Measurement of the $^{1}H(\gamma,\pi^{0})$ cross section near threshold. III. Angular coefficients

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This paper is an addendum to our previous communications on the measurement of the $^{1}H(\gamma,\pi^{0})$ reaction in the threshold region. We present the coefficients of angular distribution, not described previously, and comment in particular on the asymmetry coefficient *B*. [S0556-2813(98)04310-6]

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This Brief Report is a supplement to our previous papers on the measurement of the $^{1}H(\gamma,\pi^{0})$ reaction near threshold [1,2]. The main objective of those studies was to deduce the low-energy characteristics of the *S*- and *P*-wave photoproduction amplitudes E_{0+} and P_1, respectively, where the latter is defined in terms of the fundamental *P*-wave multipoles by $P_1=3E_{1+}+M_{1+}-M_{1-}$. Near threshold, the energy development of P_1 was described by the familiar ansatz $P_1=p_1\cdot kq$ where k and q are, respectively, the photon and pion momenta in the πN c.m. frame (expressed in units of the pion mass), and the ''reduced'' amplitude p_1 is assumed to be constant, or nearly so. In Ref. [2] the differential cross section in the c.m. frame was written

$$\frac{k}{q}\frac{d\sigma}{d\Omega}=a+b(1-\cos\theta)+c\sin^2\theta, \tag{1}$$

where θ is the pion angle. The coefficients $a-c$ are combinations of the *S*- and *P*-wave multipoles as described in Ref. [2]. The rather unorthodox functional form of Eq. (1) was dictated by the Monte Carlo model used to evaluate the response function of the π^0 spectrometer IGLOO as described in Refs. [2,3]. Actually, the Monte Carlo analysis of the pion angular distributions sidestepped the angular coefficients altogether by expressing the coefficients directly in terms of the elementary *S*- and *P*-wave amplitudes, since these were the quantities of immediate interest. This scheme also permitted energy continuity to be imposed on the *P*-wave amplitudes in a simple fashion.

The more conventional representation of the differential cross section, employed, for example, by the Mainz group [4], is

$$\frac{k}{q}\frac{d\sigma}{d\Omega}=A+B\cos\theta+C\cos^2\theta \tag{2}$$

which is related to Eq. (1) by a trivial linear transformation. Our purpose in this report is to present the coefficients $A-C$ as determined from the SAL measurements. We also use the opportunity to display the differential cross sections free from the distorting effects of the π^0 spectrometer, unlike the distributions presented in Ref. [2]. A successful algorithm for treating the distortions was invented subsequent to the publication of Ref. [2].

The $^{1}H(\gamma,\pi^{0})$ differential cross sections are displayed in Fig. 1. As in Ref. [2], each distribution is based on a grouping of four detector channels of the photon-tagging apparatus and thus subtends about 2 MeV of excitation. Each datum point subtends 10° of pion angle. The solid curves in Fig. 1 are *not* fits to the data as displayed. Rather, they represent the cross sections as predicted by the amplitudes $\mathrm{Re}\,E_{0+}$ listed in Table I of Ref. [2], $\mathrm{Im}\,E_{0+}$ as modeled there, together with the reduced *P*-wave amplitudes

$$p_1=(10.26\pm0.10)\times10^{-3}/m_{\pi},$$
$$f_0=(7.91\pm0.03)\times10^{-3}/m_{\pi} \tag{3}$$

