Meson-baryon form factors in the chiral color dielectric model

S. C. Phatak

Institute of Physics, Bhubaneswar 751 005, India (Received 5 May 1998)

The renormalized form factors for pseudoscalar meson-baryon coupling are computed in the chiral color dielectric model. This has been done by rearranging the Lippmann-Schwinger series for the meson-baryon scattering matrix so that it can be expressed as a baryon pole term with renormalized form factors and baryon masses and the rest of the terms which arise from the crossed diagrams. Thus we are able to obtain an integral equation for the renormalized meson-baryon form factors in terms of the bare form factors as well as an expression for the meson self-energy. This integral equation is solved, and renormalized meson baryon form factors and renormalized baryon masses are computed. The parameters of the model are adjusted to obtain a best fit to the physical baryon masses. The calculations show that the renormalized form factors are energy dependent and differ from the bare form factors is about 10% at zero momentum transfer. The computed form factors are soft with the equivalent monopole cutoff mass of about 500 MeV. The renormalized coupling constants are obtained by comparing the chiral color dielectric model interaction Hamiltonian with the standard form of the meson-nucleon interaction Hamiltonian. The ratio of $\Delta N \pi$ and $NN \pi$ coupling constants is found to be about 2.15. This value is very close to the experimental value. [S0556-2813(98)06109-3]

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I. INTRODUCTION

The meson-baryon form factors are interesting and useful quantities for several reasons. For one thing, we expect the form factors to have some bearing on the underlying structure of hadrons. For example, in perturbative chiral quark models [1] the pion-baryon form factors are directly related to the quark wave functions in baryons. Form factors are essential in the effective models of meson-baryon interactions since hadrons, after all, are not point particles. One also needs these form factors for a consistant description of nuclear phenomena. The meson-exchange nucleon-nucleon potentials include phenomenological vertex form factors which are presumably related to the structure of hadrons. Form factors are also needed in computation of photoproduction and electroproduction of baryon resonances since, in principle, the photon can couple to the virtual charged meson cloud in a baryon. Another technical reason for introducing the form factors is that the effective meson-baryon interaction models are, generally, not renormalizable and the form factors provide the needed cutoff functions.

Usually, the form factors employed in meson-baryon interaction models are of monopole type $[\sim 1/(1-k^2/\Lambda^2)]$ with k^2 being the four-momentum carried by the meson. It must be noted that the monopole form is, as such, purely phenomenological and is possibly not related to the underlying structure of the hadrons. The form factors used in earlier calculations [2] were "hard" with the cutoff parameters $\Lambda \ge 1$ GeV. There are, however, recent calculations [3] which indicate that softer form factors ($\Lambda \sim 500$ MeV) are required to fit Δ production on nuclei. Some indication of the value of the cutoff parameter Λ is available from low energy phenomenology. For example, Thomas and Holinde [4] argue that the 3% descrepancy between $pp\pi^0$ and $pn\pi^+$ coupling constants [5] is essentially due to the four-momentum variation of the πNN form factor between $q^2 = m_{\pi}^2$ and $q^2 = 0$. With monopole parametrization of the form factor, Λ of about 800 MeV is required to explain this descrepancy. Similar arguments have been proposed by Coon and Scadron [6] for $\Lambda \sim 800$ MeV. One should, however, note that these analyses are restricted to relatively small values of $|q^2|$ $[\sim (140 \text{ MeV})^2]$ and therefore cannot determine the form factors at larger momentum transfers. Recently, Saito and Afnan [7] have investigated the dependence of triton binding energy on the πNN form factor. They compute the threebody π - π force contribution to the triton binding energy. They claim that one can determine the πNN form factor in their model and it turns out that their renormalized form factor is softer with $\Lambda \sim 400$ MeV. On the other hand, Schultz and Holinde [8] fit πN scattering data and obtain a form factor having $\Lambda \sim 800$ MeV. Some attempts have been made to determine the pion-baryon form factor from QCD. For example, Liu et al. [9] have used quenched lattice QCD calculations and extracted a Λ of 750 MeV. Meissner [10], on the other hand, has used QCD sum rules and obtained a Λ of about 800 MeV.

In the present work, the renormalized pseudoscalar meson baryon form factors are computed by using the chiral color dielectric (CCD) model of baryons [11]. The CCD model is based on the calculation of Nielsen and Patkos [12] who showed that "coarse graining" of the QCD Lagrangian on a lattice gives rise to an effective scalar, color-neutral field called the color dielectric field. In the CCD model, the interaction of the color dielectric field with the quark and gluon fields is such that it gives rise to the confinement of these objects. Chiral symmetry is restored in the CCD model by introducing the interaction of the pseudoscalar meson octet with the quarks [11]. The CCD model has been used to compute the properties of light baryons [11] as well as πN scattering phase shifts [13]. Generally the agreement between the CCD model results and the experimental data is good. In particular, a good agreement with the πN scattering phase

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shifts is obtained in the CCD model. In the πN work, the authors showed that solving of the relativistic Lippmann-Schwinger equation for pion-nucleon scattering leads to the renormalization of baryon masses and the pion-baryon veritces are also renormalized due to multiple scattering. However, the renormalized vertex functions (or form factors) were not computed in that work. The present work is essentially an extension of the earlier work [13] but here we have concentrated on renormalized form factors and have not computed the πN scattering matrix. The basic idea here is as follows. The CCD model (or, for that matter, any other quark-based model with mesons coupling to quarks) gives bare meson-baryon form factors and bare baryon masses. These are then dressed by meson-baryon interactions. As we shall show later in the paper, computation of meson-baryon scattering (including the pole positions of stable particles) leads to the determination of dressed baryon masses as well as dressed meson-baryon form factors. This is essentially achieved by rearranging the terms in the Lippmann-Schwinger series for the meson-baryon scattering matrix so that one obtains equations for the dressed baryon masses (positions of the poles in the scattering matrix) and mesonbaryon form factors (see the following sections for details). Thus, one can compute the dressed pion baryon form factors. One should note here that these dressed (or renormalized) form factors, and not the bare form factors, should be used in nuclear physics calculations. Some saliant features, of the results of our calculation are the following.

(1) The renormalization of the form factors depends on the energy of the meson-baryon system. At zero momentum transfer, the ratio of renormalized and bare form factors varies from about 1.1 to about 1.3 as the energy is increased from nucleon mass to Δ mass.

