Scattering of dressed nucleons in nuclear matter

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The scattering of dressed nucleons in nuclear matter is studied. By casting the conventional asymptotic analysis of scattering in free space in the language of the two-body propagator, it becomes possible to develop modifications of this analysis due to the dressing of the scattering nucleons in the medium. While the scattering energy singles out a unique (on-shell) momentum characterizing the relative wave function of free or mean-field nucleons, this uniqueness is no longer maintained for dressed nucleons. The resulting distribution of momenta in the relative wave function leads to a localization in coordinate space of the influence of the scattering process which can be expressed as a healing of the wave function to the noninteracting one. An analytic approximation to the noninteracting propagator of the dressed nucleons is utilized to illustrate these points. The localization of the scattered wave implies that the particles no longer ''remember'' their scattering event beyond some finite distance. This feature suggests that the strict notion of a cross section in the medium is a tenuous concept. Approximate expressions are developed to characterize the strength of the interaction in the medium in terms of phase shifts and cross sections to facilitate comparisons with results of calculations involving mean-field nucleons. [S0556-2813(98)03811-4]

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I. INTRODUCTION

Renewed interest in the study of the interaction between nucleons in the nuclear medium has been generated by the recent experimental developments involving the $(e,e'pp)$ reaction [1,2]. The potential selectivity of this reaction for the removal of $^1S_0$ proton pairs to certain final states and the absence of large contributions from two-body currents to these transitions may allow the study of the interaction between protons in the medium at short relative distances [3]. The continuing experimental study of heavy-ion reactions at the Fermi energy relies on analyses based on transport models which contain as input the cross section of nucleons scattering in the nuclear medium. Recent theoretical work in determining these cross sections involves the scattering of mean-field (mf) nucleons in nuclear matter. Some recent issues that have emerged from this work include the enhancement of the cross section at finite temperature due to the vicinity of a pairing instability [4], the sensitivity of the cross section to the choice of the single-particle (sp) spectrum at zero temperature [5], and the density and energy dependence [6] and temperature dependence of the cross sections [7].

From the careful analysis of the $(e,e'p)$ reaction in recent years, a picture has emerged which clearly invalidates one of the assumptions of the theoretical papers which determine in-medium cross sections. This is the assumption that the scattering process in the medium takes place between nucleons which at most have a sp spectrum different from free space but are otherwise unaffected by the presence of other nucleons except for the Pauli principle. The assumption that all the sp strength is concentrated at a mf sp energy is neither borne out by experiments on nuclei [8] or by many-body calculations of the spectral function of nuclear matter [9,10]. It is unclear *a priori* whether the dressing of nucleons will have a substantial effect on the resulting cross sections. Indeed, there is no proper framework available to analyze the asymptotic behavior of the scattering wave function of dressed particles in the medium. Indeed, it may not even be possible to develop a suitable definition of a cross section in the medium. These issues are explored in the present paper.

The conceptual understanding of strongly interacting nucleons yielding a mf shell-model (Fermi-gas) picture has relied heavily on the concept of the healing of the relative wave function to the noninteracting one as discussed in Refs. [11,12]. Experimental evidence based on the $(e,e'p)$ reaction [8] has demonstrated that nucleon sp motion must be understood in terms of Landau's quasiparticle description [13]. In turn, this requires a substantial modification of the simple shell-model or Fermi-gas picture. The conventional Bethe-Goldstone propagator used to determine the effective interaction in the medium is not sufficient to generate a nucleon self-energy which realistically describes the sp strength distribution below the Fermi energy both in nuclear matter and finite nuclei. Inclusion of additional terms involving hole-hole propagation as in a Galitski-Feynman propagator is necessary to achieve a realistic spectral function [14,15]. Inclusion of hole-hole propagation destroys the healing property of the relative function since it produces a nonvanishing phase shift for energies below twice the Fermi energy [16]. The resolution of this puzzle requires consideration of the consequences of the dressing of the nucleons for the description of the scattering process in the medium. The study of the self-consistent dressing of nucleons under the influence of short-range correlations (SRC) includes the propagation of dressed particles in a ladder-diagram summation for the two-body interaction or propagator [17]. The present work is intended to provide a framework to interpret the results of such a ladder-diagram calculation of the two-body propagator which employs fully dressed sp propagators. Some preliminary and incomplete results have been discussed in Refs. [17–20].

In Sec. II the relevant results for the two-body propagator are collected which are subsequently used in Sec. III for the description of the asymptotic analysis of the scattering process of free particles, mf particles in the medium, and dressed particles. Although some results are standard, they are usually not presented in the language of the two-body propagator. By employing this description for a standard problem it becomes clear which features of this description are essentially altered by the dressing of the nucleons. The discussion includes an asymptotic analysis based on an analytic approximation of the noninteracting propagator of dressed particles. This analytic approximation is particularly helpful in understanding the required changes for the physical interpretation of the scattering process in the medium. A simple example of the scattering by a hard core is exploited to demonstrate that phase shifts necessarily become complex for dressed particles even when their interaction does not include inelastic processes. The consequences for the concept of a cross section in the medium are also explored and a set of useful expressions is introduced to characterize the scattering event of dressed particles which allows a comparison with results for mf particles. A summary and conclusions are given in Sec. IV.

II. GENERAL RESULTS FOR THE TWO-BODY PROPAGATOR

The study of scattering in the medium by means of an asymptotic analysis in coordinate space requires knowledge of the two-body propagator. For practical reasons it is appropriate to calculate the effective interaction in a partial-wave momentum representation. The present paper does not deal with an actual solution of the integral equation for the effective interaction. It will be assumed that this interaction can be obtained from a numerical calculation. The main objective of the present work is to study the consequences of propagating dressed particles for the description of the scattering process. All subsequent discussion will be based on the solution of a Lippmann-Schwinger-type scattering equation for the effective interaction which is equivalent to summing the ladder diagrams for a particular choice of noninteracting two-body propagator and two-body interaction. In order to clarify the difference between the conventional discussion of scattering in free space and the one necessary for the medium, it will be useful to cast the description completely in the language of the two-body propagator. While this is certainly not necessary for the scattering of free particles, it will provide a simple way to clarify the changes that are required to extend the description to dressed particles in the medium.

For the purpose of the present work it is sufficient to consider the two-time two-particle propagator

$$g^{\mathrm{II}}(\boldsymbol{k}_1\boldsymbol{k}_2;\boldsymbol{k}_3\boldsymbol{k}_4;t_1-t_2) = -i\langle\Psi_0^A|T\{a_{\boldsymbol{k}_2}(t_1)a_{\boldsymbol{k}_1}(t_1)a^\dagger_{\boldsymbol{k}_3}(t_2)a^\dagger_{\boldsymbol{k}_4}(t_2)\}|\Psi_0^A\rangle, \tag{1}$$

given here in the momentum representation, while spin and isospin indices are suppressed. Although this propagator depends on the conserved total momentum $\boldsymbol{K}=\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_3+\boldsymbol{k}_4$ in the medium, it is more convenient to concentrate on the case $|\boldsymbol{K}|=0$ in order to facilitate comparison with the scattering of particles in free space which exhibits no such dependence. Extension of the present work to deal with the case of nonzero total momentum requires no new steps and will not be considered here. The remaining momentum dependence of the propagator can now be associated with the relative momentum of the pair of removal operators in Eq. (1), given by $\boldsymbol{k}=\frac{1}{2}[\boldsymbol{k}_1-(-\boldsymbol{k}_1)]$, and of the pair of addition operators given by $\boldsymbol{k}'=\frac{1}{2}[\boldsymbol{k}_3-(-\boldsymbol{k}_3)]$. Since only ladder diagrams will be considered while employing a static bare nucleon-nucleon interaction, the corresponding integral equation for the propagator can be written as

$$\begin{aligned} g^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega) &= g_f^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega)+g_f^{\mathrm{II}}(\boldsymbol{k};\Omega) \\ &\quad\times\int d^3q\langle\boldsymbol{k}|V|\boldsymbol{q}\rangle g^{\mathrm{II}}(\boldsymbol{q},\boldsymbol{k}';\Omega) \\ &= g_f^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega)+g_f^{\mathrm{II}}(\boldsymbol{k};\Omega) \\ &\quad\times\langle\boldsymbol{k}|\Gamma(\Omega)|\boldsymbol{k}'\rangle g_f^{\mathrm{II}}(\boldsymbol{k}';\Omega), \end{aligned} \tag{2}$$

where

$$g_f^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega) = \delta(\boldsymbol{k}-\boldsymbol{k}')g_f^{\mathrm{II}}(\boldsymbol{k};\Omega) \tag{3}$$

is the noninteracting two-particle propagator which both in homogeneous matter and free space conserves the relative momentum as expressed by the δ function in Eq. (3). The presence of exchange terms in Eqs. (2) and (3) is hereby acknowledged but suppressed in the presentation. The second equality in Eq. (2) links the two-particle propagator with the vertex function or effective interaction Γ which contains the summation of all ladder diagrams. This result is particularly useful for the asymptotic analysis to be explored below.

