Incompressibility of hot asymmetrical nuclear matter: Lowest order constrained variational approach

M. Modarres[1,3] and G. H. Bordbar[2,3]
[1]*Department of Physics, Amir-Kabir University, Hafez Ave. 15875, Tehran, Iran*
[2]*Department of Physics, Shiraz University, Shiraz 71454, Iran*
[3]*Centre for Theoretical Physics and Mathematics AEOI, P.O. Box 11365-8486, Tehran, Iran*
(Received 16 March 1998)

The isothermal and isentropic incompressibility of asymmetrical nuclear matter are investigated in the framework of the lowest order constrained variational method. Various phenomenological nucleon-nucleon interactions, namely, the Reid, the Δ-Reid, and the new Argonne potentials are used. It is found that the temperature as well as the asymmetry parameter reduce the incompressibility of asymmetrical nuclear matter. The pressure is calculated along isentropic paths. It is shown that the equation state is much harder than the one usually assumed in the hydrodynamical simulations of supernova collapse. Various discussions are made in connection with the problem of supernova explosions and other many-body model calculations. [S0556-2813(98)03311-1]

PACS number(s): 21.65.+f, 26.60.+c, 64.70.−p

I. INTRODUCTION

After Landau introduced the concept of a neutron star as one of the possible end points of stellar evolution, the pulsars were discovered in 1967 and were interpreted as neutron stars. Then during the last two decades, the development of many-body theories and the understanding of gravitational collapse led to a better understanding of stellar evolution than that of the simple neutron fluid model originally envisaged by Landau. At present the equation of state of hot asymmetrical nuclear matter has a fundamental role in the understanding of heavy-ion collisions [1], the physical mechanism of the iron core collapse of a massive star which produces a type-II supernova [2], and the rapid cooling of the new born neutron star [2].

In the last stage of type-II supernova [1], because of electron-capture processes, a highly asymmetric nuclear matter is formed and the star reaches the proton-to-neutron ratio of $Z/N \simeq \frac{1}{2}$. This value stays almost constant during the collapse time until the core bounces and the shock wave is formed. The density varies from nuclear matter density $\simeq 0.17\ \mathrm{fm}^{-3}$ up to four times this value. On the other hand, while the temperature $\mathcal{T}$ is typically about tens of MeV, the entropy per particle is almost constant (about one in the units of Boltzmann constant k_B) [3].

In order to describe such excited nucleonic matter one needs a reliable many-body theory that is capable of giving accurate values for various thermodynamics variables. But most of the recent many-body works have been confined to zero temperature or to finite temperature with unrealistic interactions. So performing the calculation for hot asymmetrical nuclear matter at finite temperature in different thermodynamic conditions such as isothermal and isentropic paths is crucial.

Recently we have investigated various properties of cold and hot asymmetrical nuclear matter using the lowest order constrained variational (LOCV) method based on cluster expansion theory. This approach has been further generalized to include more sophisticated interactions such as the V_{14} [4], the AV_{14}, and the new Argonne AV_{18} [5] potentials, but for frozen calculations.

There are various reasons for choosing the LOCV method. (i) It is a fully self-consistent model which can use realistic potentials such as the Reid [6], the Δ-Reid [7], the V_{14} [8,9], the AV_{18} [10], etc. (ii) The convergence of cluster series have been tested by calculating the three-body cluster terms [11,4,5]. (iii) It predicts reasonably various properties of nuclear matter such as the saturation energy and density, the surface energy and the asymmetrical coefficient near the empirical values [11]. (iv) Finally, as the three-body terms are very small [point (2)], the functional minimization procedure saves a large amount of computation time compared with other variational methods, without lossing the contribution of higher cluster terms.

In this work we intend to calculate the incompressibility of asymmetrical nuclear matter along both isothermal and isentropic paths with three different potentials such as the Reid, the Δ-Reid, and the AV_{18} potentials. So the plan of this article is as follows. The lowest order constrained variational method is briefly described in Sec. II. In Sec. III the incompressibility of isothermal asymmetrical nuclear matter is calculated. Section IV is devoted to pressure and incompressibility of isentropic asymmetrical nuclear matter. Our summary and conclusion are presented in Sec. V.

