. (3.28), represents an effective two-cluster equation, and the corresponding effective multichannel potential is given exclusively by connected-type particle-exchange diagrams (Fig. 3). In the same framework, we have also given the rules to calculate the various multiparticle collision processes, including rearrangement reactions, breakups, pion-induced absorptions, and productions.

Finally, we have formulated the bound-state problem, Eq. (4.8). The equation incorporates the dynamical effect of one pion in the three-nucleon bound-state equation. From the solution of the equation it is possible to calculate the bound-state wave function in both its NNN and πNNN components, through Eqs. (4.9) and (4.10), respectively. This approach represents a formulation of the three-nucleon problem going beyond a description in terms of pure two-nucleon potentials, which is notoriously inadequate (as shown in Ref. [30] and in the references therein contained). It does not require, on the other hand, the employment of three-nucleon forces (3NF) and the associated additional fixing of new parameters (typically, 3NF cutoffs); 3NF represent an approximate, effective way to describe the underlying meson dynamics in the three-nucleon system and the separation between 2NF and 3NF requires a high level of consistency. In Eq. (4.8), *all* possible combinations of 3N diagrams reducible to two-particle interactions (πN or NN) while the dynamical pion is ''in flight'' are taken into account through the couplings with the four-body sector: these contributions obviously represent a fraction of the three-nucleon force, presumably the part with the longest range.

Of course, it is always possible to consider in principle the additional effect of a residual 3NF, representing more complex (and shorter range) diagrams with at least two dynamical pions in the intermediate states, or conversely to attempt the more ambitious program of extending the present approach to include multipion degrees of freedom. That would also reduce the effect of the main limitation implied by the approach, wherein the input interactions have to be extracted from the disconnected πNNN-NNN amplitudes, rather than from the πN subsystem amplitude.

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APPENDIX

Up to this point, the discussion has been restricted to demonstrating that Eq. (3.28), i.e., the quasiparticle version

of the original coupled equation (2.27), leads to connected-type diagrams. This appendix is devoted to the proof that also the kernel of Eq. (2.27), which solves the pion-three-nucleon problem, is connected after iterations. To achieve this goal, we consider the kernel of Eq. (2.27),

$$(K_{s,s'})_{a'a,d'd}=\sum_{c'(\subset s')}(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'c'})(t_{s'})_{c'a,d'd}G_0t_dG_0, \tag{A1a}$$

$$(K_{s,s'})_{-,d'd}=(\bar{\delta}_{ss'})(t_{s'})_{-,d'd}G_0t_dG_0, \tag{A1b}$$

$$(K_{s,s'})_{a'a,-}=\sum_{c'(\subset s')}(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'c'})(t_{s'})_{c'a,-}g_0, \tag{A1c}$$

$$(K_{s,s'})_{-,-}=(\bar{\delta}_{ss'})(t_{s'})_{-,-}g_0. \tag{A1d}$$

By calculating the cube of this kernel and subsequently showing that the result is connected, we demonstrate that after two iterations the kernel of Eq. (2.27) is connected, as happens in the standard Yakubovskĭ approach.

After some trivial manipulations, the cube of the kernel can be written as follows:

$$\begin{aligned}(K^3_{s,s'''})_{a'a,h'h}=&\sum(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'c'})(t_{s'})_{c'a,d'd}\\&\times(\bar{\delta}_{s's''}+\delta_{s's''}\bar{\delta}_{d'e'})(C1_{s'',s'''})_{e'd,h'h}\\&+\sum(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'c'})(t_{s'})_{c'a,-}\\&\times(\bar{\delta}_{s's''})(C2_{s'',s'''})_{-,h'h},\end{aligned} \tag{A2a}$$

$$\begin{aligned}(K^3_{s,s'''})_{a'a,-}=&\sum(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'c'})(t_{s'})_{c'a,d'd}\\&\times(\bar{\delta}_{s's''}+\delta_{s's''}\bar{\delta}_{d'e'})(C3_{s'',s'''})_{e'd,-}\\&+\sum(\bar{\delta}_{ss'}+\delta_{ss'}\bar{\delta}_{a'c'})(t_{s'})_{c'a,-}\\&\times(\bar{\delta}_{s's''})(C4_{s'',s'''})_{-,-},\end{aligned} \tag{A2b}$$