It is important to realize that the usual results from scattering theory are obtained in the coordinate representation. The relevant double Fourier transform of the two-particle propagator is given by

$$g^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';\Omega) = \frac{1}{(2\pi)^3}\int d^3k\int d^3k'\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}}g^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega)e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}'}. \tag{4}$$

The transform of the noninteracting propagator only involves one integration due to the presence of the δ function in Eq. (3):

$$\begin{aligned} g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';\Omega) &= \frac{1}{(2\pi)^3}\int d^3k\int d^3k'\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}}g_f^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega)e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}'} \\ &= \frac{1}{(2\pi)^3}\int d^3k\, e^{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}g_f^{\mathrm{II}}(\boldsymbol{k};\Omega). \end{aligned} \tag{5}$$

The result for Eq. (2) can then be transformed to yield

$$\begin{aligned} g^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';\Omega) &= g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';\Omega)+\int d^3r_1\int d^3r_2\, g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}_1;\Omega) \\ &\quad\times\langle\boldsymbol{r}_1|V|\boldsymbol{r}_2\rangle g^{\mathrm{II}}(\boldsymbol{r}_2,\boldsymbol{r}';\Omega) \\ &= g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';\Omega)+\int d^3r_1\int d^3r_2\, g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}_1;\Omega) \\ &\quad\times\langle\boldsymbol{r}_1|\Gamma(\Omega)|\boldsymbol{r}_2\rangle g_f^{\mathrm{II}}(\boldsymbol{r}_2,\boldsymbol{r}';\Omega). \end{aligned} \tag{6}$$

With this equation one can arrive at an asymptotic analysis and resulting definition of the cross section which is equivalent to a standard analysis involving the equation for the wave function in the case of scattering in free space or mf particles in the medium. Before this result is developed, it is useful to summarize the propagator equations in a partial-wave representation. These allow the introduction of the phase shift which contains the relevant information to obtain the asymptotic propagator or wave function in this representation.

A partial-wave decomposition of the two-body propagator in Eq. (2) yields the corresponding integral equation and the relation between the propagator and the vertex function:

$$\begin{aligned} g^{\mathrm{II}}_{JST}(kl,k'l';\Omega) &= \frac{\delta(k-k')}{k^2}\delta_{l,l'}g^{\mathrm{II}}_f(k;\Omega)+g^{\mathrm{II}}_f(k;\Omega) \\ &\quad\times\sum_{l''}\int dq\, q^2\langle kl|V^{JST}|ql''\rangle g^{\mathrm{II}}(ql'',k'l';\Omega) \\ &= \frac{\delta(k-k')}{k^2}\delta_{l,l'}g^{\mathrm{II}}_f(k;\Omega)+g^{\mathrm{II}}_f(k;\Omega) \\ &\quad\times\langle kl|\Gamma^{JST}(\Omega)|k'l'\rangle g^{\mathrm{II}}_f(k';\Omega). \end{aligned} \tag{7}$$

The appropriate notation for a partial-wave basis has been introduced in Eq. (7) in terms of l, S, J, and T representing orbital, total spin, total angular momentum, and isospin, while k and k' denote relative momentum quantum numbers. As before, only the case of zero total momentum is considered here without loss of generality. The energy Ω is conserved and must be viewed as a variable upon which the propagator depends (it also depends on the total momentum in the case of the medium). The noninteracting propagator is again denoted by g^{II}_f and may include the dressing of the individual particles when the scattering takes place in matter. The vertex function or effective interaction Γ can be obtained from the numerical solution of the ladder equation in a partial-wave momentum representation

$$\begin{aligned} \langle kl|\Gamma^{JST}(\Omega)|k'l'\rangle &= \langle kl|V^{JST}|k'l'\rangle \\ &\quad+\sum_{l''}\int_0^\infty dq\, q^2\langle kl|V^{JST}|ql''\rangle g^{\mathrm{II}}_f(q;\Omega) \\ &\quad\times\langle ql''|\Gamma^{JST}(\Omega)|k'l'\rangle. \end{aligned} \tag{8}$$

This equation has only recently been solved using fully dressed sp propagators in the medium [17,21].

The coordinate space version of Eq. (7) is obtained by a double Fourier-Bessel transform

$$\begin{aligned} g^{\mathrm{II}}_{JST}(rl,r'l';\Omega) &= \frac{2}{\pi}\int_0^\infty dk\,k^2\int_0^\infty dk'\,k'^2 j_l(kr)j_{l'}(k'r')g^{\mathrm{II}}_{JST}(kl,k'l';\Omega). \end{aligned} \tag{9}$$

The corresponding result for the noninteracting part of the propagator, represented by the first term in Eq. (7), reduces to one integral on account of the delta function which conserves relative momentum:

$$g^{\mathrm{II}}_{f,l}(r,r';\Omega)=\frac{2}{\pi}\int_0^\infty dk\,k^2 j_l(kr)j_l(kr')g^{\mathrm{II}}_f(k;\Omega). \tag{10}$$

The Fourier-Bessel transform of Eq. (7) has the form

$$\begin{aligned} g^{\mathrm{II}}_{JST}(rl,r'l';\Omega) &= \delta_{l,l'}g^{\mathrm{II}}_{f,l}(r,r';\Omega) \\ &\quad+\sum_{l''}\int_0^\infty dr_1 r_1^2\int_0^\infty dr_2 r_2^2 g^{\mathrm{II}}_{f,l}(r,r_1;\Omega) \\ &\quad\times\langle r_1 l|V^{JST}|r_2 l''\rangle g^{\mathrm{II}}_{JST}(r_2 l'',r'l';\Omega) \\ &= \delta_{l,l'}g^{\mathrm{II}}_{f,l}(r,r';\Omega) \\ &\quad+\int_0^\infty dr_1 r_1^2\int_0^\infty dr_2 r_2^2 g^{\mathrm{II}}_{f,l}(r,r_1;\Omega) \\ &\quad\times\langle r_1 l|\Gamma^{JST}(\Omega)|r_2 l'\rangle g^{\mathrm{II}}_{f,l'}(r_2,r';\Omega). \end{aligned} \tag{11}$$

When the bare two-body interaction V is local in the relative coordinate, only one integral in the first equality remains. The second equality can be used to study the asymptotic behavior of the propagator outside the range of the interaction.

III. PROPAGATOR DESCRIPTION OF SCATTERING

A. Scattering of free particles

The subsequent discussion for the scattering of dressed particles in the medium requires the consideration of the two-body propagator in the medium. For this reason it is advantageous to collect the conventional results for scattering in free space using this language. In the case of free particles the noninteracting propagator in momentum space is given by

$$g^{\mathrm{II}}_f(k;\Omega)=\frac{1}{\Omega-\hbar^2k^2/m+i\eta}. \tag{12}$$

Defining the on-shell momentum by

$$\Omega=\frac{\hbar^2k_0^2}{m}, \tag{13}$$

one can perform the relevant Fourier-Bessel transform of the noninteracting propagator in Eq. (10) analytically (see, e.g., [22]) with the well-known result

$$g^{\mathrm{II}}_{f,l}(r,r';k_0)=-ik_0\frac{m}{\hbar^2}j_l(k_0r_<)h_l(k_0r_>). \tag{14}$$

The coordinate argument in the spherical Hankel function must be the larger of r and r' and is denoted by $r_>$ while the argument of the spherical Bessel function is the smaller and denoted by $r_<$. For the current analysis it will be assumed that the interaction has a finite range, $\langle r|V|r'\rangle=0$ for r,r'

larger than some r_0. Substituting Eq. (14) in the second part of Eq. (11) in the case of an uncoupled channel for $r'>r$ and $r'>r_0$ yields

$$g^{\rm II}_{lJST}(r,r';k_0)=-ik_0\frac{m}{\hbar^2}j_l(k_0r)h_l(k_0r')+\int_0^\infty dr_1r_1^2\int_0^\infty dr_2r_2^2g^{\rm II}_{f,l}(r,r_1;k_0)\times\langle r_1|T^{JST}_l(k_0)|r_2\rangle\times\left(-ik_0\frac{m}{\hbar^2}\right)j_l(k_0r_2)h_l(k_0r')=-ik_0\frac{m}{\hbar^2}\psi^{JST}_l(r;k_0)h_l(k_0r'),\tag{15}$$

where

$$\psi^{JST}_l(r;k_0)=j_l(k_0r)+\int_0^\infty dr_1r_1^2\int_0^\infty dr_2r_2^2g^{\rm II}_{f,l}(r,r_1;k_0)\times\langle r_1|T^{JST}_l(k_0)|r_2\rangle j_l(k_0r_2),\tag{16}$$

and the conventional notation T instead of Γ has been introduced together with the replacement of Ω by k_0. This result demonstrates that under the given conditions the propagator separates as a product of a function r and a function of r'. This result can be substituted into the first part of Eq. (11) to obtain the relevant integral equation for the wave function ψ (under the condition that $r'>r_0$):

$$\psi^{JST}_l(r;k_0)=j_l(k_0r)+\int_0^\infty dr_1r_1^2\int_0^\infty dr_2r_2^2g^{\rm II}_{f,l}(r,r_1;k_0)\times\langle r_1|V^{JST}_l|r_2\rangle\psi^{JST}_l(r_2;k_0),\tag{17}$$

which can be found in standard textbooks (see, e.g., [22] for the case of a local potential). It is derived here to demonstrate the relation between the propagator and the wave function since for the case of dressed particles one has to start with the formulation in terms of propagators.