II. THE LOCV FORMALISM AT $\mathcal{T}\neq 0$

Let $\phi_i[n_i(k)]$, where $n_i(k)$ is the occupation number of the single particle states, represent the ideal Fermi-gas-type wave function, i.e., the plane wave. Then using variational techniques, we can write the wave function of interacting system as

$$\psi = F_{\mathcal{T}} \phi^{\mathcal{T}}, \tag{1}$$

where in asymmetrical nuclear matter $F_{\mathcal{T}}$ is taken to be

$$F_{\mathcal{T}}=\mathcal{S}\prod_{i>j} f(ij). \tag{2}$$

$f(ij)$ are operators that act on spin, isospin, and relative position variables of particles i and j and $\mathcal{S}$ is a symmetrizing operator, which is necessary, because the $f(ij)$ do not commute. Because of the unitariness of $F_{\mathcal{T}}$'s, one is usually faced with the problem of nonorthogonality of different states. However, since at low temperatures only the one-quasi-particle-type states are important and these states have different total momentum, they are orthogonal. $\phi^{\mathcal{T}}$ is the familiar slater determinant of the single-particle wave functions.

The occupation probability $n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)$ is the Fermi-Dirac distribution function and is written as

$$n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)=\frac{1}{1+e^{(k_B\mathcal{T})^{-1}[\epsilon_\tau(k,m_\tau^*)-\mu_\tau(\mathcal{T},\rho_\tau)]}} \tag{3}$$

with $\tau=-\frac{1}{2}$ and $+\frac{1}{2}$ for neutrons and protons. In this equation $\epsilon_\tau(k,m_\tau^*)$ are the single-particle energies and $\mu_\tau(\mathcal{T},\rho_\tau)$ are the chemical potentials associated with the neutrons and protons at a given temperature, density, and asymmetry parameter. m_τ^* stands for the effective masses.

Now, using the above trial wave function, we construct a cluster expansion for the expectation value of our Hamiltonian using the above trial wave function. We keep only the first two terms in the cluster expansion of the energy functional:

$$E([f])=\frac{1}{A}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}=E_1+E_2. \tag{4}$$

E_1 is independent of $f(ij)$ and is simply the Fermi-gas kinetic energy expression

$$E_1=\sum_{\tau=+1/2,-1/2}\epsilon_\tau \tag{5}$$

with

$$\epsilon_\tau=\sum_{k\sigma}\frac{\hbar^2k^2}{2m_\tau}n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau), \tag{6}$$

while the two-body energy E_2 is written as

$$E_2=\frac{1}{2A}\sum_{ij}\langle ij|\mathcal{V}(12)|ij\rangle_a, \tag{7}$$

where

$$\mathcal{V}(12)=-\frac{\hbar^2}{2m_\tau}[f(12),[\nabla_{12}^2,f(12)]]+f(12)V(12)f(12) \tag{8}$$

and the two-body antisymmetrized matrix element $\langle ij|\mathcal{V}(12)|ij\rangle_a$ is taken with respect to the single-particle wave functions composing $\phi^{\mathcal{T}}$.

Using projection operators $O_\alpha^{(k)}(ij)$, we write $f(ij)$ (which can also convert NN to $N\Delta$, in the case of the Δ-Reid interaction) as

$$f(ij)=\sum_\alpha f_\alpha^{(k)}(ij)O_\alpha^{(k)}(ij), \tag{9}$$

where α stands for $\{S,L,J,T,M_T;\mathcal{T}\}$, $k=1,2,3,4$, and

$$O_\alpha^{k=1,2,3,4}=1,\left(\frac{2}{3}+\frac{1}{6}S_{12}^I\right),\left(\frac{1}{3}-\frac{1}{6}S_{12}^I\right),S_{12}^{II}. \tag{10}$$

The operator S_{12}^{II} is the analog of the usual tensor operator, S_{12}^I, for the mixed $N\Delta$ channels. The value of k is set to unity for $L\neq 0$ and for the spin-triplet channels with $L\neq J\pm 1$. But for $L=J\pm 1$ it takes values of 2 and 3. Finally we have $L=0$ channels which couple the $^1S_0$ channel to the $^5D_0$ channel (for the Δ-Reid potential) where we set $k=1$ and 4.

