PHYSICAL REVIEW C VOLUME 58, NUMBER 6 DECEMBER 1998

Shadowing and antishadowing effects in a model for the $n+d$ total cross section

Ch. Elster and W. Schadow
Institute of Nuclear and Particle Physics, and Department of Physics, Ohio University, Athens, Ohio 45701

H. Kamada* and W. Glöckle
Institute for Theoretical Physics II, Ruhr-University Bochum, D-44780 Bochum, Germany
(Received 6 July 1998)

Based on the multiple scattering series incorporated in the Faddeev scheme the high-energy limit of the total $n+d$ cross section is evaluated in a nonrelativistic model system where spins are neglected. In contrast to the naive expectation that the total $n+d$ scattering cross section is the sum of two NN cross sections we find two additional effects resulting from rescattering processes. These additional terms have different signs (shadowing and antishadowing) and a different behavior as function of the energy. Our derivation of these results which are already known from Glauber theory is based on the analytical evaluation of elastic transition amplitudes in the high-energy limit. It does not depend on the diffraction-type assumptions connected with Glauber theory. In this model of spinless Yukawa type forces (with no absorption) the total $n+d$ cross section does not approach twice the NN total cross section in the high-energy limit but rather approaches the total NN cross section multiplied by a number larger than 2. Therefore, the enhancement effect resulting from rescattering is larger than the shadowing effect, which decreases faster with energy. [S0556-2813(98)04712-8]

PACS number(s): 21.45.+v, 25.10.+s, 03.65.Nk

I. INTRODUCTION

Recently it became possible to calculate the total cross section for neutron-deuteron ($n+d$) scattering with high precision in the energy regime up to 300 MeV projectile energy by solving the nonrelativistic $3N$ Faddeev equations based on modern NN forces [1]. Compared to the most naive picture, which in the high energy limit would equate the total $n+d$ cross section with the sum of the total cross sections for neutron-proton (np) and neutron-neutron (nn) scattering, the rigorously calculated result up to 300 MeV is smaller. Obviously one can expect some shadowing effect in the reaction, which would explain this result. On the other hand, rescattering of the nucleons upon each other might *a priori* also enhance the total $n+d$ cross section over the sum of the individual two nucleon cross sections, especially if the forces are attractive. In the context of Glauber theory [2–5] both features, enhancement and weakening, are present. In a model of spinless particles, which is based on the Faddeev formulation, we want to study the high-energy limit of the total $n+d$ cross section and evaluate the leading terms analytically. This approach differs from the one used in the Glauber formulation. Based on the multiple scattering series incorporated in the Faddeev framework, we calculate the first and second order terms of the series when taken in the high-energy limit and show the contributions of the two terms to the total $n+d$ cross section. Starting from a multiple scattering series implies that this is an ordering according to powers in the NN t matrix. Though we restrict ourselves to a nonrelativistic framework, we nevertheless consider the results as being instructive. The analytical steps leading to the high-energy limit, which only involve ordinary analysis, are carried out in well defined integrals. There are no *a priori* assumptions about the scattering process involved, such as, e.g., diffraction type approximations. We also take the identity of the particles explicitly into account. In this paper the complications due to the inclusion of the spin and isospin degrees of freedom are avoided so that the basic mechanism can be seen more clearly. Taking spin and isospin degrees of freedom into account will modify the results due to the interferences of spin and isospin dependent terms, as will be shown in a forthcoming article. Specifically because of those spin and isospin interference effects, which are quite involved, we want to present this more transparent case with three bosons separately.

In Sec. II we describe the Faddeev framework, its multiple scattering expansion, and the leading order terms in the NN t matrix for obtaining the total $n+d$ cross section. The high-energy limit of the corresponding expressions is carried out analytically in Sec. III. To illustrate the behavior of the leading order terms in the high energy limit numerical examples are given in Sec. IV for a superposition of Yukawa interactions. We conclude with Sec. V.

II. LEADING MULTIPLE SCATTERING TERMS FOR THE TOTAL $n+d$ CROSS SECTION

We consider three identical spinless bosons which interact via two-body forces. In our usual manner [6] to exploit the Faddeev scheme the operator for elastic scattering of a nucleon from a bound nucleon pair is given by [7]

$$U=PG_0^{-1}+PT, \tag{2.1}$$

where the three-body operator T obeys the Faddeev-type integral equation

*Present address: Institut für Kernphysik, Fachbereich 5, Technische Hochschule Darmstadt, D-64289 Darmstadt, Germany.

$$T|\Phi\rangle = tP|\Phi\rangle + tPG_0T|\Phi\rangle. \tag{2.2}$$

The channel state, which is composed of a two nucleon bound state (deuteron) and a momentum eigenstate of the projectile nucleon, is denoted by Φ. Furthermore, t is the two nucleon transition operator, G_0 the free three nucleon propagator, and P the permutation operator, which is the sum of a cyclic and an anticyclic permutation of the three particles. The elastic forward scattering amplitude is given by the matrix element $\langle\Phi|U|\Phi\rangle$. When iterating Eq. (2.2) and inserting the result into Eq. (2.1), we obtain the multiple scattering series for the elastic forward scattering amplitude

$$\langle\Phi|U|\Phi\rangle = \langle\Phi|PG_0^{-1}|\Phi\rangle + \langle\Phi|PtP|\Phi\rangle + \langle\Phi|PtPG_0tP|\Phi\rangle + \langle\Phi|PtPG_0tPG_0tP|\Phi\rangle + \cdots, \tag{2.3}$$

which is an expansion in orders of the NN t operator. Using the optical theorem one obtains for the total cross section for nucleon-deuteron $(n+d)$ scattering [7]

$$\sigma_{\text{tot}}^{nd} = -(2\pi)^3 \frac{4m}{3q_0} \text{Im}\langle\Phi|U|\Phi\rangle. \tag{2.4}$$

Here q_0 is the asymptotic momentum of the projectile nucleon relative to the bound two-body subsystem. From Eq. (2.3) follows

$$\begin{aligned} 2i\,\text{Im}\langle\Phi|U|\Phi\rangle &\equiv \langle\Phi|U|\Phi\rangle - \langle\Phi|U|\Phi\rangle^\star \\ &= \langle\Phi|P(t-t^\dagger)P + PtPG_0tP \\ &\quad - Pt^\dagger G_0^\star Pt^\dagger P + PtPG_0tPG_0tP \\ &\quad - Pt^\dagger G_0^\star Pt^\dagger G_0^\star Pt^\dagger P|\Phi\rangle + \cdots. \end{aligned} \tag{2.5}$$

Since the first term in Eq. (2.3) is real, it does not contribute to the total cross section.

For the analytical extraction of the high-energy limit it is useful to rewrite Eq. (2.5) in the following form:

$$\begin{aligned} \langle\Phi|U|\Phi\rangle - \langle\Phi|U|\Phi\rangle^\star = {} & \langle\Phi|P(t-t^\dagger)P|\Phi\rangle + \langle\Phi|P(t-t^\dagger)PG_0tP|\Phi\rangle - \langle\Phi|P(t-t^\dagger)PG_0tP|\Phi\rangle^\star \\ & + \langle\Phi|Pt^\dagger P(G_0-G_0^\star)tP|\Phi\rangle + \langle\Phi|P(t-t^\dagger)PG_0tPG_0tP|\Phi\rangle + \langle\Phi|Pt^\dagger P(G_0-G_0^\star)tPG_0tP|\Phi\rangle \\ & + \langle\Phi|Pt^\dagger PG_0^\star(t-t^\dagger)PG_0tP|\Phi\rangle + \langle\Phi|Pt^\dagger PG_0^\star t^\dagger P(G_0-G_0^\star)tP|\Phi\rangle \\ & + \langle\Phi|Pt^\dagger PG_0^\star t^\dagger PG_0^\star(t-t^\dagger)P|\Phi\rangle + \cdots. \end{aligned} \tag{2.6}$$

The next step is to explicitly evaluate the permutations given by $P \equiv P_{12}P_{23} + P_{13}P_{23}$. This specific choice of the permutation operator P corresponds to the choice of particles 2 and 3 forming the two-body bound state and 1 being the projectile, i.e., $t \equiv t_{23}$ and

$$|\Phi\rangle \equiv |\Phi\rangle_1 \equiv |\varphi_d\rangle_{23}|\mathbf{q_0}\rangle_1. \tag{2.7}$$