The asymptotic analysis of the propagator can be performed by using Eq. (14) in Eq. (11) under the assumption that the propagator will be considered for $r<r'$ while both these coordinates are larger than r_0, the range of the interaction. Values of r_1 and r_2 in Eq. (11) larger than r_0 yield no contributions to the integral. As a result, the effective interaction T has a range similar to the one of the bare interaction V. Using the relation between spherical Bessel and Hankel functions,

$$j_l(\rho)=\frac{1}{2}[h_l(\rho)+h^*_l(\rho)],\tag{18}$$

one obtains the asymptotic behavior of the propagator for the case of an uncoupled partial wave channel from the second part of Eq. (11) in the form

$$g^{\rm II}_{l,JST}(r,r';k_0)\rightarrow-i\left(\frac{m}{2\hbar^2}\right)k_0h_l(k_0r')\times\left\{h^*_l(k_0r)+h_l(k_0r)\left[1-2i\frac{m}{\hbar^2}k_0\int_0^\infty dr_1r_1^2\int_0^\infty dr_2r_2^2\langle r_1|T^{JST}_l(k_0)|r_2\rangle j_l(k_0r_1)j_l(k_0r_2)\right]\right\}=-i\frac{m}{2\hbar^2}k_0h_l(k_0r')\left\{h^*_l(k_0r)+h_l(k_0r)\left[1-2\pi i\left(\frac{mk_0}{2\hbar^2}\right)\langle k_0|T^{JST}_l(k_0)|k_0\rangle\right]\right\}.\tag{19}$$

In the last step of Eq. (19) one can return to the on-shell matrix element of the T matrix in momentum space which completely determines the outcome of the scattering process. The term in square brackets corresponds to the S-matrix element in terms of which one can define the phase shift

$$\langle k_0|S^{JST}_l(k_0)|k_0\rangle=\left[1-2\pi i\left(\frac{mk_0}{2\hbar^2}\right)\langle k_0|T^{JST}_l(k_0)|k_0\rangle\right]\equiv e^{2i\delta^{JST}_l}.\tag{20}$$

This result can be represented by

$$\tan\,\delta^{JST}_l=\frac{{\rm Im}\langle k_0|T^{JST}_l(k_0)|k_0\rangle}{{\rm Re}\langle k_0|T^{JST}_l(k_0)|k_0\rangle},\tag{21}$$

which explicitly shows that a nonzero imaginary part of the effective interaction is required to obtain a nonvanishing phase shift. In turn, this imaginary part of the interaction only appears for energies where the noninteracting propagator has a nonvanishing imaginary part. For the scattering of free particles this corresponds to all positive energies. By substituting the explicit form of the spherical Hankel functions for $l=0$ in Eq. (19) one can construct the asymptotic propagator for the $^1S_0$ channel explicitly:

$$g^{\rm II}_{l,{}^1S_0}(r,r';k_0)\rightarrow-i\frac{m}{2k_0\hbar^2}\frac{1}{rr'}e^{i(k_0r'+\delta_{{}^1S_0})}\sin(k_0r+\delta_{{}^1S_0}).\tag{22}$$

The standard result for the asymptotic wave function is contained in this equation and the imaginary part of Eq. (22) is simply the product of these wave functions as a function of r and r', respectively.

To obtain the relation between the cross section and the propagator it is necessary to return to Eq. (6) and perform the Fourier transform of the noninteracting propagator [Eq. (12)]

in Eq. (5). This Fourier transform is given by the well-known result (again replacing the energy Ω by the on-shell momentum k_0)

$$g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';k_0) = -\frac{m}{4\pi\hbar^2}\frac{e^{ik_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}. \tag{23}$$

A similar procedure as used for the asymptotic analysis in the partial wave basis can be employed to obtain the corresponding result for Eq. (6). Whereas in the former analysis the separable form of the noninteracting propagator in Eq. (14) is valid without constraint on r and r', this is not the case here since Eq. (23) only becomes separable in the case $r' \gg r$ or vice versa. In the former case one can write Eq. (23) as

$$g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';k_0) \to -\frac{m}{4\pi\hbar^2}\frac{e^{ik_0 r'}}{r'}e^{-ik_0\hat{\boldsymbol{r}}'\cdot\boldsymbol{r}}. \tag{24}$$

Substituting this result in the second part of Eq. (6) for both $r' \gg r$ and $r' \gg r_2$ demonstrates that g^{II} is separable and can be written as

$$g^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}';k_0) = -\frac{m}{4\pi\hbar^2}\frac{e^{ik_0 r'}}{r'}\psi(\boldsymbol{r};k_0) \tag{25}$$

in the asymptotic domain. By substituting this result in turn in Eq. (6) one obtains the standard integral equation for the wave function and the appropriate formulation for the asymptotic wave function to obtain the scattering amplitude:

$$\begin{aligned}\psi(\boldsymbol{r};k_0) &= e^{-ik_0\hat{\boldsymbol{r}}'\cdot\boldsymbol{r}} + \int d^3r_1 \int d^3r_2\, g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}_1;k_0) \\ &\quad\times \langle \boldsymbol{r}_1|V|\boldsymbol{r}_2\rangle \psi(\boldsymbol{r}_2;k_0) \\ &= e^{-ik_0\hat{\boldsymbol{r}}'\cdot\boldsymbol{r}} + \int d^3r_1 \int d^3r_2\, g_f^{\mathrm{II}}(\boldsymbol{r},\boldsymbol{r}_1;k_0) \\ &\quad\times \langle \boldsymbol{r}_1|T(k_0)|\boldsymbol{r}_2\rangle e^{-ik_0\hat{\boldsymbol{r}}'\cdot\boldsymbol{r}_2}.\end{aligned} \tag{26}$$

One may identify the origin of the motion in the direction of the negative z axis, meaning that $\hat{\boldsymbol{r}}'$ points in that direction, so that $\boldsymbol{k} \equiv -k_0\hat{\boldsymbol{r}}'$ points into the positive z direction. If one assumes that r is also much larger than the range of the potential and, therefore much larger than any contributing value of r_1, one can use Eq. (24) again in the second part of Eq. (26) to identify the coefficient multiplying the outgoing spherical wave $e^{ik_0 r}/r$ as the scattering amplitude (while double Fourier transforming the T-matrix element back to momentum space):

$$f_{k_0}(\theta,\phi) = -\frac{2m\pi^2}{\hbar^2}\langle \boldsymbol{k}'|T(k_0)|\boldsymbol{k}\rangle, \tag{27}$$

where θ,ϕ denote the angles associated with the direction of $\hat{\boldsymbol{r}}$ and $\boldsymbol{k}' \equiv k_0\hat{\boldsymbol{r}}$ corresponds to the momentum of the detected motion in the direction $\hat{\boldsymbol{r}}$ with the same absolute value k_0 as the initial state. The differential cross section for the direction (θ,ϕ) is then simply the square of the scattering amplitude as given by Eq. (27):

$$\frac{d\sigma}{d\Omega} = |f_{k_0}(\theta,\phi)|^2. \tag{28}$$

The present formulation is closely tailored to the conventional experimental situation where a collimated beam propagates along the z axis characterized by a given energy or momentum toward a target situated at the origin. Detection then takes place in a particular direction away from the origin characterized by angles θ and ϕ. The only difference is that the present formulation is appropriate for the corresponding center-of-mass system ($|\boldsymbol{K}|=0$).