As in our previous calculations we require the correlation functions $f_\alpha^{(1)}$, $f_\alpha^{(2)}$, and $f_\alpha^{(3)}(f_\alpha^{(4)})$ to heal to the Pauli function $f_p^{M_T}(r)$ (zero)

$$f_p^{\pm 1}(r)=\left[1-\frac{1}{2}\left(\frac{\gamma_\tau(r)}{\rho}\right)^2\right]^{-1/2},\quad \tau=\pm\frac{1}{2},\quad f_p^0(r)=1, \tag{11}$$

where ρ is the nucleonic density, i.e.,

$$\rho=\rho_{+1/2}+\rho_{-1/2}. \tag{12}$$

M_T is the total isospin projection of two nucleon states

$$\gamma_\tau=\frac{\rho}{A}\sum_{k,\sigma}n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)e^{i\mathbf{k}\cdot\mathbf{r}} \tag{13}$$

and

$$\rho_\tau=\frac{\rho}{A}\sum_{k,\sigma}n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau). \tag{14}$$

Then using the definition

$$\mathcal{R}=\frac{\rho_{+1/2}}{\rho_{-1/2}}=\frac{\rho_Z}{\rho_N}$$

(proton to neutron ratio), our asymmetry parameter can be written as

$$\beta=\frac{1-\mathcal{R}}{1+\mathcal{R}}. \tag{15}$$

The expressions for the two-body energies are given in Refs. [12] and [5]. In Ref. [5] instead of the two-body overlap integral, i.e.,

$$I_{J,M_T}(x)=\int dq P_{M_T}(q)J_J^2(xq) \tag{16}$$

which has been defined for a frozen calculation, one should use the following expression for the present case, i.e., the finite temperature calculation:

$$I_{J,M_T}(r,\rho,\mathcal{T})=(2\pi^6\rho^2)^{-1}\int \mathbf{dk_1dk_2}n_\tau(k_1,\mathcal{T},\rho_\tau,\mu_\tau)\times n_{\tau'}(k_2,\mathcal{T},\rho_{\tau'},\mu_{\tau'})J_J^2(|\mathbf{k_1-k_2}|r). \quad (17)$$

The normalization constraint that we impose on the channel two-body correlation functions $f_\alpha^{(k)}$, as well as the coupled and uncoupled differential equations coming from the Euler-Lagrange equations are similar to those described by Modarres [12] and Bordbar and Modarres [5]. The procedure for energy calculation has also been fully discussed in these references [12,5].

III. ISOTHERMAL INCOMPRESSIBILITY

At finite temperature, the isothermal incompressibility is defined as

$$\mathcal{K}^{\mathcal{T}}(\rho,\beta,\mathcal{T})=9\rho^2\left(\frac{\partial^2\mathcal{F}}{\partial\rho^2}\right)\Bigg|_{\mathcal{T}}+18\rho\left(\frac{\partial\mathcal{F}}{\partial\rho}\right)\Bigg|_{\mathcal{T}}, \quad (18)$$

where $\mathcal{F}$ is the Helmholtz free energy, i.e.,

$$\mathcal{F}=E-\mathcal{T}S, \quad (19)$$

with S the entropy per particle, being approximated by a noninteracting Fermi gas model of quasiparticles in the nuclear matter mean field [13], i.e.,

$$S=-\frac{k_B}{A}\sum_\tau\sum_k\{[1-n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)]\ln[1-n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)]+n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)\ln[n_\tau(k,\mathcal{T},\rho_\tau,\mu_\tau)]\}. \quad (20)$$

In view of the work of Friedman and Pandharipande [14] and our previous studies [15,12], we approximate the effect of mean field on the single-particle energies by introducing the effective mass $m_\tau^*(k,\mathcal{T},\rho,\mathcal{R})$,

$$\epsilon_k^\tau(k,\mathcal{T},\rho,\mathcal{R})=\frac{\hbar^2k^2}{2m_\tau^*(k,\mathcal{T},\rho,\mathcal{R})}, \quad (21)$$

where $m_\tau^*(k,\mathcal{T},\rho,\mathcal{R})$ are treated as variational parameters.

Obviously at saturation point $\mathcal{K}_0^{\mathcal{T}}$, the saturation incompressibility can be written according to Eq. (18) as follows:

$$\mathcal{K}_0^{\mathcal{T}}(\beta,\mathcal{T})=9\left(\rho^2\frac{\partial^2\mathcal{F}}{\partial\rho^2}\right)\Bigg|_{\rho_0(\beta,\mathcal{T})}. \quad (22)$$

In Fig. 1 we show the calculated isothermal incompressibility $\mathcal{K}_0^{\mathcal{T}}$ as a function of proton to neutron ratio $\mathcal{R}$ at the corresponding saturation density for various values of the temperature. It is seen that below some ratios and above some temperatures there is no minimum in free energy, so we cannot have the saturation incompressibility. This, for example, happens for $\mathcal{T}=10$ MeV with $\mathcal{R}\leqslant 0.4$. It is seen that the results of isothermal incompressibility calculation with the Δ-Reid and the AV_{18} potentials are closer to each other compared with the similar calculations performed with the Reid potential.