(2) The renormalized form factors differ from the bare ones for meson momenta of 1 GeV or smaller. There is practically no change at larger meson momenta.

(3) The form factors computed in the CCD model are soft. If one wants to fit the CCD model form factors by a monopole form in the region of 300–700 MeV, the corresponding Λ is about 500 MeV.

At this stage we would like to emphasize that our prescription of computing the renormalized form factors should be applicable to models having the basic baryon-meson interaction as an input. That is, given a model for a baryonmeson interaction Lagrangian with bare baryon masses and form factors, one should compute the renormalized form factors by fitting the data (cross sections, phase shifts, positions of the poles of the scattering matrix etc.). These renormalized form factors should then be used in other nuclear physics calculations such as NN potentials, photoproduction and electroproduction of mesons, etc. In other words, the parameters of the basic baryon-meson model are necessarily the bare parameters and should not be obtained directly from the experiments. If one does this, then one is essentially choosing dressed parameters in the Lagrangian and then one should use the model at the tree level and should not compute meson loops for meson-nucleon scattering, etc., since that would amount to double counting.

The paper is organized as follows. In Sec. II, a brief description of the CCD model is given and the bare masses and pion-baryon form factors are computed. In Sec. III, beginning with the Lippmann-Schwinger series, the integral equations for the meson-baryon form factors are obtained. In Sec. IV, the results of the calculation are presented and discussed.

II. CHIRAL COLOR DIELECTRIC MODEL

The CCD model is described and the bare masses of baryons and pseudoscalar meson-baryon vertices are computed in this section. The description of the CCD model is somewhat brief because in this work we want to focus on the computation of meson-baryon form factors. For the details of the CCD model the reader is referred to [11,13].

A. Lagrangian

The Lagrangian density of the CCD model is given by [11]

$$\mathcal{L}(x) = \overline{\psi}(x) \left\{ i \vartheta - \left(m_0 + \frac{m}{\chi(x)} U_5 \right) + \frac{g}{2} \lambda_a^c A^a(x) \right\} \psi$$
$$+ \frac{f^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) - \frac{1}{2} m_\phi^2 \phi^2(x) - \frac{1}{4} \chi^4(x)$$
$$\times [F^a_{\mu\nu}(x)]^2 + \frac{1}{2} \sigma_\nu^2 [\partial_\mu \chi(x)]^2 - U(\chi), \qquad (1)$$

where $U = e^{i\lambda_a^f \phi^a/f}$, $U_5 = e^{i\lambda_a^f \phi^a \gamma_5/f}$, and $\psi(x)$, $A_{\mu}(x)$, $\chi(x)$, and $\phi(x)$ are quark, gluon, scalar (color dielectric) and meson fields, respectively. *m* and m_{ϕ} are the masses of the quark and meson, *f* is the pion decay constant, $F_{\mu\nu}(x)$ is the usual color electromagnetic field tensor, *g* is the color coupling constant, and λ_a^c and λ_a^f are the usual Gell-Mann matrices acting in color and flavor space, respectively. The flavor symmetry breaking is incorporated in the Lagrangian through the quark mass term $[m_0 + (m/\chi)U_5]$, where m_0 = 0 for *u* and *d* quarks. So the masses of *u*, *d*, and *s* quarks are *m*, *m*, and $m_0 + m$, respectively. The meson matrix consists of a singlet η , triplet of π , and quadruplet of *K*. The self-interaction $U(\chi)$ of the scalar field is assumed to be of the form

$$U(\chi) = \alpha B \chi^{2}(x) [1 - 2(1 - 2/\alpha)\chi(x) + (1 - 3/\alpha)\chi^{2}(x)],$$
(2)

so that $U(\chi)$ has an absolute minimum at $\chi=0$ and a secondary minimum at $\chi=1$. The constant σ_v appearing in the kinetic energy term of the scalar field is $\sigma_v = \sqrt{2 \alpha B/m_{GB}^2}$, with m_{GB} being the mass of the scalar field.¹

We would like to make the following observations on the Lagrangian of Eq. (1).

¹Note that, unlike the usual definition of scalar fields, χ is dimensionless. Since σ_v has dimensions of energy, the kinetic energy term has the correct dimensions. One could define the scalar field by absorbing σ_v ($\bar{\chi} = \sigma_v \chi$). Then $U(\chi)$ will be given by $(m_{\rm GB}^2/2)\bar{\chi}^2[1-2(1-2/\alpha)\bar{\chi}/\sigma_v + (1-3/\alpha)\bar{\chi}^2/\sigma_v^2]$. The nonstandard definition of χ is chosen because χ can then be identified with the color dielectric function multiplying the color electromagnetic field tensor.

(2) The coupling of the scalar field (χ) to the quark and gluon fields is such that the quark mass becomes infinite and the dielectric function (χ^4) vanishes when χ becomes zero. This means that the quarks and gluons cannot exist in a region where χ becomes zero. That is, the confinement of quarks and gluons is included in the model.

(3) The two minima of the self-interaction of the scalar field [Eq. (2)] can be identified with the perturbative and physical vacua of the MIT bag model [14]. Thus, the CCD model is a dynamical model generating the bag.

We follow the cloudy bag model approach [1] in the present work and treat the gluon and meson interactions with the quarks perturbatively. Therefore we expand the Lagrangian in powers of 1/f and keep terms up to order 1/f in the Lagrangian of Eq. (1). With this approximation the CCD model Lagrangian becomes

$$\mathcal{L}(x) = \overline{\psi}(x) \left\{ i \vartheta - \left[m_0 + \frac{m}{\chi(x)} \left(1 + \frac{i}{f} \lambda_a \phi^a(x) \right) \right] + \frac{g}{2} \lambda_a A^a(x) \right\} \psi + \frac{1}{2} [\partial_\mu \phi_a(x)]^2 - \frac{1}{2} m_\phi^2 \phi^2(x) - \frac{1}{4} \chi^4(x) [F^a_{\mu\nu}(x)]^2 + \frac{1}{2} \sigma_v^2 [\partial_\mu \chi(x)]^2 - U(\chi).$$
(3)

The parameters of the CCD model are the quark masses $(m \text{ and } m_0)$, the "bag constant" (B), the strong coupling constant ($\alpha_s = g^2/4\pi$), the pion decay constant (f), glueball (or dielectric field) mass $(m_{\rm GB})$, and the constant α in $U(\chi)$. Of these parameters, the value of α is chosen to be 24 since from our earlier calculations [11] the results are not sensitive to it. To begin with, we choose the experimental value of the pion decay constant (f=93 MeV) in our calculations. As we shall see later, the value of f required to fit the pionnucleon coupling constant is very close to 93 MeV. Thus we are left with five free parameters to be adjusted. In our earlier calculations [11] we found that a reasonably good fit to the baryon masses is obtained for m and $B^{1/4}$ ranging between 100 and 140 MeV. We therefore choose m and $B^{1/4}$ in this range and adjust $m_{\rm GB}$, α_s , and m_0 to fit nucleon, Δ , and Λ masses. Computed masses of other octet and decuplet baryons are within a few percent of their experimental masses.

