Space correlations and the pairing interaction in nuclei

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We study the space correlations induced by the pairing interaction in a superconductive nucleus. We find that it is possible to have pairs that present short range space correlations when they are described in the full Hilbert space, while the use of a reduced Hilbert space introduces complicated structures in the space correlations. The same types of phenomena are present when a δ function is described in the full or in a reduced Hilbert space. [S0556-2813(98)01810-X]

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The importance of elementary excitations constructed by pairs of nucleons as building blocks to describe the nuclear excitations has been known for a long time [1,2]. During the 1960s it was realized that the pairing interaction was relevant in the description of two particle transfer reactions in normal and superconductive nuclei [3,4]. Nevertheless, the problem of the spatial structure of the Cooper pairs in nuclei has not attracted much attention (Refs. [5–7]). The spatial structures obtained have many peculiar features, as can be seen in the figures of Ref. [5]: the pairs are not well localized in space, they depend on the shells considered in the description of the system and the maximum of the probability does not occur for small relative distances. The relation between particle transfer reactions and correlations in space induced by the interaction has been studied before by many authors [6–9].

We will follow the nomenclature of Ref. [5]. For a pair of nucleons moving in the full coordinate space the total wave function can be written as

$$\Psi(\vec{r}_1,\chi_1;\vec{r}_2,\chi_2)=\sum_{\alpha,\beta} B_{\alpha,\beta}[\psi_\alpha(\vec{r}_1,\chi_1)\otimes\psi_\beta(\vec{r}_2,\chi_2)], \tag{1}$$

where this wave function is the general antisymmetric wave function describing these two particles. α corresponds to all the quantum numbers needed to fully describe the single particle states.

In order to display in a simple way the space distribution of the nucleon pair it is convenient to introduce the center of mass and relative coordinates and to integrate the square of the wave function over the angular and spin variables, i.e.,

$$\mathcal{S}(r,R)=\int |\Psi(\vec{r}_1,\chi_1;\vec{r}_2,\chi_2)|^2 d\hat{r}d\hat{R}d\chi_1 d\chi_2. \tag{2}$$

The variable displayed in Ref. [5] $P(r,R)$ is related to $\mathcal{S}(r,R)$ in simple terms $[P(r,R)=r^2R^2\mathcal{S}(r,R)]$.

It is convenient to use for the single particle wave functions those of the harmonic oscillator (HO). The wave functions in terms of center of mass and relative coordinates in the HO case can be written by using the Brody-Moshinsky brackets. In this case Eq. (2) yields

$$\begin{aligned}\mathcal{S}(r,R)=&\sum_{\alpha_1,\alpha_2} B_{\alpha_1,\alpha_1}B_{\alpha_2,\alpha_2}\sum_L \hat{L}^4\hat{j}_{\alpha_1}{}^2\hat{j}_{\alpha_2}{}^2\\ &\times\begin{Bmatrix} l_{\alpha_1} & l_{\alpha_1} & L\\ \frac12 & \frac12 & L\\ j_{\alpha_1} & j_{\alpha_1} & 0\end{Bmatrix}\begin{Bmatrix} l_{\alpha_2} & l_{\alpha_2} & L\\ \frac12 & \frac12 & L\\ j_{\alpha_2} & j_{\alpha_2} & 0\end{Bmatrix}\\ &\times\sum_{n,n',N,N',\lambda,\Lambda}\langle n_{\alpha_1},l_{\alpha_1},n_{\alpha_1},l_{\alpha_1},L|n,\lambda,N,\Lambda,L\rangle\\ &\times\langle n_{\alpha_2},l_{\alpha_2},n_{\alpha_2},l_{\alpha_2},L|n',\lambda,N',\Lambda,L\rangle\phi_{n\lambda}\\ &\times(r)\phi_{N\Lambda}(R)\phi_{n'\lambda}(r)\phi_{N'\Lambda}(R).\end{aligned} \tag{3}$$

We performed our study in $_{50}Sn^{114}$ using as two-body Hamiltonian the schematic pairing interaction. We choose different sets of single particle levels to study the effect of the Hilbert space size on the structure of the "Cooper pairs."

It is convenient to use the BCS approximation to treat the pairing interaction. In this case the correlated ground can be written as

$$\begin{aligned}|BCS\rangle&=\prod_{i>0}(U_i+V_i a_i^\dagger a_{\bar{i}}^\dagger)|0\rangle\\ &=\mathcal{N}*\exp\sum_i\left(\frac{V_i}{U_i}\right)a_i^\dagger a_{\bar{i}}^\dagger|0\rangle=\mathcal{N}\exp^{\Gamma^\dagger}|0\rangle,\end{aligned} \tag{4}$$

where $a_i^\dagger$ creates a particle with quantum numbers i and $\Gamma^\dagger$ creates a pair of nucleons. $\Gamma^\dagger$ can be considered as an operator that is proportional to the one that creates a "Cooper pair" and as we only want to display relative probabilities, we will not take care on the normalization of $\Gamma^\dagger$.

