Nucleon momentum and density distributions in ^{4}He considering internal rotation

G. S. Anagnostatos
Institute of Nuclear Physics, National Center for Scientific Research ''Demokritos,'' Aghia Paraskevi, Attiki, 15310 Greece

A. N. Antonov
Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

P. Ginis and J. Giapitzakis
University of Patras, School of Engineering, Patra, 26110 Greece

M. K. Gaidarov
Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

A. Vassiliou
Computer Center, National Center for Scientific Research ''Demokritos,'' Aghia Paraskevi, Attiki, 15310 Greece
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A successful, simultaneous reproduction of density and momentum distributions in ^{4}He is presented in a method accounting for nucleon correlations using only two functions ($1s$ and $1d$) and totally three parameters, while in the best fit of equal quality known to us there are 12 parameters. Further, a physical interpretation of the $1d$ state involved is given as coming from an internal rotation of nucleons. Such an interpretation provides a hint that the neutron skin and neutron halo in exotic nuclei could have a rotational origin. [S0556-2813(98)05010-9]

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I. INTRODUCTION

As is well known, the mean field approximation has been proved to be very fruitful in nuclear physics for many years. However, many recent experimental findings have pointed to the limitations of this approximation and have opened the way for investigations beyond it. Such experimental findings include the appearance of a tail in the momentum distribution, the observation of a halo in the mass distribution of exotic nuclei, and the discovery of *D*-state admixture in very light nuclei, e.g., in ^{4}He. The present work is one of the attempts beyond the mean field theory in its effort to investigate some of the above phenomena which usually are related to the study of very light nuclei. Moreover, it supports the assumption that some of these phenomena, at least for very light nuclei, have a common origin, namely, that the key for their explanation is to consider additional degrees of freedom which have not been included in the models used so far. Indeed, there are indications that for these nuclei the adiabatic assumption is not valid and as a consequence (as will become clear in the next section) the nucleon internal motion contains internal rotation (even in their ground state) beyond the usual motion described by the mean field approximation. This additional motion needs investigation beyond mean field theory. The simultaneous reproduction of experimental results of different observables in this work lends support to the present assumption that the invalidity of the adiabatic approximation and its consequence of appearance of internal rotation is the starting point for a common explanation of several from the above mentioned new phenomena in very light nuclei.

This work employs the isomorphic shell model (ISM) [1,2], which considers a different harmonic-oscillator potential for each nuclear shell. This important difference of the ISM from the conventional shell models makes it possible to incorporate short-range nucleon-nucleon correlations (SRC's) and to consider their effects on basic characteristics of nuclei, such as nucleon momentum and density distributions, energies, radii, and others. This eventually leads to an approximate simultaneous description of the momentum and density distributions in nuclei which was impossible within the models based on the mean-field approximation [3–5].

It has been shown recently [6] that, using the natural orbital representation [7] and quasiexperimental data (i.e., data whose analysis is based on certain assumptions) for the proton momentum distribution of ^{4}He [8], it is possible by employing the ISM wave functions to obtain good agreement of the calculated momentum distribution $n(k)$ with the available experimental data for ^{12}C and ^{56}Fe and with the results of other sophisticated correlated methods for ^{16}O, ^{40}Ca, and ^{208}Pb nuclei. The successful estimation of the correlated part of $n(k)$ in [6] is based on the well-known fact that the high-momentum components of the momentum distribution (at $k \geqslant 2$ fm^{-1}) (normalized to unity) are nearly the same for all nuclei with $A \geqslant 4$ [5]. This important fact leads to the necessity of studying the momentum distribution of ^{4}He by itself simultaneously with the density distribution and constitutes the subject of the present investigation.

In doing this we consider carefully the internal motion of the particles in ^{4}He whose peculiarities seems to be responsible for the mentioned high-momentum components of the momentum distribution and, in addition, they modify the density distribution and the radius. This also gives a hint why the tail of the momentum distribution in all nuclei is almost identical to that of ^{4}He and why a halo appears in some exotic nuclei. One also could find common points between this work and that of Ref. [9] describing the appearance of D-state admixture in ^{4}He.