At this stage a comment on the expansion of the Lagrangian in powers of 1/f is necessary. In particular, one may want to keep the terms up to $1/f^2$ since the meson-baryon potential, obtained in Sec. III, is $O(1/f^2)$. Indeed, Phatak, Lu, and Landau [13] had included these terms in their calculation of pion-nucleon scattering. The $O(1/f^2)$ terms are essential to have a correct behavior of S-wave meson-baryon scattering. They are, however, not so important for P-wave scattering, and because we are considering pseudoscalar mesons here, the emitted mesons are in P wave. In fact, the experimental S-wave phase shifts are small, indicating the relative weakness of $O(1/f^2)$ terms. Thus one may be justified in neglecting $O(1/f^2)$ terms.

B. Bare baryon states

The equations of motion of quark, gluon, and color dielectric fields can be obtained from the Lagrangian of Eq. (3). Since we shall be treating the gluon interactins perturbatively, the quark and color dielectric field equations of motion are solved self-consistently. The gluon as well as pseudoscalar meson interactions are then treated perturbatively. This approach is similar to the one followed in cloudy bag model calculations [1]. Thus the equations of motion for the quark and dielectric field are

$$\left(\epsilon_{i}-m_{0}-\frac{m}{\chi(r)}\right)g_{i}(r) = -f_{i}'(r) - \frac{2}{r}f_{i}(r),$$

$$\left(\epsilon_{i}-m_{0}-\frac{m}{\chi(r)}\right)f_{i}(r) = g_{i}'(r),$$
(4)

and

$$\chi''(r) + \frac{2}{r}\chi'(r) - \frac{2B\alpha}{\sigma_v^2}\chi(r)[1 - 3(1 - 2/\alpha)\chi(r) + 2(1 - 3/\alpha)\chi^2(r)] + \sum_i \frac{N_i m_i}{\sigma_v^2\chi^2(r)} \times [g_i^2(r) - f_i^2(r)] = 0.$$
(5)

We have assumed spherical symmetry in obtaining these equations. The equations of motion of quark and dielectric fields are solved self-consistently with the boundary conditions that $\chi(r), g_i(r), f_i(r) \rightarrow 0$ as $r \rightarrow \infty$ and $\chi(r)$ and $g_i(r) \rightarrow const$ and $f_i(r) \rightarrow 0$ as $r \rightarrow 0$. The bare baryon states are constructed by putting three quarks of appropriate flavor in a $1 s_{1/2}$ orbital. Thus the baryon wave function is a product of the symmetric space wave function, symmetric spin-flavor wave function, and antisymmetric color wave function [14]. The mass of this state is computed by evaluating the matrix element of the Hamiltonian

$$H_{0} = \int d^{3}x \left[\sum_{i} \Psi^{\dagger} \left(-i\vec{\alpha} \cdot \nabla + \frac{m}{\chi} + m_{0} \right) \Psi + \frac{\sigma_{v}^{2}}{2} [(\nabla \chi)^{2} + \Pi^{2}] + U(\chi) \right]$$
(6)

in the bare baryon state. Here Ψ is an annihilation operator of a quark in the state computed in Eq. (4) and II is the momentum conjugate to the dielectric field χ . The method of coherent states [15] has been used to better account for the energy associated with the dielectric field. To this the colormagnetic interaction between the quarks is added perturbatively (see Refs. [11,16] for details). The bare mass of the baryon is then

$$M_B^0 = \langle B(\vec{0}) | H_0 | B(\vec{0}) \rangle + E_M, \qquad (7)$$

where $|B(\vec{0})\rangle$ is the bare baryon state having zero momentum. This state is constructed by using the Peierls-Yoccoz projection technique [17,18]. It is convinient to express M_B^0 as

 Σ^* Ν Λ Σ Ξ Ξ^* Ω Δ $\sqrt{8}$ Ν 5 [1] $\{-\sqrt{18}\}$ Λ {3} [-2]{1} 2 $\{-2\}$ $\{-\sqrt{8}\}$ $\sqrt{\frac{32}{3}}$ [2] Σ $\{-1\}$ $\sqrt{12}$ $\{-5\}$ $\sqrt{\frac{8}{3}}$ [2] $\{-2\}$ Ξ $\{\sqrt{2}\}$ -1 [-3]2 [2] $\{-4\}$ $\sqrt{32}$ $5[\sqrt{5}]$ Δ $\sqrt{\frac{16}{3}} \left[-\sqrt{8} \right]$ $\{-\sqrt{8}\}$ Σ^* $\{\sqrt{8}\}$ $\sqrt{24}$ $\{\sqrt{10}\}$ $\{\sqrt{20}\}$ $\left| \frac{16}{3} \right|$ $\{-\sqrt{8}\} \quad [-\sqrt{8}]$ Ξ^* {4} $\sqrt{\frac{40}{3}} \ [-\sqrt{5}]$ $\{\sqrt{20}\}$ Ω $\{-4\}$ $\{\sqrt{10}\}$ $[-\sqrt{20}]$

TABLE I. The reduced matrix elements $\alpha_{BB'\phi}$. The matrix elements for *K* and η mesons are given in curly brackets and square brackets, respectively.

$$M_B^0 = m_{\rm GB} (C_1^B + \alpha_s C_2^B), \tag{8}$$

where C_1^B depends on the number of strange quarks in the baryon and C_2^B depends on the spin-flavor wave function of the baryon. (C_1^B and C_2^B are, of course, functions of other parameters of the model.) The mass M_B^0 defined above is the bare mass of baryon *B* since it does not include the meson self-energy corrections. In order to obtain the physical mass of the baryon one should determine the position of the pole of the *T* matrix in the appropriate meson-baryon channel [13]. Clearly, this mass is different from M_B^0 . In our calculations we treat m_{GB} , α_s , and m_0/m_{GB} as free parameters and obtain a best fit to the baryon masses.

