QCD sum rules for vector mesons in nuclear medium

Stefan Leupold and Ulrich Mosel
Institut für Theoretische Physik, Universität Giessen, D-35392 Giessen, Germany
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Vector mesons show up in the electromagnetic current-current correlator. QCD sum rules provide a constraint on hadronic models for this correlator. This constraint is discussed for the case of finite nuclear density concerning the longitudinal as well as the transverse part of the current-current correlator at finite three-momentum. [S0556-2813(98)01211-4]

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I. INTRODUCTION

In the last few years a lot of work has been devoted to study the behavior of vector mesons in a medium with finite baryonic density. The basic motivation was to find a sign of chiral symmetry restoration in heavy-ion collision experiments, when studying the dilepton spectra which correspond to the vector mesons. Indeed, the CERES experiments for S-Au and Pb-Au collisions show a novel feature when compared to proton-nucleus collisions, namely, an enhancement of the dilepton yield for invariant masses somewhat below the vacuum mass of the ρ meson [1–3]. Some years ago it was argued by Brown and Rho that such an enhancement might be due to the restoration of chiral symmetry [4]. In their model they assumed that the masses of the vector mesons should scale with the quark condensate, i.e., drop with rising baryonic density. If this is true, the ρ peak in the dilepton spectrum would be shifted to lower invariant mass. This might be an explanation for the observed enhancement of the dilepton yield in that region [5–7]. However, the idea of chiral symmetry restoration alone without additional model assumptions does not provide a unique picture. Other scenarios predict a rising ρ mass based on the effect that the ρ becomes degenerate with its chiral partner, the $a1$ meson [8].

Besides the problem of what the consequences of chiral symmetry restoration might be there is still the possibility that the experimental finding of the enhancement might also be explained by conventional hadronic degrees of freedom. To clarify that issue hadronic models for the in-medium behavior of vector mesons were developed by various groups, see e.g., Refs. [9–19]. Some of them predict a large peak broadening of the ρ meson or even distinct new peak structures. It was found that the enhancement in the dilepton yield might also be explained within a purely hadronic scenario, if a lot of strength is shifted to lower invariant mass [14,20]. So far we are not in a position to confirm or rule out such hadronic models by experimental data. Therefore, it is of interest to find additional model independent consistency checks which should be obeyed by arbitrary hadronic models describing vector mesons and their in-medium behavior. Such a consistency check is provided by the QCD sum rule approach.

Originally, QCD sum rules were developed for vacuum processes not as a consistency check for hadronic models, but as an alternative to them, i.e., to deduce model-independent information about hadrons from the underlying quark and gluon degrees of freedom (see, e.g., Refs. [21–23]). An important ingredient for the description of vector mesons by QCD sum rules is the assumption that the spectral functions of these resonances can be reliably approximated by δ functions (narrow width approximation). In this way, the experimentally found vector meson masses can be reproduced reasonably well. Of course, it is supported by experiments that, e.g., the respective width of the spectral function of ρ and ω meson is small as compared to the mass of the meson. However, it is important to realize that the narrow width approximation is not a result, but an ingredient of the traditional QCD sum rule approach.

In the last few years QCD sum rules were also developed for in-medium situations, i.e., for hadronic matter at finite temperature (see, e.g., Refs. [24,25]) or finite baryonic density (e.g., Refs. [26–30]). For ρ and ω mesons it was found that their masses decrease with increasing temperature and/or density, if the narrow width approximation is used for the parametrization of the respective spectral function. In contrast to the vacuum case, there is, however, no experimental support that the narrow width approximation is a reasonable assumption for the in-medium case. Indeed, some hadronic models for the ρ meson predict a very large collisional broadening already at nuclear saturation density due to the coupling of the ρ meson to resonance hole loops [13,14,17–19]. Clearly, such effects have to be taken into account for a proper modeling of the spectral function of the respective vector meson which enters the hadronic side of QCD sum rules [31]. Unfortunately, the narrow width approximation crucially influences the QCD sum rule prediction for a possible mass shift in a nuclear surrounding. If a spectral function with an appropriately chosen large width is used in the QCD sum rule approach at finite density, one could get an unshifted meson mass, in contrast to the finding utilizing the narrow width approximation. This was first discussed in Ref. [15] using a specific hadronic model and later systematically studied in Ref. [31].

This shows that QCD sum rules provide *no* model-independent prediction about a possible mass shift of vector mesons in nuclear medium. Only together with some additional assumptions (e.g., about the width of the respective vector meson) can a statement about the density dependence of the masses of the vector mesons be deduced from the sum rule analysis. Nevertheless, once a hadronic model for vector mesons has been chosen the sum rules can be used as a consistency check for this model. We believe it to be important to have such consistency checks, since it is not clear *a priori* if a hadronic model—e.g., with coupling constants derived from vacuum processes—yields a correct description of the in-medium behavior.

In most of the studies on QCD sum rules only the vector mesons which are at rest with respect to the medium were considered. On the other hand, since Lorentz invariance is broken, the behavior of vector mesons clearly depends on their velocity with respect to the surrounding. Indeed, some of the hadronic models mentioned above yield very different spectral functions for different three-momenta of the respective vector meson and for different polarizations (e.g., Ref. [17]). Only recently has the influence of finite three-momentum on the QCD sum rule prediction for vector meson masses (within the classical narrow width approximation) been explored [30]. Also at finite three-momentum, of course, the narrow width approximation is an assumption which may be justified or not, but in any case it is not a model-independent statement.

In this work we will derive QCD sum rules for ρ and ω mesons for arbitrary three-momentum of the respective vector meson with respect to the nuclear medium. The purpose is to provide a consistency check for hadronic models. This is in contrast to the traditional QCD sum rule approach which aims at a prediction for the vector meson mass assuming that the width of the vector meson is negligibly small (see especially Ref. [30] concerning the extension to nonvanishing three-momentum within the traditional QCD sum rule approach). In the application of the traditional QCD sum rule approach to nuclear matter the attention was focused on the utilization of the sum rule within the narrow width approximation [26,27,29,30], rather than on a detailed discussion of the derivation of the sum rule and of the calculation of the various condensates which contribute. In the present article we try to bridge this gap.

In the next section we introduce the basic quantity of interest, namely, the current-current correlator, and present a dispersion relation which connects the calculations for this correlator using hadronic degrees of freedom on the one hand side and quarks and gluons on the other. In Sec. III we sketch the method of operator product expansion and calculate the current-current correlator within that framework. In Sec. IV we present a QCD sum rule derived from the dispersion relation mentioned above. To get more insight into the various contributions calculated in Sec. III we discuss in Sec. V an approximation linear in the nuclear density. In Sec. VI we discuss the various approximations which have led to the results presented in the preceding sections. Finally we summarize our results in Sec. VII.

II. THE CURRENT-CURRENT CORRELATOR

The quantity we study in the following is the covariant time ordered current-current correlator:

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle. \tag{2.1}$$

Here j_μ is an electromagnetic current with the isospin quantum number of the respective vector meson

$$j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u \mp \bar{d}\gamma_\mu d), \tag{2.2}$$

where the minus sign is for the ρ meson and the plus sign for the ω. The current-current correlator enters, e.g., the cross section of $e^+e^- \to$ hadrons (see, e.g., Ref. [15]). Within a simple vector meson dominance (VMD) picture the current (2.2) can be identified with the vector meson which carries the respective isospin, e.g., for the ρ meson [32]:

$$j_\mu \overset{\text{VMD}}{=} \frac{m_\rho^2}{g_\rho}\rho_\mu, \tag{2.3}$$

where ρ_μ denotes the ρ meson field amplitude and g_ρ the coupling of the ρ meson to pions. Therefore, within simple VMD the current-current correlator is proportional to the propagator of the respective vector meson. Speaking more generally, i.e., without referring to simple VMD, the vector meson propagator is closely related to the current-current correlator.

The expectation value in Eq. (2.1) is taken with respect to the surrounding medium. We study here a (isospin neutral) homogeneous equilibrated medium with finite nuclear density and vanishing temperature. In the medium Lorentz invariance is broken. All the formulas we will present in the following refer to the Lorentz frame where the medium is at rest, i.e., where the spatial components of the baryonic current vanish.

The current-current correlator can be decomposed in the following way [32]:

$$\Pi_{\mu\nu}(q) = \Pi_T(q)T_{\mu\nu}(q) + \Pi_L(q)L_{\mu\nu}(q), \tag{2.4}$$

where we have introduced two independent projectors $L_{\mu\nu}(q)$ and $T_{\mu\nu}(q)$ which both satisfy current conservation $q^\mu L_{\mu\nu}(q) = q^\mu T_{\mu\nu}(q) = 0$ and add up to

$$T_{\mu\nu}(q) + L_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \tag{2.5}$$

The tensors T and L are transverse and longitudinal with respect to three-momentum $\vec{q}$, respectively. T is given by

$$T_{\mu\nu}(q) = \begin{cases} 0, & \mu=0 \text{ or } \nu=0, \\ -\delta^{ij} + \dfrac{q^i q^j}{\vec{q}^2}, & (\mu,\nu)=(i,j), \end{cases} \tag{2.6}$$

while L can be deduced from Eq. (2.5). The scalar functions $\Pi_{T,L}$ can be obtained using

$$\Pi_T(q^2,\vec{q}^2) = \frac{1}{2}\Pi_{\mu\nu}(q)T^{\mu\nu}(q) = \frac{1}{2}\left(\Pi^\mu_\mu + \frac{q^2}{\vec{q}^2}\Pi_{00}\right) \tag{2.7}$$

and

$$\Pi_L(q^2,\vec{q}^2) = \Pi_{\mu\nu}(q)L^{\mu\nu}(q) = -\frac{q^2}{\vec{q}^2}\Pi_{00}. \tag{2.8}$$

To get the respective last equality in the last two equations, use is made of the fact that $\Pi_{\mu\nu}(q)$ is a conserved quantity, i.e., transverse with respect to the *four-momentum* q. Note that Π_T and Π_L depend only on the invariant mass squared q^2 and on the three-momentum squared $\vec{q}^2$. The latter property is due to the remaining $O(3)$ symmetry of the equilibrated system.

At vanishing temperature the scalar functions Π_T, Π_L deduced from the time ordered current-current correlator (2.1) can be related to the commutator (spectral function)

$$\mathcal{A}_{\mu\nu}(q) := -\frac{1}{2}\int d^4x e^{iqx}\langle[j_\mu(x),j_\nu(0)]\rangle \tag{2.9}$$

in the following way (see, e.g., Ref. [33]):

$$\mathrm{Im}\,\Pi_{T,L}(q^2,\vec{q}^2) = -\mathrm{sgn}(q_0)\mathcal{A}_{T,L}(q_0,\vec{q}^2), \tag{2.10}$$

where $\mathcal{A}_T$ and $\mathcal{A}_L$ are deduced from $\mathcal{A}_{\mu\nu}$ analogously to Eqs. (2.7) and (2.8), respectively. At first sight, it seems that $\mathrm{Im}\,\Pi_{T,L}$ does not only depend on q^2 and $\vec{q}^2$ as claimed above, but also on the sign of q_0. However, it is easy to check from the definition (2.9) and the symmetry properties of the system under consideration that $\mathcal{A}_T$ and $\mathcal{A}_L$ are antisymmetric with respect to the transformation $q_0 \to -q_0$. Therefore we define

$$\mathcal{A}^+_{T,L}(q^2,\vec{q}^2) := \mathrm{sgn}(q_0)\mathcal{A}_{T,L}(q_0,\vec{q}^2). \tag{2.11}$$

Inserting this relation in Eq. (2.10),

$$\mathrm{Im}\,\Pi_{T,L}(q^2,\vec{q}^2) = -\mathcal{A}^+_{T,L}(q^2,\vec{q}^2), \tag{2.12}$$

it becomes obvious that the dependence on the sign of q_0 is only apparent.

For $q^2 \ll 0$ the current-current correlator (2.1) can be calculated using Wilson's operator product expansion (OPE) [34] for quark and gluonic degrees of freedom [21] (for in-medium calculations see, e.g., Refs. [24,26,27]). In the following we shall call the result of that calculation $\Pi^{\mathrm{OPE}}_{T,L}$. On the other hand, a hadronic model (e.g., for vector mesons [9–15,17] using one or the other form of VMD) can give an expression for the current-current correlator valid in the timelike region $q^2 > 0$. We denote the result of the hadronic model by $\Pi^{\mathrm{had}}_{T,L}$. A second representation in the spacelike region which has to match $\Pi^{\mathrm{OPE}}_{T,L}$ can be obtained from $\Pi^{\mathrm{had}}_{T,L}$ by utilizing a twice subtracted dispersion relation. We find

$$\Pi_{T,L}(q^2,\vec{q}^2) = \Pi_{T,L}(0,\vec{q}^2) + c_{T,L}(\vec{q}^2)q^2 - \frac{q^4}{\pi}\int_{-\vec{q}^2}^{\infty} ds\, \frac{\mathcal{A}^+_{T,L}(s,\vec{q}^2)}{(s-q^2-i\epsilon)(s+i\epsilon')^2} \tag{2.13}$$

with the subtraction constant

$$c_{T,L}(\vec{q}^2) = \left.\frac{\partial \Pi_{T,L}(k^2,\vec{q}^2)}{\partial(k^2)}\right|_{k^2=0}. \tag{2.14}$$

As we shall see below $\mathcal{A}^{\mathrm{had}}_{T,L}(q)$ diverges linearly with q^2. Therefore we have used above a twice subtracted dispersion relation. In the spacelike region for $Q^2 := -q^2 \gg 0$ we get the following connection between the current-current correlator calculated from OPE on the one hand side and from a hadronic model on the other:

$$\Pi^{\mathrm{OPE}}_{T,L}(Q^2,\vec{q}^2) = \Pi^{\mathrm{had}}_{T,L}(0,\vec{q}^2) - c_{T,L}(\vec{q}^2)Q^2 + \frac{Q^4}{\pi}\int_{-\vec{q}^2}^{\infty} ds\, \frac{\mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}(s,\vec{q}^2)}{(s+Q^2-i\epsilon)(s+i\epsilon')^2}, \tag{2.15}$$

where we have used Eq. (2.12) to express the spectral function in terms of the imaginary part of Π. In the next section we shall elaborate on the calculation of the left-hand side (LHS) of Eq. (2.15).