II. THE MODEL

We start by considering the Hamiltonian [10–12]:

$$H = H_0(r') + H_{\rm rot} + H, \tag{1}$$

where the three terms on the right-hand side describe the motion of the internal degrees of freedom, the rotation of the nucleus, and the coupling between the rotation and the internal motion, respectively. If the total momentum $\hat{\vec{I}}$ is written as the sum

$$\hat{\vec{I}} = \hat{\vec{R}} + \hat{\vec{J}}, \tag{2}$$

where $\hat{\vec{R}}$ is the angular momentum of the rotation and $\hat{\vec{J}}$ is the angular momentum associated with the internal degrees of freedom, the Hamiltonian (1) takes the form [10]

$$H = H_0(r') + \frac{\hbar^2}{2\Im}\hat{\vec{J}}^2 + \frac{\hbar^2}{2\Im}\hat{\vec{I}}^2 - \frac{\hbar^2}{\Im}\hat{\vec{I}}\hat{\vec{J}} + H', \tag{3}$$

For the ground state where $I=0$, the last three terms in Eq. (3) become zero and the Hamiltonian is simplified to

$$H = H_0(r') + \frac{\hbar^2}{2\Im}\hat{\vec{J}}^2, \tag{4}$$

where both terms refer to the internal motion of the nucleons. Now, the internal wave function (up to normalization factor):

$$\Psi \propto x^{\tau}_{K=0}(r'), \tag{5}$$

can be assumed to be an eigenfunction of Hamiltonian (4), where K is the projection of the total angular momentum on the axis (z') and τ stands for the rest of quantum numbers.

The existence of $J \neq 0$ implies that a sort of nonadiabaticity is included in Hamiltonian (4). Indeed, for very light nuclei it has been found [13] that the frequency of internal motion $\omega_{\rm intrinsic}$ is comparable with the frequency of rotation $\omega_{\rm rotation}$ (i.e., $\omega_{\rm int\,r} \approx \omega_{\rm rot}$), in contrast with the case of nuclei in the rare-earth region where $\omega_{\rm intrinsic} \gg \omega_{\rm rotation}$. These relationships of ω mean that while in the rare-earth region the condition for the adiabatic approximation is valid, in very light nuclei it is not.

Apparently for $I=0$ from Eq. (2) we obtain $\hat{\vec{R}} = -\hat{\vec{J}}$, i.e., if the interior angular momentum $\hat{\vec{J}}$ is different from zero, hence a rotation should exist to compensate $\hat{\vec{J}}$ and lead to $I^{\pi}=0^1$. That is, there is a collective rotation of nucleons. For reasons of parity the minimum value of $J(\neq 0)$ is 2. Indeed, this value of J can be obtained by coupling of nucleon spins in ^{4}He. It is obvious that in this case any possibility to approximate the internal motion Hamiltonian by a mean-field one is excluded. This already is an indication that correlations should be included in the Hamiltonian.

The problem of $J \neq 0$ can be approximated by using the formalism of the one-body density matrix corresponding to the correlated state of the system. It has to describe correctly as many as possible observables of the nuclear system under consideration. The natural orbital representation [7] is suitable for this purpose as explained below.

As is known, the natural orbital representation makes it possible to ''restore'' in a model-independent way the simplicity and transparency of the single-particle picture within the methods which account for nucleon-nucleon correlations. The single-particle natural orbitals (NO's) contain information on the SRC included in a given model. They form a natural orbital Fermi sea and the NO's below the corresponding Fermi-level (FL) which possess large occupation probabilities are called hole-state NO's, while those above this Fermi-level which possess small occupation probabilities are called particle-state NO's.

Let the radial function of the NO's for a state with quantum numbers nl in the coordinate and momentum space be $R_{nl}(r)$ and $R_{nl}(k)$, correspondingly (the latter being the Fourier transform of the former), and its occupation probability (occupation number) be λ_{nl}. Then, the nucleon density and momentum distributions can be written in the form

$$\begin{aligned}\rho(r) &= \frac{1}{4\pi}\sum_{nl}^{\infty} 2(2l+1)\lambda_{nl}|R_{nl}(r)|^2 \\ &= \frac{1}{4\pi}\sum_{nl}^{\rm FL} 2(2l+1)\lambda_{nl}|R_{nl}(r)|^2 \\ &\quad + \frac{1}{4\pi}\sum_{\rm FL}^{\infty} 2(2l+1)\lambda_{nl}|R_{nl}(r)|^2 \\ &\equiv \rho_h(r) + \rho_p(r),\end{aligned} \tag{6}$$