The subscripts denote which particles occupy the states. This specific choice leads to

$$|\Phi\rangle_2 \equiv P_{12}P_{23}|\Phi\rangle_1 = |\varphi_d\rangle_{13}|\mathbf{q_0}\rangle_2 \tag{2.8}$$

and

$$|\Phi\rangle_3 \equiv P_{13}P_{23}|\Phi\rangle_1 = |\varphi_d\rangle_{12}|\mathbf{q_0}\rangle_3. \tag{2.9}$$

Taking advantage of the symmetry property of the subsystem, $P_{23}|\Phi\rangle_1 = |\Phi\rangle_1$ and $P_{23}t_{23} = t_{23}P_{23}$, one obtains after some algebra

$$\langle\Phi|P(t-t^\dagger)P|\Phi\rangle = 2\,{}_2\langle\Phi|(t-t^\dagger)(1+P_{23})|\Phi\rangle_2 \tag{2.10}$$

and

$$\langle\Phi|PtPG_0tP|\Phi\rangle = 2\,{}_2\langle\Phi|tG_0t_2(|\Phi\rangle_1 + |\Phi\rangle_3) + 2\,{}_2\langle\Phi|tG_0t_3(|\Phi\rangle_1 + |\Phi\rangle_2). \tag{2.11}$$

A similar evaluation could be done for the terms third order in t given in Eq. (2.5). For our present considerations of the $n+d$ total cross section in the high energy limit we restrict ourselves to the terms given in Eqs. (2.10) and (2.11).

III. THE HIGH-ENERGY LIMIT

First we need to consider the exact momentum space representations of Eqs. (2.10) and (2.11). Equivalent decompositions of the unity operator are given by

$$\mathbf{1} = \int d^3p\,d^3q\,|\mathbf{pq}\rangle_i\,{}_i\langle\mathbf{pq}|, \quad i = 1,2, \text{ or } 3. \tag{3.1}$$

Here $|\mathbf{pq}\rangle_i \equiv |\mathbf{p}\rangle_i|\mathbf{q}\rangle_i$ and $\mathbf{p}$ and $\mathbf{q}$ are the standard three types of Jacobi momenta for three particles [6]. The index i denotes the singled out nucleon. Inserting the unity operator into the first term in Eq. (2.10) leads to

$$\begin{aligned} {}_2\langle\Phi|t-t^\dagger|\Phi\rangle_2 = {} & \int d^3p\,d^3q \int d^3p'\,d^3q' \int d^3p''\,d^3q'' \\ & \times \int d^3p'''\,d^3q_2'''\langle\Phi|\mathbf{pq}\rangle_2\,{}_2\langle\mathbf{pq}|\mathbf{p'q'}\rangle_1 \\ & \times {}_1\langle\mathbf{p'q'}|t-t^\dagger|\mathbf{p''q''}\rangle_1\,{}_1\langle\mathbf{p''q''}|\mathbf{p'''q'''}\rangle_2 \\ & \times {}_2\langle\mathbf{p'''q'''}|\Phi\rangle_2. \end{aligned} \tag{3.2}$$

Further, one has

$$ {}_i\langle \mathbf{pq}|\Phi\rangle_i = \delta^3(\mathbf{q}-\mathbf{q_0})\langle \mathbf{p}|\varphi_d\rangle. \tag{3.3}$$

The standard relations among the different sets of Jacobi momenta give

$$ {}_1\langle \mathbf{pq}|\mathbf{p}'\mathbf{q}'\rangle_2 = \delta^3(\mathbf{p}-\tfrac{1}{2}\mathbf{q}-\mathbf{q}')\,\delta^3(\mathbf{p}'+\mathbf{q}+\tfrac{1}{2}\mathbf{q}'). \tag{3.4}$$

In addition one has

$$ {}_1\langle \mathbf{pq}|t(E)|\mathbf{p}'\mathbf{q}'\rangle_1 = \langle \mathbf{p}|t\left(E-\frac{3}{4m}q^2\right)|\mathbf{p}'\rangle\,\delta^3(\mathbf{q}-\mathbf{q}'). \tag{3.5}$$

Employing all the above given relations, it is straightforward to arrive at the following expression for Eq. (3.2):

$$\begin{aligned} {}_2\langle\Phi|t-t^\dagger|\Phi\rangle_2 &= \int d^3q\langle\varphi_d|-\mathbf{q}-\tfrac{1}{2}\mathbf{q_0}\rangle \\ &\quad\times\langle\tfrac{1}{2}\mathbf{q}+\mathbf{q_0}|t-t^\dagger|\tfrac{1}{2}\mathbf{q}+\mathbf{q_0}\rangle\langle-\mathbf{q}-\tfrac{1}{2}\mathbf{q_0}|\varphi_d\rangle \\ &= \int d^3k\langle\varphi_d|-\mathbf{k}\rangle\langle-\mathbf{k}|\varphi_d\rangle \\ &\quad\times\langle\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{k}|(t-t^\dagger)(\varepsilon)|\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{k}\rangle. \end{aligned}\tag{3.6}$$

The total energy is $E=\epsilon_d+(3/4m)q_0^2$, where ϵ_d is the deuteron binding energy. When expressed in terms of $\mathbf{k}=\mathbf{q}+\frac{1}{2}\mathbf{q_0}$ one obtains for the energy argument ε of the t matrices

$$\begin{aligned}\varepsilon\equiv E-\frac{3}{4m}q^2 &= \epsilon_d+\frac{3}{4m}q_0^2-\frac{3}{4m}(\mathbf{k}-\tfrac{1}{2}\mathbf{q_0})^2\\ &= \epsilon_d+\frac{1}{m}(\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{k})^2-\frac{1}{m}k^2.\end{aligned}\tag{3.7}$$

If the projectile momentum q_0 is sufficiently large in comparison to the dominant deuteron momenta contributing to the integral in Eq. (3.6) and thus $\varepsilon\approx(1/m)(\frac{3}{4}\mathbf{q_0}+\frac{1}{2}\mathbf{k})^2$, one encounters on shell NN forward scattering amplitudes under the integral. The permutation operator P_{23} in Eq. (2.10) leads to the necessary symmetrization of the t-matrix elements

$$\langle\mathbf{q}|(t-t^\dagger)\left(\varepsilon=\frac{1}{m}q^2\right)|\mathbf{q}\rangle_s, \tag{3.8}$$

where

$$|\mathbf{q}\rangle_s=|\mathbf{q}\rangle+|-\mathbf{q}\rangle, \tag{3.9}$$

and $\mathbf{q}=\frac{3}{4}\mathbf{q}_0+\frac{1}{2}\mathbf{k}$. Using the Lippmann-Schwinger equation for t, with the two-body force V as driving term, and $\mathbf{q}\equiv|q|\hat{\mathbf{q}}$ one has

$$\begin{aligned}\langle\mathbf{q}|(t-t^\dagger)\left(\varepsilon=\frac{1}{m}q^2\right)|\mathbf{q}\rangle_s &= -2\pi i\frac{m}{2}q\int d\hat{q}'\langle\mathbf{q}|V|q\hat{\mathbf{q}}'\rangle^{(+)\,(+)}\langle q\hat{\mathbf{q}}'|V|\mathbf{q}\rangle_s\\ &= -2\pi i\frac{m}{2}q\int d\hat{q}'\langle\mathbf{q}|t|q\hat{\mathbf{q}}'\rangle_s\langle\mathbf{q}|t|q\hat{\mathbf{q}}'\rangle^*\\ &= -\pi i\frac{m}{2}q\int d\hat{q}'|\langle\mathbf{q}|t|q\hat{\mathbf{q}}'\rangle_s|^2.\end{aligned}\tag{3.10}$$

This expression is directly related to the two-body total cross section

$$\sigma_{\rm tot}^{NN}=\left(\frac{m}{2}\right)^2(2\pi)^4\int d\hat{q}'\left|\langle\mathbf{q}|t|q\hat{\mathbf{q}}'\rangle_s\frac{1}{\sqrt{2}}\right|^2. \tag{3.11}$$

