Isotopic spin effect in two-pion and three-pion Bose-Einstein correlations

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Bose-Einstein correlations of pions are calculated in the presence of both statistical and coherent pion production. An isotopic spin conservation is taken into account for the coherent component. Additional contributions due to the isospin effect in pion correlation functions are found. Moreover, the experimental data of two-particle Bose-Einstein correlations for neutral pions are analyzed in this scheme.
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I. INTRODUCTION

Bose-Einstein (BE) correlations of identical particles in multiple production processes have been extensively studied in the last years because they give information on the space-time region of interactions [1,2]. The basic effect is analogous to Hanbury-Brown–Twiss (HBT) interferometry in optics [3] and suggests statistical production of the particles (mainly π mesons). The possible presence of a coherent pionic component (for example, in the case of disoriented chiral condensate formation) modifies the HBT effect. This modification in particle physics was taken formerly to be in close analogy with optics [4].

On the other hand, pions (contrary to photons) are subjected to isotopic spin (and electric charge) conservation and so they cannot be emitted independently. While a corresponding change of the statistical part is not essential for large multiplicities (one must only ensure a symmetry between isotopic spin components [5]), the coherent part changes substantially when isotopic spin conservation is taken into account [6–8]. So we reconsider BE correlations of pions in the presence of both statistical and coherent components, taking into account isotopic spin conservation in the coherent part [9]. That will result in the appearance of an additional contribution to the BE correlation function of neutral pions.

Experimental data of BE correlations of neutral pions in π-Xe collisions, reported [10], are analyzed by our BE correlation formula.

We take annihilation (creation) operators a_i of the pionic fields as a sum of statistical (b_i) and coherent (c_i) parts:

$$a_i(\mathbf{k}) = b_i(\mathbf{k}) + c_i(\mathbf{k}), \quad i = +,-,0. \tag{1}$$

Normalizations can be chosen in such a way that the invariant single-particle inclusive momentum density for every kind of pions is

$$\rho(\mathbf{k}) = \frac{1}{\sigma}\frac{d\sigma}{dk} = \langle a^\dagger(\mathbf{k}) a(\mathbf{k})\rangle = \rho^{\mathrm{ch}}(\mathbf{k}) + \rho^c(\mathbf{k}),$$

$$\rho^{\mathrm{ch}}(\mathbf{k}) = \langle b^\dagger(\mathbf{k}) b(\mathbf{k})\rangle, \quad \rho^c(\mathbf{k}) = \langle c^\dagger(\mathbf{k}) c(\mathbf{k})\rangle, \tag{2}$$

with

$$dk = \frac{d^3k}{2E(2\pi)^3},$$

where the isotopic spin index is abbreviated, and brackets mean an averaging of matrix elements over the pure pionic state for c operators and a statistical averaging for b operators. We use also the traditional chaoticity parameter

$$p(\mathbf{k}) = \rho^{\mathrm{ch}}(\mathbf{k})/\rho(\mathbf{k}), \tag{3}$$

which gives a share of the statistical (chaotic) component in the single-particle density. If statistical and coherent parts have different momentum distributions, then the chaoticity parameter depends on momentum $\mathbf{k}$.

II. TWO-PION AND THREE-PION BE CORRELATION FUNCTIONS

Consider now the two-particle inclusive momentum density of identical pions:

$$\rho_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{\sigma}\frac{d^2\sigma}{dk_1 dk_2} = \langle a^\dagger(\mathbf{k}_1) a^\dagger(\mathbf{k}_2) a(\mathbf{k}_1) a(\mathbf{k}_2)\rangle. \tag{4}$$