and

$$\begin{aligned}n(k) &= \frac{1}{4\pi Z}\sum_{nl}^{\infty} 2(2l+1)\lambda_{nl}|R_{nl}(k)|^2 \\ &= \frac{1}{4\pi Z}\sum_{nl}^{\rm FL} 2(2l+1)\lambda_{nl}|R_{nl}(k)|^2 \\ &\quad + \frac{1}{4\pi Z}\sum_{\rm FL}^{\infty} 2(2l+1)\lambda_{nl}|R_{nl}(k)|^2 \\ &\equiv n_h(k) + n_p(k),\end{aligned} \tag{7}$$

where $\rho_h(r)$, $n_h(k)$, and $\rho_p(r)$, $n_p(k)$ are the hole-state (h) and the particle-state (p) contributions to density and momentum distributions.

The normalizations for Eqs. (6) and (7) are

$$\int \rho(r)d\vec{r} = Z, \quad \int n(k)d\vec{k} = 1. \tag{8}$$

The first moments of the distributions $\rho(r)$ and $n(k)$, namely, the mean-square radius and the mean-kinetic energy, are also divided into hole- and particle-state contributions, similar to these distributions themselves in Eqs. (6) and (7):

$$\begin{aligned}\langle r^2\rangle &= \frac{1}{Z}\int \rho(r)r^2 d\vec{r} = \frac{1}{Z}\int \rho_h(r)r^2 d\vec{r} \\ &\quad + \frac{1}{Z}\int \rho_p(r)r^2 d\vec{r} \equiv \langle r^2\rangle_h + \langle r^2\rangle_p,\end{aligned} \tag{9}$$

$$\langle T\rangle=\int \frac{\hbar^2k^2}{2m}\, n(k)d\vec{k}=\int \frac{\hbar^2k^2}{2m}\, n_h(k)d\vec{k}+\int \frac{\hbar^2k^2}{2m}\, n_p(k)d\vec{k}\equiv\langle T\rangle_h+\langle T\rangle_p . \quad (10)$$

It has been shown in different theoretical correlation methods and for different systems of particles (for instance, in [14] for Fermi-liquid drops of ^{3}He and in Jastrow-type correlation method [15] for ^{4}He, ^{16}O, and ^{40}Ca) that the particle-state NO's are much more localized in the coordinate space than the unoccupied mean-field orbitals and, hence, they have significantly larger high-momentum components. Due to the SRC's small but nonzero occupation probabilities appear for the particle-state orbitals. At the same time it was shown [15] that hole-state NO's are close to occupied mean-field orbitals (shell model or Hartree-Fock ones) in coordinate and momentum spaces. Thus, a conclusion has been drawn, that SRC's do not affect significantly the hole-state orbitals in nuclei, but they affect the hole-state occupation probabilities, which are close to (but less than) unity. These properties of hole and particle NO's and their occupation probabilities have been used for the analysis of $n(k)$ in [6]. Following them, it can be stated that hole-state contributions to density $[\rho_h(r)]$, to momentum distribution $[n_h(k)]$, to mean-square radius ($\langle r^2\rangle_h$), and to mean-kinetic energy ($\langle T\rangle_h$) correspond to shell-model predictions, while particle-state contributions present SRC effects on these quantities. Specifically, concerning the mean-kinetic energy, it has been shown [16,5] that SRC's lead to a substantial increase of $\langle T\rangle$ values with respect to their Hartree-Fock ones, a fact which is related to the high-momentum tail of the momentum distribution. It was shown, in addition, that the increase of $\langle T\rangle$ and of the mean-removal energy when SRC's are included leads to a good agreement with the empirical binding energy per nucleon, which is not the case for the Koltun sum rule within the Hartree-Fock approximation [5].

