Revised Q values for the superallowed positron decays of ^{10}C and ^{14}O

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The original data for precise determinations of the $^{10}B(p,n)^{10}C$ and $^{14}N(p,n)^{14}O$ threshold energies have been reanalyzed to fully incorporate the effects of proton beam energy spread, nonuniform proton energy loss in the targets, and atomic ionization. The results have been used to give Q values for the 0^+ to 0^+, $T=1$ superallowed positron decays of ^{10}C and ^{14}O.
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As emphasized recently by Towner and Hardy [1], the determination with high precision of the intensities, or Ft values, of nuclear positron decays between 0+, $T=1$ members of isospin triplets, continues to shed light on the validity of the conserved vector current theory of the weak interaction, and to provide a test of the unitarity of the Cabibbo-Kobayashi-Maskawa matrix. At present, there are nine such decays for which the precision is at the 0.1% level.

One of the limitations to our knowledge of the Ft values comes from uncertainties in the calculation of the charge-dependent corrections which modify the simple theory of the transitions. Indeed, Wilkinson has proposed, Ref. [2], that the suspect parts of the Z-dependent calculations be omitted entirely, and that, without them, the Ft values could be extrapolated down to $Z=0$. This would then be the most trustworthy value to take into weak interaction studies. Alternatively, one can take the best corrections available and average the Ft values. Whatever the relative merits of the different approaches, the decays of the two nuclei with the lowest atomic number, namely ^{10}C and ^{14}O, take on a particular importance.

For the ^{10}C superallowed decay, with a quoted Ft of 3074.0(54) s [1], i.e., an error of 0.18%, the contributing errors from the experimental components are: branching ratio 0.15%, half-life 0.06%, and energy, via the F value, 0.04%. This last is actually a reflection of the uncertainty in the transition Q-value energy, 0.005%, or 90 eV in 1900 keV, where the energy is derived directly from a measurement of the threshold energy of the $^{10}B(p,n)^{10}C$ reaction made at our laboratory in Auckland (Baker *et al.* [3]), 4876.91(10) keV.

Subsequent to the work reported in [3], we have developed a method for considering various small, principally atomic, effects which influence the shape of the (p,n) yield curve near threshold, and which therefore bear on the extraction of the threshold energy. These are discussed in detail in Ref. [4] and their application is illustrated in Ref. [5]. Since the effects are expected to cause a shift of the order of 100 eV to the threshold, and hence to the positron Q value, they should be incorporated. In addition, although the error in the final Ft value, with or without the radiative corrections, is currently dominated by the uncertainty in the branching ratio, an improvement in the latter seems imminent [6], and so the energy should be properly established.

The Ft value for the decay of ^{14}O is given in Ref. [1] as 3069.2(28) s. Here there is no corresponding difficulty in a branching ratio value, but the decay energy is averaged over six reported values, which are mutually somewhat inconsistent. Of these six the most significant contributor to the mean decay Q value is that from this laboratory, 2830.31(8) keV (White *et al.* [7]), which similarly was derived from a determination of the energy of the threshold for the corresponding $^{14}N(p,n)^{14}O$ reaction and again, this should be corrected.

The application of the corrections alluded to above is made particularly direct by virtue of the accessibility of the original data for all the $^{10}B(p,n)^{10}C$ and $^{14}N(p,n)^{14}O$ runs. As emphasized in Ref. [4], the most reliable way of applying the corrections is by incorporating the effects in the analysis of the (p,n) yield data, rather than by simply shifting the uncorrected threshold energy.

In a typical (p,n) energy threshold measurement, the yield of the reaction is measured at around 20–25 proton energies E, spanning a range from a few keV below to a few keV above the threshold. In the simplest approach to extraction of the threshold energy E_0, the neutron production is assumed to be s wave, which leads to a $(E-E_0)^{1/2}$ dependency for the cross section. Since the targets are almost always thick to the incoming beam, where ''thick'' means the energy of a proton has dropped below E_0 before it has left the target, the dependency of the yield $Y(E)$ on the energy E becomes $Y(E)=Y_{\rm norm}(E)=\alpha_1+\alpha_2(E-E_0)^{3/2}$, on the assumption that the protons lose energy uniformly as they pass through the target material. In this expression, α_1 represents a constant, energy-independent background, which can sometimes be zero. So the extraction of the threshold energy from a yield curve is effected by fitting the data points to the expression and obtaining the three parameters α_1, α_2, and E_0.