Splitting the statistical average of the operator product into a product of pair expectations (with particle permutation), we get a two-particle density where BE statistics is taken into account:

$$\begin{aligned}\rho_2(\mathbf{k}_1,\mathbf{k}_2) = {} & \rho(\mathbf{k}_1)\rho(\mathbf{k}_2) + |\langle b^\dagger(\mathbf{k}_1) b(\mathbf{k}_2)\rangle|^2 \\ & + 2\,\mathrm{Re}[\langle b^\dagger(\mathbf{k}_1) b(\mathbf{k}_2)\rangle \langle c^\dagger(\mathbf{k}_2) c(\mathbf{k}_1)\rangle] \\ & + \langle c^\dagger(\mathbf{k}_1) c^\dagger(\mathbf{k}_2) c(\mathbf{k}_1) c(\mathbf{k}_2)\rangle \\ & - \langle c^\dagger(\mathbf{k}_1) c(\mathbf{k}_1)\rangle \langle c^\dagger(\mathbf{k}_2) c(\mathbf{k}_2)\rangle. \end{aligned} \tag{5}$$

The second term on the right hand side of Eq. (5) is the usual effect of BE statistics (HBT effect), the third term represents an interference effect of statistical and coherent production [4], and the last two terms represent a pure coherent contribution to the correlation function. This last contribution vanishes in optics (for photons) but survives in the case of coherent pion production due to the pion correlation arising through isotopic spin conservation. A brief report on two-particle BE correlations was done in Ref. [9].

The three-pion inclusive momentum density of identical pions is given by

$$\rho_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=\frac{1}{\sigma}\frac{d^3\sigma}{dk_1dk_2dk_3}$$
$$=\langle a^\dagger(\mathbf{k}_1)a^\dagger(\mathbf{k}_2)a^\dagger(\mathbf{k}_3)a(\mathbf{k}_1)a(\mathbf{k}_2)a(\mathbf{k}_3)\rangle$$
$$=I_1+I_2+I_3, \tag{6}$$

where

$$I_1=\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)\rho(\mathbf{k}_2)+\sum_{(1,2,3)}\{|\langle b^\dagger(\mathbf{k}_1)b(\mathbf{k}_2)\rangle|^2+2\,\mathrm{Re}[\langle b^\dagger(\mathbf{k}_1)b(\mathbf{k}_2)\rangle\langle c^\dagger(\mathbf{k}_2)c(\mathbf{k}_1)\rangle]\}\rho(\mathbf{k}_3)$$
$$+2\,\mathrm{Re}[\langle b^\dagger(\mathbf{k}_1)b(\mathbf{k}_2)\rangle\langle b^\dagger(\mathbf{k}_2)b(\mathbf{k}_3)\rangle\langle b^\dagger(\mathbf{k}_3)b(\mathbf{k}_1)\rangle]+2\sum_{(1,2,3)}\mathrm{Re}[\langle b^\dagger(\mathbf{k}_1)b(\mathbf{k}_2)\rangle\langle b^\dagger(\mathbf{k}_2)b(\mathbf{k}_3)\rangle\langle c^\dagger(\mathbf{k}_3)c(\mathbf{k}_1)\rangle],$$
$$I_2=\sum_{(1,2,3)}[\langle c^\dagger(\mathbf{k}_1)c^\dagger(\mathbf{k}_2)c(\mathbf{k}_1)c(\mathbf{k}_2)\rangle-\langle c^\dagger(\mathbf{k}_1)c(\mathbf{k}_1)\rangle\langle c^\dagger(\mathbf{k}_2)c(\mathbf{k}_2)\rangle]\langle b^\dagger(\mathbf{k}_3)b(\mathbf{k}_3)\rangle$$
$$+2\sum_{(1,2,3)}\mathrm{Re}\{[\langle c^\dagger(\mathbf{k}_1)c^\dagger(\mathbf{k}_2)c(\mathbf{k}_3)c(\mathbf{k}_2)\rangle-\langle c^\dagger(\mathbf{k}_1)c(\mathbf{k}_3)\rangle\langle c^\dagger(\mathbf{k}_2)c(\mathbf{k}_2)\rangle]\langle b^\dagger(\mathbf{k}_3)b(\mathbf{k}_1)\rangle\},$$
$$I_3=\langle c^\dagger(\mathbf{k}_1)c^\dagger(\mathbf{k}_2)c^\dagger(\mathbf{k}_3)c(\mathbf{k}_1)c(\mathbf{k}_2)c(\mathbf{k}_3)\rangle-\langle c^\dagger(\mathbf{k}_1)c(\mathbf{k}_1)\rangle\langle c^\dagger(\mathbf{k}_2)c(\mathbf{k}_2)\rangle\langle c^\dagger(\mathbf{k}_3)c(\mathbf{k}_3)\rangle. \tag{7}$$