III. RESULTS OF CALCULATIONS AND DISCUSSION

The point proton density and momentum distributions of ^{4}He have been calculated according to Eqs. (6) and (7) using the corresponding wave functions and occupation numbers for $1s$ and $1d$ states. In addition here, we calculate the folded charge density distribution

$$\rho_{\rm ch}(\vec{r})=\int \rho_{\rm pr}(\vec{r}-\vec{r}\,')\rho(\vec{r}\,')d\vec{r}\,', \quad (11)$$

where $\rho(r')$ is taken from Eq. (6) and the charge distribution of a proton is taken to be [17]

$$\rho_{\rm pr}(r)=\frac{1}{(\pi\alpha_{\rm pr}^2)^{3/2}}\exp(-r^2/\alpha_{\rm pr}^2), \quad (12)$$

with $r_{\rm pr}({\rm rms})=(\frac{3}{2})^{1/2}\alpha_{\rm pr}\approx 0.8$ fm [17].

After integration over angles in Eq. (11) we get

$$\rho_{\rm ch}(r)=\frac{2}{\sqrt{\pi}\alpha_{\rm pr}r}\, e^{r^2/\alpha_{\rm pr}^2}\int_0^\infty dr'r'\rho(r')e^{r'^2/\alpha_{\rm pr}^2}\sinh\left(\frac{2rr'}{\alpha_{\rm pr}^2}\right), \quad (13)$$

from which we obtain the expression for the charge rms radius

$$\langle r_{\rm ch}^2\rangle=\frac{8\sqrt{\pi}}{Z}\frac{1}{\alpha_{\rm pr}}\int_0^\infty dr r^3 e^{-r^2/\alpha_{\rm pr}^2}\times\int_0^\infty dr'r'\rho(r')e^{-r'^2/\alpha_{\rm pr}^2}\sinh\left(\frac{2rr'}{\alpha_{\rm pr}^2}\right)\equiv\langle r_{\rm ch}^2\rangle_h+\langle r_{\rm ch}^2\rangle_p . \quad (14)$$

Figure 1(a) presents the calculated point $[\rho(r)]$ and charge $[\rho_{\rm ch}(r)]$ density distributions together with the experimental charge distribution [18] for comparison. Figure 1(b) presents the calculated proton momentum distribution $n(k)$ and the corresponding quasi-experimental data from [8]. Our curves present the results of the simultaneous fitting of $\rho_{\rm ch}(r)$ and $n(k)$ to the corresponding data. The best values of the totally three parameters involved are $(\hbar\omega)_{1s}=22.50$ MeV, $(\hbar\omega)_{1d}=150$ MeV, $\lambda_{1s}=0.86$, $\lambda_{1d}=0.028$, where $2\lambda_{1s}+10\lambda_{1d}=2$. The comparison of our results with the data from both Figs. 1(a) and 1(b) is satisfactory. This is becoming more interesting if one considers that only two functions (namely, $1s$ and $1d$) and totally three parameters are employed. Indeed, this fit is at least comparable in quality with that of any other previous publication (e.g., [19]) with simultaneous fitting of density and momentum distributions and, most importantly, it uses the smallest number of parameters. Specifically, here only three parameters are used, while in [19] which is the best fit we know there are 12 parameters. It is farther interesting to notice in Fig. 1(a) the great similarity of our point proton density distribution with that of [20] for ^{4}He.

The use of only (two functions and) three parameters in this work has the additional advantage that it leads to an understanding of the physics hidden behind these parameters. Indeed, it relates our results with the internal (collective) rotation of nucleons in ^{4}He mentioned in Sec. II as explained below.

Inspired by Refs. [9] and [13] we interpret the $1d$-particle state involved in the present analysis as coming from the internal rotation of nucleons in ^{4}He. Indeed, the large $(\hbar\omega)$ value of this state implies strong localization apparent from the shift of the $1d$ wave function towards the origin and from its very small width. This is further seen by comparing our $(\hbar\omega)_{1d}=150$ MeV with that of Ref. [2] equal to 9.32 MeV. In order to test this interpretation we substituted our $1d$ wave function with a simple Gaussian function imitating a rotation and we repeated our procedure. Similar results were obtained when $\hbar\omega$ takes on larger values than that stated above. Given that $J^\pi=2^+$ for both the $1d$ and the rotation, the above close similarity of results gives us a basis to interpret the $1d$-natural orbital as due to internal rotation of nucleons in ^{4}He. In such a case the high-momentum components of the momentum distribution and the maximum of the point proton density distribution are due to the internal rotation of