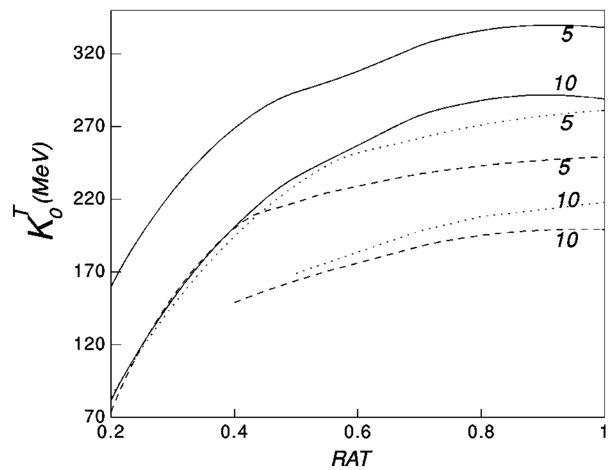

FIG. 1. Isothermal incompressibility versus the ratio $\mathcal{R}$ for different temperatures and potentials: Reid (full curves), the Δ-Reid (dashed curves), and the AV_{18} (dotted curves) potentials.

In Fig. 2 the ratio of the isothermal incompressibility $\mathcal{K}_0^{\mathcal{T}}(\beta,\mathcal{T})$ of hot asymmetric nuclear matter to the incompressibility $\mathcal{K}_0^{\mathcal{T}}(\beta,0)$ of cold asymmetric nuclear matter for $\mathcal{R}=\frac{1}{2}(\beta=0.33)$ is plotted against different values of the temperature $\mathcal{T}$. The results of Bombaci *et al.* [16,17] (BKL) and Vinas *et al.* [18] (VBTS) are also given for comparison. BKL have done a preliminary investigation within a finite temperature Brueckner-Bethe-Goldstone (BBG) theory with the Paris two-nucleon potential in which the long range correlations due to hole-hole (HH) propagation in the intermediate states have been ignored. VBTS have performed a Hartree-Fock (HF) calculation using the phenomenological Skyrme SKM* interaction which yields a good fit to the saturation properties of cold nuclear matter. For $\mathcal{T}\leqslant 6$ MeV, we get similar results to those of BKL and VBTS. But for $\mathcal{T}\geqslant 6$ MeV our calculations show a weaker

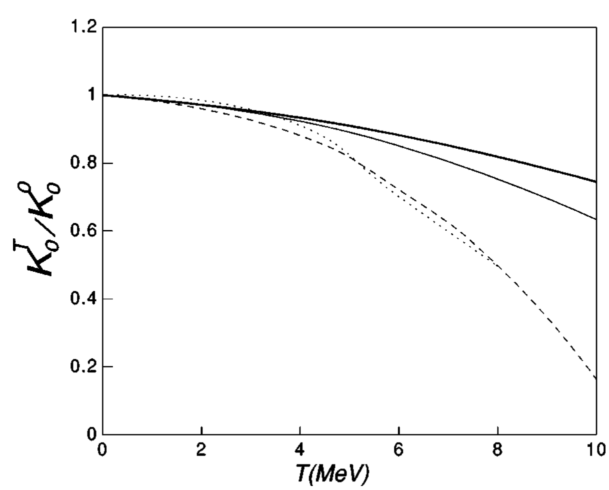

FIG. 2. The ratio of isothermal incompressibility $\mathcal{K}_0^{\mathcal{T}}$ calculation at saturation point to the corresponding calculation at zero temperature $\mathcal{K}_0^0$ for asymmetric nuclear matter: AV_{18} (full curve) and the Δ-Reid (heavy curve) potentials. The results of Refs. [16] (BKL) (dotted curve) and [18] (VBTS) (dashed curve) are also plotted.

temperature dependence than those of BKL and VBTS. This is expected since (i) we use more realistic interactions such as the Δ-Reid and the AV_{18} potentials in which a strong spin dependence, especially via the tensor force in the $^3S_1-^3D_1$ and the $^1S_0-^5D_0$ channels has been built in. So we get more attraction even when the temperature is as high as 10 MeV and this will cause a small but finite variation of the incompressibility at these temperatures. (ii) As stated before, since BKL have not taken into account the HH terms and VBTS have performed a simple HF calculation with the zero range SKM* potential, their results should not be comparable with our calculation.