III. OPERATOR PRODUCT EXPANSION

Within the method of operator product expansion we have to calculate the current-current correlator (2.1) for large spacelike momenta $Q^2 = -q^2$. Here the relevant length scale for the x integration given by the inverse of $1/\sqrt{Q^2}$ is small. This defines the hard scale in our problem. Suppose that the distance x is much smaller than the typical length of the system (soft scale). The latter might be characterized, e.g., by the average particle distance in the medium or $1/\Lambda_{\mathrm{QCD}}$ as the scale where nonperturbative effects appear. If x is that small it is reasonable to assume that a product of local operators A and B, i.e.,

$$A(x)B(0), \tag{3.1}$$

should look like a local operator, since the system cannot resolve such small distances x. Thus we find

$$A(x)B(0) \approx \sum_n C_n(x)\mathcal{O}_n, \tag{3.2}$$

where C_n denotes c-number functions (Wilson coefficients) and $\mathcal{O}_n$ *local* operators. The only dependence on the system under consideration enters via the respective matrix elements of the local operators $\mathcal{O}_n$. Thus, the dependence on the soft scale is entirely given by the local operators. On the other hand, the Wilson coefficients C_n can be calculated independently from the system under consideration. Since the operators $\mathcal{O}_n$ are local, the dependence on the hard scale (here given by the x dependence) enters only the Wilson coefficients. Thus we have achieved a separation of the hard from the soft scale.

In our case, the operators A and B are the currents j_μ and j_ν, respectively. The Fourier transformation which appears in Eq. (2.1) does not touch the local operators $\mathcal{O}_n$, but only changes the x dependence of C_n into a q dependence. From the line of arguments one can already guess that the Wilson coefficients finally yield a power series in $1/Q^2$ (corrected by logarithms from renormalization). The expectation values of the local operators $\mathcal{O}_n$ (condensates) show up as coefficients of that series. On dimensional grounds it is obvious that the higher the dimension (in terms of masses) of a condensate is, the more it is suppressed by powers of $1/Q^2$.

In the following we will consider condensates up to dimension 6. In vacuum only scalar condensates contribute.

For the case at hand, however, the condensates might also carry spin, since Lorentz invariance is broken. It is common practice to classify the condensates by their dimensionality d and their twist τ. The latter is defined as the difference of dimension d and spin s, i.e., $\tau = d - s$. We decompose the current-current correlator (2.1) in the following way:

$$\Pi^{\rm OPE}_{\mu\nu} \approx \Pi^{\rm scalar}_{\mu\nu} + \Pi^{d=4,\tau=2}_{\mu\nu} + \Pi^{d=6,\tau=2}_{\mu\nu} + \Pi^{d=6,\tau=4}_{\mu\nu}, \tag{3.3}$$

where we have neglected contributions from higher-dimensional condensates. We will discuss the various contributions separately in the following subsections.

A. Scalar condensates

The contribution of the scalar condensates to the current-current correlator for the system with finite nuclear density is formally identical to the vacuum case. The only difference is that in the former case the expectation value is taken with respect to the medium. The latter case was already discussed in the original paper by Shifman *et al.* [21]. The result is (see, e.g., Ref. [24] for details)

$$\Pi^{\rm scalar}_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) Q^2 R^{\rm scalar}(Q^2) \tag{3.4}$$

with

$$R^{\rm scalar}(Q^2) \approx -\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{m_q}{2Q^4}\langle \bar{u}u + \bar{d}d\rangle + \frac{1}{24Q^4}\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle$$
$$-\frac{\pi\alpha_s}{2Q^6}\langle (\bar{u}\gamma_\mu\gamma_5\lambda^a u \mp \bar{d}\gamma_\mu\gamma_5\lambda^a d)(\bar{u}\gamma^\mu\gamma_5\lambda^a u \mp \bar{d}\gamma^\mu\gamma_5\lambda^a d)\rangle$$
$$-\frac{\pi\alpha_s}{9Q^6}\left\langle (\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d)\sum_{q=u,d,s}\bar{q}\gamma^\mu\lambda^a q\right\rangle, \tag{3.5}$$

where we have neglected contributions quadratic in the light current quark mass m_q as well as differences in the masses of up and down quark. This is reasonable, since the hard scale $\sqrt{Q^2}$ is typically of the order of 1 GeV (the order of magnitude of the considered vector meson), while the masses of up and down quarks are of the order of a few MeV. Again the minus sign refers to the ρ meson and the plus sign to the ω. To simplify Eq. (3.5) we assume that the quark condensates of up and down quarks are approximately the same. Furthermore, we replace the four-quark condensates by products of two-quark condensates. Since it is not clear how accurate the assumption of ground state saturation (Hartree approximation) is, we multiply the result with a (still to be determined) factor κ. We end up with

$$R^{\rm scalar}(Q^2) \approx -\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{m_q}{Q^4}\langle\bar{q}q\rangle + \frac{1}{24Q^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{112\pi\alpha_s}{81Q^6}\kappa\langle\bar{q}q\rangle^2. \tag{3.6}$$

Note that there is no difference between ρ and ω any more, since there are no terms like $\langle\bar{u}d\rangle$ in an isospin neutral medium.

Of course, the crucial question is how to evaluate the expectation values with respect to the nuclear medium. If the density is small, it is reasonable to approximate the medium by a Fermi gas of free nucleons, i.e.,

$$\langle\mathcal{O}\rangle \approx \langle\mathcal{O}\rangle_0 + 4\int_{|\vec{k}|\le k_F}\frac{d^3k}{(2\pi)^3\,2E_k}\langle N(\vec{k})|\mathcal{O}|N(\vec{k})\rangle, \tag{3.7}$$

where $\langle\mathcal{O}\rangle_0$ denotes the vacuum expectation value of an arbitrary operator $\mathcal{O}$, k_F the Fermi momentum, $E_k = \sqrt{m_N^2+\vec{k}^2}$ the energy of a nucleon, and $|N(\vec{k})\rangle$ a single (isospin averaged) nucleon state with momentum $\vec{k}$ normalized according to

$$\langle N(\vec{k})|N(\vec{k}')\rangle = (2\pi)^3\,2E_k\delta(\vec{k}-\vec{k}'). \tag{3.8}$$

We will use the approximation (3.7) throughout this work and comment on it in Sec. VI.

If $\mathcal{O}$ is a scalar operator, the expectation value $\langle N(\vec{k})|\mathcal{O}|N(\vec{k})\rangle$ is a scalar as well and therefore independent of the momentum of the nucleon. Thus we get

$$\langle\mathcal{O}_{\rm scalar}\rangle \approx \langle\mathcal{O}_{\rm scalar}\rangle_0 + 4\langle N(0)|\mathcal{O}_{\rm scalar}|N(0)\rangle\int_{|\vec{k}|\le k_F}\frac{d^3k}{(2\pi)^3\,2E_k}. \tag{3.9}$$

For the evaluation of the condensates in Eq. (3.6) we need to know the expectation values of the quark and gluon condensate with respect to single nucleon states. The former can be related to the nucleon sigma term [26][1]

$$\langle N(0)|\bar{q}q|N(0)\rangle = \frac{m_N\sigma_N}{m_q}, \tag{3.10}$$

[1] Note that the normalization of the nucleon state in Ref. [26] is different from ours.

TABLE I. Parameters used in the calculation of the OPE contributions to the current-current correlator. See main text for details.

α_s	0.36	σ_N [GeV]	0.045	c_1 [GeV^2]	0.005
m_q [GeV]	0.006	$m_N^{(0)}$ [GeV]	0.75	c_2 [GeV^2]	0.011
$\langle\bar{q}q\rangle_0$ [GeV^3]	-0.0156			c_3 [GeV^2]	0.035
$\langle(\alpha_s/\pi)G^2\rangle_0$ [GeV^4]	0.012			K_{ud}^1 [GeV^2]	-0.088
κ	2.36				

while the latter can be calculated from the trace anomaly of QCD

$$\left\langle N(0)\left|\frac{\alpha_s}{\pi}G^2\right|N(0)\right\rangle=-\frac{16}{9}m_N m_N^{(0)}. \qquad (3.11)$$

Here σ_N denotes the nucleon sigma term and $m_N^{(0)}$ the nucleon mass in the chiral limit.

Finally, we have to make a choice for the parameter κ which parametrized the deviation of the four-quark condensate from the product of two-quark condensates. Even for the vacuum case, the question about the value for κ is not settled yet (see, e.g., Refs. [26,24,27,15,23]). In addition, κ might depend on the nuclear density. For simplicity, we take in the following the vacuum value for κ also for arbitrary finite densities, but note that this introduces an uncertainty into the evaluation of the OPE. All the parameters not specified so far are taken from Ref. [15] and listed in Table I. We discuss this choice in Sec. VI.

Using Eqs. (2.7), (2.8), (3.4), (3.6), (3.9)–(3.11) we are able to calculate the contribution of the scalar condensates to the LHS of Eq. (2.15). In the next subsection we discuss the contribution of the twist-2 condensates with $d=4$.

B. Twist-2 spin-2 condensates

In vacuum only scalar condensates contribute to the current-current correlator since there is no Lorentz vector which can account for the spin of a nonscalar condensate. Contrary to the vacuum case, in a nuclear medium the baryonic current can yield the spin. Using the approximation (3.7) we find for spin-2 condensates

$$\langle\mathcal{O}_{\mu\nu}\rangle\approx 4\int_{|\vec{k}|\leq k_F}\frac{d^3k}{(2\pi)^3\,2E_k}\,\langle N(\vec{k})|\mathcal{O}_{\mu\nu}|N(\vec{k})\rangle. \qquad (3.12)$$

In this approximation the four-momentum of the nucleon accounts for the spin of the condensate. Thus we get

$$\langle N(\vec{k})|\mathcal{O}_{\mu\nu}|N(\vec{k})\rangle\sim k_\mu k_\nu-\frac{g_{\mu\nu}}{4}m_N^2=:S_{\mu\nu}(k). \qquad (3.13)$$

Note that the nonscalar operators are traceless with respect to the Lorentz indices.

Expectation values of twist-2 condensates with respect to single nucleon states as they appear in Eq. (3.12) are thoroughly studied in deep inelastic scattering (DIS), albeit for a somewhat different kinematical situation. We can utilize the results obtained there for our case at hand—as already pointed out in Refs. [26,27,30]. Therefore, we will not give a detailed calculation for the contributions of these condensates to the current-current correlator, but instead present a recipe how to deduce the necessary information from the DIS calculations of Refs. [35–37].

The twist-2 operators of dimensionality 4 which contribute are given by

$$\mathcal{ST}\, i(\bar{u}\gamma_\mu D_\nu u+\bar{d}\gamma_\mu D_\nu d) \qquad (3.14)$$

and

$$\mathcal{ST}\, G^{\kappa}_{\mu}G_{\kappa\nu}. \qquad (3.15)$$

Here $\mathcal{ST}$ denotes an operator producing an expression which is symmetric and traceless with respect to the Lorentz indices μ and ν. D_ν is the covariant derivative and $G_{\kappa\nu}$ the gluonic field strength tensor. In principle, composite operators mix under the renormalization group, if they have the same quantum numbers [38]. To study that mixing we have to decompose Eq. (3.14) in a flavor singlet part which mixes with the gluonic operator (3.15) and a flavor nonsinglet part which does not mix. For the energy region of interest, i.e., roughly about the masses of ρ and ω meson, we have to deal with three active flavors. Therefore, we decompose Eq. (3.14) according to

$$u+d=\frac{1}{3}[2(u+d+s)+(u+d-2s)], \qquad (3.16)$$

where u is an abbreviation for $\bar{u}\gamma_\mu D_\nu u$, etc. Renormalization group mixing applies to $(u+d+s)$ and G, i.e., schematically

$$u+d\to\frac{1}{3}[2(u+d+s+G)+(u+d-2s)]=u+d+\frac{2}{3}G. \qquad (3.17)$$

The contribution of the twist-2 spin-2 condensates to the current-current correlator (2.1) can be written as

$$\Pi_{\mu\nu}^{d=4,\tau=2}(q)=4\int_{|\vec{k}|\leq k_F}\frac{d^3k}{(2\pi)^3\,2E_k}\left(\frac{1}{2}\right)^2\sum_{\psi=u,d}T_{\mu\nu}^{(\psi)s=2,\tau=2}(q,k) \qquad (3.18)$$

with the twist-2 spin-2 contribution to the forward scattering amplitude between a nucleon and a quark current,

$$T_{\mu\nu}^{(\psi)s=2,\tau=2}(q,k) := i\int d^4x e^{iqx} \times\langle N(\vec{k})|T\overline{\psi}(x)\gamma_\mu\psi(x)\overline{\psi}(0)\gamma_\nu\psi(0)|N(\vec{k})\rangle_{s=2,\tau=2}. \tag{3.19}$$

The latter quantity is calculated in Ref. [35] for the DIS case. Note that the factor 1/2 in the definition of the current (2.2)—which enters Eq. (2.1) quadratically—is not contained in the definition of the forward scattering amplitude, but is given in Eq. (3.18) explicitly.