We adopt here the usual convention for the total cross section for two identical particles [8] and obtain

$$\langle\mathbf{q}|(t-t)^\dagger)\left(\varepsilon=\frac{1}{m}q^2\right)|\mathbf{q}\rangle_s=-iq\frac{2}{m}\frac{1}{(2\pi)^3}\sigma_{\rm tot}^{NN}. \tag{3.12}$$

As a consequence Eq. (3.6) supplemented by the symmetrization as given in Eq. (2.10) and approximated in the energy argument of the two-body t matrix can be rewritten as

$$\begin{aligned}{}_2\langle\Phi|(t-t^\dagger)(1+P_{23})|\Phi\rangle_2 &= -\frac{2i}{m}\frac{1}{(2\pi)^3}\int d^3k\langle\varphi_d|-\mathbf{k}\rangle\\ &\quad\times\langle-\mathbf{k}|\varphi_d\rangle|\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{k}|\sigma_{tot}^{NN}\left(\frac{1}{m}(\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{k})^2\right).\end{aligned}\tag{3.13}$$

If the projectile momentum q_0 is sufficiently large compared to the typical deuteron momenta the function $|\frac{3}{4}\mathbf{q_0}+\frac{1}{2}\mathbf{k}|\sigma_{\rm tot}^{NN}[(1/m)(\frac{3}{4}\mathbf{q_0}+\frac{1}{2}\mathbf{k})^2]$ is expected to vary slowly over the range of the $\mathbf{k}$ values contributing to the integral in Eq. (3.13). Using the normalization of the two-body bound state, expanding the function at $\mathbf{k}=0$ and knowing that the contribution of the first derivative vanishes, one obtains in the limit for large $\mathbf{q_0}$

$$ {}_2\langle\Phi|(t-t^\dagger)(1+P_{23})|\Phi\rangle_2 \overset{q_0\gg k}{\to} -\frac{2i}{m}\frac{1}{(2\pi)^3}\frac{3}{4}q_0\,\sigma_{\rm tot}^{NN}\left[\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right]\int d^3k\langle\varphi_d|-\mathbf{k}\rangle\langle-\mathbf{k}|\varphi_d\rangle $$

$$ =-\frac{2i}{m}\frac{1}{(2\pi)^3}\frac{3}{4}q_0\sigma_{\rm tot}^{NN}\left[\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right]. \tag{3.14} $$

The arguments leading to Eq. (3.14) are similar to those used in arriving at the method of optimum factorization successfully applied in intermediate-energy pion-nucleus and nucleon-nucleus scattering [9,10]. The $n+d$ total cross section, Eq. (2.4), thus gives in the first order term in t of Eq. (2.6) in the high-energy limit

$$ \sigma_{\rm tot}^{nd}|_{\rm 1st\ order}\to 2\sigma_{\rm tot}^{NN}. \tag{3.15} $$

This result corresponds to the naive expectation that at high energies the projectile nucleon sees the individual nucleons inside the deuteron as if they were independent particles. It should be pointed out that due to the optical theorem $\sigma_{\rm tot}^{NN}$ is $O(t^2)$, though the expression of Eq. (3.15) has been derived from the terms linear in t in Eq. (2.6).

Let us now consider the contributions to the total $n+d$ cross section second order in t as given in Eq. (2.5). A straightforward but somewhat tedious algebra using Eq. (2.11) leads to the exact form

$$ \langle\Phi|P\tau PG_0tP|\Phi\rangle=2\int d^3q\int d^3q'\frac{\langle\varphi_d|\mathbf{q}\rangle\langle\mathbf{q}'|\varphi_d\rangle}{-|\epsilon_d|-(1/m)(q^2+q'^2+\mathbf{q}\cdot\mathbf{q}')+(3/2m)(\mathbf{q}+\mathbf{q}')\cdot\mathbf{q_0}+i\varepsilon} $$

$$ \times\langle\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{q}|\tau(\epsilon_1)|\tfrac{3}{4}\mathbf{q_0}-\tfrac{1}{2}\mathbf{q}-\mathbf{q}'\rangle\langle-\tfrac{3}{4}\mathbf{q_0}+\mathbf{q}+\tfrac{1}{2}\mathbf{q}'|t(\epsilon_3)|\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{q}'\rangle_s $$

$$ +2\int d^3q\int d^3q'\frac{\langle\varphi_d|\mathbf{q}\rangle\langle\mathbf{q}'|\varphi_d\rangle}{-|\epsilon_d|-(1/m)(q^2+q'^2+\mathbf{q}\cdot\mathbf{q}')+(3/2m)(\mathbf{q}+\mathbf{q}')\cdot\mathbf{q_0}+i\varepsilon} $$

$$ \times\langle\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{q}|\tau(\epsilon_1)|-\tfrac{3}{4}\mathbf{q_0}+\tfrac{1}{2}\mathbf{q}+\mathbf{q}'\rangle\langle\tfrac{3}{4}\mathbf{q_0}-\mathbf{q}-\tfrac{1}{2}\mathbf{q}'|t(\epsilon_2)|-\tfrac{3}{4}\mathbf{q_0}-\tfrac{1}{2}\mathbf{q}'\rangle_s. \tag{3.16} $$

The quantity τ represents either $t-t^\dagger$ or $t^\dagger$, and thus acts in the subsystem 1=(23). Again, the subscript s indicates the symmetrized state as given in Eq. (3.9). The energy arguments in the t matrices are

$$ \epsilon_1=\epsilon_d+e\frac{1}{m}\left(\frac{3}{4}\mathbf{q}_0\right)^2+\frac{3}{4m}\mathbf{q}\cdot\mathbf{q}_0-\frac{3}{4m}q^2 \tag{3.17} $$

and

$$ \epsilon_3=\epsilon_2=\epsilon_d+\frac{1}{m}\left(\frac{3}{4}\mathbf{q}_0\right)^2+\frac{3}{4m}\mathbf{q}'\cdot\mathbf{q}_0-\frac{3}{4m}q'^2. \tag{3.18} $$

It should be pointed out that the occurrence of the symmetrized state incorporates the forward and backward scattering amplitudes.

For the limit $q_0\to\infty$ one can again neglect the variations of the t-matrices under the integrals and obtains in this limit

$$ \langle\Phi|P\tau PG_0tP|\Phi\rangle \approx 2\left\langle\frac{3}{4}\mathbf{q_0}\left|\tau\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right)\right|\frac{3}{4}\mathbf{q_0}\right\rangle_s \times\left\langle\frac{3}{4}\mathbf{q_0}\left|t\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right)\right|\frac{3}{4}\mathbf{q_0}\right\rangle_s\times I, \tag{3.19} $$

where

$$ I=\int d^3q\int d^3q'\frac{\langle\varphi_d|\mathbf{q}\rangle\langle\mathbf{q}'|\varphi_d\rangle}{-|\epsilon_d|-(1/m)(q^2+q'^2+\mathbf{q}\cdot\mathbf{q}')+(3/2m)(\mathbf{q}+\mathbf{q}')\cdot\mathbf{q_0}+i\varepsilon}. \tag{3.20} $$

The interesting insight will now arise from the quantity I, which we still need to consider in the high-energy limit. These considerations are most transparent if one works in configuration space. The momentum space form will be given in the Appendix. Let

$$\langle \mathbf{q}|\varphi_d\rangle \equiv \frac{1}{(2\pi)^{3/2}}\int d^3r e^{i\mathbf{q}\cdot\mathbf{r}}\langle \mathbf{r}|\varphi_d\rangle \quad (3.21)$$

with

$$\langle \mathbf{r}|\varphi_d\rangle \equiv \frac{1}{\sqrt{4\pi}}\varphi_d(r). \quad (3.22)$$

Then the quantity I from Eq. (3.20) becomes

$$I=\frac{1}{(2\pi)^3}\frac{1}{4\pi}\int d^3q\int d^3q'\int d^3r\int d^3r' e^{i\mathbf{q}\cdot\mathbf{r}}e^{-i\mathbf{q}'\cdot\mathbf{r}'}\frac{\varphi_d(r)\,\varphi_d(r')}{-|\epsilon_d|-(1/m)(q^2+q'^2+\mathbf{q}\cdot\mathbf{q}')+(3/2m)(\mathbf{q}+\mathbf{q}')\cdot\mathbf{q_0}+i\varepsilon}. \quad (3.23)$$