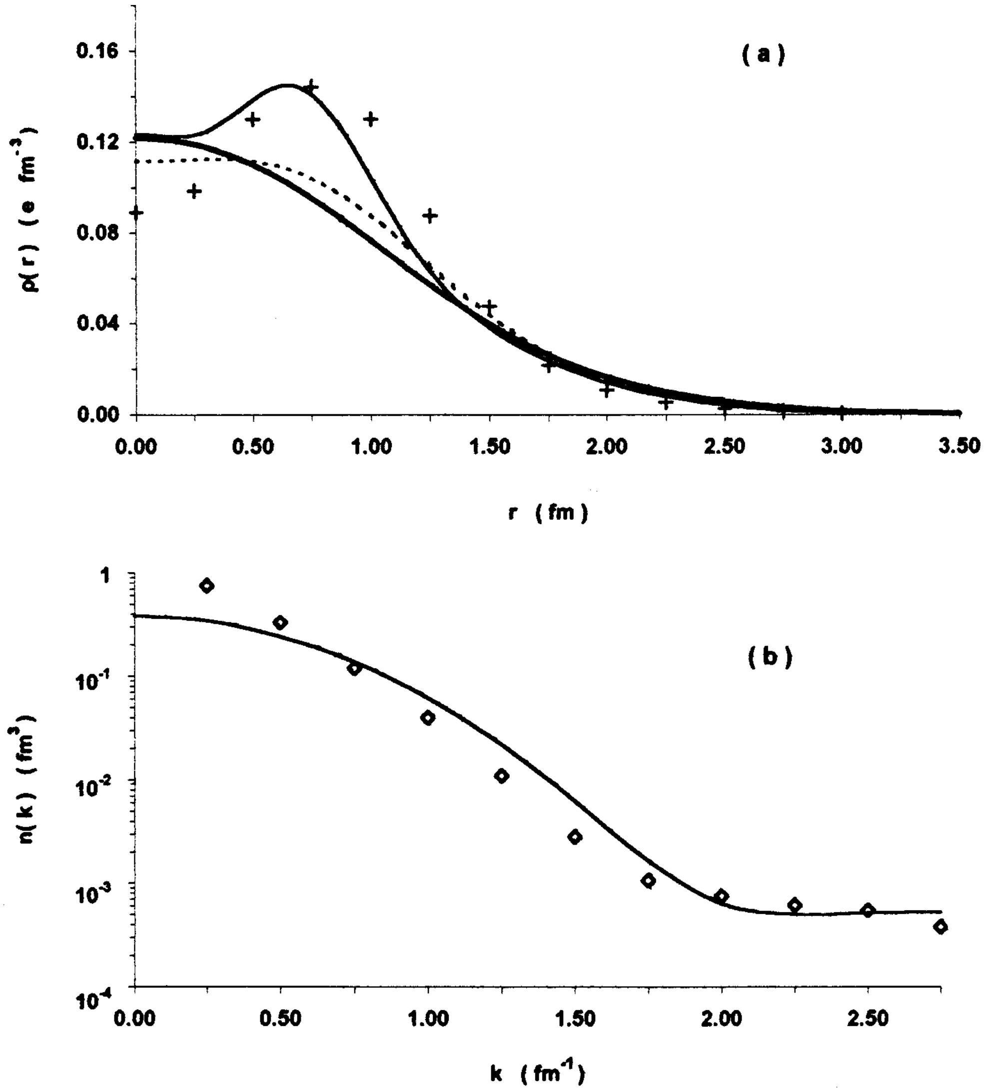

FIG. 1. (a) Density distribution in ^{4}He. Continuous thick and thin lines stand for the present predictions of charge and point proton density distributions, respectively, while the broken line and crosses (which correspond to the upper limit of the experimental errors from [20]) for the corresponding experimental quantities. (b) Momentum distribution in ^{4}He. The continuous line stands for the present prediction, while squares stand for the quasiexperimental values.

nucleons. Hence, our analysis supports that the Hamiltonian of ^{4}He is that given by Eq. (4), i.e., despite the fact that for the g.s. of ^{4}He the total angular momentum $I=0$, there is an internal angular momentum $J=2$. This J value comes from coupling of the nucleon spins. The rotation discussed above results from Eq. (2) for ^{4}He g.s. (i.e., $0=\vec{R}+\vec{J}$) and intends to compensate the internal angular momentum J in such a way that $I=0$. The d-state to s-state ratio (i.e., $10\lambda_{1d}/2\lambda_{1s}$) here is 0.16. Despite the fact that the rotation in ^{4}He supported by Ref. [9] comes from a different reasoning, we think that both works contain approximately the same physics.

