Effective spin dependent interactions for reactions: The case of $NN \rightarrow N\Delta$

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A general formalism to discuss the role of noncentral forces in reactions with arbitrary spins is outlined. The formalism is applied to the particular case of Δ excitation in inelastic nucleon-nucleon scattering. [S0556-2813(98)03510-9]

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The role of noncentral forces in elastic scattering of particles with arbitrary spin, s on spin zero targets was discussed by Johnson [1] in terms of irreducible tensor operators $\tau_q^k(\mathbf{S})$ of rank k constructed out of the spin operator $\mathbf{S}$ of the spin s particle. The formalism was extended recently [2] to elastic scattering $s_1+s_2 \rightarrow s_1+s_2$ of particles/nuclei with arbitrary spins s_1 and s_2. The purpose of the present paper is to introduce the notion of an effective interaction for reaction processes $s_1+s_2 \rightarrow s_3+s_4$ which is expressible in terms of irreducible tensor operators $\tau_q^k(\mathbf{S}_i)$, $i=1,\cdots,4$ in the spin spaces. In particular we consider the case of $NN \rightarrow N\Delta$, which is of topical interest, and make explicit the form of the effective interaction which is expected to be of use in discussing Δ excitation in nuclei.

Using the same notations as in [3], we first of all note that $S_\mu^\lambda(s_2,s_1)$ defined therein reduces to $\tau_\mu^\lambda(\mathbf{S})$ when $s_1=s_2=s$. Moreover, one can define a projection operator

$$\mathcal{P}^{lsj}(s_1,s_2) = \sum_{k_1=0}^{2s_1} \sum_{k_2=0}^{2s_2} \sum_{k=0}^{2s} G_{k_1k_2k}(l,s,j)(S^k(l,l)\cdot \tau^{(k_1k_2)k}(\mathbf{S}_1,\mathbf{S}_2)), \tag{1}$$

which selects the channel spin s, and the total angular momentum j, from two-particle states with spins s_1,s_2 and having relative angular momentum l. The geometrical factors are given by

$$G_{k_1k_2k}(l,s,j) = (-1)^{l-s-j}[j]^2[s]^2[l]^{-1}[s_1]^{-1}[s_2]^{-1} \times W(llss;kj)(-1)^{k_1+k_2+k}[k_1][k_2] \times \begin{Bmatrix} s_1 & s_2 & s \\ s_1 & s_2 & s \\ k_1 & k_2 & k \end{Bmatrix}. \tag{2}$$

The scattering matrix in spin space for any reaction $s_1+s_2 \rightarrow s_3+s_4$ may be expressed, using Eq. (1), in the form

$$\mathcal{M} = \sum_{l',s',j,l,s} \sum_{\lambda} (-1)^{l'+s-j} W(s'l'sl;j\lambda) \times [j]^2[l']^{-1}[s']^{-1}\mathcal{P}^{l's'j}(s_3,s_4) \times \mathcal{M}^j_{l's';ls}(E)(S^\lambda(l',l)\cdot S^\lambda(s',s))\mathcal{P}^{lsj}(s_1,s_2), \tag{3}$$

where $\mathcal{M}^j_{l's';ls}(E)$ denote partial wave reaction amplitudes [3] at c.m. energy E. We rewrite Eq. (3) in the form

$$\mathcal{M} = \sum_{l',s',j,l,s} \sum_{\{\alpha\},K} G_{\{\alpha\}}(l's';j;ls) \times \mathcal{M}^j_{l's';ls}(E)(\mathcal{S}^{\{\alpha\}K}\cdot S^K(l',l)), \tag{4}$$

where $\{\alpha\}$ denotes $k_3k_4k';\lambda\Lambda;k_1k_2k$, the geometrical factors are

$$G_{\{\alpha\}}(l's';j;ls) = G_{k_3k_4k'}(l',s',j)G_{k_1k_2k}(l,s,j) \times [j]^2[s']^{-1}[l][k'][k][\lambda][\Lambda] \times (-1)^{l'+s-j}W(s'l'sl;j\lambda) \times W(l\lambda l'k';l'\Lambda)W(lkl'\Lambda;lK), \tag{5}$$

and the irreducible tensors of rank K in spin space are given by

$$\mathcal{S}_Q^{\{\alpha\}K} = [(\tau^{(k_3k_4)k'}(\mathbf{S}_3,\mathbf{S}_4)\otimes S^\lambda(s',s))^\Lambda \otimes \tau^{(k_1k_2)k}(\mathbf{S}_1,\mathbf{S}_2)]_Q^K. \tag{6}$$