C. Bare meson-baryon vertex

The meson baryon interaction Hamiltonian obtained from the Lagrangian of Eq. (3) is

$$H_{\text{int}} = \frac{i}{f} \int d^3x \frac{m}{\chi(x)} \bar{\Psi}(x) \gamma_5 \lambda_i \Psi(x) \phi_i(x).$$
(9)

In order to compute the bare meson-baryon form factors we quantize the meson fields and evaluate the matrix element of H_{int} between bare baryon states defined in the preceding subsection. With this, the interaction Hamiltonian can be written as

$$H_{\rm int} = \frac{i}{f} \sum_{B,B'} |B'\rangle \langle B| \int \frac{d^3k d^3x}{\sqrt{16\pi^3 \omega_{\phi}(k)}} e^{i\vec{k}\cdot\vec{x}}$$

$$\times \left[\langle B' | \bar{\Psi}(x) \gamma_5 \lambda_i \Psi(x) \frac{m}{\chi(x)} a_i(\vec{k}) | B \rangle + \text{H.c.} \right].$$
(10)

It is convenient to use an angular momentum formalism for evaluating the spin-flavor matrix elements. The details of the calculation are described in Appendix A. The end result of the calculation is

$$H_{\text{int}} = \frac{i}{f} \sum_{B,B'} |B'\rangle \langle B| \frac{\alpha_{BB'\phi}}{\sqrt{16\pi^3}} \langle T_B, T_{\phi}, t_B, \mu | T_{B'}, t_{B'} \rangle \\ \times \langle S_B, 1, s_B, \nu | S_{B'}, s_{B'} \rangle (-1)^{\nu + i_{\phi}} \\ \times \int \frac{d^3k}{\sqrt{\omega_{\phi}(k)}} [a_{-\mu}(\vec{k}) - a_{\mu}^{\dagger}(\vec{k})] u_{BB'\phi}^0(k) k_{1,-\nu},$$
(11)

where the quantities in angular brackets $\langle \cdots \rangle$ are the usual SU(2) Clebsch-Gordon coefficients, S_B and T_B are spin and isospin of baryon B, s_B and t_B are their projections, μ is the isospin projection of meson ϕ , and $\alpha_{BB'\phi}$ are the reduced matrix elements. The phase factor i_{ϕ} is defined in Appendix A. $\alpha_{BB'\phi}$ are defined in Table I. The bare form factor $u_{BB'\phi}^0(k)$ is evaluated in the "brick-wall" frame so that the baryons $|B\rangle$ and $|B'\rangle$ have momenta $|\vec{k}/2\rangle$ and $|-\vec{k}/2\rangle$, respectively. The momentum states of the baryons are constructed using the Peierls-Yoccoz projection technique [17,16].



FIG. 1. *s*-channel (a) and *u*-channel (b) pole terms (direct and crossed terms, respectively) contributing to the meson-baryon potential.

III. MESON-BARYON SCATTERING

Given the meson-baryon interaction Hamiltonian of the previous section, one can compute the tree-level mesonbaryon scattering matrix. It involves the pole and crossed terms shown in Fig. 1. These two terms constitute the driving term or potential for meson-baryon scattering. To obtain a unitary scattering matrix and to include multiple scattering effects the relativistic Lippmann-Schwinger equation

$$T_{\beta\alpha}(\vec{k}',\vec{k}) = V_{\beta\alpha}(\vec{k}',\vec{k}) + \sum_{\gamma} \int d^3p \frac{V_{\beta\gamma}(\vec{k}',p)T_{\gamma\alpha}(p,k)}{E + i\epsilon - E_{\gamma}(p)}$$
(12)

is solved. In this equation the meson-baryon channel (including the spins and flavors of the particles) is represented by the subscripts and $E_{\gamma}(p) = \sqrt{p^2 + m_{B_{\gamma}}^2} + \omega_{\phi_{\gamma}}(p)$. The potential *V* above is computed from the diagrams of Fig. 1 (see Phatak, Lu, and Landau [13] for details).

We would like to make several points regarding the T-matrix equation given above [Eq. (12)].

(1) The *T*-matrix given above describes the scattering of one of the octet of the pseudoscalar mesons from the octet or decuplet of baryons. Thus it represents resonance production,

production of strange mesons and baryons, etc.

(2) The channel energies E_{γ} in the Lippmann-Schwinger equation above are defined in terms of the physical baryon masses since these are the physically observable states. However, the masses of baryons in the intermediate state of the pole and crossed terms are the bare masses defined in the previous section.

(3) The poles of the *T* matrix in the appropriate spinflavor channel of the meson-baryon system should coincide with the physical masses of the baryons. This means that the parameters of the CCD model (basically the glueball mass $m_{\rm GB}$, strong coupling constant α_s and m_0) are to be adjusted to obtain the best fit to the physical baryon masses defined by the pole positions of the *T* matrix.

Given the potential $V_{\beta\alpha}(\vec{k}',\vec{k})$, one can solve the Lippmann-Schwinger equation and obtain the phase shifts and cross sections for meson-baryon scattering. For this purpose, it is best to expand the T-matrix equation in spinisospin channels of the meson-baryon system [13] and solve the partial-wave Lippmann-Schwinger equation. As mentioned earlier, when one solves the Lippmann-Schwinger equation renormalization of baryon masses as well as the form factors is already included. However, one cannot compute the renormalized form factors using this procedure. Since we are interested in computing the renormalized form factors in the present work, we do not want to solve the Lippmann-Schwinger equation as such. Instead, we express the Lippmann-Schwinger equation as a multiple scattering series. From this series we obtain integral equations for the renormalized form factors by summing up parts of the series. We also obtain expressions for the renormalized masses. The details of this exercise are outlined in Appendix B. The result is an integral equation for the renormalized form factor $u_{BB'\phi}(k)$:

$$u_{BB'\phi}(k) = u_{BB'\phi}^{0}(k) + \sum_{\phi'B'',B'''} (-1)^{I} \hat{S}_{B''} \hat{S}_{B'''} \hat{T}_{B''} \frac{\alpha_{BB''\phi'} \alpha_{B''B''\phi} \alpha_{B''B''\phi} \alpha_{B''B''\phi}}{12\pi^{2} f^{2} \alpha_{BB'\phi}} W(T_{B}T_{B''}T_{B''}T_{B'}; T_{\phi}T_{\phi'}) \\ \times W(S_{B}S_{B''}S_{B''}S_{B'}; 11) \int \frac{k'^{4} dk' u_{BB''\phi'}^{0}(k') u_{B''B''\phi}^{0}(k) u_{B''B'\phi}(k) u_{B''B'\phi}(k')}{\omega_{\phi'}(k')[E - E_{B''}^{0} - \omega_{\phi'}(k') - \omega_{\phi}(k)][E - E_{B''} - \omega_{\phi'}(k')]},$$
(13)

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where $\hat{a} = \sqrt{2a+1}$, $E_B^0(k) = \sqrt{k^2 + (M_B^0)^2}$, $E_B(k) = \sqrt{k^2 + M_B^2}$, $\omega_{\phi}(k) = \sqrt{k^2 + m_{\phi}^2}$, M_B and M_B^0 are renormalized and bare masses of baryon *B*, respectively, $I = \Delta_{\phi} + \Delta_{\phi'} - S_B - S_{B'} - T_B - T_{B'}$, S_B and T_B are the spin and isospin of baryon *B*, respectively, and $W(\cdots)$'s are the usual Rakah coefficients. The equation for the renormalized propagator of a baryon *B* is

$$G(E) = \frac{1}{E - M_B^0} + \frac{1}{E - M_B^0} \Sigma(E) G(E) = \frac{1}{E - M_B^0 - \Sigma(E)},$$
(14)

$$\Sigma(E) = \sum_{B'\phi} \frac{(-1)^{T_{B'} + S_{B'} - T_B - S_B - \Delta}}{12\pi^2 f^2} \frac{\hat{S}_{B'}\hat{T}_{B'}}{\hat{S}_B \hat{T}_B} \alpha_{BB'\phi} \alpha_{B'B\bar{\phi}} \times \int \frac{k^4 dk u^0_{BB'\phi}(k) u_{B'B\bar{\phi}}(k)}{\omega_{\phi}(k) [M_B - E_{B'} - \omega_{\phi}(k)]},$$
(15)

where Δ is 0, -1/2, 1/2, and 0 for π , *K*, \overline{K} , and η , respectively. The renormalized propagator has a pole at $E = M_B^0 + \Sigma(E)$. Thus the renormalized mass of baryon M_B is given by

where the self-energy $\Sigma(E)$ is

$$M_B = M_B^0 + \Sigma(M_B). \tag{16}$$



FIG. 2. The graphical depiction of the *T*-matrix equation. The first line corresponds to Eq. (17) of the text. The second line depicts the integral equation for T^C . The thick (thin) lines represent baryons with physical (bare) masses and the dashed lines represent mesons. Solid circles represent the renormalized form factor.

Note that Eqs. (13)-(16) are coupled equations since renormalized masses and form factors appear in these equations. Thus these equations need to be solved self-consistently.

We can now use the renormalized form factors and propagators in the expression for the scattering matrix. We then get

$$T_{B\phi,B'\phi'}(\vec{k},\vec{k}';E) = T^{P}_{B\phi,B'\phi'}(\vec{k},\vec{k}';E) + T^{C}_{B\phi,B'\phi'}(\vec{k},\vec{k}'),$$
(17)

where

$$T^{P}_{B\phi,B'\phi'}(\vec{k},\vec{k}';E) = \sum_{B''} \alpha_{BB''\bar{\phi}} \alpha_{B''B'\phi'} \frac{u_{BB''\bar{\phi}}(k)u_{B''B'\phi'}(k')}{E - M_{B''}}$$
(18)

and

$$T^{C}_{B\phi,B'\phi'}(\vec{k},\vec{k}') = V^{C}_{B\phi,B'\phi'}(\vec{k},\vec{k}') + \sum_{B'',\phi''} \int d^{3}p \\ \times \frac{V^{C}_{B\phi,B''\phi''}(\vec{k},\vec{p})T^{C}_{B''\phi'',B'\phi'}(\vec{p},\vec{k}')}{E - E_{B}(p) - \omega_{\phi}(p)}.$$
(19)

Here $V_{B\phi,B'\phi'}^C(\vec{k},\vec{k}')$ is the crossed term shown in Fig. 1. A diagramatic representation of the *T* matrix is shown in Fig. 2.

The interpretation of Eq. (17) is obvious. The first term represents the pole or resonance contribution which is given in terms of *physical* masses of resonances and *renormalized* form factors or vertex functions. The rest of the contribution is lumped into the second term which can be considered as a background. Clearly, if our model is extended to include higher order terms or exchanges of heavier mesons, these will predominantly affect the background term.

Usually the *T* matrix is computed by solving the Lippmann-Schwinger equation using a suitable numerical technique (e.g., matrix inversion method [19]) or by summing the Lippmann-Schwinger series numerically to a desired accuracy. Alternatively, one can now compute the renormalized form factors and propagators and use them to evaluate the *T* matrix. The computation of the pole term of the scattering matrix is very simple in this case. For the crossed term $T^{C}_{B\phi,B'\phi'}(\vec{k},\vec{k}')$, however, one has to solve an integral equation [Eq. (19)]. Our computed iteratively and the convergence is rapid. We also find that the dominant

contribution to the scattering matrix comes from the pole term. One should note that the computation of the renormalized form factor already includes a large number of terms involving the crossed diagram of the meson-baryon potential. It is therefore not surprising that $T^{C}_{B\phi,B'\phi'}(\vec{k},\vec{k}')$ appearing in the *T*-matrix equation [Eq. (19)] is somewhat less important, particularly near the resonance.

Note that pole term of the *T* matrix, $T^P_{B\phi,B'\phi'}(\vec{k},\vec{k}';E)$, defined in Eq. (18) has a singularity at $E = M_{B''}$. If this singularity occurs at physical energy, it means that the particle B'' can decay and that should be reflected in the mass of the particle. That is, the physical mass defined above should have an imaginary part corresponding to the width of the baryon B''.