In Eq. (7), I_1 is the usual three-particle inclusive momentum density in the presence of chaotic and coherent components, where BE correlations are taken into account [12]. I_2 and I_3 are composed of two-particle and three-particle inclusive densities of coherent components, respectively. These terms stem from the isotopic spin conservation of coherent components.

Two-pion and three-pion BE correlation functions are defined, respectively, by the equations

$$\frac{N^{(2i)}}{N^{\mathrm{BG}}}=\frac{\rho_2(\mathbf{k}_1,\mathbf{k}_2)}{\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)},\quad \frac{N^{(3i)}}{N^{\mathrm{BG}}}=\frac{\rho_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)\rho(\mathbf{k}_2)}.$$

III. ISOSPIN EFFECT IN THE COHERENT CONTRIBUTION

Consider now the form of the coherent contribution. A typical ansatz for a coherent pion state with fixed isospin is given by a zero isospin projection of the standard coherent state [6,7]. It has the form

$$|f\rangle=\frac{1}{\sqrt{N}}\int d\Omega\,\exp\left[-\frac{\langle n^c\rangle}{2}+\int dk f(\mathbf{k})\mathbf{e}\mathbf{c}^\dagger(\mathbf{k})\right]|0\rangle,$$

$$\mathbf{e}=(e_+,e_-,e_0)=\left(\frac{\sin\theta}{\sqrt{2}}e^{i\phi},\frac{\sin\theta}{\sqrt{2}}e^{-i\phi},\cos\theta\right), \tag{8}$$

where N is normalization factor and

$$\langle n^c\rangle=\int dk|f(\mathbf{k})|^2. \tag{9}$$

Integration in Eq. (8) is performed over directions of the unit vector $\mathbf{e}$ in three-dimensional isotopic space for $\pi^\pm$ and π^0 mesons, respectively, and $|f(\mathbf{k})|^2$ gives the momentum distribution of the coherently produced pions. If we consider the space-time region in which pions are produced, then $f(\mathbf{k})$ is the mass-shell Fourier transform of the space-time region $f(x)$:

$$f(\mathbf{k})=\int d^4x e^{-ikx}f(x)|_{k_0=E_k}.$$

This function is not necessarily real:

$$f(\mathbf{k})=|f(\mathbf{k})|e^{i\varphi(\mathbf{k})}. \tag{10}$$

The normalization N should be determined from the condition $\langle f|f\rangle=1$, and we get

$$N=\frac{8\pi^2}{\langle n^c\rangle}(1-e^{-2\langle n^c\rangle}).$$

We suggest below that the average number of pions is large, $\langle n^c\rangle\gg 1$; then,

$$N\approx 8\pi^2/\langle n^c\rangle. \tag{11}$$

Now it is necessary to calculate the matrix elements in Eqs. (5) and (7) over the pionic state of Eq. (8). To do that we act by c_i operators to the right and by $c_i^\dagger$ operators to the left. For large $\langle n^c\rangle$ the calculation is greatly simplified because in this case one may use the steepest descent method in the course of integration over the spherical angles $\theta_1,\phi_1,\theta_2,\phi_2$ entering the states. Then we get the main contribution from the region $\theta_1\approx\theta_2,\phi_1\approx\phi_2$ and reduce the matrix elements to the average over a single solid angle Ω. The resulting expressions for matrix elements take simple forms

