Electromagnetic transitions of the $K=1$ band in the SU(3) limit of the neutron-proton interacting boson model

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$M1$ and $E2$ transition probabilities from the $K=1$ band, which is lowest in energy among the mixed-symmetry bands, to fully symmetric states are derived analytically within the framework of the SU(3) limit of the neutron-proton interacting boson model. To derive the electromagnetic transitions in closed forms, the tensorial characters of the electromagnetic transition operators, which are taken in terms of SU(3) generators, are investigated in the pure SU(3) limit. The properties of $M1$ and $E2$ intrinsic matrix elements in the classical limit of the present model and the $E2/M1$ mixing ratios for transitions of the $K=1$ band to fully symmetric states are also studied. [S0556-2813(98)01010-3]

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I. INTRODUCTION

The interacting boson model, first proposed by Arima and Iachello [1], has been successful in explaining a large variety of collective properties in even-even nuclei. The original version of the interacting boson model (IBM-1) does not distinguish the proton boson from the neutron boson. Motivated by microscopic considerations of the model, the neutron-proton interacting boson model (IBM-2), in which proton and neutron bosons are treated separately, was introduced [2,3]. Within the framework of the IBM-2, a new quantum number F spin was introduced to specify the proton-neutron symmetric properties of the wave function [2–4]. Fully symmetric (FS) IBM-2 states with a maximal F spin of $F=F_{\max}$, which are lowest in energy, are identical to the IBM-1 states. In the IBM-2, a new class of mixed-symmetry (MS) states with $F\neq F_{\max}$ is predicted. The discovery of low-lying collective 1^+ states in several deformed nuclei in the rare-earth region confirmed the existence of MS states with $F=F_{\max}-1$ [5,6]. This 1^+ state corresponds to the bandhead of the $K=1$ excitation mode in the SU(3) limit of the IBM-2. $M1$ transition strengths to the 1^+ level in deformed nuclei, which are described by the SU(3) limit of the IBM-2, have been extensively analyzed [7–11], and F-spin symmetry has been suggested to be strongly connected to the investigation of $M1$ transitions [12–15]. In particular, Van Isacker *et al.* [11] studied in detail the electromagnetic properties of 1^+, 2^+, and 3^+ states with $F=F_{\max}-1$ via the algebraic approach.

In the present work, we extend the previous studies for the electromagnetic transitions of MS states within the framework of the SU(3) limit of the IBM-2 and derive algebraically the $M1$ and $E2$ transitions from the lowest MS $K=1$ band to FS states to which the ground-state, the β, and the γ bands belong. In order to express $M1$ and $E2$ transition rates in closed forms, we take $M1$ and $E2$ transition operators in terms of the SU(3) generators and determine the tensor character for the SU(3) generators in a given group chain. In the process of reducing the matrix elements, knowledge of the U(6)⊃SU(3) isoscalar factors and the U(6)-reduced matrix elements of the one-body boson operator is needed. Although those are not known in general, the results which were obtained in studying the intrinsic states of the SU(3) limit of the IBM-2 [16,17] are available for our calculations. From the electromagnetic transition rates and the $E2/M1$ mixing ratios for transitions from the $K=1$ band to FS states, we investigate some of properties for the $M1$ and $E2$ intrinsic matrix elements in the classical limit, that is, for a large boson number.

Since the present analysis is restricted to an exact SU(3) symmetry of the IBM-2, the results obtained in this paper provide simple insight into the electromagnetic properties for MS states without complete numerical calculations. Therefore, analytic formulas for the $M1$ and $E2$ transition rates in the SU(3) limit can be useful for a preliminary approach of electromagnetic properties in the range of deformed nuclei.

