Differences among ground-state-equivalent effective nuclear interactions

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We obtain a correlation between the incompressibility K_{v} and the anharmonicity K' of nuclear saturation curves for current Lagrangians and Skyrme-Gogny effective forces—all practically equivalent for the description of binding energies and radii of nuclei. We illustrate explicitly why a change in K_{v} can be compensated by modifying K' and/or the surface thickness t of large nuclei. A realistic value of $t=2.3$ fm fixes K_{v} to a value of about (210 ± 30) MeV, consistent with recent breathing-mode analyses. We refer to previous nonrelativistic Thomas-Fermi approaches that came to the same conclusion. [S0556-2813(98)00511-1]

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Ground-state properties of nuclei, in particular energies and radii, are very well described by the Hartree-Fock (HF) or the extended Thomas-Fermi (ETF) approximation using Skyrme (Sk) or Gogny (G) effective nucleon-nucleon interactions. Also Thomas-Fermi (TF) approaches with generalized Seyler-Blanchard (SB) interactions turned out to be successful. In more than a decade, relativistic mean-field (RMF) approaches in the form of the nonlinear σ-ω-ρ model have become nearly equivalent. They have a realistic field-theoretical background with meson fields mediating the nucleon-nucleon interaction. Some phenomenological terms nonlinear in the meson fields, however, have to be taken into account in the nuclear Lagrangian.

The nuclear energy-density functionals in Sk-G and SB as well as in RMF descriptions contain open parameters representing the strengths of the effective nucleon-nucleon interaction or the coupling constants, respectively, between mesons and nucleons that are fitted to well-known ground-state properties of finite nuclei. With the parameters obtained from the fits one then can get in the model of infinite nuclear matter (INM) and semi-infinite nuclear matter (SINM) the nuclear saturation density and the leading terms in the nuclear mass formula

$$E=a_{\mathrm{v}}(1+\alpha I^2)A+a_{\mathrm{sf}}(1+\beta I^2)A^{2/3}+\cdots, \tag{1}$$

where $I=(N-Z)/(N+Z)$. If masses and radii were carefully taken into account in the fits, the resulting INM and SINM properties should come out nearly equal for different parametrizations of a given functional *Ansatz* for the interaction or the Lagrangian, respectively. The values of the coefficients in Eq. (1) obtained from effective Sk-G, SB, and RMF should not differ much if the *Ansätze* themselves are realistic. We denote parametrizations obtained from masses and radii ground-state equivalent. It cannot be expected that these ground-state-equivalent parametrizations are totally equivalent.

The resulting INM compressibility modulus K_{v} at saturation density ρ_0,

$$K_{\mathrm{v}}=9\rho_0^2\frac{d^2(E/A)}{d\rho^2}\bigg|_{\rho=\rho_0}, \tag{2}$$

has drawn a lot of interest. It is a quantity that experimentally only can be extracted indirectly from small amplitude isoscalar density vibrations (breathing modes) of finite nuclei using either microsopic approaches with effective interactions in a random phase approximation (RPA) or macroscopic models for the breathing modes, e.g., the scaling model, where the finite-nucleus compressibility modulus K_{A} is droplet-model-like expanded into a series of volume, surface, etc., terms. Investigations along the given line fix this fictitious INM incompressibility K_{v} to a value of around 220 MeV [1, and references quoted therein], [2,3].

Ground-state-equivalent effective interactions or Lagrangians with equally good fits to masses and radii may lead to quite different K_{v} values. The S3 interaction with $K_{\mathrm{v}}=355$ MeV and the NL-SH Lagrangian [6] with $K_{\mathrm{v}}=355$ MeV give excellent fits to nuclear masses and radii similar to other Skyrme interactions such as, e.g., SkM* with an essentially different $K_{\mathrm{v}}=217$ MeV or the NL1 Lagrangian with $K_{\mathrm{v}}=211$ MeV. The same holds for SB interactions with $K_{\mathrm{v}}=234$ MeV and $K_{\mathrm{v}}=301$ MeV.

In the present investigation we try to understand how parametrizations, equivalent for masses and radii, can lead to such different predictions for the breathing modes. In particular we point to the fact that also the surface diffusenesses might come out to be different. A decrease of the surface diffuseness with the incompressbility K_{v} is found for a series of current interactions. Our conclusion is that only parametrizations that take into account the surface density are reliable, leading to K_{v} values low enough to be consistent with breathing-mode analyses.