B. Scattering of mean-field particles in the medium

To obtain the phase shifts and cross sections for particles propagating in the medium with mf sp energies one can proceed in similar fashion. A useful reference is the work of Bishop *et al.* [16] where the introduction of the phase shift for the case of hole-hole propagation is discussed. The corresponding mf propagator in the medium, also known as the Galitski-Feynman propagator, is given in momentum space by

$$g_{\mathrm{mf}}^{\mathrm{II}}(k;\Omega) = \frac{\theta(k-k_F)}{\Omega - 2\epsilon(k) + i\eta} - \frac{\theta(k_F-k)}{\Omega - 2\epsilon(k) - i\eta}, \tag{29}$$

where, without essential loss of generality, the case of zero center-of-mass momentum is considered. The sp energy $\epsilon(k)$ can deviate from the simple kinetic energy spectrum and therefore yield a different relation between the energy Ω and the on-shell momentum k_0:

$$\Omega \equiv 2\epsilon(k_0). \tag{30}$$

Nevertheless, the uniqueness of k_0 for a given energy is still preserved. Although one can no longer evaluate the noninteracting propagator in coordinate space completely analytically from Eq. (10), the separability of the propagator is maintained for the contribution of the pole term as in Eq. (14) (with a different constant prefactor), while the remaining term vanishes asymptotically for r sufficiently different from r'. A discussion of a similar result for the Fourier transform of the mf propagator given in Eq. (29) can be found in Ref. [23] for the Bethe-Goldstone propagator. As a result, one preserves the integral equations for the wave function either in a partial wave basis as in Eq. (16) or for the wave function in coordinate space as in Eq. (26) in the case of mf propagators. The only difference with the free scattering case involves the use of the mf equivalents of the noninteracting propagators in coordinate space in Eqs. (17) and (26). This result is due to the uniqueness of the on-shell momentum at a given energy which guarantees that the noninteracting wave function is a plane wave or spherical Bessel function (in a partial-wave basis). One can therefore proceed with a similar asymptotic analysis as for free particles, yielding a corresponding definition of the phase shifts as in Eq. (20) in terms of the on-shell scattering matrix. The result of Eq. (21) also remains valid in this case. The presence of a nonvanishing phase shift therefore continues to be linked to the nonvanishing of the imaginary part of the noninteracting propagator. In the case of mf scattering the corresponding energy domain resides above $2\epsilon(k=0)$ which corresponds to the lowest energy of two-hole states. As in the case of noninteracting particles, the presence of bound states has specific consequences for the behavior of the phase shift at the corresponding thresholds in the energy variable [16]. While in free space this threshold corresponds to zero energy and the presence of a bound state is reflected in the phase shift going to π when the scattering energy goes to zero, the corresponding threshold in the medium is $2\epsilon_F$. If the interaction is sufficiently attractive, the phase shift may approach π on both sides of $2\epsilon_F$. This feature is intimately related to the presence of a pairing instability in normal Fermi systems with attractive effective interactions on the Fermi surface. The phase shift can also approach $-\pi$ when a bound state below the hole-hole continuum [i.e., below $2\epsilon(k=0)$] appears due to a repulsive interaction. This possibility is realized in liquid ^{3}He at sufficiently high density for mf particles [16]. If the interaction is not sufficiently attractive to yield pairing, the phase shift close to twice the Fermi energy will vanish. Finally, the result for the scattering amplitude is also preserved in the form of Eq. (27), yielding corresponding results for the cross section [Eq. (28)].

All these modest modifications of the quantities that characterize the scattering process, as compared to the case of free-particle scattering, are related to the continued one-to-one relation of the energy with a unique relative momentum for which the noninteracting propagator has an imaginary part. This on-shell momentum emerges as the momentum that characterizes the plane-wave (or spherical Bessel function) function describing the relative motion. The plane-wave character of the wave function allows for a conventional interpretation of the scattering process as in free space.

Since the correlated wave function does not heal to an uncorrelated one when hole-hole propagation is included, the usual discussion of healing must be modified. The standard interpretation of the validity of the shell model is couched in terms of the healing of the relative wave function to the noninteracting one. This interpretation was put forward in Ref. [11] and is based on the use of the Bethe-Goldstone propagator in describing the effective interaction. Since this propagator excludes the propagation of two holes in Eq. (29), the correction to the relative wave function in Eq. (17) due to the strong interaction potential heals within a characteristic healing distance to the spherical Bessel function provided that the energy is less than twice the Fermi energy. In scattering language this simply states that there is no phase shift for energies less than $2\epsilon_F$ when the Bethe-Goldstone propagator is employed since no corresponding imaginary part of the propagator exists [12].

An apparent contradiction arises with this interpretation when one realizes that it is not permitted to neglect the propagation of the hole-hole term in Eq. (29) since it is essential for the understanding of the fragmentation of the sp strength below the Fermi energy [24,25]. Inclusion of hole-hole propagation yields a nonvanishing phase shift below $2\epsilon_F$ [16] which is at odds with the healing interpretation of the relative wave function. On the other hand, this contribution to the effective interaction is responsible for the presence of an imaginary part of the nucleon self-energy which is required in order to describe the experimental situation in nuclei as obtained from the $(e,e'p)$ reaction [8]. Evidently it is not possible to propagate mf nucleons which can generate a realistic sp strength distribution and, at the same time, obtain the healing of the relative wave function to the noninteracting one which supposedly underlies the success of the mf picture.

While recent $(e,e'p)$ experiments have sharpened the range of the validity of the sp picture in terms of the more appropriate Landau quasiparticle description [13] which is adequately described by microscopic theory [14,15], the paradox at the level of the relative wave function remains. A clue to the solution of this puzzle is provided by noting the inconsistency of the description of the sp strength in terms of a realistic spectral function and the construction of the effective interaction by means of a mf propagator. Clearly, if the dressing effect of the nucleon is substantial—and experiment [8] indicates it is—then one is forced to consider the construction of the effective interaction in terms of dressed nucleons. The consequences of this extension for the description of the scattering process in matter and the resolution of the healing paradox will be taken up in the following.

C. Scattering of dressed particles in the medium

The propagation of dressed nucleons requires a different treatment of the description of the scattering process. The main ingredient for this change is the form of the noninteracting propagator which is given in the medium by

$$g_f^{\mathrm{II}}(\boldsymbol{k},\boldsymbol{k}';\Omega)=\int_{\epsilon_F}^{\infty}d\omega\int_{\epsilon_F}^{\infty}d\omega'\,\frac{S_p(\boldsymbol{k},\omega)S_p(\boldsymbol{k}',\omega')}{\Omega-\omega-\omega'+i\eta}-\int_{-\infty}^{\epsilon_F}d\omega\int_{-\infty}^{\epsilon_F}d\omega'\,\frac{S_h(\boldsymbol{k},\omega)S_h(\boldsymbol{k}',\omega')}{\Omega-\omega-\omega'-i\eta}. \tag{31}$$

The particle and hole spectral functions S_p and S_h, respectively, describe the distribution of the sp strength for a given momentum over the energy. These distributions are continuous and have sizable peaks either above or below the Fermi energy, corresponding to a momentum state above or below k_F, at the so-called quasiparticle energy. For $k_F=1.36\ \mathrm{fm}^{-1}$, corresponding to normal density, the strength contained in the peak for momenta close to k_F is typically only 70% [9,10]. From the rest of the strength about 10% is found below the Fermi energy, another 10% in the first 100 MeV above the Fermi energy, and the remaining 10% is spread thinly towards even higher energy as a result of the short-range and tensor correlations in the nuclear interaction [9]. First attempts to incorporate these features in the solution of the ladder equation have been explored in Refs. [17–20]. A more complete presentation is in preparation [21]. The details of such a calculation are not pursued here; instead, the consequences for the interpretation of the scattering process in the medium for such medium-modified particles are studied.

It should be noted that the noninteracting propagator in Eq. (31) becomes the familiar mf Galitski-Feynman propagator [see Eq. (29)] when mf spectral functions are inserted which are characterized by a δ-function peak of strength 1 at a sp energy either above the Fermi energy ($k>k_F$) or below ($k<k_F$). The difference between the Galitski-Feynman propagator and the dressed propagator is qualitatively differ-

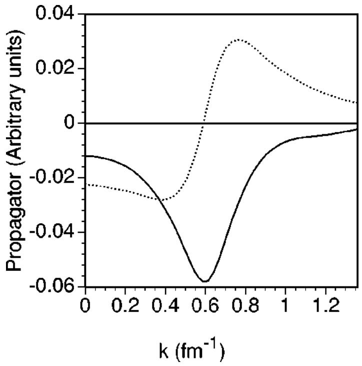

FIG. 1. Real (dotted line) and imaginary part (solid line) of the noninteracting two-particle propagator of dressed particles at an energy corresponding to an on-shell momentum of 0.5 fm^{-1} for the mf propagator.

ent for the imaginary part and quantitatively for the real part. These features are demonstrated in Figs. 1 and 2. In Fig. 1 both the real and imaginary parts of the dressed propagator [Eq. (31)] are shown as a function of the relative momentum for zero total momentum while the energy corresponds to an on-shell momentum $k_0 = 0.5$ fm^{-1} in the Galitski-Feynman case using a sp spectrum from Ref. [24]. In the latter case the imaginary part of the propagator contains a δ function only at one momentum corresponding to 0.5 fm^{-1}. It arises on account of the vanishing of the denominator in the hole-hole term at the on-shell momentum. The solid curve in Fig. 1 corresponds to the imaginary part of the dressed propagator shown here for momenta up to k_F. The position of the peak in the imaginary part of this propagator deviates slightly from 0.5 fm^{-1} (see Ref. [21] for a more detailed discussion) but, more importantly, there is a substantial spreading in the imaginary part containing even small high-momentum components (not shown in Fig. 1). This spreading is a critical feature which completely alters the conventional picture of the scattering process. In Fig. 2 the real parts of the dressed and mf propagator are compared for momenta below k_F for an energy corresponding to an on-shell momentum of 0.8 fm^{-1}. The dotted line corresponds to the mf propagator and also indicates the pole present at 0.8 fm^{-1}. The dressed propagator shows a less dramatic momentum dependence and is in general substantially reduced from the mf propagator except for high momenta where both coincide [21].