$$\langle c_i^\dagger(\mathbf{k}_1)c_i(\mathbf{k}_1)\rangle = r_i|f(\mathbf{k}_1)|^2,$$

$$\langle c_i^\dagger(\mathbf{k}_1)c_j^\dagger(\mathbf{k}_2)c_i(\mathbf{k}_1)c_j(\mathbf{k}_2)\rangle = r_{ij}|f(\mathbf{k}_1)f(\mathbf{k}_2)|^2,$$

$$\langle c_i^\dagger(\mathbf{k}_1)c_j^\dagger(\mathbf{k}_2)c_k^\dagger(\mathbf{k}_2)c_i(\mathbf{k}_1)c_j(\mathbf{k}_2)c_k(\mathbf{k}_2)\rangle = r_{ijk}|f(\mathbf{k}_1)f(\mathbf{k}_2)f(\mathbf{k}_3)|^2, \tag{12}$$

where

$$r_{j_1\cdots j_n} = \frac{1}{4\pi}\int d\Omega|e_{j_1}|^2\cdots|e_{j_n}|^2. \tag{13}$$

The factors r_i, r_{ij}, and r_{ijk} can be easily calculated from Eq. (13):

$$r_+ = r_- = r_0 = \frac{1}{3},$$

$$r_{++} = r_{--} = \frac{2}{15}, \quad r_{00} = \frac{1}{5},$$

$$r_{+++} = r_{---} = \frac{2}{35}, \quad r_{000} = \frac{1}{7}. \tag{14}$$

IV. ANOTHER APPROACH TO THE ISOSPIN EFFECT IN THE COHERENT CONTRIBUTION

In order to compare the results obtained by the steepest descent method in the previous section, we also calculate two- and three-particle densities of coherent components using an isosinglet state with fixed multiplicity $(2n)$ [6,11]:

$$|\psi_{2n}\rangle = \frac{1}{\sqrt{(2n+1)!}}[a_+^\dagger a_-^\dagger - a_0^{\dagger 2}]^n|0\rangle. \tag{15}$$

Single-particle and two-particle densities are shown in [11]:

$$\langle a_+^\dagger a_+\rangle = \langle a_-^\dagger a_-\rangle = \langle a_0^\dagger a_0\rangle = \frac{2n}{3},$$

$$\langle a_+^{\dagger 2}a_+^2\rangle = \langle a_-^{\dagger 2}a_-^2\rangle = \frac{8n(n-1)}{15}, \quad \langle a_0^{\dagger 2}a_0^2\rangle = \frac{2n(6n-1)}{15}, \tag{16}$$

where

$$\langle F\rangle = \langle\psi_{2n}|F|\psi_{2n}\rangle.$$

After some calculations, the three-pion densities given from the isosinglet state (15) are evaluated as

$$\langle a_+^{\dagger 3}a_+^3\rangle = \langle a_-^{\dagger 3}a_-^3\rangle = \frac{16n(n-1)(n-2)}{35},$$