The strategy to utilize the DIS results for the forward scattering amplitude (3.19) is the following: We make a general ansatz for this amplitude, specify it to the DIS case, match it with the calculations of Ref. [35], and determine in this way the unknown quantities of the general ansatz.

The Lorentz structure of the forward scattering amplitude (3.19) must be built up from the tensor $g_{\mu\nu}$, the four-momentum q of the quark current, and the tensor $S_{\mu\nu}(k)$, defined in Eq. (3.13). In addition, the amplitude has to obey current conservation. Finally, it must be symmetric with respect to an exchange of the Lorentz indices. A general ansatz which fulfills all requirements is

$$T_{\mu\nu}^{(\psi)s=2,\tau=2}(q,k) = B_1[q^4 S_{\mu\nu}(k) - q^2 q_\mu q^\alpha S_{\nu\alpha}(k) - q^2 q_\nu q^\alpha S_{\mu\alpha}(k) + g_{\mu\nu}q^2 q^\alpha q^\beta S_{\alpha\beta}(k)] + B_2(q_\mu q_\nu - g_{\mu\nu}q^2)q^\alpha q^\beta S_{\alpha\beta}(k) \tag{3.20}$$

with so far arbitrary coefficients B_1 and B_2 which might depend on q^2 and $q\cdot k$.

The kinematical situation of DIS is such that both $-q^2$ and $q\cdot k$ are large with the Bjorken variable $x=-q^2/(2q\cdot k)$ fixed. In this limit only the $k_\mu k_\nu$ term of $S_{\mu\nu}(k)$ has to be taken into account.[2] We get

$$T_{\mu\nu}^{(\psi)s=2,\tau=2\,\mathrm{DIS}}(q,k) = -q^2(q\cdot k)^2(B_1 d_{\mu\nu} + B_2 e_{\mu\nu}) \tag{3.21}$$

with the tensors [35]

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \tag{3.22}$$

and

$$d_{\mu\nu} = -\frac{k_\mu k_\nu}{(q\cdot k)^2} + \frac{k_\mu q_\nu + k_\nu q_\mu}{q\cdot k} - g_{\mu\nu}. \tag{3.23}$$

By comparison of Eq. (3.21) with Eq. (2.4) of Ref. [35] we find

TABLE II. Relevant coefficient functions $C_{i,n}^j$ taken from Ref. [36] and moments of parton distributions A_n^j calculated from Ref. [37] for $\mu^2=1$ GeV^2.

$C_{2,2}^q$	1.013	$C_{2,4}^q$	1.171	$C_{2,6}^q$	1.316
$C_{L,2}^q$	0.050	$C_{L,4}^q$	0.030	$C_{L,6}^q$	0.022
$C_{2,2}^G$	−0.042	$C_{2,4}^G$	−0.063	$C_{2,6}^G$	−0.060
$C_{L,2}^G$	0.057	$C_{L,4}^G$	0.023	$C_{L,6}^G$	0.012
A_2^{u+d}	1.12	A_4^{u+d}	0.11	A_6^{u+d}	0.03
A_2^G	0.83	A_4^G	0.04	A_6^G	0.01

$$B_1 = -\frac{4}{q^6}\sum_i C_{2,2}^i A_2^i \tag{3.24}$$

and

$$B_2 = -\frac{4}{q^6}\sum_i C_{L,2}^i A_2^i, \tag{3.25}$$

with the process independent coefficient function $C_{r,n}^i$ and the $(n-1)$th moment A_n^i of the distribution of the parton i in the nucleon. Expressions for the former can be found, e.g., in Ref. [36] including α_s corrections.[3] The latter is given by [30]

$$A_n^\psi = 2\int_0^1 dx x^{n-1}[\psi(x,\mu^2) + \overline{\psi}(x,\mu^2)] \tag{3.26}$$

for quarks and

$$A_n^G = 2\int_0^1 dx x^{n-1} G(x,\mu^2) \tag{3.27}$$

for gluons. The parton distributions ψ, $\overline{\psi}$, and G at the renormalization scale μ^2 are parametrized in Ref. [37].

Now we collect all the obtained information to get the contribution of the twist-2 spin-2 condensates to the current-current correlator:

$$\Pi_{\mu\nu}^{d=4,\tau=2}(q) = \int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3\, 2E_k}\, T_{\mu\nu}^{(u+d)s=2,\tau=2}(q,k) \tag{3.28}$$

with

[2] Note that this is not true for the case we are actually interested in. Because the three-momentum $\vec{q}$ might be small, we also have to take into account the $g_{\mu\nu}$ term. However, for the determination of the coefficients B_1 and B_2 this does not matter.

[3] Note that our notation (basically adopted from Ref. [35]) is somewhat different from the one of Ref. [36]: Our coefficient function $C_{2,n}^j$ is identical to $C_{2,j}^N$ of Ref. [36] with $n=N$. The longitudinal coefficient functions differ by a factor 2: $C_{L,n}^j = 2C_{L,j}^N$, where again the former is our coefficient function and the latter the one of Ref. [36].

$$T_{\mu\nu}^{(u+d)s=2,\tau=2}(q,k)$$
$$= -\frac{4}{q^4}\,[q^2 S_{\mu\nu}(k) - q_\mu q^\alpha S_{\nu\alpha}(k) - q_\nu q^\alpha S_{\mu\alpha}(k)$$
$$+ g_{\mu\nu} q^\alpha q^\beta S_{\alpha\beta}(k)]\left(C^q_{2,2}A_2^{u+d} + \frac{2}{3}C^G_{2,2}A_2^G\right)$$
$$- \frac{4}{q^6}(q_\mu q_\nu - g_{\mu\nu}q^2)q^\alpha q^\beta S_{\alpha\beta}(k)$$
$$\times\left(C^q_{L,2}A_2^{u+d} + \frac{2}{3}C^G_{L,2}A_2^G\right), \tag{3.29}$$

where we have taken into account that the gluonic contribution enters with a factor 2/3 according to Eq. (3.17). The coefficient functions and the moments of the parton distributions are listed in Table II. We note in passing that the momentum integrations in Eq. (3.28) can be performed analytically. Since it is not illuminating to present the lengthy result of these integrations we stick to the compact form given by Eqs. (3.28) and (3.29).

C. Twist-2 spin-4 condensates

The twist-2 spin-4 condensates can be treated in the same way as the twist-2 spin-2 condensates. We use the approximation (3.7) to find

$$\langle \mathcal{O}_{\mu\nu\kappa\lambda}\rangle \approx 4\int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3\,2E_k}\,\langle N(\vec{k})|\mathcal{O}_{\mu\nu\kappa\lambda}|N(\vec{k})\rangle. \tag{3.30}$$

For the expectation value with respect to a single nucleon state we get the decomposition

$$\langle N(\vec{k})|\mathcal{O}_{\mu\nu\kappa\lambda}|N(\vec{k})\rangle \sim k_\mu k_\nu k_\kappa k_\lambda$$
$$- \frac{1}{8}(k_\mu k_\nu g_{\kappa\lambda} m_N^2 + 5 \ \ \text{permutations})$$
$$+ \frac{1}{48}(g_{\mu\nu}g_{\kappa\lambda} m_N^4 + 2 \ \ \text{permutations})$$
$$=: S_{\mu\nu\kappa\lambda}(k). \tag{3.31}$$

The relevant operators are

$$\mathcal{ST}\, i(\bar{u}\gamma_\mu D_\nu D_\kappa\, D_\lambda u + \bar{d}\gamma_\mu D_\nu\, D_\kappa\, D_\lambda d) \tag{3.32}$$

and

$$\mathcal{ST}\, G^\alpha_\mu D_\nu\, D_\kappa G_{\alpha\lambda}\,. \tag{3.33}$$

By comparison with the DIS calculations we find

$$\Pi_{\mu\nu}^{d=6,\tau=2}(q) = \int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3\,2E_k}\, T_{\mu\nu}^{(u+d)s=4,\tau=2}(q,k) \tag{3.34}$$

with

$$T_{\mu\nu}^{(u+d)s=4,\tau=2}(q,k)$$
$$= -\frac{16}{q^{10}}(q_\mu q_\nu q^\alpha q^\beta - g_\mu{}^\alpha q^2 q_\nu q^\beta - g_\nu{}^\alpha q^2 q_\mu q^\beta$$
$$+ g_\mu{}^\alpha g_\nu{}^\beta q^4) q^\gamma q^\delta S_{\alpha\beta\gamma\delta}(k)\left(C^q_{2,4}\,A_4^{u+d} + \frac{2}{3}C^G_{2,4}\,A_4^G\right)$$
$$- \frac{16}{q^{10}}(q_\mu q_\nu - g_{\mu\nu}q^2)q^\alpha q^\beta q^\gamma q^\delta S_{\alpha\beta\gamma\delta}(k)$$
$$\times\left((C^q_{L,4} - C^q_{2,4})A_4^{u+d} + \frac{2}{3}(C^G_{L,4} - C^G_{2,4})A_4^G\right), \tag{3.35}$$

where the tensor $S_{\alpha\beta\gamma\delta}(k)$ is defined in Eq. (3.31) and the coefficient functions and parton distribution moments are listed in Table II.

D. Twist-4 spin-2 condensates

Finally we turn to the higher twist condensates. For our calculation up to dimensionality $d=6$ we need condensates with twist $\tau=4$ and spin $s=2$ (besides the higher twist condensates which are scalar and have already been discussed in Sec. III A). These condensates are [39–42]

$$\mathcal{O}^1_{\mu\nu} := \mathcal{ST}\, g^2\,\frac{1}{4}\,(\bar{u}\gamma_\mu\gamma_5\lambda^a u \mp \bar{d}\gamma_\mu\gamma_5\lambda^a d)$$
$$\times(\bar{u}\gamma_\nu\gamma_5\lambda^a u \mp \bar{d}\gamma_\nu\gamma_5\lambda^a d), \tag{3.36}$$

$$\mathcal{O}^2_{\mu\nu} := \mathcal{ST}\, g^2\,\frac{1}{4}\,(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d)\sum_{q=u,d,s}\bar{q}\gamma_\nu\lambda^a q, \tag{3.37}$$

$$\mathcal{O}^g_{\mu\nu} := \mathcal{ST}\, ig\,\frac{1}{4}\,(\bar{u}\{D_\mu, \tilde{G}_{\nu\alpha}\}\gamma^\alpha\gamma_5 u + \bar{d}\{D_\mu, \tilde{G}_{\nu\alpha}\}\gamma^\alpha\gamma_5 d), \tag{3.38}$$

$$\mathcal{ST}\, g\,\frac{1}{4}\,(\bar{u}[D_\mu, G_{\nu\alpha}]\gamma^\alpha u + \bar{d}[D_\mu, G_{\nu\alpha}]\gamma^\alpha d), \tag{3.39}$$

and

$$\mathcal{ST}\,\frac{1}{4}\,(m_u\bar{u}D_\mu D_\nu u + m_d\bar{d}D_\mu D_\nu d), \tag{3.40}$$

where the minus sign in Eq. (3.36) corresponds to the ρ and the plus sign to the ω meson. For an unknown reason the condensate given in Eq. (3.39) actually does not contribute to the current-current correlator [40]; we have listed it here for the sake of completeness, only. In the following we will neglect the condensate (3.40), since it is proportional to the very small light quark masses and therefore suppressed. Additionally, we will neglect the contribution of the strange quarks to the nucleon expectation values of the operators given above [41].