The effective noncentral interaction for reactions is now readily given by taking the Fourier transform of Eq. (4) and is expressed in the elegant form

$$\langle \mathbf{r}'|V_{\text{eff}}|\mathbf{r}\rangle = \sum_{\{\alpha\},K} (\mathcal{S}^{\{\alpha\}K}\cdot V^{\{\alpha\}K}). \tag{7}$$

The irreducible tensors $V_Q^{\{\alpha\}K}$ of rank K in configuration space are given by

$$V_Q^{\{\alpha\}K} = \sum_{l',s',j,l,s} G_{\{\alpha\}}(l's';j;ls)\langle r'|V_{l's';j;ls}|r\rangle \times \langle \hat{\mathbf{r}}'|S_Q^K(l',l)|\hat{\mathbf{r}}\rangle, \tag{8}$$

in terms of relative coordinates $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and $\mathbf{r}'=\mathbf{r}_3-\mathbf{r}_4$, respectively, in the initial and final states of the reaction. The radial terms in Eq. (8) are given by

$$\langle r'|V_{l's';j;ls}|r\rangle = \frac{2}{\pi}(i)^{l'-l} \times \int E^2 dE p' p j_{l'}(p'r')\mathcal{M}^j_{l's';ls}(E) j_l(pr), \tag{9}$$

in terms of spherical Bessel functions j_l, $j_{l'}$ and c.m. momenta p and p', respectively, in the initial and final states. The result, Eq. (7), is quite general and is applicable to any reaction process with spins $s_1+s_2\to s_3+s_4$. It may be used to identify the effective spin-dependent nonlocal interaction in each case.

We now specialize the above considerations to discuss the particular case of $NN\to N\Delta$. In the case of target Δ excitation process, wherein the spin spaces of the projectile and ejectile are identical, the irreducible tensors in Eq. (6) may be recoupled to express the spin tensors $\mathcal{S}_Q^{\{\alpha\}K}$ in the much simpler form

$$\mathcal{S}_Q^{\{\alpha\}K} = \sum_{\xi_1=0}^{1}\sum_{\xi_2=1}^{2} \mathcal{G}\left(\sigma_P^{\xi_1}\otimes S_T^{\xi_2}\left(\frac{3}{2},\frac{1}{2}\right)\right)_Q^K, \tag{10}$$

where

$$\mathcal{G} = \sum_{\lambda_1,\lambda_2,\eta_1,\eta_2} (-1)^{s+\lambda_1+\lambda_2+k_3+k_4} \begin{Bmatrix} \frac{1}{2} & \frac{3}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & s \\ \lambda_1 & \lambda_2 & \lambda \end{Bmatrix} \times \begin{Bmatrix} k_3 & k_4 & k' \\ \lambda_1 & \lambda_2 & \lambda \\ \eta_1 & \eta_2 & \Lambda \end{Bmatrix} \begin{Bmatrix} \eta_1 & \eta_2 & \Lambda \\ k_1 & k_2 & k \\ \xi_1 & \xi_2 & K \end{Bmatrix}$$
$$\times W\left(\frac{1}{2}\lambda_1\frac{1}{2}k_3;\frac{1}{2}\eta_1\right) W\left(\frac{1}{2}\frac{1}{2}\xi_1\eta_1;k_1\frac{1}{2}\right) \times W\left(\frac{1}{2}\lambda_2\frac{3}{2}k_4;\frac{3}{2}\eta_2\right) W\left(\frac{1}{2}\frac{1}{2}\xi_2\eta_2;k_2\frac{3}{2}\right)$$
$$\times [k'][k][k_1][k_2][k_3][k_4][\lambda_1]^2[\lambda_2]^2[\lambda] \times [\xi_1][\xi_2][\eta_1]^2[\eta_2]^2[s']^2[s], \tag{11}$$