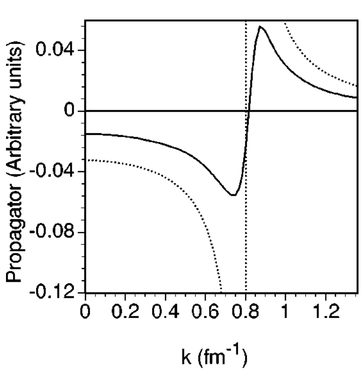

FIG. 2. Comparison of the real part of the mf (dotted line) and dressed (solid line) two-particle propagator at an on-shell momentum of 0.8 fm^{-1} indicated by the vertical dotted line.

In order to perform the analysis of the scattering process it will be illustrative to use an analytical approximation to the noninteracting propagator of dressed particles [Eq. (31)] which contains the essential new features. In addition, only the case of zero total momentum of the propagating pair will be considered in the following. As a result, the noninteracting propagator contains equal and opposite momenta for the two particles (holes). Since the spectral functions do not depend on the direction of the momentum, one can rewrite Eq. (31) for the present purposes as

$$g_f^{\mathrm{II}}(k,\Omega)=\int_{\epsilon_F}^{\infty}d\omega\int_{\epsilon_F}^{\infty}d\omega'\,\frac{S_p(k,\omega)S_p(k,\omega')}{\Omega-\omega-\omega'+i\eta}-\int_{-\infty}^{\epsilon_F}d\omega\int_{-\infty}^{\epsilon_F}d\omega'\,\frac{S_h(k,\omega)S_h(k,\omega')}{\Omega-\omega-\omega'-i\eta}. \tag{32}$$

The momentum k not only corresponds to the absolute value of the sp momenta but also represents the relative momentum for the case of zero total momentum.

Introducing a two-body self-energy term for purely practical reasons, one can attempt to write this noninteracting propagator as

$$g_f^{\mathrm{II}}(k,\Omega)=\frac{\pm 1}{\Omega-\Sigma^{\mathrm{II}}(k,\Omega)}, \tag{33}$$

where the sign is determined by whether the energy Ω is above $(+)$ or below $(-)$ $2\epsilon_F$. By assuming that this *ad hoc* self-energy Σ^{II} has a slowly varying imaginary part as a function of the relative momentum k one can expand the self-energy at the momentum k_0 for which

$$\Omega\equiv\mathrm{Re}\,\Sigma^{\mathrm{II}}(k_0,\Omega). \tag{34}$$

Noting that the expansion is in the square of the momentum one obtains a complex pole approximation (CPA) to the propagator by only keeping the real and imaginary part of Σ^{II} at k_0^2 and the first derivative of the real part. The resulting propagator has the form

$$g_{f,\mathrm{CPA}}^{\mathrm{II}}(k,\Omega)=\frac{m}{\hbar^2}\,\frac{\pm c}{k_0^2-k^2\pm i\gamma}, \tag{35}$$

where the constant c is obtained from

$$c=\frac{\hbar^2}{m}\left(\frac{\partial\ \mathrm{Re}\,\Sigma^{\mathrm{II}}}{\partial k^2}\bigg|_{k_0^2}\right)^{-1} \tag{36}$$

and γ from

$$\gamma=|\mathrm{Im}\,\Sigma^{\mathrm{II}}(k_0,\Omega)|\left(\frac{\partial\ \mathrm{Re}\,\Sigma^{\mathrm{II}}}{\partial k^2}\bigg|_{k_0^2}\right)^{-1}. \tag{37}$$

Typical values of c at low energies correspond to 0.5 whereas it rises slowly to 1 for higher energies. This feature is closely related to the pattern of the distribution of the sp strength. The quasiparticle pole strength at $k_F = 1.36$ fm^{-1} is about 0.7, and so for a two-particle propagator close to these energies a factor of $(0.7)^2$ is expected. For higher momenta

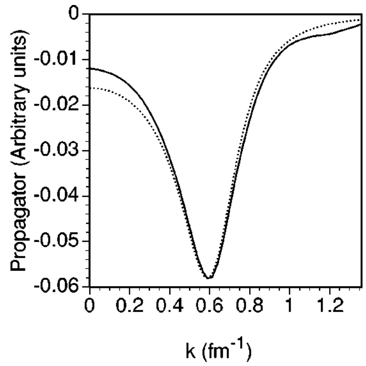

FIG. 3. Comparison for the imaginary part of the noninteracting two-particle propagator of dressed nucleons between the complete (numerical) result given by the solid line and the simple CPA [see Eq. (35)] given by the dotted line for momenta below k_F. This result is obtained for an energy below $2\epsilon_F$ corresponding to an on-shell momentum of 0.5 fm^{-1} in the mf case.

the strength in the peak grows back to 1, yielding a propagator which is more of the mf or even free-particle kind. It is also apparent that this factor of about 0.5 can be identified from Fig. 2. It should be noted that the CPA is obtained after first numerically calculating the noninteracting propagator of the dressed particles [21]. In Fig. 3 the quality of this CPA to the propagator can be judged by comparing it to the numerically calculated result for the imaginary part of the propagator at an energy below $2\epsilon_F$ corresponding to an on-shell momentum of 0.5 fm^{-1} in the mf case. Also for the real part of the propagator one obtains a very satisfactory description for momenta below k_F as shown in Fig. 4.

The CPA result for the propagator cannot be used to solve any of the integral equations [Eq. (8) for the effective interaction, for example] since it is only a good approximation to the noninteracting propagator close to the peak of the imaginary part. The full solution of Eq. (8) also requires an accurate representation of the high-momentum components of the propagator in order to properly include the effect of short-range correlations in the interaction or wave function. The CPA result does provide a reasonable representation of the long-range part of the propagator and therefore can be profitably used to discuss the asymptotic analysis of the scattering process. Indeed, using the CPA to the dressed propagator one can repeat the steps involved in the Fourier-Bessel transform leading to Eq. (14). For free or mf particles the integral in Eq. (10) yields the product of a spherical Bessel function and one of the spherical Hankel functions with as argument the real on-shell momentum k_0 [see Eqs. (13) and (30)]. This on-shell momentum is real since the corresponding noninteracting propagators [Eqs. (12) and (29)] can only have a vanishing denominator for a real momentum. Since Eq. (10) can be calculated by a contour integral for Eqs. (12) and (29) (at least for the long-range part), as well as for Eq. (35), it is clear that the presence of a nonvanishing imaginary part for the pole of Eq. (35), due to the nonvanishing of γ, will lead to a complex on-shell momentum which will be denoted by κ_0. Using the CPA [Eq. (35)] for $\Omega<2\epsilon_F$ one obtains, from Eq. (10),

$$g^{\mathrm{II}}_{l,\mathrm{CPA}}(r,r';\Omega)=-ic\frac{m}{\hbar^2}\kappa_0 j_l(\kappa_0 r_<)h_l^*(\kappa_0 r_>). \quad (38)$$

The momentum argument of the spherical Bessel and Hankel functions, κ_0, is now complex; its real and imaginary parts κ_0^R and κ_0^I, respectively, are easily obtained from k_0 and γ [see Eqs. (34) and (37)] by determining the zeros of the denominator of Eq. (35). Equation (38) contains the Hankel function h_l^* due to the different boundary condition associated with hole-hole propagation for energies below $2\epsilon_F$. This leads to a pole in the upper half of the complex k plane in contrast to the case of particle-particle or free-particle scattering. As a result, κ_0^I is negative for $\Omega<2\epsilon_F$ and its magnitude can become as large as 0.2–0.3 fm^{-1} [21]. The resulting propagator for $l=0$ can be written as (for $r<r'$)

$$g^{\mathrm{II}}_{l=0,\mathrm{CPA}}(r,r';\Omega)=\frac{-icm}{2\hbar^2(\kappa_0^R+i\kappa_0^I)}\left(\frac{e^{i\kappa_0^R r}e^{-\kappa_0^I r}}{r}-\frac{e^{-i\kappa_0^R r}e^{\kappa_0^I r}}{r}\right)\frac{e^{-i\kappa_0^R r'}e^{\kappa_0^I r'}}{r'}. \quad (39)$$

A comparison between this analytical result and the numerical Fourier-Bessel transform of the dressed noninteracting propagator which it approximates is shown in Fig. 5 for the imaginary part. For fixed r', corresponding to the location of the maximum in Fig. 5, both propagators are shown as a function of r without the factor $1/rr'$. While confirming the validity of the CPA result, Fig. 5 also demonstrates that the propagator for dressed particles is radically different from the noninteracting or mf one [see Eq. (14)] due to the presence of damping terms related to the nonzero value of κ_0^I. As noted before, there is no longer a unique on-shell momentum. Indeed, the complex pole at κ_0 in the CPA propagator is just a simple (and approximate) representation of this feature. As a consequence, the relative wave function of the dressed particles contains a spread in momentum states. This, in turn, must yield a localization of the corresponding wave function in coordinate space. This is exhibited in the propagator Eq. (39) which represents the probability amplitude for removing a pair with relative distance r' and adding the pair after propagation at r (without interaction between

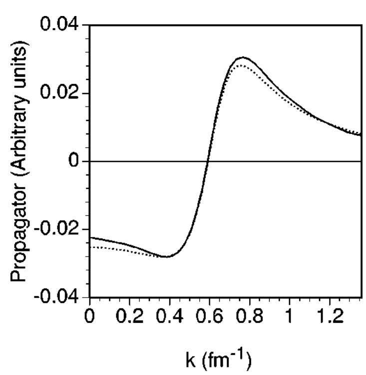

FIG. 4. Comparison for the real part of the noninteracting two-particle propagator of dressed nucleons between the complete (numerical) result given by the solid line and the simple CPA [see Eq. (35)] given by the dotted line for momenta below k_F. This result is obtained for an energy below $2\epsilon_F$ corresponding to an on-shell momentum of 0.5 fm^{-1} in the mf case.