As in the previous subsections we use the Fermi gas approximation which for the case of spin-2 condensates we have already given in Eq. (3.12). We can also use the Lorentz decomposition from Eq. (3.13):

$$\langle N(\vec{k})|\mathcal{O}^j_{\mu\nu}|N(\vec{k})\rangle =: A^j S_{\mu\nu}(k), \tag{3.41}$$

where the index $j=1,2,g$ denotes the twist-4 operators specified in Eqs. (3.36)–(3.38).[4]

In Ref. [41] it is thoroughly discussed how the expectation values of these condensates with respect to single nucleon states can be extracted from DIS experiments. To determine the contribution of these condensates to the current-current correlator we follow the same strategy as described in Sec. III B, i.e.,

$$\Pi^{d=6,\tau=4}_{\mu\nu}(q) = 4\int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3\, 2E_k}\, T^{s=2,\tau=4}_{\mu\nu}(q,k) \tag{3.42}$$

with the twist-4 spin-2 contribution to the forward scattering amplitude between a nucleon and the respective isospin current (2.2)

$$T^{s=2,\tau=4}_{\mu\nu}(q,k) := i\int d^4x e^{iqx}\langle N(\vec{k})|Tj_\mu(x)j_\nu(0)|N(\vec{k})\rangle_{s=2,\tau=4}. \tag{3.43}$$

With the general ansatz [see Eq. (3.20)]

$$\begin{aligned} T^{s=2,\tau=4}_{\mu\nu}(q,k) = {} & \tilde{B}_1[q^4 S_{\mu\nu}(k) - q^2 q_\mu q^\alpha S_{\nu\alpha}(k) \\ & - q^2 q_\nu q^\alpha S_{\mu\alpha}(k) + g_{\mu\nu} q^2 q^\alpha q^\beta S_{\alpha\beta}(k)] \\ & + \tilde{B}_2(q_\mu q_\nu - g_{\mu\nu}q^2) q^\alpha q^\beta S_{\alpha\beta}(k) \end{aligned} \tag{3.44}$$

we get for the kinematical situation of DIS

$$T^{(\psi)s=2,\tau=2\ \ \mathrm{DIS}}_{\mu\nu}(q,k) = -q^2(q\cdot k)^2(\tilde{B}_1 d_{\mu\nu} + \tilde{B}_2 e_{\mu\nu}) \tag{3.45}$$

which has to be compared with Eq. (2) from Ref. [41]. We end up with

$$\tilde{B}_1 = \frac{4}{q^8}\left(A^1 + \frac{5}{8}A^2 + \frac{1}{16}A^g\right) \tag{3.46}$$

and

$$\tilde{B}_2 = \frac{4}{q^8}\left(\frac{1}{4}A^2 - \frac{3}{8}A^g\right). \tag{3.47}$$

Based on a flavor decomposition new parameters K^j_ψ are introduced in Ref. [41] in terms of which the coefficients A^j can be expressed. We refer to Ref. [41] for details and only give the final result for our isospin averaged coefficients A^j:

$$A^1 = \frac{1}{4}[K^1_u + K^1_d - (1\pm 1)K^1_{ud}], \tag{3.48}$$

$$A^2 = \frac{1}{4}(K^2_u + K^2_d), \tag{3.49}$$

$$A^g = \frac{1}{4}(K^g_u + K^g_d), \tag{3.50}$$

where the plus sign in Eq. (3.48) corresponds to the ρ and the minus sign to the ω meson. The parameters K^j_ψ are given by the expectation values of the twist-4 condensates (3.36)–(3.38) with respect to a single proton state p [see Eqs. (3.36)–(3.38), (3.41)]:

$$\langle p(k)|\mathcal{ST}g^2\bar{u}\gamma_\mu\gamma_5\lambda^a u(\bar{u}\gamma_\nu\gamma_5\lambda^a u + \bar{d}\gamma_\nu\gamma_5\lambda^a d)|p(k)\rangle =: K^1_u S_{\mu\nu}(k), \tag{3.51}$$

$$\langle p(k)|\mathcal{ST}g^2\bar{u}\gamma_\mu\lambda^a u(\bar{u}\gamma_\nu\lambda^a u + \bar{d}\gamma_\nu\lambda^a d)|p(k)\rangle =: K^2_u S_{\mu\nu}(k), \tag{3.52}$$

$$\langle p(k)|\mathcal{ST}ig\bar{u}\{D_\mu,\tilde{G}_{\nu\alpha}\}\gamma^\alpha\gamma_5 u|p(k)\rangle =: K^g_u S_{\mu\nu}(k) \tag{3.53}$$

and respective definitions for K^j_d, $j=1,2,g$. Furthermore,

$$\langle p(k)|\mathcal{ST}g^2\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{d}\gamma_\nu\gamma_5\lambda^a d|p(k)\rangle =: K^1_{ud}S_{\mu\nu}(k). \tag{3.54}$$

Since the flavor structure of $K^{1,2,g}_d$ and $K^{1,2,g}_u$ are governed by the d quark and the u quark, respectively, it seems reasonable to assume that the ratio is always the same [41]:

$$\frac{K^1_d}{K^1_u} = \frac{K^2_d}{K^2_u} = \frac{K^g_d}{K^g_u} =: \beta. \tag{3.55}$$

Within that assumption, the ratio β and the quantities $\tilde{B}_1$ and $\tilde{B}_2$ for ρ and ω can be determined from the DIS data. As expected from the valence quark decomposition of the proton one finds

$$\beta \approx 0.5 \tag{3.56}$$

within a small error. Furthermore we get

$$\tilde{B}_1 = \frac{1}{q^8}[18(c_1 - c_2) - (1\pm 1)K^1_{ud}] \tag{3.57}$$

and

$$\tilde{B}_2 = \frac{1}{q^8}6c_3, \tag{3.58}$$

where the constants c_i, $i=1,2,3$, as well as K^1_{ud} can be found in Table I.

In total, the contribution of the twist-4 spin-2 operators is given by Eqs. (3.42), (3.44), (3.57), (3.58). We note that a difference between the ρ and the ω meson within the OPE up to dimensionality $d=6$ only shows up for the twist-4 condensates and is expressed here in terms of the quantity K^1_{ud}.

To summarize, we have presented in this section the operator product expansion of the current-current correlator (2.1) including condensates up to dimensionality $d=6$. The

[4]The coefficients A^j should not be confused with the parton distribution moments A^i_n.

general form for the transverse and longitudinal part of the current-current correlator which enter Eq. (2.15) is given by

$$\Pi^{\mathrm{OPE}}_{T,L}(Q^2,\vec{q}^2)=Q^2\left[-\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right)+\sum_n \frac{c^n_{T,L}(\vec{q}^2)}{Q^{2n}}\right], \tag{3.59}$$

where the coefficients $c^n_{T,L}$ have to be deduced from the various contributions discussed in this section. We note that contrary to the vacuum case [21,23] and to the in-medium case with vanishing three-momentum $\vec{q}$ [26,24,43,27,29,15,31] for the general case $\vec{q}\neq 0$ there are not only $1/Q^4$ and $1/Q^6$ terms in Eq. (3.59) but also higher order terms, even when we restrict the OPE to $d\leqslant 6$ condensates. Appropriate powers of $\vec{q}$ in the numerator serve to achieve the correct overall dimension. We will come back to that point in Sec. V.

IV. QCD SUM RULE

In the last section we have calculated the LHS of Eq. (2.15) within the operator product expansion and some additional assumptions. We postpone the discussion of these assumptions to Sec. VI and present here some general ideas about the calculation of the RHS of Eq. (2.15) and about the use of this equation.

If one has a model at hand which yields the current-current correlator for arbitrary positive energy and arbitrary three-momentum, one could directly use Eq. (2.15) to judge the reliability of this model. In practice, however, the situation is such that one might have a model for the lowest hadronic resonance in the respective isospin channel, i.e., for ρ and ω, respectively, but one usually has no model which remains valid for arbitrary high energies. In the dispersion integral of Eq. (2.15) higher lying resonances are suppressed, but only by a factor $1/s^3$. Clearly, it is desirable to achieve a larger suppression of the part of the hadronic spectral function on which one has less access. With this goal, a Borel transformation [21,38] can be applied to Eq. (2.15). For an arbitrary function $f(Q^2)$ the Borel transformation is defined as

$$f(Q^2)\overset{\hat{B}}{\rightarrow}\tilde{f}(M^2) \tag{4.1}$$

with

$$\hat{B}:=\lim_{\substack{Q^2\to\infty,N\to\infty\\ Q^2/N=:M^2=\text{fixed}}}\frac{1}{\Gamma(N)}(-Q^2)^N\left(\frac{d}{dQ^2}\right)^N, \tag{4.2}$$

where M is the so-called Borel mass.

We will apply the Borel transformation to [see Eqs. (2.15), (3.59)]

$$-\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right)+\sum_n \frac{c^n_{T,L}(\vec{q}^2)}{Q^{2n}}$$
$$=\frac{\Pi^{\mathrm{had}}_{T,L}(0,\vec{q}^2)}{Q^2}-c_{T,L}(\vec{q}^2)$$
$$+\frac{Q^2}{\pi}\int_{-\vec{q}^2}^{\infty}ds\,\frac{\mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}(s,\vec{q}^2)}{(s+Q^2-i\epsilon)(s+i\epsilon')^2}. \tag{4.3}$$

Therefore we need to know the Borel transforms of $f(Q^2)=(Q^2+s)^{-\beta}$ and $f(Q^2)=\ln Q^2$. From the definition (4.2) it is easy to derive [38]

$$f(Q^2)=(Q^2+s)^{-\beta}\Rightarrow\tilde{f}(M^2)=\frac{1}{\Gamma(\beta)}\frac{1}{M^{2\beta}}e^{-s/M^2} \tag{4.4}$$

and

$$f(Q^2)=\ln Q^2\Rightarrow\tilde{f}(M^2)=-1. \tag{4.5}$$

Applying the Borel transformation to Eq. (4.3) we get

$$\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)+\sum_n\frac{1}{\Gamma(n)}\frac{c^n_{T,L}(\vec{q}^2)}{M^{2n}}$$
$$=\frac{\Pi^{\mathrm{had}}_{T,L}(0,\vec{q}^2)}{M^2}$$
$$-\frac{1}{\pi M^2}\int_{-\vec{q}^2}^{\infty}ds\,\frac{\mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}(s,\vec{q}^2)}{s+i\epsilon}e^{-s/M^2}. \tag{4.6}$$

It is useful to write the RHS of the last equation in a form, where it is more obvious that this expression is actually real valued. To this aim we split $1/(s+i\epsilon)$ into a principal value and a δ function:

$$\frac{1}{s+i\epsilon}=\frac{s}{s^2+\epsilon^2}-i\pi\delta(s). \tag{4.7}$$

Using this decomposition we find the QCD sum rule

$$\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)+\sum_n\frac{1}{\Gamma(n)}\frac{c^n_{T,L}(\vec{q}^2)}{M^{2n}}$$
$$=\frac{\mathrm{Re}\,\Pi^{\mathrm{HAD}}_{T,L}(0,\vec{q}^2)}{M^2}$$
$$-\frac{1}{\pi M^2}\int_{-\vec{q}^2}^{\infty}ds\,\mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}(s,\vec{q}^2)\,\frac{s}{s^2+\epsilon^2}\,e^{-s/M^2}. \tag{4.8}$$

We observe that higher resonance states are now exponentially suppressed. Additionally we find a $1/s$ suppression. The latter is due to the fact that we have applied the Borel transformation to $1/Q^2$ times Eq. (2.15) instead of directly applying it to Eq. (2.15). On the one hand, such an additional suppression factor is desirable. On the other hand, we have to pay a price for this, namely, that the subtraction constant $\Pi^{\rm had}_{T,L}(0,\vec{q}^2)$ has not dropped out in contrast to the other subtraction constant $c_{T,L}(\vec{q}^2)$. Had we applied the Borel transformation to $1/Q^4$ times Eq. (2.15), the latter would also have survived. This is of course easy to understand from the point of view of subtracted dispersion relations: the suppression of high-energy contributions has to be compensated for by a more detailed knowledge of the function at the subtraction point. We note that it is easy to get from Eq. (4.8) also the direct Borel transformation of Eq. (2.15) without the $1/Q^2$ factor. We simply have to multiply Eq. (4.8) by $(-M^2)$ and differentiate with respect to M^2 afterwards. Using this recipe the subtraction constant $\Pi^{\rm had}_{T,L}(0,\vec{q}^2)$ obviously would drop out. Also the $1/s$ suppression in the integral of Eq. (4.8) would disappear.

Having achieved a reasonable suppression of the energy region above the lowest lying resonance the integral in Eq. (4.8) is no longer sensitive to the details of the hadronic spectral function in that region. For high energies the quark structure of the current-current correlator is resolved. QCD perturbation theory becomes applicable yielding

$$\mathrm{Im}\,\Pi^{\rm had}_{T,L}(s,\vec{q}^2)=-\frac{s}{8\pi}\left(1+\frac{\alpha_s}{\pi}\right)\quad\text{for large } s. \tag{4.9}$$

These considerations suggest the ansatz

$$\mathrm{Im}\,\Pi^{\rm had}_{T,L}(s,\vec{q}^2)=\Theta(s_0-s)\mathrm{Im}\,\Pi^{\rm res}_{T,L}(s,\vec{q}^2)+\Theta(s-s_0)\,\frac{-s}{8\pi}\left(1+\frac{\alpha_s}{\pi}\right), \tag{4.10}$$

where s_0 denotes the threshold between the low energy region described by a spectral function for the lowest lying resonance $\mathrm{Im}\,\Pi^{\rm res}$ and the high-energy region described by a continuum calculated from perturbative QCD. Of course, the high-energy behavior given in Eq. (4.10) is only an approximation on the true spectral function for the current-current correlator. Also the rapid crossover in Eq. (4.10) from the resonance to the continuum region is not realistic. However, exactly here the suppression factors discussed above should become effective making a more detailed description of the cross-over and the high-energy region insignificant.