The calculated falloff of the incompressibility with temperature can be fitted with a parabolic equation as has been used by BKL and Baron *et al.* [19] (BCK), i.e.,

$$\mathcal{K}_0^{\mathcal{T}}(\beta,\mathcal{T})=\mathcal{K}_0^{\mathcal{T}}(\beta,0)[1-\mathcal{A}^{\mathcal{T}}(\beta)\mathcal{T}^2]. \tag{23}$$

At $\beta=0.33$ we find $\mathcal{A}^{\mathcal{T}}(\beta)=4.38\times10^{-3}$ MeV^{-2} which can be compared with the value of 7.87×10^{-3} MeV^{-2} of BKL. A similar behavior has been found by other groups, such as Huang *et al.* [20] and Stocker [21]. We hope that an exact calculation along the lines followed by BKL will yield results close to what we report here, since at zero temperature we get similar results to their full BBG calculation [4].

IV. ISENTROPIC INCOMPRESSIBILITY

In the collapsing supernova core, the entropy per particle is very low, about the order of unity (in the k_B units) and nearly constant during all stages, until the shock wave starts to form [3]. So the properties of hot asymmetric nuclear matter along an isentropic path rules the collapsing process (the details of the above evolution have been discussed by Bethe *et al.* [3]). This means that we should consider an adiabatic process of a highly ordered system during the period of collapse. In this situation there is no heat flow to or from the system.

In order to perform calculations at constant entropy, we vary the temperature until we get the required constant entropy through Eqs. (14) and (20). Then we can evaluate the energy per particle via Eq. (4) and the calculated temperature.

The temperature of asymmetric nuclear matter as a function of density along three different constant entropy paths is given in Fig. 3. As usual, the proton to neutron ratio is taken to be $\mathcal{R}=\frac{1}{2}$ ($\beta=0.33$). In this calculation we do not vary the effective masses (m_τ^*) in Eq. (12), since we found that the internal energy does not change with these parameters. The reason is obvious from Eq. (20), in which we intend to keep the entropy constant by the varying temperature $\mathcal{T}$ and effective masses m_τ^*. This means that $n_\tau(k)$ should remain constant as well and hence $\mathcal{T}$ and m_τ^* must vary such that their products remain constant. This is what one should expect in the adiabatic processes for the single particle states [22]. The results of Fig. 3 are relatively independent of potentials, since they are produced through Eqs. (12), (14) and (20).

In this figure we also present the result of BKL. Because of the above discussion, by changing the effective masses we can get their result ($m^*\simeq0.6m$). But this will not have any effect on the internal energy calculation (BKL reported the same value for the effective mass).

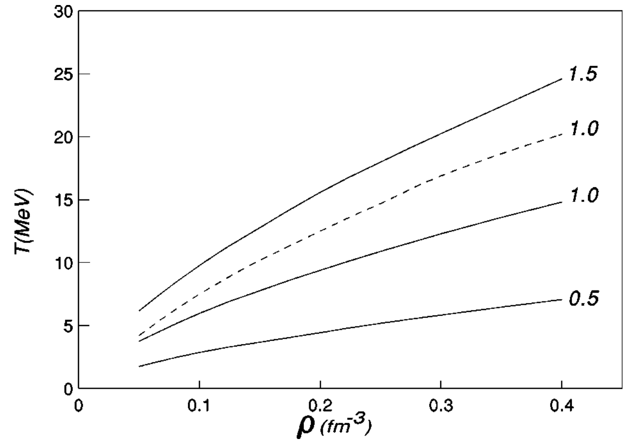

FIG. 3. Temperature versus density for isentropic calculation of asymmetric nuclear matter ($\beta=0.33$) (full curves). The dashed curve is the result of Ref. [16] (BKL).

From the calculated binding energies we can find $\mathcal{K}_0^S$, the isentropic incompressibility at saturation density. The results are plotted in Fig. 4 against the proton to neutron ratio $\mathcal{R}$ for three values of entropy. Unlike the isothermal case we get very similar results for two different types of potential. But the ratio ($\mathcal{R}$) dependence is the same as our isothermal calculation.