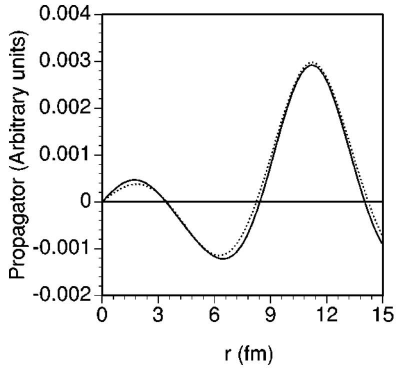

FIG. 5. Comparison between the analytical approximation and the complete numerical result for the dressed noninteracting two-particle propagator in coordinate space [Eq. (10)] for a value of r' for which both propagators have a maximum (around 11 fm). Indeed, for $r=r'$ the damping is least effective [see Eq. (39)]. Shown is the imaginary part for an energy below $2\epsilon_F$ (also used in Fig. 3) for the CPA propagator (dotted line) and the complete result (solid line). The corresponding propagators in momentum space are shown in Fig. 3.

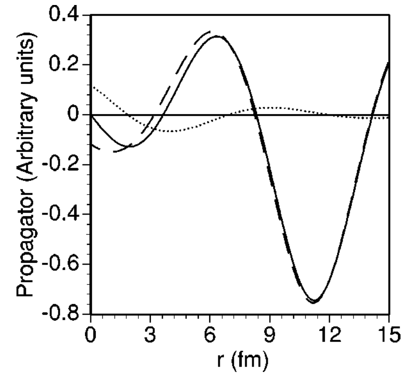

FIG. 6. Decomposition of the two-particle CPA propagator in coordinate space into ingoing (dashed line) and outgoing wave (dotted line) for the same value of r' and energy as in Fig. 5. Also shown is the sum of both contributions (solid line). The result for the total differs from the corresponding result in Fig. 5 by an overall negative constant. The ingoing and outgoing waves correspond to the two terms in Eq. (38).

the pair included yet). This amplitude peaks at $r=r'$ (see Fig. 5) and is exponentially damped with the decay constant $|\kappa_0^I|$. This feature has interesting physical consequences since it means that if the separation distance between the scatterers is too large, there is little probability that they will actually interact because this requires a small relative distance. Indeed, taking r' to much larger values than in Fig. 5 yields a negligible contribution to the noninteracting propagator near small r where the interaction will act to modify the wave function. Clearly this effect is governed by the size of κ_0^I, the imaginary part of the pole of the CPA. It should be noted that this value will only become very small when the scattering energy approaches $2\epsilon_F$. Just as in the case of the sp motion, this means that the noninteracting wave function will tend to a plane wave again only in this limit. The corresponding result for Fig. 5 then yields a simple sine wave characterized by κ_0^R which approaches k_F. For all other energies damping does occur sufficiently rapidly to warrant the following observation: since only the part of the wave which returns from the scattering can be affected and this part always decreases with increasing r, only that part of the noninteracting wave can be influenced by the scattering which is exponentially damped in r. This is illustrated in Fig. 6 where a decomposition of the CPA propagator shown in Fig. 5 is presented in terms of the incoming and outgoing waves. For values of r' outside the range of the interaction as in Fig. 6 this implies that even a substantial modification of the outgoing wave will hardly affect the total propagator and the wave function must automatically heal, according to the value of κ_0^I, to the noninteracting one.

It is possible to proceed with an asymptotic analysis of the scattering process using the CPA propagator. A procedure to deal with the analysis of the complete numerical propagator will be outlined later. By following the steps leading to Eq. (19) in the case when the noninteracting propagator is given by the CPA result Eq. (38), one obtains the asymptotic propagator in the following form (for an uncoupled channel and energy Ω above $2\epsilon_F$)

$$\begin{aligned} g^{\mathrm{II}}_{l,JST}(r,r';\Omega) &\rightarrow -i\left(\frac{mc}{2\hbar^2}\right)\kappa_0 h_l(\kappa_0 r') \\ &\quad\times\left\{h_l^*(\kappa_0 r)+h_l(\kappa_0 r)\left[1-2i\frac{mc}{\hbar^2}\kappa_0\int_0^\infty dr_1 r_1^2\int_0^\infty dr_2 r_2^2\langle r_1|\Gamma_l^{JST}(\Omega)|r_2\rangle j_l(\kappa_0 r_1)j_l(\kappa_0 r_2)\right]\right\} \\ &= -i\frac{mc}{2\hbar^2}\kappa_0 h_l(\kappa_0 r')\{h_l^*(\kappa_0 r)+h_l(\kappa_0 r)e^{2i\delta_l^{JST}}\}. \end{aligned} \tag{40}$$

A simple example for a hard-core potential will be used to illustrate some features in more detail. The term in brackets in Eq. (40) corresponds to the asymptotic wave function including the effect of the potential in terms of a phase shift as in Eq. (19) for free or mf particles. For a hard-core potential with hard-core radius r_0 one must require this correlated wave function to vanish at r_0 also in the case of (dropping the JST subscript) Eq. (40):

$$0=h_l^*(\kappa_0 r_0)+h_l(\kappa_0 r_0)e^{2i\delta_l}. \tag{41}$$

This boundary condition yields an expression for the phase shifts given by

$$\tan\,\delta_l = \frac{j_l(\kappa_0 r_0)}{n_l(\kappa_0 r_0)}. \quad (42)$$

In the limit that κ_0^I vanishes, corresponding to free or mf particles, one recovers the usual result in terms of the ratio of spherical Bessel and Neumann functions with real arguments. For the $l=0$ case one obtains

$$\tan\,\delta_0 = -\tan\,\kappa_0 r_0, \quad (43)$$

yielding a real part

$$\delta_0^R = -\kappa_0^R r_0 \quad (44)$$

and an imaginary part

$$\delta_0^I = -\kappa_0^I r_0 \quad (45)$$

of the phase shift. Somewhat surprisingly a complex phase shift appears. Its role becomes clear when one considers the asymptotic propagator explicitly. For energies Ω above $2\epsilon_F$ ($\kappa_0^I > 0$) the corresponding result for $l=0$ in the CPA, inserting the result for the real and imaginary parts of the phase shift [Eqs. (44) and (45)], yields

$$g_{l=0}^{\rm II}(r,r';\Omega) \rightarrow i\,\frac{cm}{2(\kappa_0^R + i\kappa_0^I)\hbar^2}\,\frac{1}{rr'}\,e^{i\kappa_0^R r'}e^{-\kappa_0^I r'} \times\{-e^{-i\kappa_0^R r}e^{\kappa_0^I r} + e^{i\kappa_0^R (r-2r_0)}e^{\kappa_0^I (2r_0 - r)}\}. \quad (46)$$

It is clear that Eq. (46) vanishes for $r=r_0$. Note, however, that this can only be achieved by the presence of a complex phase shift. The incoming wave given by the first term in the bracket needs to be exactly compensated by the outgoing wave at r_0. Only shifting the oscillatory character of the wave function by the real part of the phase shift does not suffice; an additional enhancement provided by the imaginary part of the phase shift is required to achieve cancellation at r_0 since the incoming wave has a larger amplitude than the outgoing part (without the phase shift). The result is a shift of the complete wave function as appropriate for a hard-core potential. This is illustrated in Fig. 7 where the solid line represents the uncorrelated outgoing wave and the dashed line includes the real and imaginary parts of the phase shift. Results for values of $k_0 = 0.6\ \text{fm}^{-1}$ and $\gamma = 0.2\ \text{fm}^{-1}$ [see Eqs. (34) and (37)] yield phase shifts of $\delta_0^R = -0.3$ and $\delta_0^I = -0.08$ for a hard-core radius of 0.5 fm used in Fig. 7. The results for the complete asymptotic propagator including the complex phase shift (dashed line) and the noninteracting CPA propagator (solid line) are shown in Fig. 8. As in Fig. 6 the unaffected incoming wave dominates both propagators (wave functions) while both outgoing waves (shown in Fig. 7) are damped exponentially. The dashed line vanishes at the hard-core radius of 0.5 fm as required. More importantly, even though a phase shift will exist representing the effect of the scattering interaction, the asymptotic wave function nevertheless heals to the noninteracting one as shown in Fig 8. This same feature is observed for the complete numerical calculation including a realistic interaction [21].