We can then use this two particle wave function as the one of Eq. (1). Thus $B_{\alpha,\beta}=\delta_{\alpha,\beta}(V_\alpha\sqrt{2j_\alpha+1}/U_\alpha)$ and Eq. (3) can be used to evaluate $\mathcal{S}(r,R)$.

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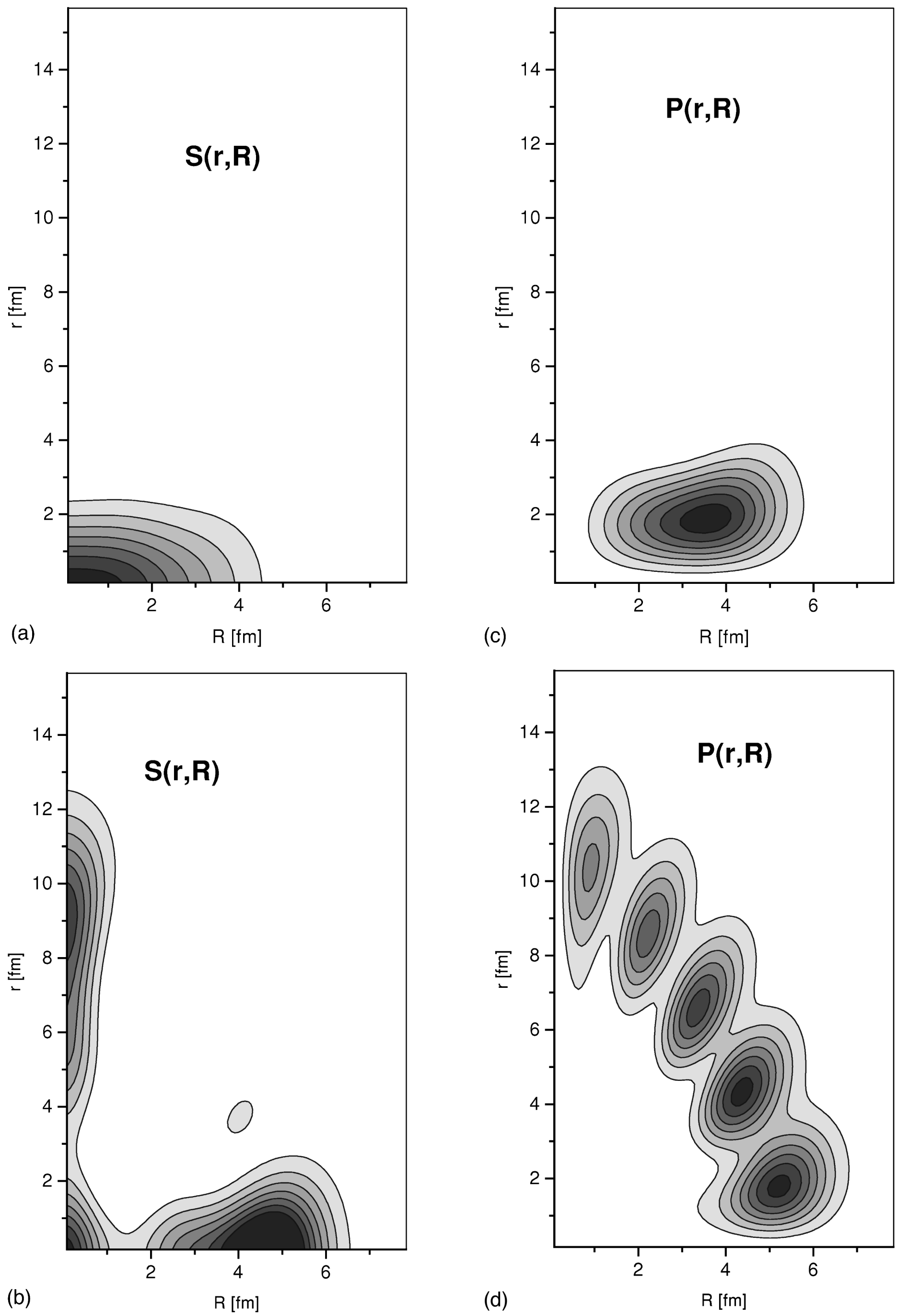

FIG. 1. Countor plots of $\mathcal{S}(r,R)$ [(a) and (b)] and $P(r,R)$ [(c) and (d)]. In (a) and (c) we use the larger Hilbert space ($N=0$ to $N=6$ with $G_n=14.5$ MeV/nucleon) while in (b) and (d) we use the reduced one (five levels with $G_n=21$ MeV/nucleon).