Since the denominator depends on $\mathbf{q_0}$, the quantity whose infinite limit we want to consider, it is natural to split the vector $\mathbf{q}$ ($\mathbf{q}'$) in components parallel and perpendicular to $\mathbf{q_0}$, and consider $\mathbf{q_0}$ pointing into the z direction. For the vector $\mathbf{r}$ ($\mathbf{r}'$) we define a similar decomposition

$$\mathbf{q}\equiv(\mathbf{q}_\perp,q_z), \quad (3.24)$$

$$\mathbf{r}\equiv(\mathbf{r}_\perp,z),$$

and obtain

$$I=\frac{m}{(2\pi)^3}\frac{1}{4\pi}\int d^2q_\perp\int d^2q'_\perp\int d^3r\varphi_d(r)\int d^3r'\varphi_d(r')e^{i\mathbf{q}_\perp\cdot\mathbf{r}_\perp}e^{-i\mathbf{q}'_\perp\cdot\mathbf{r}'_\perp}\int dq_z\int dq_{z'}e^{iq_zz}e^{-iq'_zz'}$$
$$\times\frac{1}{-m|\epsilon_d|-q_\perp^2-q'^2_\perp-\mathbf{q}_\perp\cdot\mathbf{q}'_\perp-q_z^2-q'^2_z-q_zq'_z+\frac{3}{2}q_0(q_z+q'_z)+i\varepsilon}. \quad (3.25)$$

Further we substitute

$$q_z+q'_z=s, \quad (3.26)$$

$$\tfrac{1}{2}(q_z-q'_z)=s',$$

so that the denominator takes the form

$$D=-\tfrac{3}{4}(s^2-2q_0s+\tfrac{4}{3}\alpha^2-i\varepsilon), \quad (3.27)$$

with

$$\alpha^2=m|\epsilon_d|+q_\perp^2+q'^2_\perp+\mathbf{q}_\perp\cdot\mathbf{q}'_\perp+s'^2>0\,. \quad (3.28)$$

For large q_0 one obtains

$$D\approx-\tfrac{3}{4}(s-2q_0-i\varepsilon)\left(s-\frac{2}{3}\frac{\alpha^2}{q_0}+i\varepsilon\right)$$
$$\approx-\tfrac{3}{4}(s-2q_0-i\varepsilon)(s+i\varepsilon) \quad (3.29)$$

and thus encounters two poles in the complex s plane. The integrals over s and s' can be carried out analytically, leading to

$$\int_{-\infty}^{\infty}ds\int_{-\infty}^{\infty}ds' e^{i(z+z')s'}e^{(i/2)(z-z')s}\frac{-4/3}{(s-2q_0-i\varepsilon)(s+i\varepsilon)}$$
$$=-\frac{2i}{3q_0}(2\pi)^2\delta(z+z')[\theta(z)e^{2iq_0z}+\theta(-z)]. \quad (3.30)$$

Inserting this into Eq. (3.25) and carrying out the integrations over $\mathbf{q}_\perp$ and $\mathbf{q}'_\perp$ yields

$$I\approx m\,(2\pi)^3\frac{-2i}{3q_0}\int d^2r_\perp\int d^2r'_\perp\int_{-\infty}^{\infty}dz\int_{-\infty}^{\infty}dz'$$
$$\times\varphi_d(\sqrt{r_\perp^2+z^2})\,\varphi_d(\sqrt{r'^2_\perp+z'^2})$$
$$\times\delta(\mathbf{r}_\perp)\,\delta(\mathbf{r}'_\perp)\,\delta(z+z')[\theta(z)e^{2iq_0z}+\theta(-z)]\frac{1}{4\pi}. \quad (3.31)$$

The δ functions under the integral show that contributions to I only arise if the two nucleons in the deuteron sit behind each other with respect to the projectile momentum $\mathbf{q_0}=q_0\hat{z}$. This coincides with our naive understanding of shadowing. The term containing e^{2iq_0z} falls off fastest in Eq. (3.31) and will be neglected. Thus, one arrives at

$$I \approx -2im(2\pi)^3 \frac{1}{3q_0}\int_0^\infty dz\, \varphi_d^2(z)\frac{1}{4\pi}$$
$$= -\frac{2im}{3q_0}(2\pi)^3 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}. \tag{3.32}$$

The corresponding algebraic steps performed in momentum space are shown in the Appendix and yield the result

$$I \to -\frac{2im}{3q_0}(2\pi)^3 \int_0^\infty dp p^2\, \varphi_d(p) \int_0^\infty dp' p'^2\, \varphi_d(p') \frac{1}{p_>}\frac{1}{4\pi}, \tag{3.33}$$

where $p_> = \max(p,p')$.

We now return to Eq. (3.19), set $\tau = t - t^\dagger$, use the relation given in Eq. (3.12) to the total NN cross section and obtain

$$\langle \Phi | P(t-t^\dagger) P G_0 t P | \Phi \rangle$$
$$\to -2\, \sigma_{tot}^{NN} \left\langle \frac{3}{4}\mathbf{q_0} \left| t\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right) \right| \frac{3}{4}\mathbf{q_0} \right\rangle_s$$
$$\times \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}. \tag{3.34}$$

Finally subtracting the conjugate complex according to Eq. (2.6) and using the optical theorem in the two-body subsystem

$$\langle \tfrac{3}{4}\mathbf{q}_0 | t | \tfrac{3}{4}\mathbf{q}_0 \rangle_s - \langle \tfrac{3}{4}\mathbf{q}_0 | t | \tfrac{3}{4}\mathbf{q}_0 \rangle_s^* = -2i \frac{3q_0}{4m}\left(\frac{1}{2\pi}\right)^3 \sigma_{\rm tot}^{NN}, \tag{3.35}$$

leads to

$$\langle \Phi | P(t-t^\dagger) P G_0 t P | \Phi \rangle - \langle \Phi | P(t-t^\dagger) P G_0 t P | \Phi \rangle^\star$$
$$\to i \frac{3q_0}{m}\frac{1}{(2\pi)^3}(\sigma_{\rm tot}^{NN})^2 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}. \tag{3.36}$$

Because of $(\sigma_{\rm tot}^{NN})^2$ this expression is of order $O(t^4)$.

It remains now to discuss the last term of second order in t in Eq. (2.6) in the high-energy limit. Here we encounter $G_0 - G_0^\star = -2i\pi\delta(E-H_0)$ and obtain

$$\langle \Phi | P t^\dagger P (G_0 - G_0^\star) t P | \Phi \rangle$$
$$\to -4i\pi \left| \left\langle \frac{3}{4}\, \mathbf{q_0} \left| t\left(\frac{1}{m}\left(\frac{3}{4}\, q_0\right)^2\right) \right| \frac{3}{4}\mathbf{q_0} \right\rangle_s \right|^2$$
$$\times \int d^3q \int d^3q' \langle \varphi_d | \mathbf{q} \rangle \langle \mathbf{q}' | \varphi_d \rangle$$
$$\times\, \delta\bigg(-|\epsilon_d| - \frac{1}{m}(q^2 + q'^2 + \mathbf{q}\cdot\mathbf{q}')$$
$$+ \frac{3}{2m}\mathbf{q_0}\cdot(\mathbf{q}+\mathbf{q}')\bigg). \tag{3.37}$$

The integral term can simply be related to the expression I from Eq. (3.20). Similarly to the Appendix we work in momentum space and obtain

$$I' \equiv \int d^3q \int d^3q' \langle \varphi_d | \mathbf{q} \rangle \langle \mathbf{q}' | \varphi_d \rangle$$
$$\times\, \delta\left(-|\epsilon_d| - \frac{1}{m}(q^2 + q'^2 + \mathbf{q}\cdot\mathbf{q}') + \frac{3}{2m}\mathbf{q_0}\cdot(\mathbf{q}+\mathbf{q}')\right)$$
$$= \int d^2q_\perp \int d^2q'_\perp \int dq_z \int dq_{z'}\, \varphi_d$$
$$\times (\sqrt{q_\perp^2 + q_z^2})\, \varphi_d(\sqrt{q'^2_\perp + q'^2_z}) \frac{1}{4\pi}$$
$$\times\, \delta\bigg(-|\epsilon_d| - \frac{1}{m}(q_\perp^2 + q'^2_\perp + \mathbf{q}_\perp \cdot \mathbf{q}'_\perp$$
$$+ q_z^2 + q'^2_z) + \frac{3}{2m}q_0(q_z + q'_z)\bigg). \tag{3.38}$$