The price we have to pay for the simple decomposition (4.10) is the appearance of a new parameter s_0, the continuum threshold, which in general depends on the three-momentum $\vec{q}$ and on the nuclear density. We will elaborate later on the determination of s_0.

Inserting Eq. (4.10) into Eq. (4.8) yields

$$\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)(1-e^{-s_0(\vec{q}^2)/M^2})+\sum_n\frac{1}{\Gamma(n)}\frac{c^n_{T,L}(\vec{q}^2)}{M^{2n}}=\frac{\mathrm{Re}\,\Pi^{\rm res}_{T,L}(0,\vec{q}^2)}{M^2}-\frac{1}{\pi M^2}\int_{-\vec{q}^2}^{s_0(\vec{q}^2)}ds\ \mathrm{Im}\,\Pi^{\rm res}_{T,L}(s,\vec{q}^2)\frac{s}{s^2+\epsilon^2}\,e^{-s/M^2}. \tag{4.11}$$

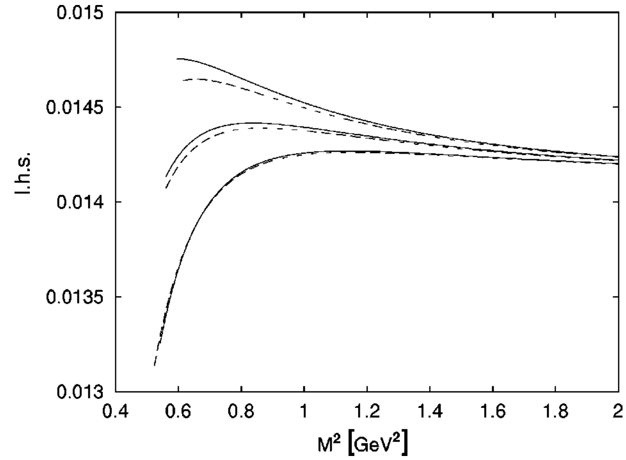

FIG. 1. Transverse part of the LHS of Eq. (4.8) as a function of the Borel mass squared M^2 for three-momenta $|\vec{q}|=0$, 0.5, 1 GeV (top to bottom) and for ρ (full lines) and ω mesons (dashed lines).

Obviously, the exponential suppression in Eq. (4.11) works only if the Borel mass M is not too large. On the other hand, the OPE expression on the LHS of Eq. (4.11) gives a reliable prescription, if M is not too small, since we have neglected higher order condensates which are accompanied by higher orders in $1/M^2$. At best, the sum rule (4.11) is valid inside of a Borel window

$$M^2_{\rm min}\leq M^2\leq M^2_{\rm max}, \tag{4.12}$$

where $M^2_{\rm min}$ has to be determined such that the neglected condensates do not spoil the validity of the LHS of Eq. (4.11), while $M^2_{\rm max}$ has to be determined such that the suppression of the details in the high-energy structure of the current-current correlator becomes effective. Of course, it is not clear *a priori*, if such a Borel window exists at all. It might happen that $M^2_{\rm min}$ is larger than $M^2_{\rm max}$. In this worst case, the sum rule (4.11) would be useless.

The strategy to determine the Borel window is as follows [23].

(i) For $M^2_{\rm min}$ we require that for this Borel mass the absolute value of the contribution of the $d=6$ condensates is a certain percentage p of the total absolute value of the LHS of Eq. (4.8). Since the $d=6$ condensates have the highest order in mass which is taken into account, one might expect that the relative contribution of the neglected condensates is much less than p. Following Ref. [23] we take $p=10\%$, i.e.,

$$\left|\sum_n\frac{1}{\Gamma(n)}\frac{c^{n,d=6}_{T,L}(\vec{q}^2)}{(M^2_{\rm min})^n}\right|=0.1\left|\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)+\sum_n\frac{1}{\Gamma(n)}\frac{c^n_{T,L}(\vec{q}^2)}{(M^2_{\rm min})^n}\right|. \tag{4.13}$$

(ii) For $M^2_{\rm max}$ we require that for this Borel mass the absolute value of the continuum contribution to the integral in Eq. (4.8) is a certain percentage p' of the total absolute value of the integral. Again we follow Ref. [23] and take $p'=50\%$, i.e.,

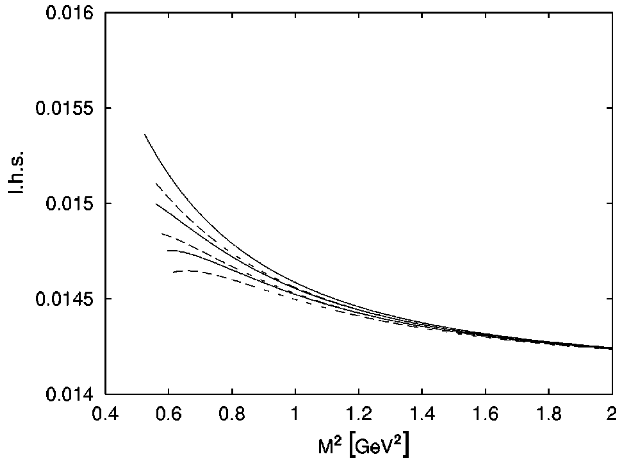

FIG. 2. Longitudinal part of the LHS of Eq. (4.8) as a function of the Borel mass squared M^2 for three-momenta $|\vec{q}|=0$, 0.5, 1 GeV (bottom to top) and for ρ (full lines) and ω mesons (dashed lines).

$$\left| \int_{-\vec{q}^2}^{\infty} ds\,\Theta(s-s_0) \mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}(s,\vec{q}^2)\,\frac{s}{s^2+\epsilon^2}\,e^{-s/M^2_{\mathrm{max}}} \right| = 0.5 \left| \int_{-\vec{q}^2}^{\infty} ds\;\mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}(s,\vec{q}^2)\,\frac{s}{s^2+\epsilon^2}\,e^{-s/M^2_{\mathrm{max}}} \right|. \tag{4.14}$$

By insertion of the decomposition (4.10) we get

$$\frac{1}{8\pi}\left(1+\frac{\alpha_s}{\pi}\right)e^{-s_0(\vec{q}^2)/M^2_{\mathrm{max}}} = -\int_{-\vec{q}^2}^{s_0(\vec{q}^2)} ds\;\mathrm{Im}\,\Pi^{\mathrm{res}}_{T,L}(s,\vec{q}^2)\,\frac{s}{s^2+\epsilon^2}\,e^{-s/M^2_{\mathrm{max}}}. \tag{4.15}$$

We note in passing that the sign of $\mathrm{Im}\,\Pi^{\mathrm{had}}_{T,L}$ and therefore also the sign of $\mathrm{Im}\,\Pi^{\mathrm{res}}_{T,L}$ is always negative [see Eqs. (4.9), (2.12)].

Obviously, the lower limit of the Borel window depends only on the condensates calculated in Sec. III. In contrast to that, the upper limit depends on the choice for the continuum threshold s_0 and on the hadronic model which yields $\Pi^{\mathrm{res}}_{T,L}$. In general, both limits may depend on the three-momentum $\vec{q}$ and on the nuclear density.

Figure 1 shows the transverse component of the LHS of Eq. (4.8) as a function of the Borel mass squared, M^2, for various values of three-momentum squared, $\vec{q}^2$, and for ρ and ω meson. For the nuclear density we have chosen the nuclear saturation density of 0.17 fm^{-3}. Figure 2 shows the same for the longitudinal component. On the left hand side both figures start with $M^2=M^2_{\mathrm{min}}$ as deduced from Eq. (4.13). Obviously, the difference between ρ and ω meson is only very small and vanishes with rising M^2. The latter observation can be easily understood recalling that the only difference in the OPE's for ρ and ω comes from the twist-4 spin-2 condensates which are suppressed at least by a factor $1/M^6$. Hence, the suppression becomes more effective with rising M^2. Note that the small difference between ρ and ω does not necessarily mean that there is not much difference in their spectral functions. It only means that the integrated quantity given in Eq. (4.11) (to be rigorous, the RHS of that equation) is nearly the same for both mesons. Nonetheless, the fact that the sum rule (4.8) is nearly insensitive to the choice for the meson provides a strong constraint on a hadronic model which aims at a description of ρ and ω on the same footing, like, e.g., Ref. [15].

We also observe that the dependence of the LHS of the sum rule (4.8) on the three-momentum $\vec{q}$ is rather weak. This also constrains the hadronic model. Again, we stress that this does not mean that the dependence of the spectral function on the three-momentum is weak.

Suppose now that one has a hadronic model at hand which yields at least the imaginary part $\mathrm{Im}\,\Pi^{\mathrm{res}}_{T,L}(s,\vec{q}^2)$ for the respective isospin channel at finite nuclear density. Examples can be found in Refs. [9–15,17]. Then, for given nuclear density and three-momentum $\vec{q}$ one can utilize the sum rule (4.11) and the results shown in Figs. 1 and 2 as a consistency check for the hadronic model in the following way (see also Ref. [31]).

(i) Choose a continuum threshold s_0 and a subtraction constant $\mathrm{Re}\,\Pi^{\mathrm{res}}_{T,L}(0,\vec{q}^2)$.

(ii) Calculate the limits of the Borel window according to Eqs. (4.13) and (4.15).

(iii) Calculate the relative deviation r of the RHS from the LHS of the sum rule (4.11), averaged over the Borel window, i.e., schematically

$$r=\int_{M^2_{\mathrm{min}}}^{M^2_{\mathrm{max}}} d(M^2)\,|1-\mathrm{RHS}/\mathrm{LHS}|/\Delta M^2 \tag{4.16}$$

with

$$\Delta M^2 = M^2_{\mathrm{max}} - M^2_{\mathrm{min}}. \tag{4.17}$$

The LHS function can be taken from Figs. 1 and 2.

(iv) Tune the ‘‘fit parameters’’ s_0 and $\mathrm{Re}\,\Pi^{\mathrm{res}}_{T,L}(0,\vec{q}^2)$ such that the deviation r becomes minimal.

If this optimal r is reasonably small and the Borel window not too small, one might conclude that the considered hadronic model is in agreement with the QCD sum rule for the chosen nuclear density and three-momentum $\vec{q}$.

We close this section with some remarks on the respective size of the ‘‘fit parameters’’ s_0 and $\mathrm{Re}\,\Pi^{\mathrm{res}}_{T,L}(0,\vec{q}^2)$. Clearly, the more fit parameters we have the less restrictive is the sum rule for the hadronic model which should be checked. At least, it is therefore important to get an idea about the size and the possible influence of the fit parameters.

In vacuum the continuum threshold s_0 turns out to be about $1-1.6$ GeV^2 [31,15,23]. At least it has to be below the exited states of ρ and ω. Model calculations suggest that the threshold decreases with increasing density [26,29,15,31].

Concerning the subtraction constant $\mathrm{Re}\,\Pi^{\mathrm{res}}_{T,L}(0,\vec{q}^2)$ it is important to note that within the Fermi gas approximation it can be rigorously calculated for vanishing three-momentum [27,15]. Here it turned out that it is so small that it would not change the results drastically, if it is simply neglected. Unfortunately, the expectation that it could be neglected also for

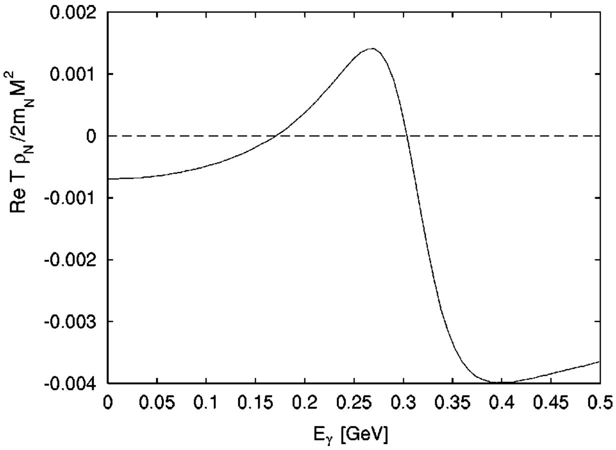

FIG. 3. Real part of isospin averaged γN forward scattering amplitude as a function of the photon energy in the rest frame of the nucleon [rescaled with $\rho_N/(2m_N M^2)$, see text for details].

finite three-momentum is presumably not justified. If we use the full electromagnetic current in Eq. (2.2) instead of a part of it with well-defined isospin, then within the Fermi gas approximation the transverse part of the subtraction constant would simply be the real part of the forward scattering amplitude $T(\vec{q},\vec{k})$ of a (real) photon with momentum $\vec{q}$ on a nucleon with momentum $\vec{k}$ averaged over the Fermi sphere, i.e.,

$$\mathrm{Re}\,\Pi_T^{\mathrm{em}}(0,\vec{q}^2)=4\int_{|\vec{k}|\le k_F}\frac{d^3k}{(2\pi)^3\,2E_k}\,\mathrm{Re}\,T(\vec{q},\vec{k})$$
$$\approx\frac{\rho_N}{2m_N}\,\mathrm{Re}\,T(\vec{q},0), \tag{4.18}$$

where we have used the linear density approximation (cf. next section) to obtain the last expression. It is reasonable to assume that $\mathrm{Re}\,\Pi_T^{\mathrm{res}}(0,\vec{q}^2)$ is of the same order of magnitude as $\mathrm{Re}\,\Pi_T^{\mathrm{em}}(0,\vec{q}^2)$. Thus, to get an idea about the size of the corresponding quantity in the sum rule (4.11) we have plotted in Fig. 3 the quantity

$$\frac{1}{M^2}\,\frac{\rho_N}{2m_N}\,\mathrm{Re}\,T(\vec{q},0) \tag{4.19}$$

as a function of the photon energy $E_\gamma=|\vec{q}|$ for nuclear saturation density and a typical value for the Borel mass, $M=1$ GeV. We have used the model for photoproduction presented in Ref. [44] to obtain the real part of the isospin averaged photon-nucleon forward scattering amplitude. Comparing the absolute sizes in Figs. 1 and 3 we find indeed that the subtraction constant might be negligible for $\vec{q}=0$, but not for arbitrary three-momentum. Especially, if we are interested in the dependence on the three-momentum we have to take the subtraction constant into account, since the variation of the curves in Fig. 1 with three-momentum is of the same order of magnitude as the quantity plotted in Fig. 3. Concerning the longitudinal part of the subtraction constant we cannot compare with photon-nucleon scattering, since there are no real longitudinal photons. Therefore, we refrain from presenting any estimates for this case.