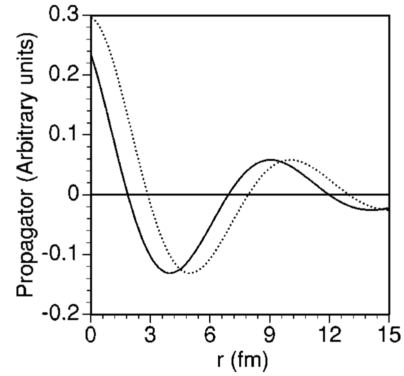

FIG. 7. Comparison of the outgoing two-particle wave function with (dotted line) and without (solid line) complex phase shift for a hard-core radius of 0.5 fm corresponding to the second term in Eq. (46) for values of the energy and r' used in Fig. 5.

The above observations allow for the resolution of the paradox related to the healing properties of wave functions in the medium. This property has been considered the physical justification of the mf-like properties observed in nuclei in the presence of strong short-range interactions. There is overwhelming experimental evidence [8] that sp motion in nuclei must be described in terms of dressed nucleons with substantial fragmentation of the strength. The original discussion of the healing properties of the relative wave function of particles in the medium [11] used a Bethe-Goldstone propagator involving mf nucleons to arrive at the healing property of the relative wave function. If the nucleons are dressed particles, a Bethe-Goldstone propagator does not suffice to generate a self-energy that will describe the sp strength distribution. Instead a Galitski-Feynman propagator must be employed. This will generate quite a reasonable description of the sp strength including the quasiparticle features for nucleons at the Fermi surface [9]. The description of the scattering process is, however, modified by employing a Galitski-Feynman propagator. Whereas it was possible to obtain healing with a Bethe-Goldstone propagator due to a vanishing phase shift for scattering energies below $2\epsilon_F$, this is no longer possible with a Galitski-Feynman propagator. A nonvanishing phase shift is obtained [16] and the asymptotic relative wave function of mf particles does not heal. The present work discusses the consequences of scattering dressed particles and demonstrates that this dressing of nucleons automatically leads to a localization of the relative wave functions in coordinate space. The results for the CPA propagator analysis for hard-core scattering indicate that even with sizable phase shifts the localization of the wave function leads to the desired healing property of the wave function since the part of the wave function affected by the scattering event is exponentially damped. Also for the complete numerical propagator the same features are observed [21]. Even in the presence of strong interaction processes the resulting picture of the nuclear medium is a tranquil one in which the dressed particles no longer remember their scattering event beyond some finite distance and their wave functions heal to the corresponding noninteracting ones. This appears to be a satisfactory picture of a correlated medium in which particles do not carry the information of their interaction indefinitely around unlike a description of scattering using a mf Galitski-Feynman propagator.

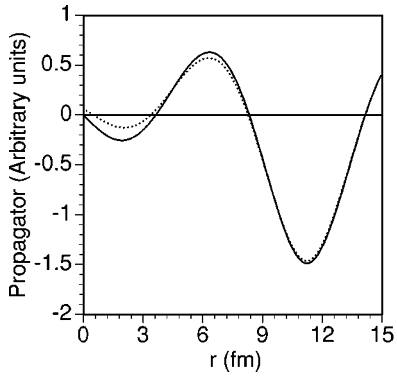

FIG. 8. Comparison of the imaginary part of the total noninteracting two-particle propagator (solid line) with the asymptotic one (dotted line) for a hard-core radius of 0.5 fm for values of the energy and r' used in Fig. 5. The asymptotic wave function [Eq. (46)] does not vanish inside the hard-core radius. The exact wave function vanishes of course for $r<r_0$.

The preceding discussion has focused on an analytically solvable model. First, the CPA result was developed to obtain a sufficiently realistic approximation to the propagator of two dressed particles. Second, a hard-core scattering problem for such a propagator was analyzed. A generalization of the discussion to the case of the complete propagator will be outlined now but results for a realistic interaction will be presented in detail in Ref. [21]. Results of such calculations completely confirm the analysis given here. As in the case of the CPA result shown in Fig. 6, it is possible to separate the ingoing and outgoing parts of the numerical noninteracting propagator in coordinate space. This can be written schematically as

$$g^{\mathrm{II}}_{f,l}=g^{\mathrm{II}}_{f,l}(\mathrm{in})+g^{\mathrm{II}}_{f,l}(\mathrm{out}). \tag{47}$$

The corresponding results are similar to those presented in Fig. 6. The equation for the propagator [Eq. (11)] can be written for an uncoupled channel as

$$g^{\mathrm{II}}_{l,JST}=g^{\mathrm{II}}_{f,l}+\Delta g^{\mathrm{II}}_{l,JST}, \tag{48}$$

where Δg^{II} contains the contribution due to the interaction Γ which can only affect the outgoing wave. By using Eq. (47) it is possible to identify a phase shift similar to Eq. (40):

$$e^{2i\delta_l^{JST}}=\frac{g^{\mathrm{II}}_{f,l}(\mathrm{out})+\Delta g^{\mathrm{II}}_{l,JST}}{g^{\mathrm{II}}_{f,l}(\mathrm{out})}. \tag{49}$$

Two remarks are in order here. First, because of the localization of the propagator, this result for the phase shift must be calculated for r' not too far away from the origin in order to generate a nonvanishing outgoing wave. Second, and relatedly, one must expect some r' dependence of this definition of the phase shift since the dressed noninteracting propagator does not completely separate into a product of a function of r and a function of r' as in the CPA result of Eq. (38). More importantly, these observations and the healing property of the propagator imply that a conventional derivation and definition of the cross section for the scattering process is not possible. No outgoing wave reaches asymptotically meaningful distances with damping constants as large as tenths of a fermi. As for the definition of the phase shift in the partial-wave basis above, one may infer the corresponding result for the scattering amplitude by considering the CPA result of the noninteracting propagator in coordinate space. Equation (5) can be calculated analytically using Eq. (35). For $\Omega>2\epsilon_F$, Eq. (5), this yields

$$g^{\mathrm{II}}_{f,\mathrm{CPA}}(\boldsymbol{r},\boldsymbol{r}';\Omega)=-\frac{mc}{4\pi\hbar^2}\frac{e^{i\kappa_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}. \tag{50}$$

For a derivation of the scattering amplitude one requires a meaningful separability of this propagator as in Eq. (24) contingent on the condition that either r' be much larger than r or vice versa. In the present case one cannot make this assumption without running into a vanishing propagator due to the nonvanishing presence of the imaginary part of κ_0. This observation does not change for the complete numerical propagator. As a result, there is no asymptotic (large distance) notion of a cross section as in the case of conventional scattering experiments. Only a local modification of the wave function is possible with a rapid healing to the noninteracting wave. Even for energies close to $2\epsilon_F$ where the imaginary part of κ_0 becomes small, the phase shift vanishes (or approaches π [16]) and no asymptotically relevant cross section can be identified. Similar conclusions are reached for the complete numerical propagator [21].

The above discussion does not imply that the local interaction between dressed particles is small. It does mean that one has to be cautious with the notion of a cross section of particles in the medium. In order to provide a way to assess the strength of the interaction of dressed particles it is convenient to generate a quantity which will yield the conventional cross section in the limit of mf or free-particle scattering. In addition, it is useful to provide a similar quantity for the phase shifts in order that one can make meaningful comparisons for the results involving mf or free particles [26,27]. Although the quantities introduced below are approximate, they do provide physically meaningful generalizations. The first step involves the practical observation that in most cases the imaginary part of κ_0 which characterizes the damping of the wave function is considerably smaller than its real part [21]. Only for energies of two particles deep in the Fermi sea do the real and imaginary parts of κ_0 become comparable [21]. Considering the identity (for $r<r'$)

$$\begin{aligned}
-ik_0 j_l(k_0r)j_l(k_0r')
&=\frac{-ik_0}{2}\{j_l(k_0r)h_l(k_0r')+j_l(k_0r)h_l^{*}(k_0r')\}\\
&=\frac{1}{\pi}\int_0^\infty dk k^2\,\frac{j_l(kr)j_l(kr')}{k_0^2-k^2+i\eta}\\
&\quad-\frac{1}{\pi}\int_0^\infty dk k^2\,\frac{j_l(kr)j_l(kr')}{k_0^2-k^2-i\eta}\\
&=i\frac{2}{\pi}\int_0^\infty dk k^2 j_l(kr)j_l(kr')\,\mathrm{Im}\left\{\frac{1}{k_0^2-k^2+i\eta}\right\}\\
&=i\frac{2\hbar^2}{\pi m}\int_0^\infty dk k^2 j_l(kr)j_l(kr')\,\mathrm{Im}\{g^{\mathrm{II}}_f(k;\Omega)\},
\end{aligned} \tag{51}$$

which is valid for vanishing η in the case of particles propagating in free space. Invoking the smallness of the imaginary part of κ_0 with respect to its real part, one may heuristically approximate the product of the spherical Bessel functions appearing in the first line of Eq. (40) using the identity given in Eq. (51). This approximation is appropriate for a pole in the complex momentum plane not too far from the real axis ($|\kappa_0^I| \ll \kappa_0^R$ for the CPA result) but also makes sense for r not too different from r' since the damping effect is smallest there. Since the integral in Eq. (51) contains real spherical Bessel functions, one can use the transformation to momentum space for both integrals in the first line of Eq. (40). The resulting asymptotic propagator for the CPA result then reads

$$g^{\mathrm{II}}_{l,JST}(r,r';\Omega) \to -i\left(\frac{mc}{2\hbar^2}\right)\kappa_0 h_l(\kappa_0 r')\Bigg\{h_l^*(\kappa_0 r)+h_l(\kappa_0 r) \times\left[1+2i\int_0^\infty dk k^2 \mathrm{Im}\{g_f^{\mathrm{II}}(k;\Omega)\} \times\langle k|\Gamma_l^{JST}(\Omega)|k\rangle\right]\Bigg\}. \tag{52}$$