Then using Eq. (3.29) for the argument of the δ function we arrive at the dominant term

$$I' \approx m \int d^2q_\perp \int d^2q'_\perp \int ds' \int ds\, \tfrac{4}{3}$$
$$\times \frac{1}{2q_0}\delta(s)\, \varphi_d(\sqrt{q_\perp^2 + (s' + \tfrac{1}{2}s)^2})$$
$$\times \varphi_d(\sqrt{q'^2_\perp + (s' - \tfrac{1}{2}s)^2}) \frac{1}{4\pi}$$
$$= \frac{2m}{3q_0}\int d^2q_\perp \int d^2q'_\perp \int ds'$$
$$\times \varphi_d(\sqrt{q_\perp^2 + s'^2})\, \varphi_d(\sqrt{q'^2_\perp + s'^2}) \frac{1}{4\pi}$$
$$= -\frac{1}{i\pi} I. \tag{3.39}$$

The last step follows from Eq. (A1), if one evaluates this integral in a different way. Thus we end up with

$$\langle \Phi | P t^\dagger P (G_0 - G_0^\star) t P | \Phi \rangle$$
$$\to -2i \frac{4m}{3q_0}(2\pi)^3 \left| \left\langle \frac{3}{4}\mathbf{q_0} \left| t\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right) \right| \frac{3}{4}\mathbf{q_0} \right\rangle_s \right|^2$$
$$\times \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}. \tag{3.40}$$

Adding the above expression to Eq. (3.36) and using Eq (2.4) gives the asymptotic contribution of the terms second order in t of Eq. (2.6) to the total $n+d$ cross section as

$$\sigma_{\rm tot}^{nd}|_{\rm 2nd\ order} = -2(\sigma_{\rm tot}^{NN})^2 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi} + (2\pi)^6 \left(\frac{4m}{3q_0} \right)^2 \left| \left\langle \frac{3}{4}\mathbf{q_0} \right| t\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2 \right) \left| \frac{3}{4}\mathbf{q_0} \right\rangle_s \right|^2 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}. \quad (3.41)$$

If we use the optical theorem in the two-body system for the imaginary part occurring in

$$\left| \left\langle \frac{3}{4}\mathbf{q_0} \right| t\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2 \right) \left| \frac{3}{4}\mathbf{q_0} \right\rangle_s \right|^2,$$

we obtain

$$\sigma_{\rm tot}^{nd}|_{\rm 2nd\ order} \rightarrow -(\sigma_{\rm tot}^{NN})^2 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi} + (2\pi)^6 \left(\frac{4m}{3q_0} \right)^2 \left\{ \mathrm{Re} \left\langle \frac{3}{4}\mathbf{q_0} \right| t\left[\frac{1}{m}\left(\frac{3}{4}q_0\right)^2 \right] \left| \frac{3}{4}\mathbf{q_0} \right\rangle_s \right\}^2 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}. \quad (3.42)$$

In summary, we obtain in the high-energy limit for the total $n+d$ cross section based on the terms of first and second order in t in the expression for $\mathrm{Im}\langle\Phi|U|\Phi\rangle$

$$\begin{aligned}\sigma_{\rm tot}^{nd} &= \sigma_{\rm tot}^{nd}|_{\rm 1st\ order} + \sigma_{\rm tot}^{nd}|_{\rm 2nd\ order}\\ &= 2\,\sigma_{\rm tot}^{NN}\\ &\quad + (2\pi)^6 \left(\frac{4m}{3q_0} \right)^2 \left\{ \mathrm{Re} \left\langle \frac{3}{4}\mathbf{q_0} \right| t\left(\frac{1}{m}\left(\frac{3}{4}q_0\right)^2 \right) \left| \frac{3}{4}\mathbf{q_0} \right\rangle_s \right\}^2\\ &\quad \times \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi} - (\sigma_{\rm tot}^{NN})^2 \left\langle \varphi_d \left| \frac{1}{r^2} \right| \varphi_d \right\rangle \frac{1}{4\pi}\\ &= 2\,\sigma_{\rm tot}^{NN} + O(t^2) - O(t^4). \end{aligned} \quad (3.43)$$

As mentioned earlier, the first term, obtained from the first order term in the two-nucleon t matrix within the multiple scattering expansion gives twice the NN total cross section, thus considering the two nucleons in the deuteron as being free particles. The terms of second order in t within the multiple scattering expansion of Eq. (2.6) give rise to two correction terms. The positive term in Eq. (3.43), being of $O(t^2)$, enhances the contribution of the first term. It is tempting to consider this term as an antishadowing effect. The third, negative term in Eq. (3.43), which is due to $(\sigma_{\rm tot}^{NN})^2$ of $O(t^4)$, reduces the value of the two total NN cross sections and its action is naturally called a shadowing effect. In the Glauber approximation [2–5] one arrives formally at the same result. However, it should be pointed out that Eq. (3.43) contains the symmetrized two-body t-matrix elements and thus implicitly forward and backward amplitudes. We think that our derivation based on the multiple scattering expansion of the Faddeev formulation together with the momentum space treatment is a viable alternative. It uses standard and transparent evaluations of integrals, which could also be calculated numerically without going to the high-energy limit.

IV. APPLICATION

In this section we want to give a numerical illustration of the results derived in the previous section. We use two NN model forces of Malfliet-Tjon type [11], one being purely attractive and the other having attractive and repulsive parts. Both are of Yukawa type and given as

$$V(\mathbf{q}',\mathbf{q}) = \frac{1}{2\pi^2}\left(\frac{V_R}{(\mathbf{q}'-\mathbf{q})^2+\mu_R^2} - \frac{V_A}{(\mathbf{q}'-\mathbf{q})^2+\mu_A^2} \right). \quad (4.1)$$

The parameters are given in Table I. As we saw in the last section, there are two momenta which control the asymptotic expansion. The first is the initial projectile momentum $\mathbf{q_0}$, which appears as $\frac{3}{4}\mathbf{q_0}$ in the three-body context. The second is the ''typical'' deuteron momentum. For the two model potentials we display in Fig. 1 the corresponding deuteron wave functions. We see that in both cases the wave functions drop by about a factor of 10 within 0.6 fm^{-1}. Thus we expect that for $\frac{3}{4}q_0$ sufficiently larger than this value the asymptotic expressions derived in the previous section should be valid. In order to get quantitative insight into the onset of the validity of the asymptotic expressions one should evaluate the terms in the multiple scattering series exactly. Strictly spoken this amounts to solving the Faddeev equation, Eq. (2.2). As a first step we exactly evaluate here the term in Eq. (2.10), which is in first order in t. Using Eq. (3.6) one finds the following exact form:

TABLE I. Parameters of the Malfliet-Tjon type potentials. As conversion factor we use units such that $\hbar c = 197.3286$ MeV fm$=1$.

	V_A	μ_A [MeV]	V_R	μ_R [MeV]	$\langle\varphi_d\|\frac{1}{r^2}\|\varphi_d\rangle$ [fm^{-2}]
MT-IV	0.3303	124.91			0.6343
MT-III	3.1769	305.86	7.291	613.69	0.3090

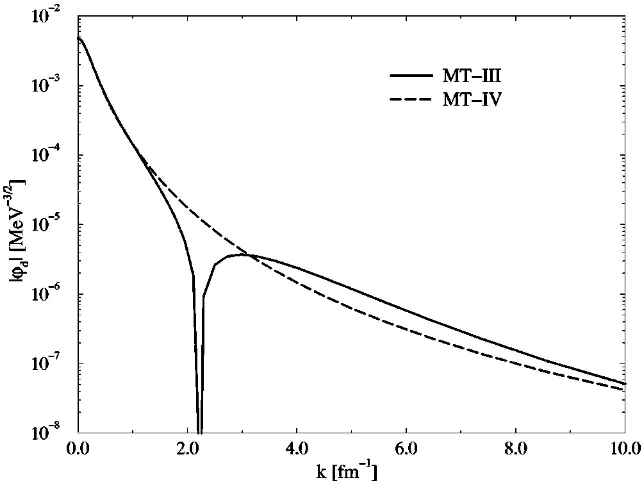

FIG. 1. The absolute values of the bound state wave functions $\varphi_d(k)$ as calculated from the two potential models MT-III and MT-IV given in Table I.