Of course, the most fortunate situation would be if the considered hadronic model already provides a value for $\mathrm{Re}\,\Pi_{T,L}^{\mathrm{res}}(0,\vec{q}^2)$. Then, only the continuum threshold s_0 would remain as a fit parameter.

V. LINEAR DENSITY APPROXIMATION

Obviously, the contributions of the OPE presented in Sec. III are quite unillustrative, simply due to its complexity. To get more insight in the various contributions we restrict ourselves in this section to the parts which are at most linear in the nuclear density, i.e., cubic in the Fermi momentum. Recalling that we have evaluated all in-medium condensates using the Fermi gas approximation (3.7) we have neglected nucleon-nucleon correlations anyway which are quadratic in the density. Thus, the results presented in Sec. III are at best correct up to Fermi momentum to the power of 5, i.e., up to $o(\rho_N^{5/3})$. Therefore, we do not lose too much information, if we restrict ourselves here to the linear density approximation. To put it in physical terms, what we neglect further on is the Fermi motion of the nucleons. Anyway, for concrete calculations we can use the full results presented above. We note in passing that actually all the required integrals over the Fermi sphere can be calculated analytically. The results, however, are lengthy and unillustrative. Thus, for pedagogical reasons it is useful to discuss the linear density case

$$4\int_{|\vec{k}|\le k_F}\frac{d^3k}{(2\pi)^3\,2E_k}\,\langle N(\vec{k})|\mathcal{O}|N(\vec{k})\rangle$$
$$\to\frac{\rho_N}{2m_N}\langle N(0)|\mathcal{O}|N(0)\rangle. \tag{5.1}$$

In this case, the coefficients $c_{T,L}^n(\vec{q}^2)$ introduced in Eq. (3.59) are given by

$$c_{T,L}^n(\vec{q}^2)=c_{T,L}^{n,d=4}(\vec{q}^2)+c_{T,L}^{n,d=6}(\vec{q}^2) \tag{5.2}$$

with the contributions from the $d=4$ condensates to the transverse part

$$\begin{aligned}c_T^{n=2,d=4}=&\frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0+m_q\langle\bar{q}q\rangle_0\\&+\rho_N\left\{-\frac{m_N^{(0)}}{27}+\frac{\sigma_N}{2}+\frac{m_N}{4}\left[A_2^{u+d}\left(C_{2,2}^q-\frac{3}{2}C_{L,2}^q\right)\right.\right.\\&\left.\left.+\frac{2}{3}A_2^G\left(C_{2,2}^G-\frac{3}{2}C_{L,2}^G\right)\right]\right\},\end{aligned} \tag{5.3}$$

$$\begin{aligned}&c_T^{n=3,d=4}(\vec{q}^2)\\&=-\rho_N\vec{q}^2\,\frac{m_N}{2}\left[A_2^{u+d}(C_{2,2}^q-C_{L,2}^q)+\frac{2}{3}A_2^G(C_{2,2}^G-C_{L,2}^G)\right],\end{aligned} \tag{5.4}$$

and to the longitudinal part

$$c_L^{n=2,d=4}=c_T^{n=2,d=4}, \tag{5.5}$$

$$c_L^{n=3,d=4}(\vec{q}^2)=\rho_N\vec{q}^2\frac{m_N}{2}\left[A_2^{u+d}C_{L,2}^q+\frac{2}{3}A_2^G C_{L,2}^G\right], \tag{5.6}$$

and the corresponding contributions from the $d=6$ condensates,

$$\begin{aligned}c_T^{n=3,d=6}=&-\frac{112}{81}\kappa\alpha_s\pi\langle\bar{q}q\rangle_0^2+\rho_N\Bigg\{-\frac{112}{81}\frac{\sigma_N}{m_q}\kappa\alpha_s\pi\langle\bar{q}q\rangle_0\\&-\frac{5m_N^3}{12}\left[A_4^{u+d}\left(C_{2,4}^q-\frac{3}{2}C_{L,4}^q\right)\right.\\&\left.+\frac{2}{3}A_4^G\left(C_{2,4}^G-\frac{3}{2}C_{L,4}^G\right)\right]\\&+\frac{9m_N}{2}\left[c_1-c_2-\frac{1}{2}c_3-\frac{1}{18}(1\pm1)K_{ud}^1\right]\Bigg\},\end{aligned} \tag{5.7}$$

$$\begin{aligned}c_T^{n=4,d=6}(\vec{q}^2)=\rho_N\vec{q}^2\Bigg\{&\frac{9m_N^3}{4}\left[A_4^{u+d}\left(C_{2,4}^q-\frac{10}{9}C_{L,4}^q\right)\right.\\&\left.+\frac{2}{3}A_4^G\left(C_{2,4}^G-\frac{10}{9}C_{L,4}^G\right)\right]\\&-9m_N\left[c_1-c_2-\frac{1}{3}c_3-\frac{1}{18}(1\pm1)K_{ud}^1\right]\Bigg\},\end{aligned} \tag{5.8}$$

$$\begin{aligned}&c_T^{n=5,d=6}(\vec{q}^2)\\&=-2\rho_N\vec{q}^4m_N^3\left[A_4^{u+d}(C_{2,4}^q-C_{L,4}^q)+\frac{2}{3}A_4^G(C_{2,4}^G-C_{L,4}^G)\right],\end{aligned} \tag{5.9}$$

and

$$c_L^{n=3,d=6}=c_T^{n=3,d=6}, \tag{5.10}$$

$$\begin{aligned}&c_L^{n=4,d=6}(\vec{q}^2)\\&=\rho_N\vec{q}^2\Bigg\{\frac{m_N^3}{2}\left[A_4^{u+d}(C_{2,4}^q-5C_{L,4}^q)+\frac{2}{3}A_4^G(C_{2,4}^G-5C_{L,4}^G)\right]\\&\quad+3m_Nc_3\Bigg\},\end{aligned} \tag{5.11}$$

$$c_L^{n=5,d=6}(\vec{q}^2)=2\rho_N\vec{q}^4m_N^3\left[A_4^{u+d}C_{L,4}^q+\frac{2}{3}A_4^GC_{L,4}^G\right]. \tag{5.12}$$

All coefficients which are not given explicitly above vanish. As always, the $\pm$ sign which accompanies the K_{ud}^1 term corresponds to ρ and ω.

We immediately observe that the scalar condensates contribute only to the three-momentum independent coefficients $c_{T,L}^{n=2,d=4}$ and $c_{T,L}^{n=3,d=6}$. These coefficients are identical for transverse and longitudinal part, since at $\vec{q}=0$ we cannot distinguish between transverse and longitudinal directions [24].

Obviously, there are contributions from numerous condensates to each of the coefficients presented above. On inspection of the parameters given in Tables I and II we can work out which condensates dominate which coefficient. Neglecting less important condensates we find

$$c_T^{n=2,d=4}=c_L^{n=2,d=4}\approx\frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0+\rho_N\frac{m_N}{4}A_2^{u+d}, \tag{5.13}$$

$$c_T^{n=3,d=4}(\vec{q}^2)\approx-\rho_N\vec{q}^2\frac{m_N}{2}A_2^{u+d}, \tag{5.14}$$

$$|c_L^{n=3,d=4}(\vec{q}^2)|\ll|c_T^{n=3,d=4}(\vec{q}^2)|, \tag{5.15}$$

$$\begin{aligned}c_T^{n=3,d=6}=c_L^{n=3,d=6}\approx&-\frac{112}{81}\kappa\alpha_s\pi\langle\bar{q}q\rangle_0^2\\&-\rho_N\frac{112}{81}\frac{\sigma_N}{m_q}\kappa\alpha_s\pi\langle\bar{q}q\rangle_0,\end{aligned} \tag{5.16}$$

$$\begin{aligned}c_T^{n=4,d=6}(\vec{q}^2)\approx\rho_N\vec{q}^2\Bigg\{&\frac{9m_N^3}{4}A_4^{u+d}-9m_N\bigg[c_1-c_2-\frac{1}{3}c_3\\&-\frac{1}{18}(1\pm1)K_{ud}^1\bigg]\Bigg\},\end{aligned} \tag{5.17}$$

$$c_T^{n=5,d=6}(\vec{q}^2)\approx-2\rho_N\vec{q}^4m_N^3A_4^{u+d}, \tag{5.18}$$

$$c_L^{n=4,d=6}(\vec{q}^2)\approx\rho_N\vec{q}^2\left(\frac{m_N^3}{2}A_4^{u+d}+3m_Nc_3\right), \tag{5.19}$$

$$|c_L^{n=5,d=6}(\vec{q}^2)|\ll|c_T^{n=5,d=6}(\vec{q}^2)|. \tag{5.20}$$

Especially we have neglected all α_s corrections to the twist-2 condensates, i.e., we have approximated $C_{2,n}^q$ by 1 and neglected all other $C_{r,n}^i$ (see Ref. [36]). For vanishing three-momentum $\vec{q}$ the density dependent terms are dominated by the twist-2 spin-2 quark condensate and the scalar four-quark condensate. Concerning the $\vec{q}^2$-terms it is remarkable that the twist-4 condensates are equally important as the twist-2 spin-4 quark condensates. Of course, both are suppressed for large Borel masses as compared to the $(d=4)$ terms. Thus, for the transverse part the twist-2 spin-2 quark condensate governs the $\vec{q}^2$-terms. For the longitudinal direction the situation is more involved, since the $(d=4)$ coefficient given in Eq. (5.6) is quite small. Therefore, we have competing contributions from Eqs. (5.6) and (5.19).

We stress that in principle it is not necessary to perform the approximations which have led from Eqs. (5.3)–(5.12) to (5.13)–(5.20). Of course, one can use the exact expressions for the coefficients. The purpose here was only to figure out the condensates which have the most influence on the coefficients.

Since we have nonvanishing coefficients up to $n=5$ we find contributions up to $o(1/Q^{10})$. One may suspect that it is inconsistent to keep terms of order $o(1/Q^8)$ and higher, since we have neglected $(d=8)$ condensates which would contribute at $o(1/Q^8)$. However, this is misleading, since the dependence on the three-momentum $\vec{q}$ is different in both cases. Schematically the neglected higher order condensates would contribute as

$$\frac{(d=8) \text{ condensate}}{Q^8}\left(\#+\#\frac{\vec{q}^2}{Q^2}+\#\frac{\vec{q}^4}{Q^4}+\cdots\right), \quad (5.21)$$

where # denotes arbitrary dimensionless numbers which do not depend on q. Thus, e.g., the $\vec{q}^2$ terms of the neglected condensates are actually $o(1/Q^{10})$, while the corresponding terms of the condensates taken into account are $o(1/Q^8)$.