As discussed in detail in Ref. [4], there are three obvious physical effects which are not taken into account in the sample treatment outlined above. Firstly, the incoming proton beam is not monoenergetic. For the data whose results are reported in Ref. [3] the beam energy distribution Y_g was as illustrated by the points in Fig. 1, where the continuous line represents a Gaussian distribution with a full width at

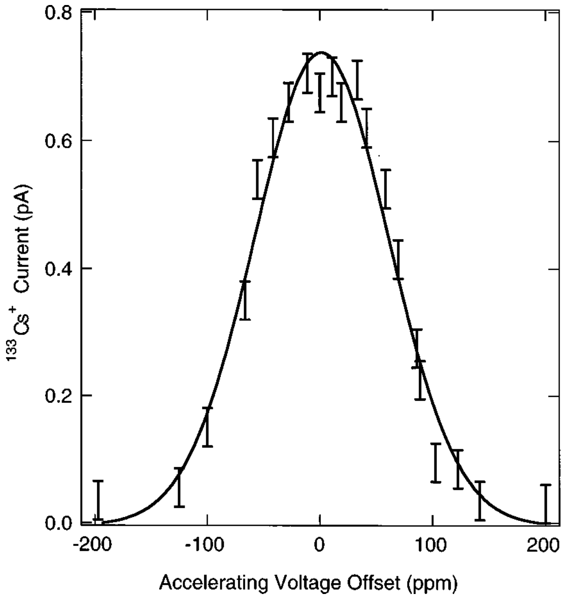

FIG. 1. Energy distribution of the $^{133}Cs^{+}$ and the proton beams for the $^{10}B(p,n)^{10}C$ measurements.

half maximum of 740 eV. Secondly, the protons, in slowing down, do not lose energy uniformly, but rather the probability for losing energy Q is proportional to $1/Q^2$. This favors small energy losses and so leads to an energy distribution, Y_Q, in the target which has more higher energy protons than a uniform one, and this is illustrated in Fig. 2. Thirdly, there is a finite probability that a proton initiating a (p,n) reaction also ionizes the atom. In Fig. 3 are shown the differential probabilities, Y_K, Y_{L1}, and Y_{L2} for ejection of an electron from the atomic subshells, summed, as a function of the proton energy lost. The "edges" correspond to the *K*, *L1*, and *L2* electron binding energies of boron.

An expression corresponding to Y_{norm}, but which also includes the above effects is obtained as follows: a yield is calculated according to an $(E-E_0)^{1/2}$ energy dependency, in 1 eV steps, and this is convolved with Y_q. The result is then convolved with each of Y_K, Y_{L1}, and Y_{L2} in turn, and then this is finally convolved with Y_g, to give a function $Y_{\mathrm{corr}}(E)$ to which the data may be fitted as $Y(E)=\alpha_1+\alpha_2 Y_{\mathrm{corr}}(E-E_0)$ and E_0 extracted. Both $Y_{\mathrm{norm}}(E)$ and $Y_{\mathrm{corr}}(E)$ are illustrated in Fig. 4. They have equal area and an assumed threshold energy of 0 keV. Their difference is also shown, increased by a factor of 50 for clarity.

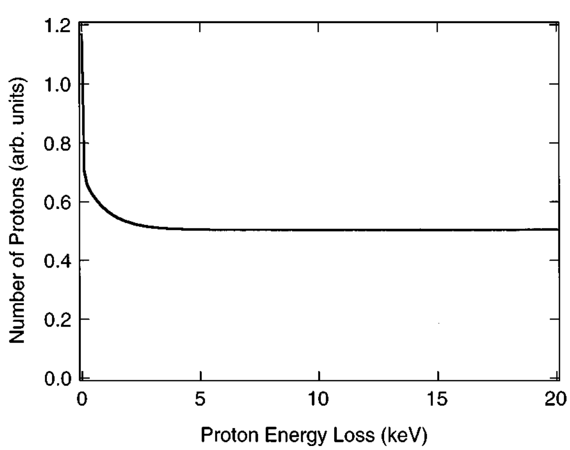

FIG. 2. Energy-loss distribution of an initially monoenergetic proton beam of 4.88 MeV in a tantalum nitride target which is 20 keV thick.