In Table I the isentropic incompressibility $\mathcal{K}_0^S$ of asymmetric nuclear matter at $\beta=0.33$ for different values of total entropy per nucleon as well as its ratio respect to the corresponding calculation for the cold asymmetric nuclear matter are compared with those of BKL. We find an overall agreement with them especially for the Δ-Reid interaction. Looking at Figs. 1 and 2 and Table I we can argue that the incompressibility for the isothermal paths are more sensitive to potentials and the many-body techniques than for isentropic paths.

BCK and BKL also calculated the pressure density of hot asymmetric nuclear matter at constant entropy ($S=1$) with $\beta=0.33$, i.e.,

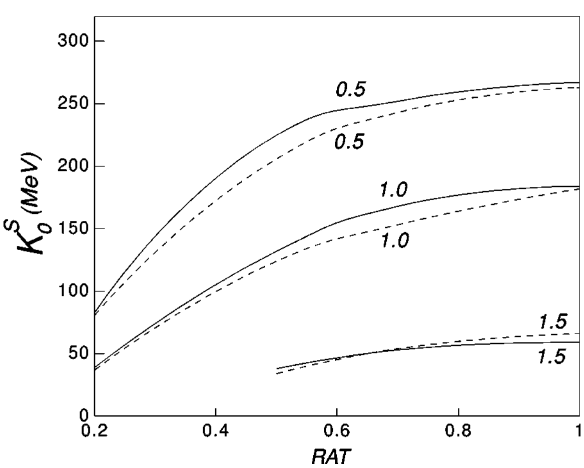

FIG. 4. Isentropic incompressibility of asymmetric nuclear matter at saturation density versus ratio $\mathcal{R}$ at different entropies. Full curve (AV_{18} potential) and dashed curve (Δ-Reid potential).

TABLE I. The result of the isentropic incompressibility calculation at the saturation point of asymmetrical nuclear matter $\mathcal{K}_0^S(\beta,S)$ at $\beta=0.33$ for different values of entropy and potentials. In the second column the ratio between $\mathcal{K}_0^S(\beta,S)$ and the incompressibility of cold asymmetric nuclear matter is tabulated. The results of Ref. [16] (BKL) are given for comparison.

	AV_{18}		Δ-Reid		BKL	
S	$\mathcal{K}_0^S$	$\mathcal{K}_0^S/\mathcal{K}_0^0$	$\mathcal{K}_0^S$	$\mathcal{K}_0^S/\mathcal{K}_0^0$	$\mathcal{K}_0^S$	$\mathcal{K}_0^S/\mathcal{K}_0^0$
0.5	225	0.872	206	0.936	134	0.931
1.0	132	0.511	124	0.563	99	0.688
1.5	38	0.147	34	0.154	34	0.236

$$P(\rho,\beta=0.33,S=1)/\rho=\rho\left.\frac{\partial E}{\partial\rho}\right|_s. \tag{24}$$

Our LOCV results with the AV_{18} and the Δ-Reid potentials are plotted in Fig. 5 versus density. The BKL and BCK pressure density calculations are also given for comparison. It is seen that we get much harder equation of state than those of BCK and BKL. The BCK pressure has been calculated by adding [17] the thermal pressure of the two component degenerate free Fermi gas and using an effective mass $m^*\simeq 0.66m$ to the zero temperature phenomenological nuclear matter equations of state with $\mathcal{K}_0(\beta=0.33)=90$ MeV and adiabatic index [19] $\gamma=3.0$, i.e.,

$$p(\rho)=\frac{\mathcal{K}_0(\beta)}{9\gamma}\rho_0(\beta)[u^\gamma-1], \tag{25}$$

where $u=\rho/\rho_0(\beta)>1$. So neither our LOCV calculation nor those of BKL results can support the soft nuclear matter equation of state which has been used in the hydrodynamical calculations of supernova explosions.

V. SUMMARY AND CONCLUSION

In this work we have computed the equation of state of asymmetrical nuclear matter at finite temperature along both isothermal and isentropic paths. Three different nucleon-nucleon potentials, namely, the Reid, the Δ-Reid, and the new AV_{18} interactions were used. The density and temperature dependence of isothermal and isentropic incompressibility of asymmetrical nuclear matter were calculated and an overall agreement was found between our results and those of Bombaci *et al.* [16,17]. We have found that the temperature as well as the asymmetry parameter could reduce the incompressibility of asymmetrical nuclear matter. It appears that the equation of state that is currently being used in the supernova explosion models is much softer than what we get here. One can argue that there are still missing effects, such as three-body forces, relativistic effects, etc., which have not been taken into account in our calculation, and can change our predictions. But considering these effects still we believe that one should use harder equation of state than what is given by Eq. (25) for the type-II supernova explosions.