$$\langle\Phi|P(t-t^\dagger)P|\Phi\rangle=\frac{8i}{\pi}\int d^3p\,\varphi_d^2(2|\mathbf{p}-\tfrac{3}{4}\mathbf{q_0}|)\,\mathrm{Im}\langle\mathbf{p}|t(\varepsilon)|\mathbf{p}\rangle_s, \tag{4.2}$$

where

$$\varepsilon=\epsilon_d+\frac{1}{m}p^2-\frac{4}{m}(\mathbf{p}-\tfrac{3}{4}\mathbf{q_0})^2. \tag{4.3}$$

Introducing as variables explicitly the magnitudes of the momentum vectors $\mathbf{p}$ and $\mathbf{q_0}$ and the angle between them, $x=\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}_\mathbf{0}$, we obtain for the $n+d$ total cross section in the first order in t

$$\sigma_{\mathrm{tot}}^{nd}|_{\text{1st order}}=-(2\pi)^3\frac{4m}{3q_0}8\int_{-1}^{1}dx\int_0^\infty dp\,p^2\varphi_d^2 \times(2\sqrt{p^2+(\tfrac{3}{4}q_0)^2-\tfrac{3}{2}pq_0x})\,\mathrm{Im}\,t_s(p,p,\varepsilon). \tag{4.4}$$

The integral over the imaginary part of the forward scattering amplitude receives contributions from the region $\varepsilon>0$ and the deuteron pole. The condition $\varepsilon>0$ limits the integration regions in x as well as p. The x integration has to be carried out only between

$$x_{\min}=\sqrt{\frac{3}{4}+\frac{1}{3}\frac{m|\epsilon_d|}{q_0^2}}$$

and $x_{\max}=1$, and p values given by

$$p_{\max,\min}=q_0x\pm\sqrt{q_0^2x^2-\tfrac{3}{4}q_0^2-\tfrac{1}{3}m|\epsilon_d|}. \tag{4.5}$$

The pole contribution is obtained from an integration over x with similar boundaries at fixed p values.

The off-shell NN scattering amplitude is determined by firstly solving the two-body Lippmann-Schwinger equation exactly in three dimensions, i.e., without partial wave expansion [12]. The quantities entering the integral in Eq. (4.4) are off-shell t-matrix elements for equal magnitudes of the momenta. Thus the integration requires a two-dimensional interpolation of the imaginary part of the forward scattering amplitude in the variables p and ε. We use standard b splines for the numerical calculation [13].

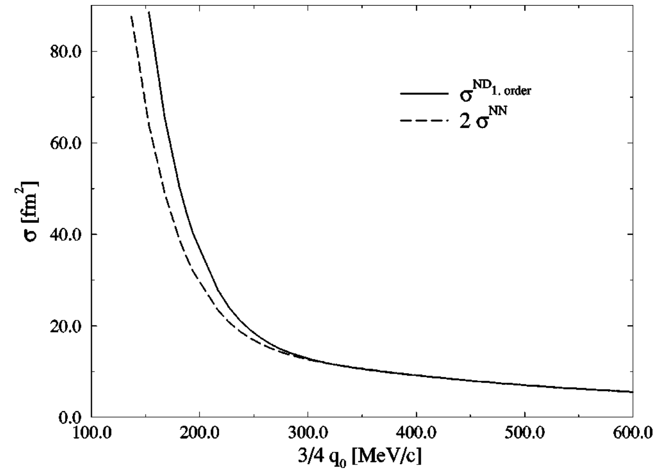

FIG. 2. The contribution of the first order term of the multiple scattering series for the $n+d$ total cross section calculated exactly from the MT-III potential model as function of the asymptotic momentum $\frac{3}{4}q_0$ (solid line). The dashed line represents the corresponding contribution of the first order term in the high-energy limit. The variable $\frac{3}{4}q_0=\sqrt{(m/2)E_{\mathrm{lab}}}$ is chosen such that the kinetic laboratory energies for the two and three body systems are the same.

The first order term of the total $n+d$ cross sections for the two potential models MT-III and MT-IV is shown in Figs. 2 and 3 as function of $\frac{3}{4}q_0$ and compared to its asymptotic limit $2\sigma_{\mathrm{tot}}^{NN}$ given in Eq. (3.15). A closer inspection reveals that this limit is reached at nucleon laboratory energies of about 300 (600) MeV within 1% for MT-III (MT-IV). This energy is fairly high so that the simple potential picture will no longer be valid and relativistic features will be important, including meson production. These absorption processes will

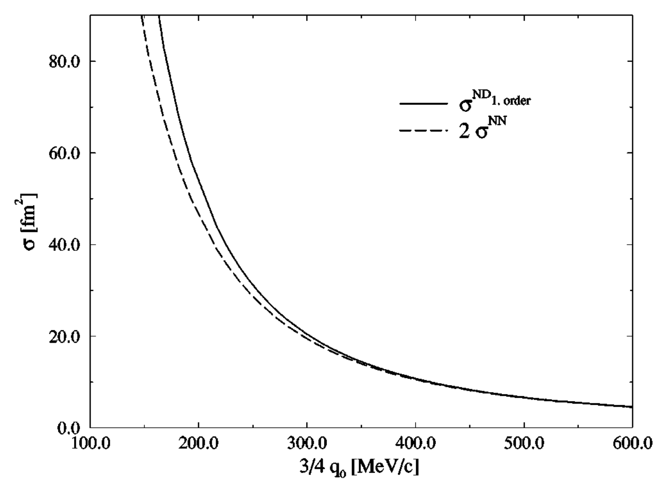

FIG. 3. Same as Fig. 2, except that the two-body potential employed is the MT-IV potential.

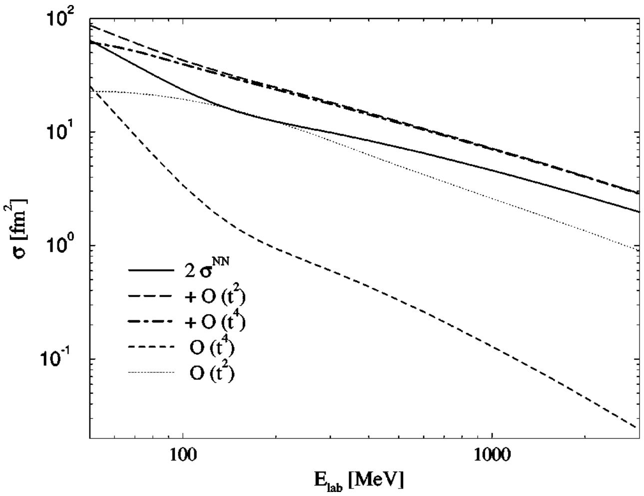

FIG. 4. Contributions to the $n+d$ total cross sections as given by the MT-III potential as function of the corresponding NN laboratory energy. The solid line shows twice the total NN cross section. Successively added to this is the positive contribution (antishadowing) of $O(t^2)$ (long dashed) and the negative contribution (shadowing) of $O(t^4)$ (dash-dotted). The magnitudes of those contributions are shown separately as dotted line for the positive and as short dashed line for the negative term.

of course change our results as is known from studies within the Glauber formalism [3,4]. Nevertheless we think it should be of theoretical interest to see the results within a pure potential picture. We leave the exact solution of the Faddeev equation, Eq. (2.2), to a future study and estimate now the resulting rescattering terms via the asymptotic expressions contained in Eq. (3.43). They are expected to be a good approximation around 300 MeV and above for the potential containing the realistic feature of repulsion. The deuteron matrix elements for the two potentials entering in the asymptotic expression of Eqs. (3.32) and (3.43) are given in Table I.