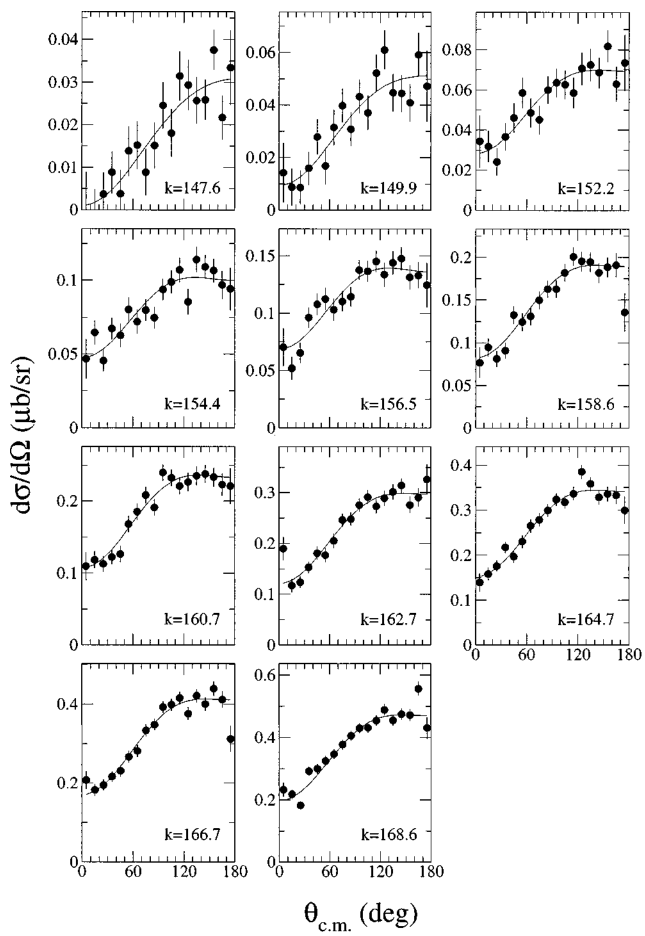

FIG. 1. Differential cross sections for the reaction $^{1}H(\gamma,\pi^{0})$ in the πN c.m. frame, corrected for the distorting effects of the π^0 spectrometer. Mean photon energies in the laboratory frame are indicted. Each distribution subtends about 2 MeV of excitation. The curves are *not* fits to the cross sections as displayed, but are calculated from the photopion amplitudes as deduced in our earlier papers (see text).

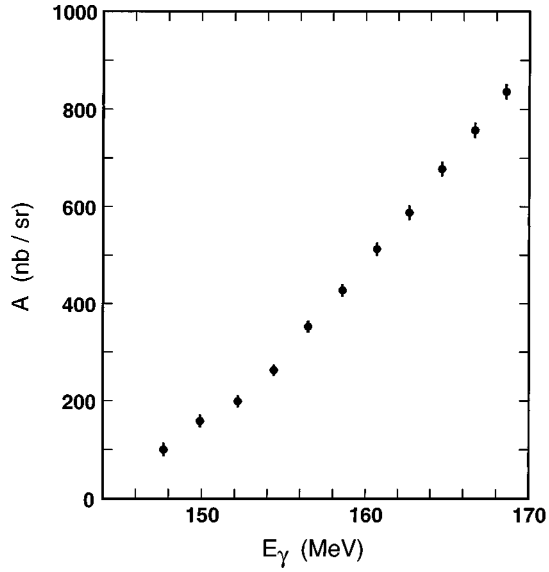

FIG. 2. Angular distribution coefficient A [Eq. (2)] deduced from the differential cross sections of Fig. 1.

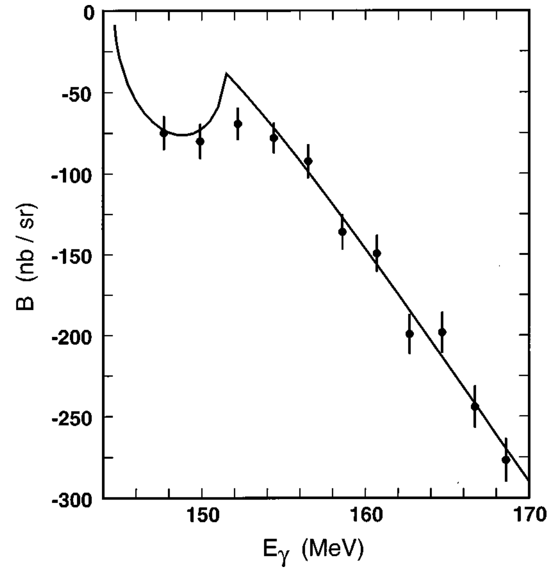

FIG. 3. Angular distribution coefficient B [Eq. (2)]. The curve is given by Eq. (5) using the reduced amplitude p_1 of Eq. (3), deduced in Ref. [2] prior to spectrometer compensation, together with the theoretical model for E_{0+} described in the text.

as deduced in Ref. [2] by a simultaneous fit to all distributions *prior* to the removal of the IGLOO distortions. The quantity f_0 is a composite P-wave amplitude introduced for analytical convenience (see Ref. [2]). The S- and P-wave parameters which generate the curves in Fig. 1 were deduced from a Monte Carlo analysis of the original pion angular distributions. The excellent agreement evident in Fig. 1 is thus an affirmation of the amplitudes deduced in our earlier work, and of the procedure for addressing the π^0 spectrometer response.

The angular coefficients A–C are determined by least-squares fits of Eq. (2) to the cross sections depicted in Fig. 1. The distributions are treated independently, that is we do *not* enforce a continuity with energy as was done in Ref. [2] and which lead to the reduced amplitudes of Eq. (3). The resulting angular coefficients are displayed in Figs. 2–4 and are tabulated in Table I.

A general discussion of these results in terms of fundamental theory will not be given here. However, we will comment on the coefficient B since, among the coefficients, it alone directly reflects the interference between the S-wave and P-wave photoproduction amplitudes. The coefficient B is given by the general expression

$$B = 2\ \mathrm{Re}(E_{0+}P_1^*), \tag{4}$$

where the amplitude P_1 has been defined previously in terms of the elementary P-wave multipoles. At low energy the latter are essentially real quantities, and invoking the low-energy ansatz, we write

$$B = 2(\mathrm{Re}\,E_{0+})\cdot p_1 kq, \tag{5}$$

where p_1 is the reduced P-wave amplitude. The solid curve in Fig. 3 follows from Eq. (5) using the reduced amplitude p_1 of Eq. (3) together with the qualitative estimate of $\mathrm{Re}\,E_{0+}$ as provided by the ''subtracted'' Lippmann-Schwinger formulation described in Ref. [5]. In that construct there are two free parameters—the threshold value of E_{0+} defining the subtraction constant, and the mass parameter α which occurs in the momentum representation of the $\pi^+ n \to \pi^0 p$ charge exchange amplitude. The threshold amplitude is constrained by experiment, $E_{0+}(\mathrm{thr}) = -1.3\times 10^{-3}/m_\pi$ [1,4]. With $\alpha = 250$ MeV/c (which falls within the range of the phenomenological estimates of Ref. [5]), the Lippmann-Schwinger formulation gives a very good description of $\mathrm{Re}\,E_{0+}$ as demonstrated for example in Refs. [2,5].