$$\langle a_0^{\dagger 3}a_0^3\rangle = \frac{4n(n-1)(10n+1)}{35}. \tag{17}$$

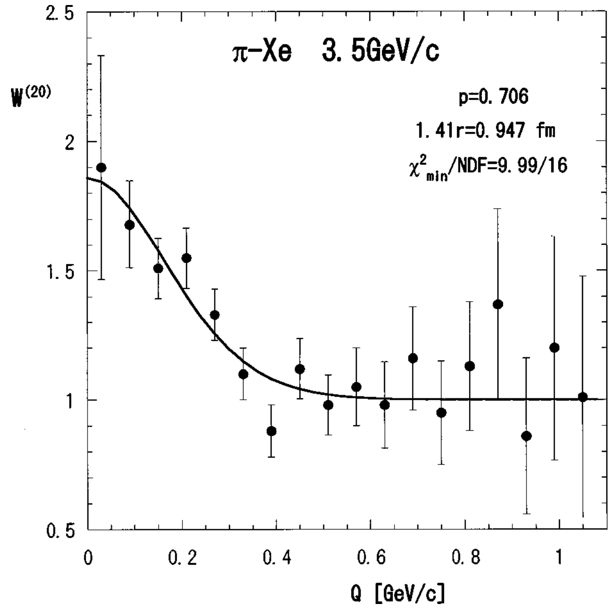

FIG. 1. Two-particle BE correlations for neutral pions in π-Xe collisions. Data are taken from Ref. [10]. The solid curve is drawn from Eq. (23) with $\sqrt{2}r = 0.947$ fm and $p = 0.706$, which are determined by the minimum chi-squared fit ($\chi^2_{\rm min}$/NDF=9.99/16). Q denotes $Q_{2\pi}$.

In the large n limit, the results shown in Eqs. (16) and (17) are essentially the same as those given in Eq. (14) in the previous section.

V. ISOSPIN EFFECT IN BE CORRELATION FUNCTIONS

To examine the isotopic spin effect to the BE correlation functions, we calculate two- and three-particle BE correlation functions in the symmetric representation [12]. In the following, the chaoticity parameter and the phase factor in the momentum density are assumed to be constant: $p_\alpha = p$, $\varphi_\alpha = \varphi$, $\alpha = 1,2,3$.

The correlation of chaotic components in n-particle BE correlation functions is parametrized as

$$\langle b^\dagger(\mathbf{k}_1)b(\mathbf{k}_2)\rangle = p\sqrt{\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)}E_{n\mathrm{B}}, \tag{18}$$

where

$$E_{n\mathrm{B}} = \exp\left[-\frac{2}{n(n-1)}r^2Q_{n\pi}^2\right],$$

$$Q_{n\pi} = \sqrt{-\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(k_i-k_j)^2}. \tag{19}$$

In the derivation of $E_{2\mathrm{B}}$, the Gaussian source function with a parameter of source size $\sqrt{2}r$ is assumed (see Ref. [12]). Then two- and three-particle BE correlation functions with constant $p_\alpha = p$ are given, respectively, as

$$\frac{N^{(2i)}}{N^{\rm BG}} = 1 + 2p(1-p)E_{2\mathrm{B}} + p^2E_{2\mathrm{B}}^2 + \beta_{ii}(1-p)^2,$$

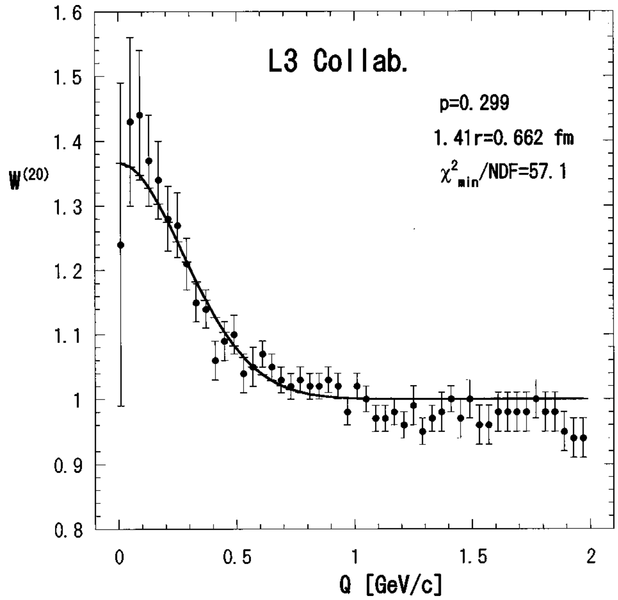

FIG. 2. Two-particle BE correlations for neutral pions in e^+e^- collisions. Data are taken from Ref. [14]. The solid curve is drawn from Eq. (23) with $p=0.299$ and $\sqrt{2}r=0.662$ fm, which are determined by the minimum chi-squared fit ($\chi^2_{\rm min}$/NDF=57.1/48). Q denotes $Q_{2\pi}$.