while the indices P and T have to be interchanged in Eq. (10) to discuss the case of projectile Δ excitation process. Observing that

$$\langle \hat{\mathbf{r}}'|S_Q^K(l',l)|\hat{\mathbf{r}}\rangle = (-1)^l[l'](Y_{l'}(\hat{\mathbf{r}}')\otimes Y_l(\hat{\mathbf{r}}))_Q^K \tag{12}$$

and defining

$$V_Q^K(\xi_1,\xi_2) = \sum_{\{\alpha\}} \mathcal{G} V_Q^{\{\alpha\}K}, \tag{13}$$

we may rewrite Eq. (7) for $NN\to N\Delta$ in the form

$$\langle \mathbf{r}'|V_{\text{eff}}|\mathbf{r}\rangle = \sum_{\xi_1=0}^{1}\sum_{\xi_2=1}^{2}\sum_{K=0}^{3} [(\sigma_P^{\xi_1}\otimes S_T^{\xi_2}(\tfrac{3}{2},\tfrac{1}{2}))^K\cdot V^K(\xi_1,\xi_2)] \tag{14}$$

$$= \langle \mathbf{r}'|\sum_{i=\mathrm{I}}^{\mathrm{VIII}} V_i|\mathbf{r}\rangle, \tag{15}$$

which enables us to identify the different forms of the effective spin-dependent interaction for $NN\to N\Delta$, which are listed in Table I.

TABLE I. Effective spin-dependent interactions for $NN\to N\Delta$.

V_i	ξ_1	ξ_2	K	
V_{I}	0	1	1	$[S^1(\frac{3}{2},\frac{1}{2})\cdot V^1(0,1)]$
V_{II}	0	2	2	$[S^2(\frac{3}{2},\frac{1}{2})\cdot V^2(0,2)]$
V_{III}	1	1	0	$[(\boldsymbol{\sigma}\cdot S^1(\frac{3}{2},\frac{1}{2}))V^0(1,1)]$
V_{IV}	1	1	1	$[(\boldsymbol{\sigma}\otimes S^1(\frac{3}{2},\frac{1}{2}))^1\cdot V^1(1,1)]$
V_{V}	1	1	2	$[(\boldsymbol{\sigma}\otimes S^1(\frac{3}{2},\frac{1}{2}))^2\cdot V^2(1,1)]$
V_{VI}	1	2	1	$[(\boldsymbol{\sigma}\otimes S^2(\frac{3}{2},\frac{1}{2}))^1\cdot V^1(1,2)]$
V_{VII}	1	2	2	$[(\boldsymbol{\sigma}\otimes S^2(\frac{3}{2},\frac{1}{2}))^2\cdot V^2(1,2)]$
V_{VIII}	1	2	3	$[(\boldsymbol{\sigma}\otimes S^2(\frac{3}{2},\frac{1}{2}))^3\cdot V^3(1,2)]$

The meson exchange models extensively used in the discussion of Δ excitation in nuclear processes [4] draw upon π and ρ exchange mechanisms to provide, respectively, the spin longitudinal (LO) and spin transverse (TR) components of the basic $NN\to N\Delta$ interaction [5]. However, on expressing the LO and TR components in the form

$$(\boldsymbol{\sigma}\cdot\mathbf{q})(S^1\cdot\mathbf{q}) = \frac{1}{3}(\boldsymbol{\sigma}\cdot S^1)q^2 + (\boldsymbol{\sigma}\otimes S^1)^2\cdot(\mathbf{q}\otimes\mathbf{q})^2, \tag{16}$$

$$(\boldsymbol{\sigma}\times\mathbf{q})\cdot(S^1\times\mathbf{q}) = \frac{2}{3}(\boldsymbol{\sigma}\cdot S^1)q^2 - (\boldsymbol{\sigma}\otimes S^1)^2\cdot(\mathbf{q}\otimes\mathbf{q})^2, \tag{17}$$

we note that only two, viz. V_{III} and V_{V} listed in Table I come into play in these simple models, requiring more complicated mechanisms to generate noncentral interactions involving other spin tensors. It is interesting to look for empirical evidence for the existence of the other (which are as many as six) different forms in $NN\to N\Delta$ through appropriate spin measurements.

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