By applying Eqs. (14) and (10) we calculate the first moments of the charge density and momentum distributions, namely the charge rms radius and the mean-kinetic energy. Their hole-state and particle-state components are also computed. Their numerical values are

$$\langle r_{\rm ch}^2\rangle^{1/2}=\sqrt{(1.71)^2+(0.47)^2}=1.77 \ \ \text{fm}, \tag{15}$$

$$\langle T\rangle=(14.51)+(36.75)=51.26 \ \ \text{MeV}. \tag{16}$$

The square of our rms charge radius 1.77 fm has to be reduced by the negative contribution of the charge distribution of a neutron $(0.34)^2$. This reduction leads to $\langle r_{\rm ch}^2\rangle^{1/2}=1.74$ fm, which compares well with the corresponding experimental value 1.71 fm [18]. As seen from Eq. (15), there is a component of the radius (0.47 fm) coming from the rotation which, however, for ^{4}He is rather small.

The part of kinetic energy due to $1s$ state, i.e., $\langle T\rangle_h=14.51$ MeV, can be compared to 17.1 MeV coming from shell model calculations [5], if the corresponding occupation probabilities are considered. Indeed, by considering that in shell model $\lambda_{1s}=1.00$, while here $\lambda_{1s}=0.86$, one can obtain $\langle T\rangle_{1s}=17.1\times0.86=14.71$ MeV, which is in good agreement with the present value. The rather large values of total kinetic energy, i.e., $\langle T\rangle=51.76$ MeV, and its part due to $1d$ state, i.e., $\langle T\rangle_p=36.75$ MeV, show that the explanation of the high momentum tail in $n(k)$ requires rather strong SRC to be included.

Concerning the binding energy of ^{4}He by consulting Ref. [2] we found that the depth of the harmonic-oscillator central potential needed to reproduce the experimental binding energy is about 107 MeV. This is a rather large value, but it is justified in our analysis since it has to compensate the negative contribution of the rotational kinetic energy or, in other words of the rather significant contribution of the d state.

IV. CONCLUSIONS

In the present work a good approximation of a simultaneous reproduction of the charge density and momentum distributions in ^{4}He has been obtained, together with the rms charge radius and the mean-kinetic energy. By applying natural orbital (NO) representation it was possible to include in our calculations the effect of short-range correlations in the quantities considered. Only two functions corresponding to $1s$ and $1d$ states and totally three parameters were sufficient for our task. The first function $(1s)$ is the expected one from the mean-field approximation, while the second $(1d)$ is the particle-state wave function. The latter is responsible for the (rather small) increase of the radius and for the observed bump of the point proton density distribution, but also for the high-momentum components of the momentum distribution and for the large value of the mean-kinetic energy.

The $1d$ function has been interpreted here as coming from the internal (collective) rotation of nucleons in ^{4}He, which is also supported in Ref. [9]. Specifically, the nucleon spins in the g.s. of ^{4}He are considered to couple to an internal angular momentum $J=2$ and, as a consequence, a rotation $\vec{R}$ of nucleons (or of the two deuterons in [9]) results which compensate $\vec{J}$ and leads to a total angular momentum $I=0$. This rotation is a new degree of freedom beyond the mean-field approximation which affects the numerical values of many observables, e.g., of density and momentum distributions, radii, kinetic energy, etc.

The increase of the nuclear radius due to internal rotation [see Eq. (15)] gives a hint that the neutron skin and neutron halo observed in exotic nuclei could have a rotational origin. Such work is in progress. According to Ref. [6], one can calculate the momentum distribution of nuclei beyond ^{4}He by employing the present results. Such an effort is also in progress.

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