The renormalized form factors are computed by solving Eq. (13) above iteratively. We find that the iterative procedure does converge and the convergence is reached within 15 iterations. Note that the integrand of the form factor integral equation has singularities due to the energy denominators and the integral should be regularized properly. We have used a principal value prescription for this purpose. The results of the calculation are discussed in the following section.

IV. RESULTS AND DISCUSSIONS

In this section we shall discuss the results of our calculation. The procedure adopted for the calculation is as outlined below. The coupled quark and dielectric field equations are first solved and bare baryon states are constructed. The bare baryon masses are computed by evaluating the matrix element of the Hamiltonian [Eq. (6)] in the momentumprojected bare baryon states and including the color magnetic interaction. The baryon masses are essentially functions of m_{GB} , α_s , and m_0 (*m* and $B^{1/4}$ are kept fixed for a given parameter set). Also, using the momentum-projected baryon wave functions the bare form factors (vertex functions) for baryon-meson coupling are computed. These are then used in Eq. (13) and the renormalized form factors are computed. In principle, one could use the matrix inversion method [19] for this purpose. However, we find that the dimensionality of the matrices is prohibitively large because we are considering coupling to all the octets of pseudoscalar mesons. We therefore solve Eq. (13) iteratively. We find that convergence is reached after 10-15 iterations. Using the renormalized form factors the renormalized masses are computed by using Eq. (16). The parameters of the CCD model (m_{GB} , α_s , and m_0) are then adjusted to fit the renormalized masses to the physical masses.

For a comparison with the experimental data we consider the πNN and $\pi N\Delta$ coupling constants at nucleon and delta masses, respectively. Conventionally, in terms of the nonrelativistic πNN interaction Hamiltonian, the πNN coupling constant $f_{\pi NN}$ is defined as [1]

$$H_{\rm int} = \frac{i f_{\pi NN}}{m_{\pi} \sqrt{2} \pi} \int \frac{d^3 k}{\sqrt{2 \omega_{\pi}(k)}} v(k) [\sigma \cdot \vec{k} \, \vec{\tau} \cdot \vec{a}(\vec{k}) + \text{H.c.}],$$
(20)

where σ and τ are nucleon spin and isospin operators, respectively, v(k) is the form factor defined to be unity when $k = im_{\pi}$, and $\vec{a}(\vec{k})$ is the annihilation operator for the pion





FIG. 3. Renormalized and bare form factors vs momentum in units of the glueball mass. The different curves are for $u_{NN\pi}$ (solid line), $u_{NN\pi}^0$ (dotted line), and $u_{N\Delta\pi}$ (dashed line). $u_{N\Delta\pi}^0$ is the same as $u_{NN\pi}^0$.

field. The experimental value of $f_{\pi NN}$ is 0.279 ($f_{\pi NN}^2$ = 0.078). Comparing the interaction Hamiltonian of Eq. (11), we have $f_{\pi NN} = \alpha_{NN\pi} u_{NN\pi} (k=0) m_{\pi} / 6 \sqrt{\pi} f$, where the renormalized form factor is calculated at $E = m_N$, the nucleon mass. Similarly, $f_{\pi N\Delta} = \alpha_{N\Delta\pi} u_{N\Delta\pi} (k=0) m_{\pi} \sqrt{2} / 5 \sqrt{\pi} f$.

Computations have been done for a number of parameter sets of the CCD model. The results are quite similar for all these sets as long as $m_{\rm GB}$ is between 800 and 1200 MeV. Therefore we have chosen one of the representative parameter sets ($m_{\rm GB} = 978.6$, $\alpha_s = 0.438$, $B^{1/4} = 122.3$, m = 122.3, and $m_0 = 210.1$) for a detailed discussion. The pion decay constant f has been fixed at its physical value (93 MeV). The bare and renormalized πNN and $\pi N\Delta$ form factors evaluated at nucleon delta masses are displayed in Fig. 3. It is clear from this figure that the renormalized form factors differ from the bare form factors at three-momentum transfers smaller than 1 GeV. An inspection of Eq. (13) shows why this happens. As the momentum transfer increases, the first term in the denominator of the integrand also increases because of an increase in $\omega_{\phi}(k)$. As a result, the contribution of the integral to the form factor decreases with an increase in the momentum transfer. Even at momentum transfers smaller than 1 GeV, the change in the form factor is about 10% for $u_{NN\pi}$ and 30% for $u_{N\Delta\pi}$. Thus one could conclude that the effects of renormalization are relatively minor.

We can extract equivalent monopole form factors from the computed renormalized form factors. If, for example, $u_{NN\pi}$ is expanded in power series near k=0, one obtains $\Lambda \sim 800$ MeV. This value of Λ is about same as the one obtained by various authors [4,6,8–10]. However, one must note that as a function of momentum transfer, the monopole form factor falls off more slowly than the CCD model form factor. We find that a better fit to the CCD model form factor is obtained by using a dipole or Gaussian form. In fact, we find that if we replace the bare CCD model form factor by a dipole or Gaussian form factor, we get practically the same results. If, on the other hand, one wants that the monopole



Energy (MeV)

FIG. 4. The ratio of renormalized and bare form factor at zero momentum transfer as a function of energy. The solid curve is for $NN\pi$ and the dashed curve is for $N\Delta\pi$.

form factor should be close to $u_{NN\pi}(k)$ for $k \sim 300-500$ MeV, $\Lambda \sim 500$ MeV.

The computed value of the πNN coupling constant is 0.254. This value is about 9% smaller than the experimental $f_{NN\pi}$. This, by itself, can be considered as a success of the CCD model. The $f_{NN\pi}$ as well as the baryon masses can be fitted by decreasing the pion decay constant f to 86 MeV (a 7% decrease). It may be mentioned here that this change in f does not affect the computed properties of baryons very much.