The S-matrix element (and phase shift) can then be written for an uncoupled channel in the following way:

$$S_l(\Omega)=1+2i\int_0^\infty dk k^2 \mathrm{Im}\{g_f^{\mathrm{II}}(k;\Omega)\}\langle k|\Gamma_l^{JST}|k\rangle \equiv e^{2i\delta_l^{JST}}. \tag{53}$$

This result reduces to Eq. (20) for free or mf particles. The result of Eq. (53) can also be used to calculate phase shifts for the complete propagator. A consequence of the approximation contained in using Eq. (53) is that the phase shift δ_l^{JST} remains real, a reasonable approximation at most energies [21] considering the smallness of $|\kappa_0^I|$ compared to κ_0^R which determines the strength of δ_l^I with respect to δ_l^R [see Eqs. (45) and (44)]. As a result, the phase shifts can be fruitfully compared with results for mf or free particles. Detailed results using Eq. (53) for a realistic interaction will be presented in Ref. [21]. Equation (53) is exact for noninteracting particles and for dressed particles includes the physically reasonable expectation that the distribution over the momenta as contained in the imaginary part of the propagator will feature in determining the scattering process. While this approximation does not make sense at large distance scales, it provides, locally, a very reasonable generalization of the phase shift and cross section. The corresponding ''short-distance'' approximation to the scattering amplitude yields the result

$$f^S_{m_s' m_s}(\theta,\phi)=4\pi\sum_{ll'J}\sum_{mm'M} i^{l'}(-i)^l Y_{lm_l}(\hat{\mathbf{r}})Y^*_{l'm_l'}(\hat{\mathbf{z}}) \times(lm_l S m_s|JM) \times(l'm_l' S m_s'|JM)\int_0^\infty dk\ \mathrm{Im}\{g_f^{\mathrm{II}}(k;\Omega)\} \times\langle k(lS)J|\Gamma(\Omega)|k(l'S)J\rangle, \tag{54}$$

where a coupling to total spin S and projections m_s, m_s' for initial and final spin states has been included together with the usual decomposition in partial waves. In the case of free or mf particle scattering the δ function of the imaginary part of g_f^{II} yields the conventional result. For the case of a central interaction and free particles Eq. (54) reduces to (suppressing spin indices)

$$f(\theta,\phi)=\sum_l \frac{2l+1}{k_0}\left\{\frac{-mk_0\pi}{2\hbar^2}\right\}\langle k_0|T_l(k_0)|k_0\rangle P_l(\cos\theta) = \sum_l \frac{2l+1}{k_0} e^{i\delta_l}\sin\delta_l P_l(\cos\theta), \tag{55}$$

where the addition theorem for spherical harmonics and the δ function of the imaginary part of the propagator have been used to obtain the first equality and the second equality of Eq. (20) to obtain the second equality of this result. For the total cross section (in the neutron-proton case) one obtains

$$\sigma_{\mathrm{tot}}=\pi\sum_{Sll'J}(2J+1)\left|\int_0^\infty dk\ \mathrm{Im}\{g_f^{\mathrm{II}}(k;\Omega)\} \times\langle k(lS)J|\Gamma(\Omega)|k(l'S)J\rangle\right|^2, \tag{56}$$

which for a central interaction and free particles reduces to the standard result

$$\sigma_{\mathrm{tot}}=\frac{4\pi}{k_0^2}\sum_l (2l+1)\sin^2\delta_l. \tag{57}$$

Equation (56) demonstrates that a sensible cross section will be obtained in the case of dressed particles at all energies for which a nonvanishing imaginary part of the propagator exists. For two particles deep in the Fermi sea, for example, Eq. (56) avoids the divergence associated with the k_0^{-2} term in Eq. (57). The formulation of the cross section in terms of Eq. (56) provides a reasonable way to assess the strength of the interaction between dressed particles in the medium in terms of the square of the relevant transition matrix element (Γ) multiplied by an appropriate measure of the density of states represented by the imaginary part of the noninteracting propagator. Indeed one may anticipate that the cross section defined in Eq. (56) will be smaller than the corresponding one for mf particles since the density of states

$$\rho(\Omega)=-\frac{1}{\pi}\int_0^\infty dk k^2 \mathrm{Im}\{g_f^{\mathrm{II}}(k;\Omega)\} \tag{58}$$

for dressed particles is substantially smaller than for mf particles for energies both below and above $2\epsilon_F$. This feature is closely related to the distribution of the sp strength which for a given momentum contains only about 70% in the quasiparticle peak while the admixture of other momentum states at the corresponding energies does not nearly compensate for this loss. The corresponding cross sections will therefore be reduced with respect to those for mf particles. In addition, a considerable reduction of the effects of pairing, for example for the $^3S_1$-$^3D_1$, channel can be anticipated as a result of this reduction of the density of states [21].

IV. SUMMARY AND CONCLUSIONS

The main purpose of the present work is to provide some conceptual understanding of the scattering process of nucleons in nuclear matter when the dressing of the nucleons is taken into account. By employing the formulation of the two-body propagator it is shown how the usual asymptotic analysis of the free-particle scattering can be recovered. With minor modifications one obtains similar results for the scattering of mf particles in the medium which employs the Galitski-Feynman propagator in the ladder equation [16]. Both for free particles and mf particles in the medium the asymptotic wave function is characterized by a unique (on-shell) momentum which is related to the scattering energy. In both cases this results in plane-wave functions (or spherical Bessel functions in a partial-wave basis).

With the propagation of dressed nucleons one is forced to abandon this feature completely. When nucleons are described in terms of spectral functions, their relative wave function at a given energy contains contributions from all momenta. Although there is a range of momenta close to the former on-shell momentum which is dominant at a given energy, this distribution is sufficiently broad to yield a localized wave function in coordinate space. A simple complex pole approximation to the noninteracting propagator has been introduced to clarify some of these features. While it provides a good approximation to the numerical propagator for the dominant momenta at a given energy, it cannot be used to solve scattering equations since it lacks accuracy at high momenta which are necessary for the description of short-range effects. It can be used, however, to illustrate the changes that are required in the asymptotic analysis of the scattering process. Since the wave function of the dressed particles is damped, the incoming wave part dominates and the outgoing wave can only be affected when propagation is started at not too large relative distances. In addition to generating a complex phase shift, illustrated for the case of a hard-core potential, it is shown that the correlated wave function must heal to the noninteracting one. This feature is also recovered in a complete numerical calculation of the interacting propagator [21].

With this observation one completes the evolution of the picture of the effect of correlations on the properties of interacting nucleons. While it was originally thought that a Bethe-Goldstone propagator would suffice to generate this healing property (no phase shift for energies below $2\epsilon_F$) [11] and explain the simple shell-model picture then available, our current understanding of the sp properties of the nucleon in the medium coincides more with a Landau quasiparticle picture. As a consequence, one is required to include at least hole-hole terms in the nucleon self-energy to describe the sp strength distribution. In turn, this leads to a Galitski-Feynman propagator for the scattering of nucleons and a phase shift below $2\epsilon_F$ [16] indicating no healing. The picture comes full circle when dressed nucleons are used to describe the scattering in the medium, both allowing an understanding of the sp properties of correlated nucleons and maintaining the healing property of the relative wave function albeit in an updated form. Because of the localization of the relative wave function of dressed particles, one can no longer associate macroscopic distance scales with the scattering process in the medium and the strict concept of a cross section is no longer valid. Expressions are proposed to characterize the strength of the interaction in terms of quantities that will yield the correct results for phase shifts and cross sections in the limit of mf or free particles. These expressions involve the weighting of the transition matrix element (effective interaction) by the relevant measure of the density of states given by the imaginary part of the noninteracting propagator. A discussion of the results for phase shifts, cross sections, and correlated wave functions will be presented in Ref. [21] for a realistic interaction.

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