The energy e per nucleon in isospin symmetric homogeneous INM can be written as a function of the density ρ_{c} by Taylor expanding around $\rho_{\mathrm{c}}=\rho_0$ with $\epsilon=(\rho_{\mathrm{c}}-\rho_0)/\rho_0$:

$$e(\rho_{\mathrm{c}})=a_{\mathrm{v}}+\frac{1}{18}K_{\mathrm{v}}\epsilon^2-\frac{1}{162}K'\epsilon^3+\cdots, \tag{3}$$

where the coefficient K' is defined by

$$K'\equiv-27\rho_0^3\frac{d^3e(\rho_{\mathrm{c}})}{d\rho_{\mathrm{c}}^3}\bigg|_{\rho_{\mathrm{c}}=\rho_0}. \tag{4}$$

Instead of K' the quantity S,

TABLE I. INM and SINM coefficients for different RMF, Skyrme, generalized Skyrme, Gogny, and generalized Seyler-Blanchard (Thomas-Fermi) parameter sets. The Sk surface properties were provided to us by Ref. [11]; the generalized SB properties were extracted from Refs. [12,13].

		a_v (MeV)	ρ_0 (fm^{-3})	K_v (MeV)	K' (MeV)	t (fm)	σ (MeV fm^{-2})	a_{sf} (MeV)
RMF (Hartree)	NL-Z	-16.18	0.1508	172.8	422.5	2.29	1.038	17.72
	NL1	-16.42	0.1518	211.1	32.7	2.24	1.098	18.66
	NLC	-15.77	0.1485	224.5	278.1	2.07	1.021	17.61
	NL3	-16.24	0.1482	271.5	-203.0	1.99	1.069	18.46
	NL-RA	-16.25	0.1570	320.5	-216.2	1.88	1.169	19.43
	NL-SH	-16.35	0.1460	355.3	-601.6	1.83	1.092	19.05
Skyrme (HF)	SkM*	-15.77	0.1603	216.6	386.0	2.45	1.074	17.60
	SkKM	-15.85	0.1607	220	292.2	2.43	1.072	17.54
	S3	-15.85	0.1453	355.4	-101.4	2.01	1.079	18.88
generalized Skyrme (HF)	SkK200	-15.85	0.1554	200	1189.1	2.25	1.053	17.62
	SkK220	-15.82	0.1536	220	616.4	2.17	1.038	17.50
	SkK240	-15.79	0.1519	240	434.7	2.08	1.025	17.41
Gogny (HF)	D1S	-16.02	0.166	209	543.4	2.46	1.138	18.21
	D1	-16.32	0.166	228	456.0	2.45	1.268	20.29
	D250	-15.86	0.1589	252.7	353.8	2.30	1.182	19.48
	D300	-16.23	0.1571	303.1	230.4	2.02	1.239	20.58
generalized SB (TF)	Set 1	-16.24	0.1611	234	252.7	2.4	1.14	18.63
	Set 2	-16.53	0.1654	301.3	194.6	2.09	1.26	20.27

$$S \equiv k_{F_0}^3 \left. \frac{d^3 e(k_F)}{dk_F^3} \right|_{k_F=k_{F_0}}, \tag{5}$$

is often used when the dependence of e on the Fermi momentum k_F is considered. It is connected to K' by the relation $S=-K'+6K_v$.

The energy density of a finite nucleus with density $\rho=\rho(\mathbf{r})$ in a local density approximation (LDA) is given by $e(\rho)\,\rho$. This is the main part of the energy density. Corrections come from density-gradient terms when the finite range of the nucleon-nucleon interaction is taken care of. In nuclei with mostly $\rho<\rho_0$ (i.e., $\epsilon<0$) an increase in K_v in Eq. (3) can be compensated for by a suitable decrease of the anharmonicity K'. Anyhow, increasing K_v must be balanced by higher-order terms since the whole expansion for $e(\rho_c)$ should go to zero for $\rho_c \rightarrow 0$.

In the following we consider ground-state-equivalent interactions in current use. They are all fitted at least to experimental masses and radii, and therefore they should have practically the same mass-formula coefficients a_v, a_{sf} etc., and ρ_0. In Table I we present a collection of properties obtained in relativistic Hartree approximation for different RMF parameter sets [4–8] as well as for Sk-G parametrizations that are summarized in Ref. [9] for the Sk and in Ref. [1] for the G functional, respectively, and for the SB interaction in the TF approach [12,13]. In addition to mass-formula coefficients and saturation densities ρ_0 we also present the surface tensions σ, the surface-energy mass-formula coefficients a_{sf}, and the 90%–10% surface thicknesses t obtained from SINM calculations.