Let us now consider the ratio of $\pi N\Delta$ and πNN coupling constants evaluated at nucleon and delta masses, respectively. This ratio is $6\sqrt{2}/5 = 1.697$ for bare coupling constants, as dictated by SU(3)_{flavor} symmetry. For renormalized coupling constants this ratio is about 2.15, which is very close to the experimental value of 2.14 [20]. We believe that this is a very important result emerging from our calculation. This result implies that, to begin with, the quark model value of the ratio, which is about 25% smaller than the experimental value, is itself a good starting point. In other words, the bare coupling constants in the CCD model themselves are in qualitative agreement with the experimental values. The renormalization of these quantities, which should be done in any case, is able to produce a very good agreement with the experimental result. Incidently, we would like to note that when f = 86 MeV the πNN coupling constant agrees with the experimental value and the ratio of $\pi N\Delta$ and πNN coupling constants still remains close to 2.15.

The energy dependence of $f_{\pi}NN$ and $f_{\pi}N\Delta$ is shown in Fig. 4. The figure shows that the renormalized coupling constants increase with energy in the energy range shown.

V. CONCLUSIONS

The chiral color dielectric model has been used in this work to compute renormalized meson-baryon form factors. This has been done by writing the Lippmann-Schwinger equation in a series form and deriving the integral equation for the renormalized form factor. We find that the renormalization effects are primarily restricted to meson momenta smaller than 1 GeV. The increase in the form factor at zero meson momentum is about 10% at nucleon mass and about 30% at Δ mass. The renormalized NN π coupling constant is about 10% smaller than the experimental value if f, the pion decay constant, is chosen to be 93 MeV. The ratio of $N\Delta\pi$ and $NN\pi$ coupling constants is found to be 2.15, which is close to the experimental value. This ratio for bare coupling constants is 1.697. Thus the renormalization effects yield good agreement between experiment and theory. The computed form factors are soft with the equivalent monopole parameter Λ of about 800 MeV if fitting is done at small momentum transfers. However, the CCD model form factors decrease faster than the monopole form factor for large momenta and the CCD form factors are closer to Gaussian or dipole forms.

Strictly speaking, the computations done in this work are not fully relativistic although relativistic expressions for energies of the baryons are used. For a completely relativistic calculation, one will need to use relativistic propagators for baryons and use a relativistic integral equation, such as the Blankenbeckler-Sugar equation, instead of the relativized Lippmann-Schwinger equation that is used in the present work. Indeed it is technically possible to do this, with some increase in the complexity of the problem. However, we feel that the approach followed in the present work is reasonable since we have restricted ourselves to kinetic energies small in comparison with the baryon masses. Extension of the present calculation to higher energies would require the above-mentioned relativistic generalizations as well as inclusion of heavier baryon resonances.

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APPENDIX A: BARE FORM FACTORS

The meson-baryon interaction Hamiltonian is

$$H_{\text{int}} = \frac{i}{f} \int d^3x \frac{m}{\chi(x)} \bar{\Psi}(x) \gamma_5 \lambda_i \Psi(x) \phi_i(x), \qquad (A1)$$

where λ_i are the SU(3) Gell-Mann matrices acting on the flavor coordinate of the quarks. In terms of the pseudoscalar octet fields ϕ_i , we define the π , K, and η fields as

$$\phi_{\pi^{\mp}} = \phi_{1,\pm 1} = \pm \frac{1}{\sqrt{2}} (\phi_1 \pm i \phi_2),$$

$$\phi_{\pi^0} = \phi_{1,0} = \phi_3,$$

$$\phi_{K^-} = \phi_{1/2,1/2}^{(1)} = \frac{1}{\sqrt{2}} (\phi_4 + i \phi_5),$$

$$\phi_{-+} = \phi_{-+}^{(2)} = -\frac{1}{\sqrt{2}} (\phi_4 - i \phi_5),$$

(A2)

$$\phi_{K^{+}} = \phi_{1/2,-1/2}^{(2)} = \frac{1}{\sqrt{2}} (\phi_4 - i \phi_5),$$

$$\phi_{\bar{K}^0} = \phi_{1/2,-1/2}^{(1)} = \frac{1}{\sqrt{2}} (\phi_6 + i \phi_7),$$

$$\phi_{K^0} = \phi_{1/2,1/2}^{(2)} = \frac{1}{\sqrt{2}} (\phi_6 - i \phi_7),$$

FIG. 5. Diagrams (up to order $1/f^6$) contributing to the *T* matrix. Thick lines represent physical baryons, thin lines represent bare baryons, and dashed lines represent mesons.

$$\phi_{\eta} = \phi_{0,0} = \phi_8$$
.

An extra negative sign for the π^- field is introduced to conform with the usual definition of vectors in a spherical basis and is different from the usual definition of pion fields. With these definitions of meson fields,

$$\lambda_a \phi_a = (-1)^{\mu} \tau_{\mu} \phi_{1,-\mu} + v_{1/2,\mu} \phi_{1/2,-\mu}^{(1)} + u_{1/2,\mu} \phi_{1/2,-\mu}^{(2)} + \lambda_8 \phi_{0,0}.$$
(A3)

Quantizing the meson fields we have

$$\phi(x) = \frac{1}{\sqrt{8\pi^3}} \int \frac{d^3k}{\sqrt{2\omega_i(k)}} \left[e^{i\vec{k}\cdot\vec{x}} a_i(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}} \overline{a}_i^{\dagger}(\vec{k}) \right], \tag{A4}$$

where a_i is the annihilation operator of meson *i* and \overline{a}_i^{\dagger} is the creation operator of the meson conjugate to *i*. The commutation relations for the meson field operators are

$$\begin{split} & [a_{1,\mu}(\vec{k}), a_{1,\nu}^{\dagger}(\vec{k}')] = (-1)^{\mu} \delta_{\mu,\nu} \delta(\vec{k} - \vec{k}'), \\ & [a_{1/2,\mu}^{(i)}(\vec{k}), a_{1/2,\nu}^{(j)\dagger}(\vec{k}')] = \delta_{i,j} \delta_{\mu,\nu} \delta(\vec{k} - \vec{k}'), \end{split}$$

and $[a_{0,0}(\vec{k}), a_{0,0}^{\dagger}(\vec{k}')] = \delta(\vec{k} - \vec{k}')$. Computation of the matrix element of the interaction Hamiltonian between bare baryon states gives

$$H_{\text{int}} = \sum_{B,B',i} |B'\rangle \langle B| \frac{i}{f\sqrt{8\pi^3}} \int \frac{d^3k}{\sqrt{2\omega_i(k)}} [a_i(\vec{k}) - \bar{a}_i^{\dagger}(\vec{k})] \\ \times u^0_{BB'i}(k) \langle B'|\lambda_i \vec{\sigma}|B\rangle \cdot \vec{k}.$$
(A5)