Spherical HO levels including corrections due to the centrifugal and spin-orbit interactions were used as single-particle levels [10,11]. We used these single-particle levels starting at $N=2n+l=0$ up to $N=6$ (2 shells above the Fermi energy), except for the levels close to the Fermi surface, were we used those from Ref. [12]. Those substitutions were done so that the sum of the single particle energy times the degeneracy of the levels involved remains constant.

The contour plot of the functions $\mathcal{S}(r,R)$ and $P(r,R)$ depend strongly on the size of the Hilbert space considered in their evaluation. We used two different Hilbert spaces: the first one was formed by all the single particle levels from $N=0$ to $N=6$. The other was formed only by the five single particle levels that are usually considered in the description of ${}_{50}\mathrm{Sn}^{114}$. As the value of the pairing gap Δ depends on the size of the Hilbert space we used two different strengths: one is the value that gives the right Δ when using five levels ($G=21$ MeV/nucleon), the other one yields the same value for Δ when using all the levels from $N=0$ to $N=6$ ($G=14.5$ MeV/nucleon). The fact that the gaps have the same value implies that the resulting U and V factors for the levels near the Fermi surface will almost be the same, but for the levels away from the Fermi surface, the U and V factors will not be exactly 1 or zero.

In Fig. 1 we display the contour plot of the functions $\mathcal{S}(r,R)$ [(a) and (b)] and $P(r,R)$ [(c) and (d)]. In (a) and (c)

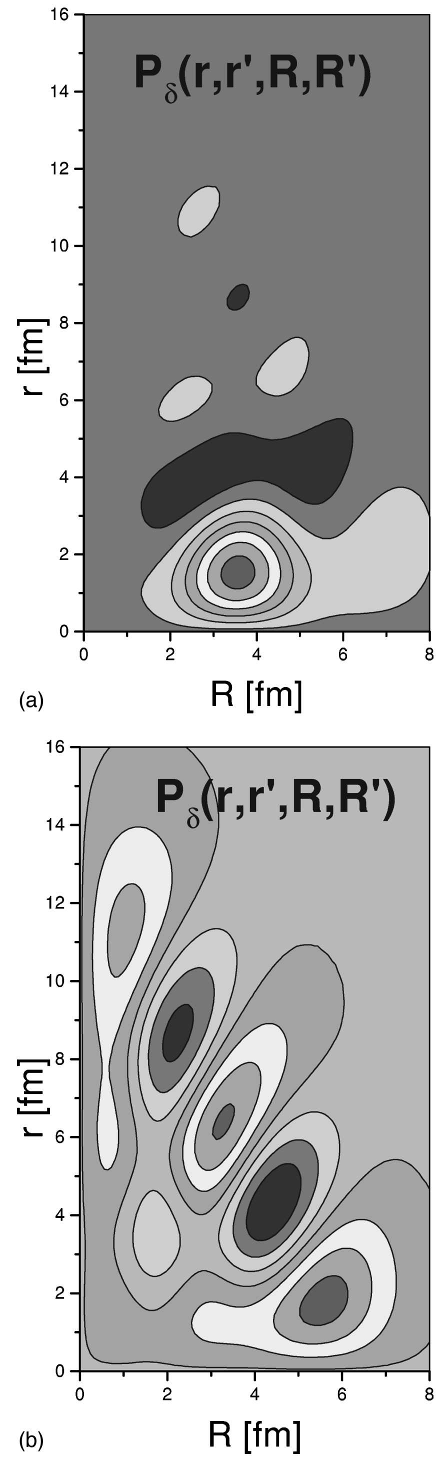

FIG. 2. Countor plots of $P_\delta(r,r',R,R')$ using $r'=0.01\mathbf{b}$ and $R'=2.75\mathbf{b}$. In (a) we use the larger Hilbert space ($N=0$ to $N=6$) while in (b) we use the reduced one (five levels).

we use the larger Hilbert space with $G=14.5$ MeV/nucleon, while in (b) and (d) we use the reduced one with $G=21$ MeV/nucleon.

It is found that once the system is superconductive the results are almost independent of the strength of the pairing interaction. It can also be seen that the square of the wave function $\mathcal{S}(r,R)$ has less structure than the probability $P(r,R)$. On the other hand, the consideration of a larger Hilbert space decreases dramatically the appearance of complicated structures. In the larger space the pairs have strong short range correlations not only in the square of the wave function, but also in the probability distribution. The maxima of the former take place for reasonable values not only for the relative distances but also for the value of the center of mass.