To work out the dependence of the current-current correlator on the three-momentum $\vec{q}$ more explicitly we study Eq. (4.8) in the vicinity of $\vec{q}=0$. For vanishing three-momentum we find

$$\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)+\frac{c^{n=2,d=4}}{M^4}+\frac{c^{n=3,d=6}}{2M^6} = \frac{\text{Re } \Pi^{\text{had}}(0,0)}{M^2} - \frac{1}{\pi M^2}\int_0^\infty ds \text{ Im } \Pi^{\text{had}}(s,0)\frac{s}{s^2+\epsilon^2}e^{-s/M^2}. \quad (5.22)$$

We have skipped the label T,L, since there are no distinct directions at vanishing three-momentum. Next we differentiate Eq. (4.8) with respect to $\vec{q}^2$ and put $\vec{q}=0$ afterwards. This yields

$$\frac{d_{T,L}^{n=3,d=4}}{2M^6}+\frac{d_{T,L}^{n=4,d=6}}{6M^8} = \frac{d}{d(\vec{q}^2)}\left(\frac{\text{Re } \Pi_{T,L}^{\text{had}}(0,\vec{q}^2)}{M^2} - \frac{1}{\pi M^2}\int_{-\vec{q}^2}^\infty ds \text{ Im } \Pi_{T,L}^{\text{had}}(s,\vec{q}^2)\frac{s}{s^2+\epsilon^2}e^{-s/M^2}\right)\Bigg|_{\vec{q}=0} \quad (5.23)$$

with

$$d_{T,L}^{n=3,d=4}=\frac{c_{T,L}^{n=3,d=4}(\vec{q}^2)}{\vec{q}^2}, \quad d_{T,L}^{n=4,d=6}=\frac{c_{T,L}^{n=4,d=6}(\vec{q}^2)}{\vec{q}^2}. \quad (5.24)$$

In the same way we find by differentiating twice

$$\frac{d_{T,L}^{n=5,d=6}}{24M^{10}} = \frac{d^2}{d(\vec{q}^2)^2}\left(\frac{\text{Re } \Pi_{T,L}^{\text{had}}(0,\vec{q}^2)}{M^2} - \frac{1}{\pi M^2}\int_{-\vec{q}^2}^\infty ds \text{ Im } \Pi_{T,L}^{\text{had}}(s,\vec{q}^2)\frac{s}{s^2+\epsilon^2}e^{-s/M^2}\right)\Bigg|_{\vec{q}=0} \quad (5.25)$$

with

$$d_{T,L}^{n=5,d=6}=\frac{c_{T,L}^{n=5,d=6}(\vec{q}^2)}{\vec{q}^4}. \quad (5.26)$$

Let us discuss now the range of validity for the new sum rules (5.23) and (5.25). As pointed out in Sec. III it is crucial to find a non-vanishing Borel window where the validity of the sum rule is guaranteed inside of this window. To find the lower limit of this window we have compared the contribution of the highest order condensates to the LHS of Eq. (4.8) with the total result [see Eq. (4.13)]. In Eq. (5.22) we have four different orders in $1/M^2$, namely zeroth to third order.[5] Thus, it is no problem to compare the third order contribution to the total result. In Eq. (5.23) we are left with third and fourth order in $1/M^2$, only. Thus, the number of orders we have access on is already diminished. This leads to a lower limit of the Borel window of $M^2_{\text{min}}\approx 3\text{ GeV}^2$ for the transverse component of sum rule (5.23) which is already much higher than the one for sum rule (5.22): $M^2_{\text{min}}\approx 0.6\text{ GeV}^2$. For the longitudinal component of sum rule (5.23) we even find $M^2_{\text{min}}\approx 10\text{ GeV}^2$. As already discussed above the $\vec{q}^2$ part of the $(d=4)$ contribution to the longitudinal part (5.6) is quite small. Therefore, only for very large values of the Borel mass the $(d=4)$ contribution can overwhelm the $(d=6)$ contribution. Both for the longitudinal and the transverse part we find that the respective lower limit of the Borel window for sum rule (5.23) is much higher than the one for Eq. (5.22). If the upper limit of the Borel window does not rise in the same way, the sum rule (5.23) would not be as useful as Eq. (5.22). To determine the upper limit of the Borel window we would need, of course, a hadronic model. For the most simple case, the approximation of the spectral function by a δ function, it was found in Ref. [30] that the upper limit of the Borel window also rises. Thus, also the sum rule (5.23) might be useful as a consistency check for hadronic models. We note, however, that the definition of the Borel window in Ref. [30] differs from ours. For Eq. (5.25) the situation is even worse. There we have only access on one order in $1/M^2$. Thus, we cannot determine a lower limit for the Borel window. The sum rule (5.25) is therefore useless.

We stress again that the approximations performed in this section are not mandatory. The purpose of these approximations was to obtain more qualitative insight in the importance of the various contributions and in the dependence on the three-momentum $\vec{q}$. To check the consistency of a hadronic model with the QCD sum rule (4.11) the OPE coefficients should be deduced from the formulas presented in Sec. III. Only if the hadronic model is also restricted to the linear density case, would a direct comparison with the simplified expressions be appropriate.

[5]Note that the first order term vanishes. Strictly speaking it is proportional to the light current quark mass squared which is negligibly small.

VI. DISCUSSION OF THE ASSUMPTIONS

In this section we will discuss the various assumptions that have led to the condensate contributions calculated in Sec. III. The basic assumption we have made for the evaluation of in-medium condensates is the Fermi gas approximation (3.7). Clearly, this approximation is only valid for not too high densities. Of course, it would be of interest to quantify this statement. Unfortunately this is hard to do within the OPE approach, since it is not clear how to calculate expectation values with respect to multinucleon states for arbitrary nuclear densities. An idea about the importance of multinucleon states might be obtained from a comparison of the moments of parton distributions (3.26), (3.27) as deduced from DIS experiments with nucleons on the one hand side and with nuclei on the other. This might be a direction of future studies. Concerning the parton distributions in nuclei we refer to Ref. [45], and references therein.

Within the framework of a hadronic model it is tedious but possible to approximately take into account interactions of the vector meson with more than one nucleon [9–14,17]. In Ref. [17] it was found that already at nuclear saturation density it is important to account for such processes. On the other hand, presumably in every hadronic model one can distinguish between single and multinucleon interactions. Therefore, it should be possible to compare the OPE calculation with the hadronic model treated in a ''single nucleon mode.'' This comparison was, e.g., performed in Ref. [15] for vanishing three-momentum $\vec{q}$. If in the framework of a hadronic model it turns out that interactions with more than one nucleon are important for the nuclear density under consideration, then the QCD sum rule cannot serve to check the consistency of the whole hadronic model but only of its restriction to scattering processes of the vector meson with one nucleon from the Fermi sphere.

After these general considerations about the validity of the calculation of in-medium condensates we turn now to the discussion of the accuracy in the determination of the different types of condensates. Concerning the scalar condensates of Sec. III A we have neglected there terms which are quadratic in the light current quark masses m_q as well as possible differences in the condensates of up and down quarks. In view of the fact that the light current quark masses are about 6 MeV, while the Borel window for the QCD sum rule (4.8) starts at about $M^2_{\rm min}\approx 0.6\ {\rm GeV}^2$ (see Figs. 1,2), the neglect of m_q^2 terms is very well justified. Also, a possible difference between up and down quark condensate is presumably small and anyway hard to disentangle in view of the uncertainties in the determination of the (average) light quark condensate (see, e.g., discussions in Refs. [31,23]). In addition, the contribution from the two-quark condensate to Eq. (3.6) is much smaller than the one from the gluon condensate. The largest uncertainty lies in the evaluation of the four-quark condensate, i.e., in the value for κ. First, even the vacuum value is still under discussion. Second, it might very well be that the value for κ varies with nuclear density. Concerning the first problem, it is useful to choose a hadronic model which describes the data for $e^+e^-\rightarrow$hadrons reasonably well and utilize the sum rule (4.11) for the vacuum case to determine κ. This was performed in Ref. [15] and we therefore have taken the condensate values given there. For the second problem we have no solution to offer so far. Without any better knowledge we use the vacuum value for κ also at finite density. To get rid of this uncertainty one can differentiate the sum rule (4.8) with respect to the three-momentum squared $\vec{q}^{\,2}$. In this way, all scalar condensates drop out, since they do not yield $\vec{q}^{\,2}$-dependent contributions to the OPE side of the sum rule. As already discussed in Sec. V this differentiated sum rule results in a lower limit for the Borel window which is much higher than the one for the original sum rule. This might diminish or even close the Borel window, i.e., the thus obtained sum rule might be less reliable or even useless. This clearly depends on the explicit hadronic model under consideration.

A second possibility to deal with the uncertainty in the density dependence of the four-quark condensate would be to use κ as a free ''fit parameter'' in the same way as the ''fit parameters'' s_0 and ${\rm Re}\,\Pi^{\rm res}_{T,L}(0,\vec{q}^{\,2})$ as discussed after Eq. (4.17). In contrast to the latter, κ does not depend on the three-momentum $\vec{q}$. Therefore, once κ is determined from a fit of the OPE side to the hadronic model under consideration, e.g., at $\vec{q}=0$, then κ is fixed for arbitrary $\vec{q}$. Nonetheless, one might be afraid that one now has too many ''fit parameters'' so that effectively there remains no constraint on the hadronic model under consideration. Fortunately the situation is not that bad. To see this one can count the number of orders in $1/M^2$ one has on the OPE side of a sum rule. This can be seen as the number of constraints.[6] If the number of ''fit parameters'' is larger than the number of constraints one cannot learn anything from the sum rule. If the situation is the other way around one gets constraints on the hadronic model under consideration. To be more specific let us discuss as an example Eq. (5.22). For ${\rm Im}\,\Pi^{\rm had}$ we use the decomposition (4.10). On the LHS of Eq. (5.22) we find four orders in $1/M^2$. However, the size of the purely perturbative contribution, i.e., the constant term, has already been used to determine the high-energy behavior in Eq. (4.10). Therefore, we have three constraints from the remaining three powers in $1/M^2$. This has to be compared with the number of ''fit parameters.'' As already mentioned in Sec. IV for vanishing three-momentum $\vec{q}$ the quantity ${\rm Re}\,\Pi^{\rm had}_{T,L}(0,\vec{q}^{\,2})$ can be calculated rigorously within the Fermi gas approximation [27,15]. Thus, even if one uses s_0 and κ as ''fit parameters'' one is still in the situation that the number of constraints (3) is higher than the number of ''fit parameters'' (2). In addition, if a hadronic model is suggested to be reasonable by this sum rule analysis it might yield a prediction for the (possibly density dependent) value of κ. This, of course, is an interesting perspective.

The twist-2 condensates discussed in Secs. III B, III C are the best known contributions, as soon as one accepts the Fermi gas approximation discussed above. We even can use the results of DIS to get an idea about the neglected higher order condensates (see below).

The twist-4 spin-2 condensates can in principle also be deduced from DIS data. The uncertainties in their extraction are, however, quite large. We have adopted the analysis of

[6] A similar analysis was performed in Ref. [27].

Ref. [41]. There, condensates depending on the light current quark masses, strange quark contributions, and dependences on the renormalization scale were neglected. These errors are presumably smaller than the uncertainties in the extraction of these condensates from DIS data. In general, the contributions of the twist-4 spin-2 condensates are small as compared to the twist-2 spin-2 contributions. For the $\vec{q}^{\,2}$-contributions to the longitudinal direction, however, the twist-2 spin-2 contribution is suppressed by α_s. There, the twist-4 spin-2 condensates cannot be disregarded [30].

Of course, one is forced to somewhere cut off the OPE used here as an expansion in the dimensionality of the condensates. We have neglected all condensates of dimensionality 8 or higher. To get an idea about the size of the neglected condensates we calculate now the twist-2 spin-6 contribution to the current-current correlator, since we have access on that quantity utilizing the DIS results. The calculation proceeds along the lines described in Secs. III B, III C. We use the Fermi gas approximation

$$\langle \mathcal{O}_{\mu\nu\kappa\lambda\xi\chi}\rangle \approx 4\int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3\, 2E_k}\, \langle N(\vec{k})|\mathcal{O}_{\mu\nu\kappa\lambda\xi\chi}|N(\vec{k})\rangle \tag{6.1}$$

and the decomposition

$$\begin{aligned}\langle N(\vec{k})|\mathcal{O}_{\mu\nu\kappa\lambda\xi\chi}|N(\vec{k})\rangle &\sim k_\mu k_\nu k_\kappa k_\lambda k_\xi k_\chi \\ &\quad -\frac{1}{12}(k_\mu k_\nu k_\kappa k_\lambda g_{\xi\chi} m_N^2 + 14 \text{ permutations}) \\ &\quad +\frac{1}{120}(k_\mu k_\nu g_{\kappa\lambda} g_{\xi\chi} m_N^4 + 44 \text{ permutations}) \\ &\quad -\frac{1}{960}(g_{\mu\nu} g_{\kappa\lambda} g_{\xi\chi} m_N^6 + 14 \text{ permutations}) \\ &=: S_{\mu\nu\kappa\lambda\xi\chi}(k).\end{aligned} \tag{6.2}$$

The relevant operators are

$$\mathcal{ST}\, i(\bar{u}\gamma_\mu D_\nu D_\kappa D_\lambda D_\xi D_\chi u + \bar{d}\gamma_\mu D_\nu D_\kappa D_\lambda D_\xi D_\chi d) \tag{6.3}$$

and

$$\mathcal{ST}\, G^\alpha_\mu D_\nu D_\kappa D_\lambda D_\xi G_{\alpha\chi}. \tag{6.4}$$