One can see how for these interactions the anharmonicity K' of the equation of state (3) decreases with increasing incompressibility K_v. In Fig. 1 this relation between K' and the K_v values of ground-state-equivalent current interactions is displayed graphically. For each functional *Ansatz* one gets practically a linear decrease of K'. Even negative values of K' are possible.

Now, the question arises why only effective interactions with K_v around 220 MeV are able to reproduce breathing modes. Why is there not such a compensation of a K_v in-

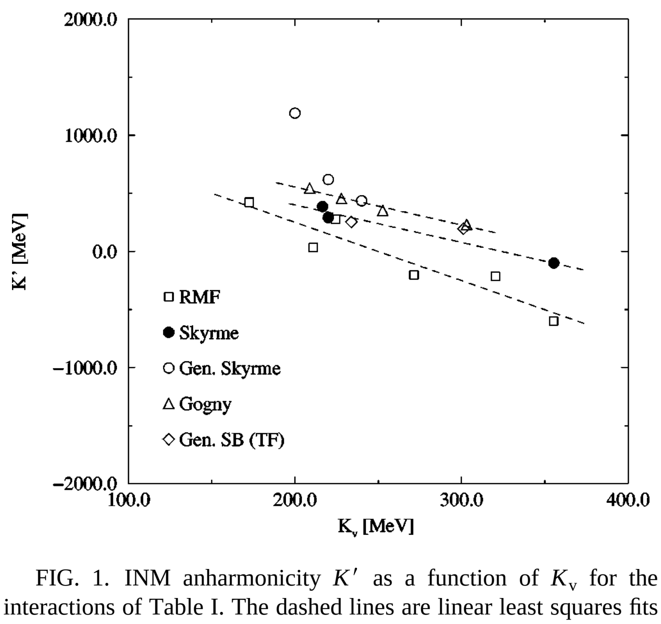

FIG. 1. INM anharmonicity K' as a function of K_v for the interactions of Table I. The dashed lines are linear least squares fits to the RMF, the Skyrme, and the Gogny values.

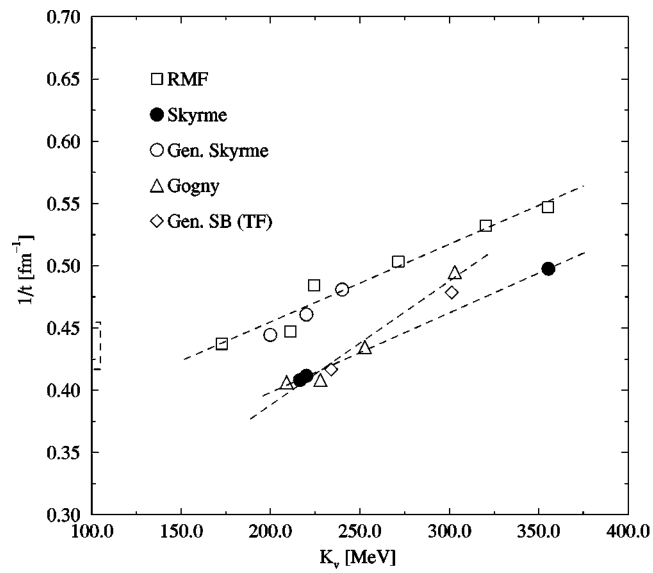

FIG. 2. $1/t$ vs K_v for the interactions of Table I. The dashed lines are obtained from linear regression to the $1/t$ values of the parameter sets for the RMF, the Skyrme, and the Gogny functional. The dashed box near the vertical axis indicates the experimental region for $1/t$ (see text for explanation).

crease by a suitable change of K' possible? How, in particular, can the unsuccessful attempt by Serr [10] be understood to change K_v and simultaneously K' in such a way that breathing modes remain unchanged? The answer is that breathing modes are density oscillations so small in amplitude that they do not feel the anharmonicity of the energy per particle curve as a function of density. The harmonic approximation is very accurate already. Therefore the breathing modes, and not the masses and radii, are a reliable probe to fix K_v.