As can be seen from Fig. 3, the experimental results for B and the predictions from Eq. (5) are in good agreement over the entire energy range. In particular, we understand the rather sudden change in slope as one approaches π^0 threshold ($E_{\mathrm{th}} = 144.7$ MeV) as a direct reflection of the deep cusp in $\mathrm{Re}\,E_{0+}$. The effect of the cusp on the B coefficient has also been discussed by Bernstein *et al.* [6] in their interpretation of the Mainz data.

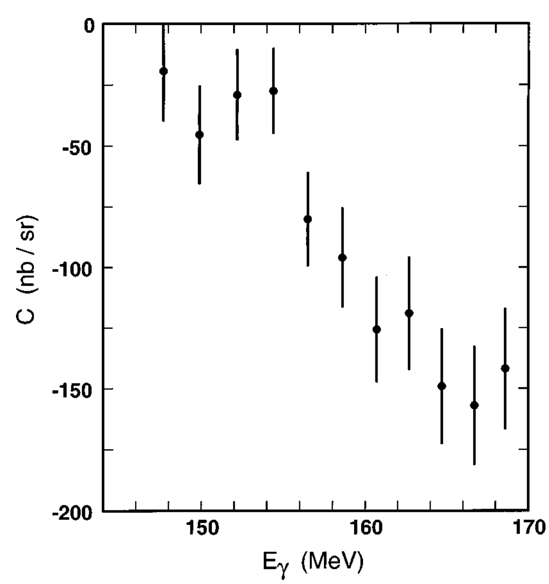

FIG. 4. Angular distribution coefficient C [Eq. (2)].

TABLE I. Angular distribution coefficients A–C of Eq. (2) as deduced from the cross sections of Fig. 1, as a function of the incident photon energy E_γ in the laboratory frame. Units are nb/sr.

E_γ (MeV)	A	B	C
147.6	100±12	−75±10	−19±20
149.9	158±11	−80±10	−45±20
152.2	199±10	−69±9	−29±18
154.4	263±10	−78±9	−28±17
156.5	352±10	−92±10	−80±19
158.6	427±11	−136±10	−96±20
160.7	512±12	−149±11	−126±21
162.7	588±13	−199±12	−119±23
164.7	677±13	−198±12	−149±23
166.7	757±13	−244±12	−157±24
168.6	836±14	−277±13	−142±25

The influence of the cusp on the experimental results in Fig. 3 is less dramatic than the prediction would suggest. We recall, however, that the experimental points each span about 2 MeV, so incorporating the appropriate response function into the prediction would soften the cusp in the direction of experiment.

Finally, let us compare the present angular coefficients A–C with those derived from the Mainz measurements as presented in Ref. [6], in the energy domain where the two analyses overlap ($E_\gamma \leq 160$ MeV). From a visual inspection of the plotted coefficients, there appears to be general agreement between the A coefficients at the 5–10 % level. The same might also be said for the C coefficients, although the much larger errors makes a quantitative comparison problematical. However, there appears to be a large discrepancy between the two estimations of the B coefficient. For example, at $E_\gamma = 160$ MeV the present results are about 50% larger than those of Ref. [6], which cannot be attributed to our coarser energy bins. Again, at $E_\gamma = 150$ MeV (i.e., the bottom of the loop in Fig. 3) the present results exceed those of Ref. [6]. However, at the cusp apex ($E_\gamma = 151.4$ MeV), the B coefficients of Ref. [6] are closer to the predicted curve in Fig. 3, perhaps because of the finer energy resolution than we have employed for the present analysis. The discrepancy between the B coefficients, especially at the higher energies where it is most severe, has not been fully resolved to date. The source of the disagreement probably lies in the different fore-back asymmetries exhibited by the respective pion angular distributions—the asymmetry appears to be more pronounced in the SAL measurements.

To summarize, the angular distribution coefficients A–C for the reaction ${}^1H(\gamma,\pi^0)$ have been extracted from the SAL measurements. The asymmetry coefficient B is in close agreement with a prediction based on a previous determination of the reduced P-wave amplitude p_1 together with a model calculation of Re E_{0+}. The coefficients A and C are in qualitative agreement with the amplitudes deduced from the Mainz measurements, but a significant discrepancy occurs between the respective estimates of the coefficient B. Tabulated values of the differential cross section, corrected for spectrometer distortion, are available on the SAL web page at URL http://sal.usask.ca

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