$$\frac{N^{(3i)}}{N^{\rm BG}}=1+6p(1-p)E_{\rm 3B}+3p^2(3-2p)E_{\rm 3B}^2+2p^3E_{\rm 3B}^3$$
$$+3\beta_{ii}(1-p)^2[1+2pE_{\rm 3B}]+\beta_{iii}(1-p)^3, \quad (20)$$

where

$$\beta_{ii}=\frac{r_{ii}}{r_i^2}-1, \quad \beta_{iii}=\frac{r_{iii}}{r_i^3}-3\beta_{ii}-1, \quad i=+,-,0. \quad (21)$$

In Eq. (20), terms with β_{ii} and β_{iii} denote additional contributions from the isotopic spin effect of coherent components. From Eqs. (14) and (21), coefficients β_{ii} and β_{iii} are calculated as

$$\beta_{++}=\beta_{--}=\frac{1}{5}, \quad \beta_{00}=\frac{4}{5}$$
$$\beta_{+++}=\beta_{---}=-\frac{2}{35}, \quad \beta_{000}=\frac{16}{35}. \quad (22)$$

It should be noted that the isotopic spin effect in BE correlation functions of neutral particles is much more enhanced than that of charged particles. This tendency is also observed in the Regge pole expansion with BE statistics [13]. In general, the coefficients β_{ij}'s depend on the total isospin of the pionic system.

As can be seen from Eq. (20), the asymptotic values (large $Q_{n\pi}$ limit) of two- and three-particle BE correlation functions do not necessarily approach 1 if the chaoticity parameter is not equal to 1. In other words, the asymptotic values of the BE functions will deviate from 1 due to the isotopic spin effect of coherent components. As the chaoticity parameter decreases from 0.7 to 0.5, the asymptotic value of the two-particle BE correlation function for neutral pions becomes from 1.07 to 1.20.

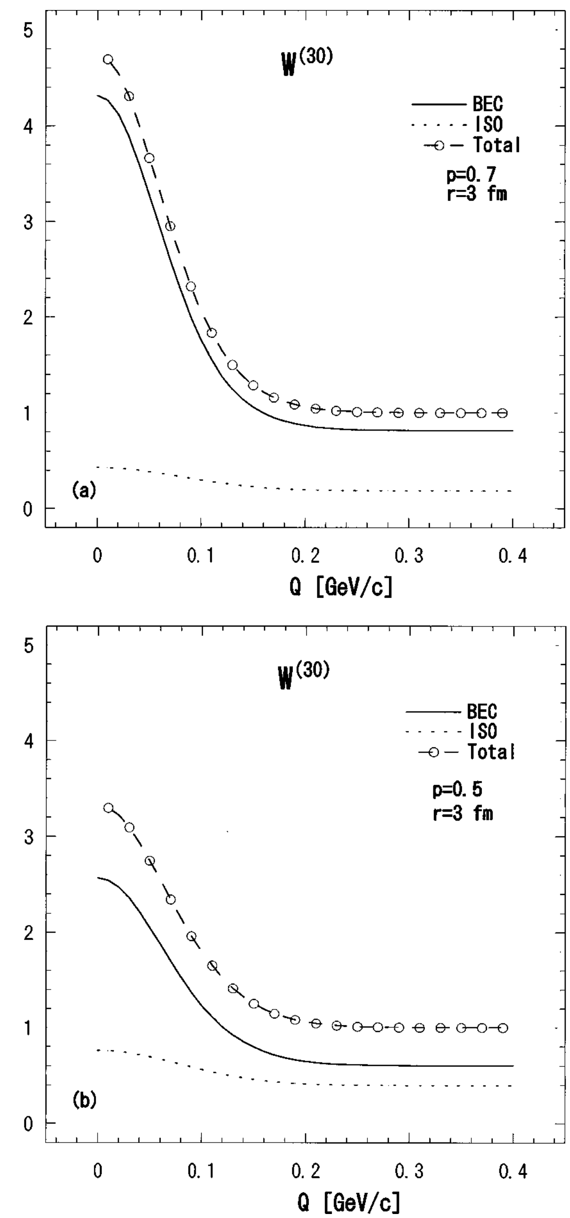

FIG. 3. Three-particle correlation functions for neutral pions calculated from Eq. (23): (a) with $r=3$ fm and $p=0.7$ and (b) with $r=3$ fm and $p=0.5$. Q denotes $Q_{3\pi}$.