There are parameter sets for Sk-G and SB interactions as well as for RMF Lagrangians where in addition to experimental masses and radii also charge densities and neutron densities are taken into consideration, in particular the surface densities with their finite surface diffusenesses. Examples are the SkM* Skyrme interaction and the NL1 Lagrangian. Remarkably, both interactions lead to an incompressibility modulus K_v of around 210 MeV consistent with the value obtained from breathing-mode analyses. The NL-SH Lagrangian with its too high K_v value was only fitted to masses and radii, not to diffraction radii and surface thicknesses (see remark in Ref. [7]). In Table I we display the SINM 90%–10% density falloff surface thicknesses t for a series of interactions. They can be studied as functions of K_v. In Fig. 2 the reciprocal $1/t$ is displayed as a function of the K_v value. For a given functional form of the interactions the increase of $1/t$ with K_v is nearly linear within the considered region of K_v values. From the experimental surface densities [14] a somewhat unsure experimental region for the SINM surface diffuseness can be extracted. From relativistic Thomas-Fermi calculations we found the t values of large nuclei to be larger by about 6% than the SINM value. We have indicated the experimentally acceptable region of SINM $1/t$ values on the vertical axis in Fig. 2. Very high values for K_v of more than 280 MeV can be definitely excluded, as well as very low values below 150 MeV. Obviously the realistic functional form of the energy density is not yet reached. What can be concluded nevertheless is that fits must take into account surface densities in order to end up with K_v values compatible with breathing modes.

TABLE II. The pocket formula values of K_v [Eq. (6)] for the interactions given in Table I. The quantities PF2 and PF3 are defined by PF2≡432 $\sigma/(5\ t\rho_0)$, PF3≡PF2−$K'/12$.

		σ/ρ_0 (MeV fm)	K_v (MeV)	PF2 (MeV)	PF3 (MeV)
RMF (Hartree)	NL-Z	6.88	172.8	260	225
	NL1	7.23	211.1	280	277
	NLC	6.87	224.5	288	264
	NL3	7.21	271.5	314	331
	NL-RA	7.45	320.5	342	360
	NL-SH	7.48	355.3	354	403
Skyrme (HF)	SkM*	6.70	216.6	236	204
	SkKM	6.67	220	237	213
	S3	7.43	355.4	319	328
generalized Skyrme (HF)	SkK200	6.78	200	260	161
	SkK220	6.76	220	269	218
	SkK240	6.75	240	280	244
Gogny (HF)	D1S	6.85	209	240	195
	D1	7.63	228	269	231
	D250	7.44	252.7	279	250
	D300	7.89	303.1	337	318
generalized SB (TF)	Set 1	7.08	234	255	234
	Set 2	7.63	301.3	316	299

Figures 1 and 2, and therefore our conclusions, follow from a survey over calculated properties. Obviously, there is a close connection between bulk and surface properties. In previous studies based on the Thomas-Fermi model using generalized Seyler-Blanchard interactions Myers and Swiatecki came to the conclusion that the surface diffuseness is a useful property to be taken into account for obtaining more reliable estimates of K_v. In particular they found a linear relationship between σ and K_v for a given value of the surface width [12,13]. In order to study this bulk-surface correlation directly we refer to a pocket formula [16] derived earlier that displays the connection between bulk and surface properties of saturated systems explicitly.

Using the stationarity of the SINM surface tension $\sigma(\rho_c)$ at saturation density ρ_0 with respect to density changes in the bulk and the stationarity of $\sigma(\rho_0)$ with respect to virtual changes of the surface density the following relation was derived after the INM energy per particle was expanded according to Eq. (3):

$$K_v = \frac{432}{5}\frac{\sigma}{\rho_0}\frac{1}{t} - \frac{1}{12}K'. \qquad (6)$$

(Note that in Ref. [16] t was defined as the total surface thickness.) It was asssumed that the SINM density falls off in the surface region linearly. Farine [17] has generalized this pocket formula (6) for more arbitrary density shapes and higher-order terms in the expansion for e. The relation (6) for the bulk quantities K_v, K', and ρ_0 and the surface quantities t and σ is checked in Table II for the RMF, Sk-G, and SB interactions. For these ground-state-equivalent interactions the ratio σ/ρ_0 is seen from Table II to be nearly constant with a standard deviation of 5%. Therefore relation (6) connects K' and K_v for these interactions in a simple way. Also a linear relation between $1/t$ and the INM incompressibility K_v is found. If one concentrates on the leading term in Eq. (6), one reproduces qualitatively the decrease of t with increasing K_v. The relation between K' and K_v (disregarding a possible change in t) can also be seen.

From our considerations we conclude that for a realistic parametrization of Sk-G and RMF interactions it is not sufficient to reproduce only masses and radii. As found by Myers and Swiatecki previously [12,13,15] in the TF model with generalized SB interactions it is important to fit also the nuclear surface density in order to reflect the compressibility properties of nuclei. Anyhow surface densities should be taken into account, in particular if an extrapolation to exotic nuclei with unusual surface properties should become realistic. For an interaction that can be extrapolated into regions of exotic nuclei with neutron halos at least the surface of normal nuclei should be reproduced. It is remarkable that if realistic surface densities are carefully taken into account, a realistic value of the INM incompressibility K_v results, consistent with breathing-mode analyses.

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