We start our investigation with the MT-III potential, which has a short range repulsion and a intermediate range attraction, and thus contains some realistic features of the NN force. In Fig. 4 the three different terms of the right-hand side of Eq. (3.43) are shown separately as function of the nucleon laboratory energy together with their partial sums building up the $n+d$ total cross section in the high-energy limit. We see that the rescattering term of $O(t^2)$ is quite large and can not be neglected in relation to the leading term $2\sigma_{\text{tot}}^{NN}$ in the whole energy range shown. For a more quantitative inspection the three different terms are listed separately in Table II for the higher energies. The first column gives twice the NN total cross section, the second column shows the positive term, which is also of $O(t^2)$ like the total NN cross section, and the third column gives the negative term, which is of $O(t^4)$, and which causes shadowing. We see from the table that the positive term of $O(t^2)$, which results from the second order in t of the multiple scattering expansion of the elastic forward scattering amplitude $\langle\Phi|U|\Phi\rangle$ is relatively large. Already at 100 MeV the shadowing term is much smaller than the other two terms and drops of course quickly due to its energy dependence $O(1/q_0^4)$. From these numbers we see that the rescattering corrections are quite large in this pure potential picture. This is even more pronounced for the purely attractive potential MT-IV as shown in Fig. 5. The corresponding terms are displayed in Table III. We see that the positive term of $O(t^2)$, which results from the second order in t of the multiple scattering expansion of the elastic forward scattering amplitude $\langle\Phi|U|\Phi\rangle$ is quite large and more important than the sum of the two NN total cross sections.

In the purely attractive potential model MT-IV we are able to rigorously provide the extreme high-energy limit of the $n+d$ total cross section. In this limit the Born term for the NN t-matrix gives the only contribution and we can replace the t-matrix element in Eq. (3.41) with the potential matrix element, which is real,

TABLE II. Contributions of the different leading order terms of the MT-III potential to the total $n+d$ cross section as function of the NN laboratory energy. The last column shows the factor $F_{\text{MT-}III}$, which multiplies σ_{tot}^{NN} and approaches a constant for higher energies.

E_{lab} [MeV]	$2\ \sigma_{\text{tot}}^{NN}$ [fm^2]	O(t^2) [fm^2]	O(t^4) [fm^2]	$F_{\text{MT-}III}=\dfrac{2\sigma_{\text{tot}}^{NN}+O(t^2)}{\sigma_{\text{tot}}^{NN}}$
50	63.945	22.845	-25.218	2.71
100	23.438	19.410	-3.388	3.66
300	9.885	8.376	-0.603	3.69
500	7.314	5.027	-0.330	3.37
1000	4.577	2.595	-0.129	3.13
1500	3.400	1.769	-0.071	3.04
2000	2.725	1.341	-0.046	2.98
2500	2.283	1.078	-0.032	2.94
3000	1.970	0.899	-0.024	2.91
3500	1.738	0.771	-0.019	2.88
4000	1.557	0.673	-0.015	2.86
4500	1.414	0.597	-0.012	2.84
5000	1.296	0.536	-0.010	2.83

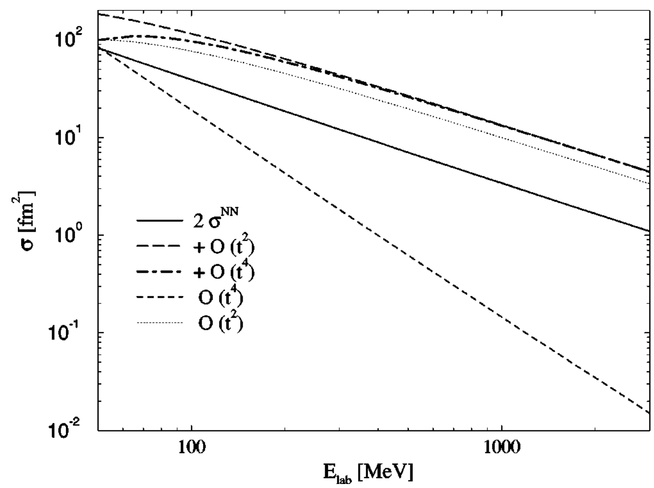

FIG. 5. Same as Fig. 4 except that the two-body potential employed is the MT-IV potential.

$$\left\langle \frac{3}{4}\mathbf{q_0}\right| t\left[\frac{1}{m}\left(\frac{3}{4}q_0\right)^2\right]\left|\frac{3}{4}\mathbf{q_0}\right\rangle_s \rightarrow \left\langle \frac{3}{4}\mathbf{q_0}\right|V\left|\frac{3}{4}\mathbf{q_0}\right\rangle_s . \tag{4.6}$$

This expression can be related to the total cross section, and one finds after a simple integration

$$\int d\hat{p}\, |\langle \tfrac{3}{4}q_0\hat{p}|V|\tfrac{3}{4}\mathbf{q_0}\rangle_s|^2 \rightarrow 4\pi\frac{8}{9q_0^2}\mu_A^2\langle \tfrac{3}{4}\mathbf{q_0}|V|\tfrac{3}{4}\mathbf{q_0}\rangle_s^2 . \tag{4.7}$$

Using the definition of the NN total cross section we obtain

$$\langle \tfrac{3}{4}\mathbf{q_0}|V|\tfrac{3}{4}\mathbf{q_0}\rangle_s^2 \rightarrow \left(\frac{3q_0}{m}\right)^2 \frac{1}{(2\pi)^5(1/2\mu_A^2)}\sigma_{\rm tot}^{NN} . \tag{4.8}$$

As a consequence one can write Eq. (3.43) as

$$\sigma_{\rm tot}^{nd} \rightarrow 2\sigma_{\rm tot}^{NN}\left[1+2\left\langle \varphi_d\left|\frac{1}{\mu_A^2 r^2}\right|\varphi_d\right\rangle\right] - (\sigma_{\rm tot}^{NN})^2\left\langle \varphi_d\left|\frac{1}{r^2}\right|\varphi_d\right\rangle\frac{1}{4\pi}. \tag{4.9}$$

For the potential MT-IV the term additive to 1 is 3.166 (using the quantities given in Table I) and therefore the total NN cross section is multiplied by 8.33 if the exact limit is reached. This number is substantially larger than the naive expectation, which would be twice the NN cross section. Numerically this asymptotic limit is reached around 4000 MeV nucleon laboratory energy within 2% as we see from the last column in Table III. There we display the ratio of the high-energy limit obtained from Eq. (3.43) and the exact limit from Eq. (4.9)

$$R \equiv \frac{2\,\sigma_{\rm tot}^{NN}+(2\pi)^6(4m/3q_0)^2(\mathrm{Re}\langle(3/4)\mathbf{q_0}|t\{1/m[(3/4)q_0]^2\}|(3/4)\mathbf{q_0}\rangle_s)^2\langle\varphi_d|1/r^2|\varphi_d\rangle(1/4\pi)}{2\,\sigma_{\rm tot}^{NN}[1+2\langle\varphi_d|1/\mu_A^2r^2|\varphi_d\rangle]}, \tag{4.10}$$

TABLE III. Contributions of the different leading order terms of the MT-IV potential to the total $n+d$ cross section as function of the NN laboratory energy. The explicit expression for R is given in Eq. (4.10). The definition for $F_{\text{MT-}IV}$ is the same as in Table II.

$E_{\rm lab}$ [MeV]	$2\,\sigma_{\rm tot}^{NN}$ [fm^2]	O(t^2) [fm^2]	O(t^4) [fm^2]	$F_{\text{MT-}IV}$	R
50	82.057	101.812	-84.392	4.48	0.54
100	39.035	76.478	-19.098	5.92	0.71
300	12.076	31.616	-1.828	7.24	0.87
500	7.040	19.590	-0.621	7.57	0.91
1000	3.411	9.989	-0.146	7.86	0.94
1500	2.242	6.693	-0.063	7.97	0.96
2000	1.669	5.035	-0.035	8.03	0.96
2500	1.329	4.035	-0.022	8.07	0.97
3000	1.097	3.349	-0.015	8.08	0.97
3500	0.946	2.894	-0.011	8.12	0.97
4000	0.817	2.513	-0.008	8.15	0.98
4500	0.729	2.240	-0.007	8.15	0.98
5000	0.661	2.034	-0.006	8.15	0.98

where contributions of $O(t^4)$ to the total cross section are neglected. The factor $F_{\text{MT-}IV}$ shown in Table III is explained below.

For the MT-III potential there is interference among the repulsive and attractive parts of the potential and the connection between the potential matrix element in forward direction and the total NN cross section is more involved. Since $\sigma_{\text{tot}}^{NN}=O(1/q_0^2)$ and the real part of the forward scattering amplitude t approaches a constant, both positive terms in Eq. (3.43) can again be combined as $\sigma_{\text{tot}}^{NN}\times F$ and we determined the factor multiplying σ_{tot}^{NN} as $F_{\text{MT-}III}=2.83$, as shown in the last column of Table II. Because of the repulsion contained in the model MT-III the asymptotic limit is reached much later compared to the purely attractive potential model MT-IV.