To compare our formulas with experimental data, the asymptotic values of the BE correlation functions should be renormalized to 1:

$$W^{(2i)}=\frac{1}{1+\beta_{ii}(1-p)^2}\frac{N^{(2i)}}{N^{\rm BG}},$$
$$W^{(3i)}=\frac{1}{1+6\beta_{ii}(1-p)^2+\beta_{iii}(1-p)^3}\frac{N^{(3-)}}{N^{\rm BG}}. \quad (23)$$

In Fig. 1, two-particle BE correlations for neutral pions observed in π-Xe collisions [10] are analyzed by the formula

(23). The data are well fitted by Eq. (23) with $p=0.706 \pm 0.162$ and $\sqrt{2}r=0.947\pm 0.087$ fm at $\chi^2_{\rm min}$/NDF=9.99/16.

For the sake of reference, we also analyze the preliminary data of two-pion BE correlations in e^+e^- collisions by the L3 Collaboration [14]. The result is shown in Fig. 2. The data can be fitted by Eq. (23) with $p=0.299\pm 0.031$ and $\sqrt{2}r=0.662\pm 0.045$ fm at $\chi^2_{\rm min}$/NDF=57.1/48. The coherent component seems to correspond to contributions from resonances [14]. Our prediction should be examined in the near future [15].

In Fig. 3, calculated results of three-particle correlation functions $W^{(30)}$ with $p=0.7$ and $p=0.5$ at $r=3$ fm for neutral pions are shown. The contribution from isotopic spin conservation in coherent components is denoted by ISO and that from the usual two-particle BE correlations by BEC. The contribution of coherent components increases as the chaoticity parameter decreases in both correlation functions.

VI. CONCLUDING REMARKS

We have calculated two-particle and three-particle BE correlation functions in the presence of chaotic and coherent components. Isotopic spin conservation of the coherent component is taken into account. Additional contributions due to isotopic spin conservation appear in the two- and three-particle correlation functions. Moreover, our results for the coherent component are the same as those of Refs. [6] and [11], as n is large in their case. Because these terms are not a function of the momentum difference, the effect of isotopic conservation will not necessarily vanish, as the momentum difference squared increases: $|Q_{n\pi}|^2\to\infty$.

We have analyzed the data of two-particle BE correlations for neutral pions in π-Xe collisions and the preliminary data of two neutral pion BE correlations in e^+e^- collisions by the L3 Collaboration [14]. Fairly speaking, it can be said that the data are described by our formula (23).

If E_{2B} and the chaoticity parameter do not depend on charge states, we can extract the following relations from Eqs. (20) and (21), respectively:

$$\frac{N^{(20)}}{N^{\rm BG}}-\frac{N^{(2-)}}{N^{\rm BG}}=\frac{3}{5}(1-p)^2>0,$$

$$W^{(20)}-W^{(2-)}=-\frac{3(1-p)^2}{5}\times\frac{2p(1-p)E_{2B}+p^2E_{2B}^2}{[1+4(1-p)^2/5][1+(1-p)^2/5]}<0.$$

The sign of the above equations alternates according to the normalization conditions. This will be an indication of isotopic spin conservation.

As shown in Eq. (20), the presence of isotopic spin and its conservation leads to additional contributions to identical pion correlations, which depend on the portion of the coherent component. These terms affect the normalization for the two-pion and three-pion BE correlations as shown in Eq. (23).

Our results on two- and three-particle correlations will be useful in estimating the chaoticity parameter more precisely.

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