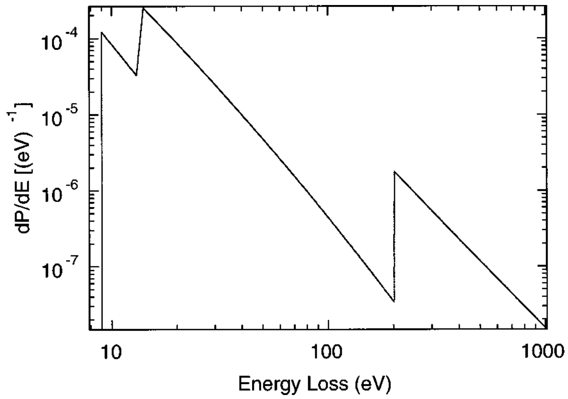

FIG. 3. Differential energy-loss probability for a proton initiating the $^{10}B(p,n)^{10}C$ reaction near threshold.

In Table I are shown the results for the eight threshold energy measurements. The values E_{norm} were obtained by analyzing the yield curves using the functional form Y_{norm} above, i.e., they are uncorrected. The E_{orig} are the values which appeared in Ref. [3] and were corrected only for a triangular beam energy distribution. (Note the unfortunate typographical error in the result for run 5.) The E_{corr} threshold energies are obtained by analyzing the yield curves using the Y_{corr} function above. The mean from these last will be taken as the result, $E_0=4877.10(10)$ keV.

Estimates, using artificial data, of the sizes of the individual corrections due to Y_g, Y_Q, and $(Y_K+Y_{L1}+Y_{L2})$ give $+0.01$, $+0.28$, and -0.09 keV, respectively, and an overall total of $+0.20$ keV, which is in reasonable agreement with the actual shift $(4877.10-4876.93=+0.17)$ keV.

There are two adjustments to the value of E_0, at the level of a few parts per million (ppm), which must be made. Since our energy measurements are based on a Josephson-derived 1-volt standard, E_0 must be reduced by 9 ppm to take into account the redefinition of the volt which has taken place since the time of the original work. Also E_0 must be reduced by a further 6 ppm to incorporate a feature of our electrical

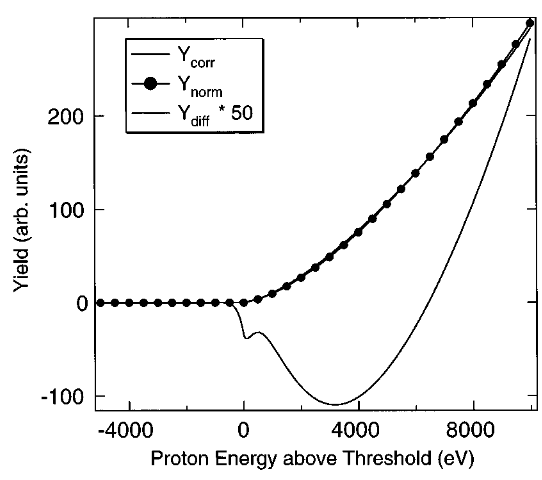

FIG. 4. Expected dependence on proton energy of the yield of the $^{10}B(p,n)^{10}C$ reaction near threshold. Y_{norm} is the simple $(E-E_0)^{3/2}$ behavior. Y_{corr} incorporates the effects discussed in the text. Y_{diff} is their difference.

TABLE I. Analysis of the yield curves of the $^{10}B(p,n)^{10}C$ reaction. The energies E_{norm} come from an $(E-E_0)^{3/2}$ analysis, E_{orig} appeared in Ref. [3] and E_{corr} are obtained using the functional form Y_{corr} (see text).

Run number	$(E_{norm}-4870)$ keV	$(E_{orig}-4870)$ keV	$(E_{corr}-4870)$ keV
1	6.77(25)	6.76(25)	6.97(24)
2	7.16(35)	6.91(30)	7.30.(34)
3	7.20(29)	7.17(30)	7.32(28)
4	6.78(28)	6.79(28)	6.93(27)
5	6.84(26)	6.85(26)	7.03(25)
6	6.84(29)	6.88(30)	7.01(29)
7	6.91(39)	7.14(31)	7.07(37)
8	7.34(47)	7.42(31)	7.59(46)
Mean	6.93(11)	6.97(10)	7.10(10)

technique whose effect had earlier been thought negligible. Finally, into the purely statistical error of 0.10 keV must be added a component of 0.08 keV representing the uncertainty in the three corrections discussed above, and one of 8 ppm, which is the systematic error in the energy measuring system, giving as the final threshold 4877.03(13) keV. In Ref. [3] the value of E_0 from these measurements was averaged with an earlier one from this laboratory, but we do not do that here, since, although the earlier results should be subject to the same kinds of correction as discussed above, the original data are not available to be reanalyzed.