In both cases the rigorous asymptotic limits are

$$\sigma_{\text{tot}}^{nd}\rightarrow\sigma_{\text{tot}}^{NN}\times F, \tag{4.11}$$

where F is either $F_{\text{MT-}III}$ or $F_{\text{MT-}IV}$. The factors F contain the deuteron matrix element $\langle\varphi_d|1/r^2|\varphi_d\rangle$ and numerical as well as potential parameter constants. The factors F are larger than 2 for both potential models, and thus the rescattering process enhances the asymptotic cross section over the sum of the two NN cross sections, the shadowing has vanished already before that.

We started from an expansion of the elastic $n+d$ forward scattering amplitude $\langle\Phi|U|\Phi\rangle$ in powers of the NN t matrix. Due to the optical theorem the term first order in t ended up as a term second order in t in the total $n+d$ cross section. The terms second order in t in the elastic forward scattering amplitude provide two terms in the total $n+d$ cross section, one of second order and one of fourth order in t. The one second order in t behaves as $O(1/q_0^2)$, whereas the one of fourth order in t decreases as $O(1/q_0^4)$ in the limit q_0 going to infinity. It is therefore natural to group the terms together according to their power in t as we did in Eq. (3.43), which coincides with their energy dependence. Thus the shadowing effect will disappear faster than the antishadowing effect. For the two model forces considered these antishadowing effects modify the naive expectation that the total cross section for $n+d$ scattering tends to twice the total NN cross section. The true asymptotic result is larger than twice the NN cross section, which means that the two nucleons in the deuteron can never be considered to be independent. Finally it is obvious that the terms in Eq. (2.6), which are third order in t can not cancel the terms $O(t^2)$ in the total $n+d$ cross section. They may, however, modify the shadowing effect, since there will be also contributions of order $O(t^4)$ to the total $n+d$ cross section. Thus the asymptotic value of the $n+d$ total cross section in our model of spinless nucleons interacting by local forces is the first term on the right hand side of Eq. (4.9) in case of the purely attractive Yukawa potential MT-IV, and $\sigma_{\text{tot}}^{NN}\times F$ with $F=2.83$ for the potential model MT-III. These are exact results for our potential models.

V. SUMMARY

In view of new precise measurements of the total $n+d$ cross section at energies above 100 MeV nucleon laboratory energy and of precise solutions of the Faddeev equations using modern NN forces [1] it is of interest to understand the leading rescattering effects which modify the naive expectation that the total $n+d$ scattering cross section is the sum of the np and nn total cross sections. In a model of spinless nucleons we investigate the first few terms of the multiple scattering series for the forward elastic $n+d$ scattering amplitude resulting from the Faddeev equations in the high energy limit. Although the treatment is purely nonrelativistic and enters far into the region, where relativity is important, we think it is interesting to know the asymptotic behavior at high energies for pure potential models without absorption. Absorption processes (particle productions) occurring in a relativistic context will, however, change the results presented here [2,4]. In accordance to the naive expectation we extract from the second order term in the multiple scattering series a shadowing effect proportional to the total NN cross section squared, which is negative and reduces the total $n+d$ cross section. However, we also find a positive term proportional to the square of the real part of the NN forward scattering amplitude, which decreases in energy only proportional to the total NN cross section, whereas the shadowing term vanishes faster. Both terms are proportional to the expectation value of $1/(\text{nucleon distance})^2$ with respect to the deuteron wave function.

In a numerical illustration using NN force models of Yukawa type the positive term has as limit the total NN cross section multiplied by a number larger than 2 (which would be the naive expectation). For the purely attractive potential this factor is 8.15, whereas for the potential with the additional repulsion this factor is 2.83. These are exact results for our choice of potential models. For both potentials the negative shadowing effect decreases faster as function of energy than the positive terms.

We expect that the asymptotic expressions given in Eq. (3.43) start to be valid around 300–600 MeV depending on the potential model employed. Strictly speaking this can only be assured, if the Faddeev equation, Eq. (2.2), is solved exactly. The estimate given in this work is based on the exact evaluation of the term first order in the NN t matrix in the $n+d$ elastic forward scattering amplitude. From this calculation we found the corresponding asymptotic limit starting to be valid at the quoted energies.

The formal expression, Eq. (3.43), has also been found in the context of the Glauber approximation [2,5]. A simple geometrical explanation of the deuteron matrix element has been given in Ref. [2]. Our derivation is different and follows simply from the elastic forward $n+d$ scattering amplitude evaluated in leading orders in the NN t matrix and taking the high-energy limit analytically. A similar investigation including spin and isospin degrees of freedom is in preparation.

ACKNOWLEDGMENTS

This work was performed in part under the auspices of the U. S. Department of Energy under Contract No. DE-FG02-93ER40756 with Ohio University, a grant of the Ohio Board of Regents Research Challenge Program, the NATO Collaborative Research Grant No. 960892, and the Research Contract No. 41324878(COSY-44) with the Forschungszentrum Jülich. Two of the authors (Ch.E. and W.G.) would like to thank the Institute for Nuclear Theory at the University ofWashington for their hospitality during some part of the work. We thank the Ohio Supercomputer Center (OSC) for the use of their facilities under Grant No. PHS206.

APPENDIX: DERIVATION OF THE MOMENTUM SPACE FORM OF THE INTEGRAL I

In this appendix we derive the limiting momentum space form of the integral I given in Eq. (3.20). Using the leading expression for the denominator as given in Eq. (3.29) the expression in Eq. (3.20) can be rewritten as

$$I \rightarrow \frac{2m}{3q_0} \int d^3q \int d^3q' \frac{\varphi_d(q)\, \varphi_d(q')}{q_z + q'_z + i\varepsilon} \frac{1}{4\pi}$$
$$= \frac{2m}{3q_0} (2\pi)^2 \int_0^\infty dq q^2 \int_0^\infty dq' q'^2 \int_{-1}^{1} dx \int_{-1}^{1} dx'$$
$$\times \frac{\varphi_d(q)\, \varphi_d(q')}{qx + q'x' + i\varepsilon} \frac{1}{4\pi}. \qquad \text{(A1)}$$

The x and x' integrals are elementary and one ends up directly with the expression given in Eq. (3.33).

[1] W. P. Abfalterer, F. B. Bateman, F. S. Dietrich, Ch. Elster, R. W. Finlay, W. Glöckle, J. Golak, R. C. Haight, D. Hüber, G. L. Morgan, and H. Witała, Phys. Rev. Lett. **81**, 57 (1998).

[2] R. J. Glauber, Phys. Rev. **100**, 242 (1955); in *Lectures in Theoretical Physics*, edited by W. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. 1.

[3] C. J. Joachain, *Quantum Collision Theory* (North-Holland, Amsterdam, 1987).

[4] H. Frauenfelder and E.M. Henley, *Subatomic Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1991), p. 168.

[5] E. S. Abers, H. Burkhardt, V. L. Teplitz, and C. Wilkin, Nuovo Cimento A **XLII**, 365 (1969).

[6] W. Glöckle, *The Quantum Mechanical Few-Body Problem* (Springer Verlag, Berlin, 1983).

[7] W. Glöckle, H. Witała, D. Hüber, H. Kamada, and J. Golak, Phys. Rep. **274**, 107 (1996).

[8] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[9] D. J. Ernst and G. A. Miller, Phys. Rev. C **21**, 1472 (1980); D. L. Weiss and D. J. Ernst, *ibid.* **26**, 605 (1982); D. J. Ernst, G. A. Miller, and D. L. Weiss, *ibid.* **27**, 2733 (1983).

[10] A. Picklesimer, P. C. Tandy, R. M. Thaler, and D. H. Wolfe, Phys. Rev. C **30**, 1861 (1984); Ch. Elster, S. P. Weppner, and C. R. Chinn, *ibid.* **56**, 2080 (1997).

[11] R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969).

[12] Ch. Elster, J. H. Thomas, and W. Glöckle, Few-Body Syst. **24**, 55 (1998).

[13] C. de Boor, *A Practical Guide to Splines* (Springer, New York, 1978).