$$\begin{aligned}(K^3_{s,s'''})_{-,h'h}=&\sum(\bar{\delta}_{ss'})(t_{s'})_{-,d'd}(\bar{\delta}_{s's''}+\delta_{s's''}\bar{\delta}_{d'e'})\\&\times(C1_{s'',s'''})_{e'd,h'h}\\&+\sum(\bar{\delta}_{ss'})(t_{s'})_{c'a,-}(\bar{\delta}_{s's''})(C2_{s'',s'''})_{-,h'h},\end{aligned} \tag{A2c}$$

$$\begin{aligned}(K^3_{s,s'''})_{-,-}=&\sum(\bar{\delta}_{ss'})(t_{s'})_{-,d'd}(\bar{\delta}_{s's''}+\delta_{s's''}\bar{\delta}_{d'e'})\\&\times(C3_{s'',s'''})_{e'd,-}+\sum(\bar{\delta}_{ss'})(t_{s'})_{c'a,-}(\bar{\delta}_{s's''})\\&\times(C4_{s'',s'''})_{-,-},\end{aligned} \tag{A2d}$$

with the $C1$, $C2$, $C3$, and $C4$ coefficients defined by

$$\begin{aligned}&(C1_{s'',s'''})_{e'd,h'h}\\&=\sum G_0t_dG_0(t_{s''})_{e'd,f'f}G_0t_f\\&\quad\times G_0(\bar{\delta}_{s''s'''}+\delta_{s''s'''}\bar{\delta}_{f'g'})(t_{s'''})_{g'f,h'h}G_0t_hG_0\\&\quad+\sum G_0t_dG_0(t_{s''})_{e'd,-}g_0(\bar{\delta}_{s''s'''})(t_{s'''})_{-,h'h}G_0t_hG_0,\end{aligned} \tag{A3a}$$

$$\begin{aligned}(C2_{s'',s'''})_{-,h'h}=&\sum g_0(t_{s''})_{-,f'f}G_0t_fG_0(\bar{\delta}_{s''s'''}+\delta_{s''s'''}\bar{\delta}_{f'g'})\\&\times(t_{s'''})_{g'f,h'h}G_0t_hG_0\\&+\sum g_0(t_{s''})_{-,-}g_0(\bar{\delta}_{s''s'''})\\&\times(t_{s'''})_{-,h'h}G_0t_hG_0,\end{aligned} \tag{A3b}$$

$$\begin{aligned}(C3_{s'',s'''})_{e'd,-}=&\sum G_0t_dG_0(t_{s''})_{e'd,f'f}G_0t_fG_0(\bar{\delta}_{s''s'''}\\&+\delta_{s''s'''}\bar{\delta}_{f'g'})(t_{s'''})_{g'f,-}g_0\\&+\sum G_0t_dG_0(t_{s''})_{e'd,-}g_0(\bar{\delta}_{s''s'''})\\&\times(t_{s'''})_{-,-}g_0,\end{aligned} \tag{A3c}$$

$$\begin{aligned}(C4_{s'',s'''})_{-,-}=&\sum g_0(t_{s''})_{-,f'f}G_0t_fG_0(\bar{\delta}_{s''s'''}+\delta_{s''s'''}\bar{\delta}_{f'g'})\\&\times(t_{s'''})_{g'f,-}g_0\\&+\sum g_0(t_{s''})_{-,-}g_0(\bar{\delta}_{s''s'''})(t_{s'''})_{-,-}g_0.\end{aligned} \tag{A3d}$$

For brevity, here and in the following we have assumed that the sums run over all indexes except those appearing explicitly on the left-hand side of each equation.

There are two ways to prove the connectedness of these coefficients. One consists in the replacement of all the two-cluster subamplitudes $\mathbf{t}_s$ contained in Eqs. (A3) with the explicit form of their driving terms, i.e., the quantities $\mathbf{v}_s$ given by Eqs. (2.11), (2.12), (2.13a), and (2.13b), and then one must systematically verify that this leads exclusively to connected-type diagrams. But there is also a more economical and perhaps more transparent method. By expanding the two-body t matrices, t_a, as a sum over a series of separable pieces, where each term is of the form given by Eq. (3.14), it is possible to reexpress the four classes of coefficients in the following manner:

$$(C1_{s''s'''})_{e'd,h'h}$$
$$= \sum G_0|d\rangle\tau_d^{(3)}(x_{s''})_{e'd,f'f}\tau_f^{(3)}(\overline{\delta}_{s''s'''}+\delta_{s''s'''}\overline{\delta}_{f'g'})$$
$$\times(x_{s'''})_{g'f,h'h}\tau_h^{(3)}\langle h|G_0+\sum G_0|d\rangle\tau_d^{(3)}$$
$$\times(x_{s''})_{e'd,-}g_0(\overline{\delta}_{s''s'''})(x^\dagger_{s'''})_{-,h'h}\tau_h^{(3)}\langle h|G_0, \quad \text{(A4a)}$$

$$(C2_{s''s'''})_{-,h'h}$$
$$= \sum g_0(x^\dagger_{s''})_{-,f'f}\tau_f^{(3)}(\overline{\delta}_{s''s'''}+\delta_{s''s'''}\overline{\delta}_{f'g'})$$
$$\times(x_{s'''})_{g'f,h'h}\tau_h^{(3)}\langle h|G_0$$
$$+\sum g_0(x_{s''})_{-,-}g_0(\overline{\delta}_{s''s'''})(x^\dagger_{s'''})_{-,h'h}\tau_h^{(3)}\langle h|G_0, \quad \text{(A4b)}$$

$$(C3_{s''s'''})_{e'd,-}$$
$$= \sum G_0|d\rangle\tau_d^{(3)}(x_{s''})_{e'd,f'f}$$
$$\times\tau_f^{(3)}(\overline{\delta}_{s''s'''}+\delta_{s''s'''}\overline{\delta}_{f'g'})(x_{s'''})_{g'f,-}g_0$$
$$+\sum G_0|d\rangle\tau_d^{(3)}(x_{s''})_{e'd,-}g_0(\overline{\delta}_{s''s'''})(x_{s'''})_{-,-}g_0, \quad \text{(A4c)}$$

$$(C4_{s''s'''})_{-,-} = \sum g_0(x^\dagger_{s''})_{-,f'f}\tau_f^{(3)}(\overline{\delta}_{s''s'''}+\delta_{s''s'''}\overline{\delta}_{f'g'})$$
$$\times(x_{s'''})_{g'f,-}g_0$$
$$+\sum g_0(x_{s''})_{-,-}g_0(\overline{\delta}_{s''s'''})(x_{s'''})_{-,-}g_0. \quad \text{(A4d)}$$

By iterating the quasiparticle expansion method it is possible to express these folded subamplitudes $\mathbf{x}_s$ (defined in Sec. III) as a sum over separable terms of the form given by Eq. (3.27). With this form, these C coefficients finally become

$$(C1_{s''s'''})_{e'd,h'h} = G_0|d\rangle\tau_d^{(3)}|(s''^{(2)})_{e'd}>\tau^{(2)}_{s''}Z^{(2)}_{s''s'''}\tau^{(2)}_{s'''}$$
$$\times\langle(s'''^{(2)})_{h'h}|\tau_h^{(3)}\langle h|G_0, \quad \text{(A5a)}$$

$$(C2_{s''s'''})_{-,h'h}$$
$$= g_0|(s''^{(2)})_-\rangle\tau^{(2)}_{s''}Z^{(2)}_{s''s'''}\tau^{(2)}_{s'''}\langle(s'''^{(2)})_{h'h}|\tau_h^{(3)}\langle h|G_0, \quad \text{(A5b)}$$

$$(C3_{s''s'''})_{e'd,-} = G_0|d\rangle\tau_d^{(3)}|(s''^{(2)})_{e'd}\rangle\tau^{(2)}_{s''}Z^{(2)}_{s''s'''}\tau^{(2)}_{s'''}$$
$$\times\langle(s'''^{(2)})_-|g_0, \quad \text{(A5c)}$$

$$(C4_{s''s'''})_{-,-} = g_0|(s''^{(2)})_-\rangle\tau^{(2)}_{s''}Z^{(2)}_{s''s'''}\tau^{(2)}_{s'''}\langle(s'''^{(2)})_-|g_0, \quad \text{(A5d)}$$

where the $Z^{(2)}_{s''s'''}$ have been defined according to Eq. (3.29). Thus, it has been demonstrated that these C coefficients are connected because they have been expressed in terms of quantities corresponding to the driving terms of the quasiparticle equation. The connected structure of these objects has been exhibited diagrammatically in Fig. 3.

It has to be noted that, for the demonstration of the connectivity of these C coefficients, it is uninfluential whether the T matrix is of rank one (separable) or not. In this second case, as long as it is representable in terms of a separable expansion, the above arguments apply to each term of the expansion, with the conclusion that these coefficients contain no disconnected contributions at all.

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