From the DIS calculations we can deduce the following contribution to the current-current correlator (2.1) which was neglected in Eq. (3.3):

$$\Pi^{d=8,\tau=2}_{\mu\nu}(q) = \int_{|\vec{k}|\le k_F} \frac{d^3k}{(2\pi)^3\, 2E_k}\, T^{(u+d)s=6,\tau=2}_{\mu\nu}(q,k) \tag{6.5}$$

with the forward scattering amplitude

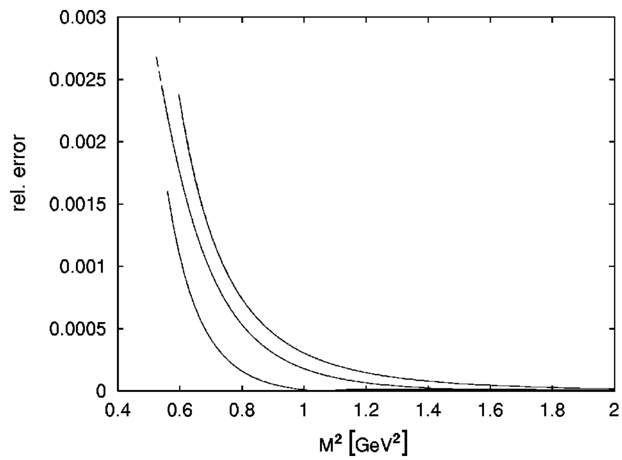

FIG. 4. Relative error made by the neglect of twist-2 spin-6 condensates in the calculation of the transverse part of the LHS of Eq. (4.8) as a function of the Borel mass squared M^2 for three-momenta $|\vec{q}|=0$ (upper line), 0.5 (lower line), 1 GeV (middle line), and for ρ (full lines) and ω mesons (dashed lines). The only noticeable difference between ρ and ω meson appears in the slightly different lower limits of the Borel window, i.e., in the starting points of the curves on the left-hand side.

$$\begin{aligned}&T^{(u+d)s=6,\tau=2}_{\mu\nu}(q,k)\\ &\quad= -\frac{64}{q^{14}}(q_\mu q_\nu q^\alpha q^\beta - g_\mu{}^\alpha q^2 q_\nu q^\beta - g_\nu{}^\alpha q^2 q_\mu q^\beta \\ &\qquad + g_\mu{}^\alpha g_\nu{}^\beta q^4) q^\gamma q^\delta q^\epsilon q^\zeta S_{\alpha\beta\gamma\delta\epsilon\zeta}(k)\\ &\qquad \times\left(C^q_{2,6}A_4^{u+d} + \frac{2}{3}C^G_{2,6}A^G_4\right)\\ &\qquad -\frac{64}{q^{14}}(q_\mu q_\nu - g_{\mu\nu}q^2) q^\alpha q^\beta q^\gamma q^\delta q^\epsilon q^\zeta S_{\alpha\beta\gamma\delta\epsilon\zeta}(k)\\ &\qquad \times\left((C^q_{L,6}-C^q_{2,6})A_4^{u+d} + \frac{2}{3}(C^G_{L,6}-C^G_{2,6})A^G_4\right).\end{aligned} \tag{6.6}$$

With this at hand we can calculate the ratio between the twist-2 spin-6 contribution to the LHS of the sum rule (4.8) and the total value for this LHS as calculated in Sec. III. We

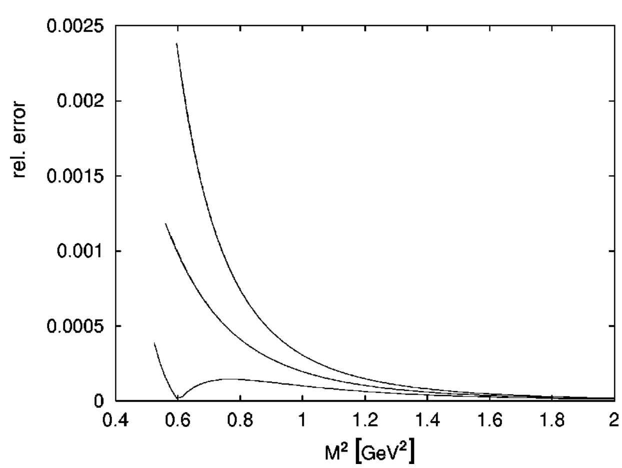

FIG. 5. Same as Fig. 4 for longitudinal part. The curves refer to $|\vec{q}|=0$, 0.5, 1 GeV (top to bottom).

have calculated that ratio in Figs. 4,5 for transverse and longitudinal directions and for ρ and ω mesons. Obviously, the obtained ratios are very small justifying at least the neglect of twist-2 spin-6 condensates and suggesting that all higher-dimensional condensates are reasonably suppressed. Of course, for a thorough discussion of the error made by neglecting higher-dimensional condensates we also would need to know the other condensates beside the twist-2 spin-6 condensates. Since we do not know, e.g., the scalar ($d=8$) condensates, etc., the error analysis presented here is only a first guess.

As long as we do not know the actual values of the neglected condensates (or at least an upper limit for them) we cannot present a rigorous proof that the OPE approach works, i.e., that the truncated series yields a reliable value for the current-current correlator in the region of interest. Indeed, in Ref. [16] it was doubted that the QCD sum rule approach provides any reliable information about the medium modifications of vector mesons (see also Refs. [46,47]). Two arguments were given there to support these doubts: The first qualitative argument concerns the connection between the mass shift of a vector meson and the forward scattering amplitude of this vector meson with a nucleon. It was argued there that the forward scattering amplitude and hence also the mass shift is a long distance property, while the OPE is only capable of short distance properties. We think that this argument is misleading, since the OPE always concerns the interplay of long and short distance properties, as already pointed out in Sec. III. Actually, in the same oversimplifying way one might argue just the other way round: The in-medium ρ mass is still large, i.e., a short distance property, and therefore can be described by the OPE approach. This shows that one needs more quantitative arguments to check the validity of the sum rule approach. A useful self-consistency check within the Borel sum rule method is the Borel stability analysis described in Sec. IV: A breakdown of the sum rule might be observed in a small or even vanishing Borel window. Indeed, this stability analysis was the key point to resolve the question, whether mass shift and/or forward scattering amplitude can be extracted within the traditional sum rule approach utilizing the narrow width approximation. We refer to Refs. [48,27,29,49] for details. In general, the Borel window can only be determined after specifying a hadronic model. Therefore, we do not discuss this point here any further. The preceding discussion has clearly emphasized the necessity to perform a Borel stability analysis.

A second, formal argument has been raised in Ref. [16] against the applicability of the OPE approach to vector mesons in nuclear medium: It was claimed there that the OPE turns out to be an expansion in the nucleon mass m_N over the invariant mass $\sqrt{Q^2}$. After Borelization this would turn into an expansion in m_N over the Borel mass M. If the latter is assumed to be of the order of the ρ meson mass, one would get an expansion parameter m_N/m_ρ which is obviously not small. Therefore, it was argued in Ref. [16] that the truncation of the OPE at the $d=6$ condensates is not appropriate. Indeed, concerning the twist-2 condensates the statement is true that the used OPE is an expansion in m_N/M. This can most easily be discussed within the linear density approximation of Sec. V. For example, for vanishing three-momentum the twist-2 spin-2 condensates contribute with a term proportional to $\rho_N m_N/M^4$ [see Eq. (5.3)] and the twist-2 spin-4 condensates with a term proportional to $\rho_N m_N^3/M^6$ [see Eq. (5.7)]. In general, the twist-2 spin-s condensates yield a contribution $\sim\rho_N m_N^{s-1}/M^{s+2}$. Since the nucleon mass m_N is large this expansion might break down for the Borel masses of interest (typically of the order of 1 GeV). However, one should not discuss the convergence of a series without looking at its coefficients. The twist-2 spin-s contribution is accompanied by the sth moment of the parton distributions. Inspecting the last two lines of Table II we find that these moments become very small with increasing s. Indeed, we have already discussed above that the twist-2 spin-6 condensates do not spoil the truncation of the OPE, in spite of the fact that they are proportional to $\rho_N m_N^5/M^8$. This shows that also the second argument raised in Ref. [16] against the QCD sum rule approach is oversimplified.

VII. SUMMARY

This work was motivated by the finding that the QCD sum rule approach provides no model-independent prediction about a possible mass shift of vector mesons in nuclear medium [15,31]. In Ref. [31] we discussed at length that the sum rule restricts the ρ meson only to a rather wide area in the (mass, width) plane without making any further statements about the specific properties of the ρ meson. Only within additional model assumptions can the behavior of the ρ meson in nuclear matter be further specified. For example, if one *assumes* that the width of the ρ meson is not increased, then the sum rule predicts a ρ mass which decreases with increasing nuclear density. However, it is also possible to *assume* instead that the ρ mass is not shifted. In this case the sum rule suggests an increasing width of the spectral function of the ρ meson.

This, however, does not mean that the sum rule approach is useless: We have presented here a QCD sum rule for ρ and ω mesons propagating with arbitrary three-momentum through nuclear matter at vanishing temperature. This sum rule provides a nontrivial consistency check for hadronic models which describe that propagation. At least as long as different hadronic models cannot be judged unambiguously by experiments such consistency checks are important to confirm or rule out hadronic models.

The main formula was given in Eq. (4.11). The OPE coefficients $c^n_{T,L}$ which appear on the LHS of this formula are defined via $\Pi^{\rm OPE}_{T,L}$ in Eq. (3.59). In view of their complexity we have not given the explicit formulas for $c^n_{T,L}$. However, they can be easily deduced in the following way from the equations presented in Sec. III: The transverse and longitudinal parts of $\Pi^{\rm OPE}_{T,L}$ are obtained from the respective last expressions of Eqs. (2.7) and (2.8). The current-current correlator with the full Lorentz structure $\Pi^{\rm OPE}_{\mu\nu}$ is decomposed in Eq. (3.3).

The scalar contribution is given in Eq. (3.4) where $R^{\rm scalar}$ can be read off from Eq. (3.6). The expectation values showing up there are decomposed in vacuum and medium-dependent expectation values in Eq. (3.9) using the Fermi gas approximation. The vacuum expectation values are listed in Table I. Finally, the medium-dependent parts of the scalar condensates are connected in Eqs. (3.10), (3.11) with parameters also listed in Table I.

The contribution from the twist-2 spin-2 condensates is given in Eqs. (3.28), (3.29) with the traceless tensor $S_{\alpha\beta}$ defined in Eq. (3.13). In the same way the contribution from the twist-2 spin-4 condensates is given in Eqs. (3.34), (3.35) with the traceless tensor $S_{\alpha\beta\gamma\delta}$ defined in Eq. (3.31). The required values for the moments of the parton distributions A_n^i and the coefficients $C_{r,n}^i$ are collected in Table II.

The contribution from the twist-4 spin-2 condensates is given in Eqs. (3.42), (3.44) where the coefficients $\tilde{B}_{1,2}$ are connected in Eqs. (3.57), (3.58) to quantities listed in Table I.

In this way, the OPE coefficients can be easily calculated. As discussed in the last section the QCD sum rule (4.11) can be used to check the consistency of a hadronic model, provided that in the latter the medium is also described by the Fermi gas approximation. Going one step further by neglecting the Fermi motion of the nucleons both in the hadronic model under consideration and in the calculation of the OPE coefficients one might also utilize the sum rule in this linear density approximation. For this case the OPE coefficients $c_{T,L}^n$ are explicitly given in Eqs. (5.2)–(5.12).

By inspecting the QCD sum rule (4.11) we observe that the hadronic model which should be checked has to yield the current-current correlator for invariant masses in the region $(-\vec{q}^2)$ to s_0. The lower limit refers to vanishing energy. For nonvanishing three-momentum $\vec{q}$ this means that we need information not only about the timelike region, but also about the spacelike region. For small three-momenta the spacelike region is dominated by the coupling of the respective vector meson to nucleon-hole states [27]. For higher three-momenta also resonance-hole loops come into play in the spacelike region (see Fig. 3 in Ref. [17]). Thus, at finite nuclear density there are important structures in the spectral function of the vector mesons also in the spacelike region. This is in contrast to the vacuum case where there is no structure below the two-pion (three-pion) threshold for the ρ (ω) meson.

Qualitatively, we have found that our sum rules are not very sensitive to the difference between ρ and ω meson as well as to a variation in the three-momentum of the vector meson with respect to the nuclear medium. This, however, does not *a priori* mean that the current-current correlator for different isospin channels and for different three-momenta should be more or less the same. In the sum rule only an integral over this correlator enters which might be the same, e.g., for ρ and ω mesons, even if the respective correlators themselves are different. Thus, on this qualitative level the sum rule approach does not rule out hadronic models which predict a different behavior of vector mesons with different three-momenta, such as, e.g., Refs. [13,14,16–18]. A quantitative analysis of these models is beyond the scope of this paper.

We believe that the QCD sum rule presented here provides an interesting and nontrivial consistency check for hadronic models which describe vector mesons in nuclear matter. We have tried to present the derivation of the sum rule in great detail to make it possible for nonexperts in OPE to utilize the sum rule for a consistency check of their hadronic models also.

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