The details of computing the bare form factor $u_{BB'i}^{0}(k)$ are given elsewhere [11]. Computing the spin-flavor matrix element $\langle B' | \lambda_i \vec{\sigma} | B \rangle$ and expressing the result in angular momentum formalism, we get

$$H_{\text{int}} = \sum \frac{i\alpha_{BB'\phi}}{4f\pi^{3/2}} \int \frac{d^3k}{\sqrt{\omega_{\phi}(k)}} [a_{\lambda,-\mu}(\vec{k}) - a^{\dagger}_{\lambda,\mu}(\vec{k})] \\ \times k_{1,-\nu} u^0_{BB'\phi}(k)(-1)^{\nu+i\phi} \langle T_B, \lambda, t_B, \mu | T_{B'}, t_{B'} \rangle \\ \times \langle S_B, 1, s_B, \nu | S_{B'}, s_{B'} \rangle.$$
(A6)

Here the summation over spin and isospins of baryons and mesons is implied. Also ϕ implies that the meson type (π , K, \overline{K} , and η) and their total isospins and projections along the *z* axis are represented by λ and μ , respectively. The



FIG. 6. Diagramatic representation of the vertex renormalization.

baryon spin (isospin) and its projection are represented by S_B (T_B) and s_B (t_B), respectively. The reduced matrix element $\alpha_{BB'\phi}$ and the phase factor i_{ϕ} are determined by explicit calculation of one matrix element, as is usually done. The phase factor i_{ϕ} is μ , 0, $1/2 - \mu$, and 0 for π , *K*, \overline{K} , and η , respectively.

APPENDIX B: RENORMALIZATION

One can derive the equations for renormalized form factors and masses by writing the *T*-matrix equation in a Lippmann-Schwinger series and summing parts of the series. This procedure, however, is somewhat cumbersome. On the other hand, the same conclusions can be arrived at by considering the graphical representation of the series. We have therefore shown the diagrams corresponding to the individual terms of the Lippmann-Schwinger series for the *T* matrix of Eq. (12) in Fig. 5. The figure shows the diagrams up to $1/f^6$. It should be clear that one can easily generalize



FIG. 7. Diagramatic representation of the renormalized propagator.

the conclusions drawn from the analysis of these diagrams to the full Lippmann-Schwinger series.

The diagrams in Fig. 5 are organized as follows. The diagrams in the first line renormalize the rightmost vertex. The diagrams in the second line renormalize the leftmost vertex and the diagrams in the third line renormalize the mass or the propagator of the intermediate baryon. The diagrams in the fourth line are the mixed diagrams representing renormalization of vertices as well as mass. Finally, the diagrams in the last line are the rest of the diagrams which do not give rise to renormalization. Generalizing the diagrams in the first line to all orders, we obtain an integral equation for the renormalization of vertex (form factor). Diagramatically, this is represented in Fig. 6 where the solid circle represents the renormalized form factor.

The corresponding integral equation obtained after some angular momentum algebra is

$$u_{BB'\phi}(k) = u_{BB'\phi}^{0}(k) + \sum_{\phi'B'',B'''} (-1)^{I} \hat{S}_{B''} \hat{S}_{B'''} \hat{T}_{B''} \frac{\alpha_{BB'''\phi'} \alpha_{B''B''\phi} \alpha_{B''B''\phi}}{12\pi^{2} f^{2} \alpha_{BB'\phi}} \\ \times W(T_{B}T_{B'''}T_{B''}T_{B'}; T_{\phi}T_{\phi'}) W(S_{B}S_{B'''}S_{B''}S_{B'}; 11) \\ \times \int \frac{k'^{4} dk' u_{BB'''\phi'}^{0}(k') u_{B''B''\phi}^{0}(k) u_{B''B''\phi'}(k')}{\omega_{\phi'}(k')[E - E_{B'''}^{0} - \omega_{\phi'}(k') - \omega_{\phi}(k)][E - E_{B''} - \omega_{\phi'}(k')]},$$
(B1)

where $\hat{a} = \sqrt{2a+1}$, $E_B^0(k) = \sqrt{k^2 + (M_B^0)^2}$, $E_B(k) = \sqrt{k^2 + M_B^2}$, $\omega_{\phi}(k) = \sqrt{k^2 + m_{\phi}^2}$, M_B and M_B^0 are renormalized and bare masses of baryon *B*, respectively, $I = \Delta_{\phi} + \Delta_{\phi'} - S_B - S_{B'} - T_B - T_{B'}$, with Δ_{ϕ} being 1, 0, 1, and 0 for π , *K*, \bar{K} , and η , respectively, S_B and T_B are the spin and isospin of baryon *B*, respectively, and $W(\cdots)$'s are the usual Rakah coefficients.

Now consider the mass renormalization. The relevent diagrams are shown in Fig. 7. Clearly, the third line in this diagram is the Schwinger-Dyson equation for the propagator with the self-energy being given by the meson loop. The thing to notice, however, is that one of the meson-baryon form factors appearing in the self-energy is a renormalized one where as the other is a bare one. Assuming that the computation is done in the frame in which the baryon is stationary, the renormalized and bare propagators are 1/[E $-M_B^0 - \Sigma(E)$] and $1/(E - M_B^0)$, respectively. The selfenergy $\Sigma(E)$ is

$$\Sigma(E) = \sum_{B'\phi} \frac{(-1)^{T_{B'} + S_{B'} - T_B - S_B - \Delta}}{12\pi^2 f^2} \times \frac{\hat{S}_{B'}\hat{T}_{B'}}{\hat{S}_B\hat{T}_B} \alpha_{BB'\phi} \alpha_{B'B\bar{\phi}} \int \frac{k^4 dk u^0_{BB'\phi}(k) u_{B'B\bar{\phi}}(k)}{\omega_{\phi}(k) [M_B - E_{B'} - \omega_{\phi}(k)]},$$
(B2)

where Δ is 0, -1/2, 1/2, and 0 for π , *K*, \overline{K} , and η , respectively. The mass of the physical baryon $M_B = M_B^0 + \Sigma (E = M_B)$.

It should be evident that using the renormalized form factors and propagators the T matrix can be written in the form given in Eqs. (16)–(18) and Fig. 2.

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