The results obtained when using the reduced Hilbert space have similar features than those obtained in Ref. [5]: the inclusion of the $0h_{11/2}$ state makes the probability distribution more asymmetric, shifting it to larger values of R and correspondingly to smaller values of r. Besides, the number of maxima is similar to N, as was noted in Ref. [5].

When using the larger Hilbert space it is difficult to compare in detail our results for ${}_{50}\mathrm{Sn}^{114}$ with those of Ref. [6] but their wave function (that includes all the shells from $N=0$ to $N=5$) has its most important feature in common with ours: it is strongly peaked for small values of r.

It was difficult to understand how an interaction of short range, as one believes the pairing interaction is, may induce spatial structures so complicated and clearly not of short range when working in a reduced Hilbert space. A possible explanation can be that the effective interaction in the reduced Hilbert space (implied by the use of only five shells close to the Fermi surface) is not really of short range but has a complicated structure. These structures are artifacts due to the fact that in a reduced Hilbert space one can only reproduce functions similar to those of the basis.

To check this possibility we consider the influence of the Hilbert space size in the representation of the $J=0$ part of a short range function, such as $\delta(\vec{r}-\vec{r}\,')\delta(\vec{R}-\vec{R}\,')$. It is

$$\delta(\vec{r}-\vec{r}\,')\delta(\vec{R}-\vec{R}\,') = \frac{1}{4}\sum_{S,S',M_S,M_{S'},\alpha\geqslant\beta,J,M} \langle \vec{r},\vec{R},S,M_S|\alpha j_\alpha \beta j_\beta,JM\rangle \times\langle \alpha j_\alpha \beta j_\beta,JM|\vec{r}\,',\vec{R}\,',S',M_{S'}\rangle. \tag{5}$$

In order to look at the existence or not of complicated structures depending on the Hilbert space size, it is convenient to use the HO wave functions as a complete basis to write down the space representation of these $\delta's$ functions. Using Eq. (2) we can integrate over all the angles and sum over all the spins using for the intermediate states two particle HO antisymmetric states. In this way we obtain for the $J=0$ part of the $\delta's$

$$\mathcal{S}_\delta(r,r',R,R') = \sum_{\alpha\geqslant\beta}\sum_{\lambda} \hat{\lambda}^4 \hat{j}_\alpha^2 \hat{j}_\beta^2 \begin{Bmatrix} l_\alpha & l_\beta & \lambda \\ \frac{1}{2} & \frac{1}{2} & \lambda \\ j_\alpha & j_\beta & 0 \end{Bmatrix}^2 \times \sum_{nlNLn'N'} \langle n_\alpha l_\alpha n_\beta l_\beta \lambda|nlNL\lambda\rangle \times\langle n_\alpha l_\alpha n_\beta l_\beta \lambda|n'lN'L\lambda\rangle \phi_{nl}(r)\phi_{n'l}(r') \times\phi_{NL}(R)\phi_{N'L}(R'). \tag{6}$$

Of course here we do not have the B_{α_1,α_2} factors as in Eq. (3) as we are considering a short range function and not the two particles wave function. In a similar way as we did with the square of the wave function, we can define $P_\delta(r,r',R,R') = rr'RR'\mathcal{S}_\delta(r,r',R,R')$.

In Fig. 2 we display $P_\delta(r,r',R,R')$ for the δ functions using both the large and the reduced Hilbert spaces considered in the model calculation of $_{50}\mathrm{Sn}^{114}$. In order to have a similar display as those in Figs. 1 and 2, we used the values $r'=0.01\mathbf{b}$ and $R'=2.75\mathbf{b}$. It is seen that in the small Hilbert space the δ functions are poorly reproduced, while in the larger space their representation is more appropriate. The similarities between the results shown in Fig. 1 and Fig. 2 are remarkable, and prove that the assumption that connects the existence of complicated structures in the two particle correlations with the reduction of the Hilbert space is basically correct.

The pairing interaction, which usually is thought of as the short range part of the interaction, is a phenomenological one and it is considered by many authors to be schematic. Nevertheless recently [13] it has been found that the pairing interaction is the most important part of the phenomenological interactions derived from several realistic forces. These authors also suggested the convenience of using a renormalization of the force as one moves to levels away from the Fermi surface. We have been able to use up to 7 major shells obtaining the same value for the gap parameter, therefore using a BCS wave function that in the region near the Fermi surface is similar for both types of Hilbert spaces. The similitude between these wave functions makes plausible the explanation that the increase in the Hilbert space size does not induce big changes in the pair wave function but allows for the cancellations of small parts that in the reduced Hilbert space seem to produce long range correlations.

One may conclude that the complicated structures found in the pair wave functions [5] are mainly due to the small size of the Hilbert space used. The pairing interaction in a larger Hilbert space [13] will in general yield structures for the pairs that are short range.

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