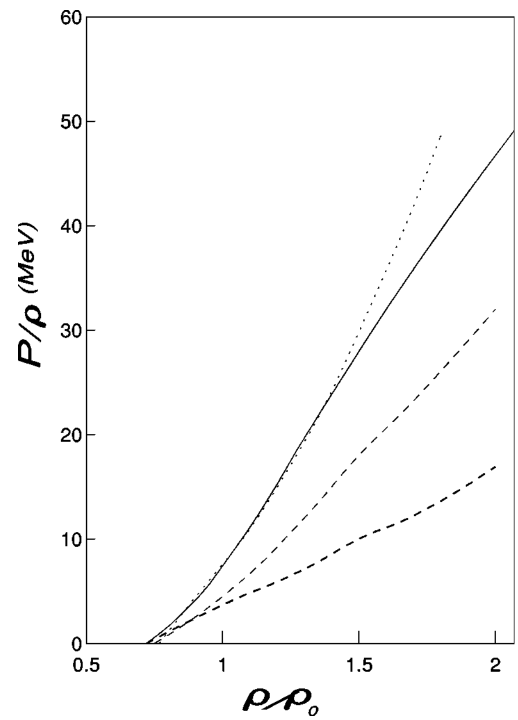

FIG. 5. Pressure density of isentropic nuclear matter versus density (in units of the saturation density at $\beta=0.33$) for different potentials. The Δ-Reid (dotted curve), the AV_{18} (full curve), dashed curve [Ref. [16], (BKL)] and heavy dashed curve [Ref. [19] (BCK)].

[1] R. Stoch *et al.*, Phys. Rev. Lett. **49**, 1236 (1982); J. J. Molitoris, D. Hahn, and H. Stocker, Nucl. Phys. **A447**, 13c (1985); H. Stocker and W. Greiner, Phys. Rep. **137**, 137 (1986); G. F. Bertsch and S. Das Gupta, *ibid.* **160**, 189 (1988).

[2] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Hansel, Phys. Rev. Lett. **66**, 2701 (1991); M. Prakash, I. Bombaci, P. J. Ellis, R. Knorren, and J. M. Lattimer, Phys. Rep. **280**, 1 (1997).

[3] H. A. Bethe, G. E. Brown, J. Applegate, and J. M. Lattimer, Nucl. Phys. **A324**, 487 (1979); H. A. Bethe, Rev. Mod. Phys. **62**, 801 (1990).

[4] G. H. Bordbar and M. Modarres, J. Phys. G **23**, 1631 (1997).

[5] G. H. Bordbar and M. Modarres, Phys. Rev. C **57**, 714 (1998).

[6] R. V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).

[7] A. M. Green, J. A. Niskanen, and M. E. Sainio, J. Phys. G **4**, 1085 (1978).

[8] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A359**, 331 (1981).

[9] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. Lett. **29**, 1207 (1984).

[10] R. B. Wiringa, V. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).

[11] M. Modarres and J. M. Irvine, J. Phys. G **5**, 511 (1979); **5**, 7 (1979).

[12] M. Modarres, J. Phys. G **23**, 923 (1997).

[13] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Body Systems* (McGraw-Hill, New York, 1971).

[14] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981).

[15] M. Modarres, J. Phys. G **19**, 1349 (1993).

[16] I. Bombaci, T. T. S. Kuo, and U. Lombordo, Phys. Lett. B **311**, 9 (1993).
[17] I. Bombaci, T. T. S. Kuo, and U. Lombordo, Phys. Rep. **242**, 165 (1994).
[18] X. Vinas, M. Baranco, J. Treiner, and S. Stringari, Astron. Astrophys. **182**, L34 (1987).
[19] E. Baron, J. Cooperstein, and S. Kahana, Phys. Rev. Lett. **55**, 126 (1985).
[20] S. W. Huang, M. Z. Fu, S. S. Wu, and S. D. Yang, Mod. Phys. Lett. A **5**, 1071 (1990).
[21] W. Stocker, Phys. Lett. **142B**, 319 (1984).
[22] C. Kittele, *Thermal Physics* (Wiley, New York, 1969).