Using this value of E_0, the Q value for the $^{10}B(p,n)^{10}C$ reaction becomes $-4430.30(12)$ keV, and that for the hypothetical electron capture decay of ^{10}C becomes 1907.86(12) keV, where 782.354 keV has been used as the mass difference (^1H-n), 1740.08(1) keV is the excitation energy of the 0^+, $T=1$ state in ^{10}B and the actual calculation has been carried through to three decimal places. The final Ft value then becomes 3075.2(55) s, as compared with the 3074.0(54) s quoted in Ref. [1].

The Q value for the superallowed positron decay of ^{14}O was obtained from a determination of the threshold energy of the $^{14}N(p,n)^{14}O$ reaction and details are given in Ref. [7]. In all, nine individual measurements were made, and these have now been analyzed in the same way as described above for $^{10}B(p,n)^{10}C$, with the details being very similar.

A slightly complicating feature is that the nitrogen targets were in the form of sputtered tantalum nitride layers in which the proportion of nitrogen to tantalum was somewhat uncertain, and this affects slightly the magnitude of the correction due to Y_Q. Accordingly, two cases were calculated, with 25% and 50% nitrogen, and the results averaged. The overall results are shown in Table II, where once again E_{norm} corresponds to the $(E-E_0)^{3/2}$ analysis (which was also the analysis reported in Ref. [7]), and E_{corr} incorporates the corrections discussed above. Estimates of the sizes of the individual corrections due to Y_g, Y_Q, and $(Y_K+Y_{L1}+Y_{L2})$ give $+0.02$, $+0.18$ and -0.13 keV, respectively, and an overall total of $+0.70$ keV, which is in good agreement with the actual shift.

TABLE II. Analysis of the yield curves of the $^{14}N(p,n)^{14}O$ reaction. The energies E_{norm} come from an $(E-E_0)^{3/2}$ analysis and were the ones reported in Ref. [7], and E_{corr} are obtained using the functional form Y_{corr} (see text).

Run number	$(E_{norm}-6300)$ keV	$(E_{corr}-6300)$ keV
1	52.77(33)	52.89(31)
2	52.89(35)	53.00(33)
3	52.57(30)	52.66(30)
4	53.11(20)	53.21(19)
5	52.59(27)	52.62(26)
6	52.96(23)	53.01(22)
7	52.94(25)	53.00(24)
8	53.12(16)	53.19(14)
9	53.21(15)	53.25(14)
Mean	53.01(7)	53.08(7)

Treating the threshold value of 6353.08(7) keV in the same way as that for $^{10}B(p,n)^{10}C$, (i.e., reduce by 15 ppm, and then increase the attributed error by 70 eV for the corrections and 10 ppm for the energy measurement systematic effects appropriate at the time), gives a final value of 6352.99(12) keV and a Q value for the $^{14}N(p,n)^{14}O$ reaction of 5925.41(11) keV. The Q value for the superallowed positron decay of ^{14}O then becomes $Q_{ec}=2830.26(11)$ keV, where the excitation energy of the 0^+, $T=1$ state in ^{14}N has been taken as 2312.798 keV. It is difficult to quote a final, revised, Ft value, to be contrasted with the 3069.2(28) s quoted in Ref. [1], because of the necessity of deciding how the inconsistent Q values referred to earlier should be combined.

The Q values of the superallowed positron decays of ^{10}B and ^{14}O, measured relative to a 1-volt standard by the Auckland Heavy Ion Source System, have been revised to be 1907.86(12) and 2830.26(11) keV, respectively, in the light of a new understanding of phenomena, the effects of which have been explicitly incorporated into an analysis of the original data. These values were originally published as 1907.72(9) and 2830.25(8) keV, and although the overall shifts seem not to be significant, that is largely due to the accidental cancellation of individual terms whose sizes are several times the ascribed errors.

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