II. ELECTROMAGNETIC TRANSITIONS FROM THE $K=1$ BAND IN THE SU(3) LIMIT OF THE IBM-2

Among the various kinds of dynamic symmetries of the IBM-2, in this paper we consider the SU(3) limit of the IBM-2, where the proton and neutron degrees of freedom are joined at the level of U(6). The group chain in this limit is given by [18]

$$\mathrm{U}_\pi(6)\otimes\mathrm{U}_\nu(6)\supset\mathrm{U}_{\pi+\nu}(6)\supset\mathrm{SU}_{\pi+\nu}(3)\supset\mathrm{O}_{\pi+\nu}(3)\supset\mathrm{O}_{\pi+\nu}(2). \tag{1}$$

The basis states of the IBM-2 span the irreducible representation (irrep) $[N_\pi]\otimes[N_\nu]$ of $\mathrm{U}_\pi(6)\otimes\mathrm{U}_\nu(6)$, where $N_\pi(N_\nu)$ is the number of proton (neutron) bosons. $\mathrm{U}_{\pi+\nu}(6)$ is characterized by the irrep $[N-f,f]$, where N is the total boson number ($N=N_\pi+N_\nu$), and the quantum numbers associated with SU(3) and its subgroups are same as those of the IBM-1 [19]. Wave functions in this limit are thus characterized by

$$|[N_\pi]\otimes[N_\nu];[N-f,f]\beta(\lambda\mu)\kappa LM\rangle, \tag{2}$$

where β and κ are labels necessary to completely specify U(6)⊃SU(3) and SU(3)⊃O(3) reductions, respectively. To simplify the notation, the labels β and κ will be omitted when U(6)⊃SU(3) and SU(3)⊃O(3) reductions are unique. The irrep $[N-f,f]$ of $\mathrm{U}_{\pi+\nu}(6)$ is related to F spin through $F=N/2-f$ [11]. FS states are characterized by the irrep $[N]$

corresponding to the maximum value of F spin ($F_{\max}=N/2$), while the lowest MS states are characterized by the irrep $[N-1,1]$ corresponding to $F=F_{\max}-1$. In this paper, we restrict only the ground-state (g), the β, the γ, and the lowest MS $K=1$ (m) bands which are most important in studying the collective properties of the low-lying levels in the IBM-2. They are denoted as

$$|g,LM\rangle=|[N_\pi]\otimes[N_\nu];[N](2N,0)LM\rangle, \tag{3a}$$

$$|\beta,LM\rangle=|[N_\pi]\otimes[N_\nu];[N](2N-4,2),\kappa=0,LM\rangle, \tag{3b}$$

$$|\gamma,LM\rangle=|[N_\pi]\otimes[N_\nu];[N](2N-4,2),\kappa=2,LM\rangle, \tag{3c}$$

$$|m,LM\rangle=|[N_\pi]\otimes[N_\nu];[N-1,1](2N-2,1)LM\rangle. \tag{3d}$$

The state given in Eq. (3d) belongs to the $K=1$ band, which is the lowest in energy among MS bands in the SU(3) limit. In the IBM-2, the generators of $SU_{\pi+\nu}(3)$ are given by

$$L_q=L_{\pi,q}+L_{\nu,q} \quad (q=0,\pm1), \tag{4a}$$

$$Q_q=Q_{\pi,q}+Q_{\nu,q} \quad (q=0,\pm1,\pm2), \tag{4b}$$

with

$$L_{\rho,q}=\sqrt{10}(d_\rho^\dagger \tilde{d}_\rho)_q^{(1)}, \tag{5a}$$

$$Q_{\rho,q}=(d_\rho^\dagger s_\rho+s_\rho^\dagger \tilde{d}_\rho)_q^{(2)}-\frac{\sqrt{7}}{2}(d_\rho^\dagger \tilde{d}_\rho)_q^{(2)}, \tag{5b}$$

where ρ corresponds to π (proton) or ν (neutron) bosons. For analytic calculations of matrix elements, it is necessary to know the tensorial properties of the operators within the group chain given in Eq. (1). The one-body boson operator transforms as a $T^{[2,14]}$ tensor under U(6), and L_q (Q_q) transforms as the irreducible tensor operator $T_{1q}^{(11)}$ ($T_{2q}^{(11)}$) under SU(3) and its subgroups [11]. The phase and normalization factor for the tensor operator can be determined from the definition of the irreducible tensor operator $T_{\kappa LM}^{(\lambda\mu)}$ under SU(3)⊃O(3) in the usual way [20]:

$$[G_q,T_{\kappa LM}^{(\lambda\mu)}]=\sum_{\kappa'L'}\langle(\lambda\mu)\kappa'L'M+q|G_q|(\lambda\mu)\kappa LM\rangle T_{\kappa'L'M+q}^{(\lambda\mu)}, \tag{6}$$

where $G_q=L_q$ and Q_q. The matrix element of L_q can be easily calculated from the reduced matrix element $\langle(\lambda\mu)\kappa'L'||L||(\lambda\mu)\kappa L\rangle=\sqrt{L(L+1)(2L+1)}\delta_{LL'}\delta_{\kappa\kappa'}$. The matrix element of Q_q can be calculated by using the Elliott matrix elements [21] and the Vergados expansion coefficients [22]. On the other hand, via the Wigner-Eckart theorem, the generators of SU(3) can be put into the form [20]

$$[T_{lq}^{(11)},T_{\kappa LM}^{(\lambda\mu)}]=\sum_{\kappa'L'}\sum_{\xi=1}^{2}\langle LM,lq|L'M+q\rangle\langle(\lambda\mu)\kappa L;(11)l||(\lambda\mu)\kappa'L'\rangle_\xi\langle(\lambda\mu)||T^{(11)}||(\lambda\mu)\rangle_\xi T_{\kappa'L'M+q}^{(\lambda\mu)}. \tag{7}$$

The additional quantum number ξ is necessary, since $(\lambda\mu)$ occurs twice in the Kronecker product $(\lambda\mu)\otimes(11)$ when $\mu\neq0$. Hecht [23] defined the quantum number ξ as a special choice of the tensor operator with nonzero reduced matrix elements for only one state $\xi=1$ and expressed the reduced matrix element of $T^{(11)}$ in the intrinsic scheme as

$$\langle(\lambda\mu)||T^{(11)}||(\lambda\mu)\rangle_\xi=\left(\frac{\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu}{3}\right)^{1/2}\delta_{\xi1}. \tag{8}$$

Vergados [22] defined the phases so as to ensure that the SU(3)⊃O(3) isoscalar factor (ISF) is real and obtained the SU(3)⊃O(3) ISF's extensively. In the present work, we follow the phase convention adopted by Vergados. The reduced matrix element of $T^{(11)}$ in the present model is the same as the corresponding matrix element in the intrinsic scheme [Eq. (8)] except for a $(\lambda\mu)$-dependent phase factor. The phase of the reduced matrix element in Eq. (7) is required to be positive for $\mu=0$ and negative for $\mu=1,2$ for consistency with the definition of the SU(3)⊃O(3) ISF's.

Comparing the results of Eq. (6) with (7), the tensor character of the SU(3) generator under SU(3)⊃O(3) can be obtained. Considering together with tensorial properties under U(6), SU(3) generators are expressed as the tensor forms

$$L_q=2T_{1q}^{[2,14](11)} \quad (q=0,\pm1), \tag{9a}$$

$$Q_q=\sqrt{\frac{3}{2}}\,T_{2q}^{[2,14](11)} \quad (q=0,\pm1,\pm2). \tag{9b}$$

The matrix element of the tensor operator can be calculated analytically by applying the generalized Wigner-Eckart theorem. The matrix element involves the generalized Clebsch-Gordan coefficient, which is written as the product of U(6)⊃SU(3) ISF, SU(3)⊃O(3) ISF, and the ordinary Clebsch-Gordan coefficient according to Racah's factorization lemma. The reduced matrix element of the ρ-boson SU(3) generator between the FS state and $K=1$ band is written as

$$\langle [N_\pi]\otimes[N_\nu];[N](\lambda\mu)\kappa L||T^{[2,1^4](11)}_{\rho,l}||[N_\pi]\otimes[N_\nu];[N-1,1](2N-2,1)L'\rangle$$
$$=\sqrt{2L+1}\langle [N-1,1](2N-2,1);[2,1^4](11)||[N](\lambda\mu)\rangle\langle (2N-2,1)L';(11)l||(\lambda\mu)\kappa L\rangle$$
$$\times\langle [N_\pi]\otimes[N_\nu];[N]||||T^{[2,1^4]}_{\rho}||||[N_\pi]\otimes[N_\nu];[N-1,1]\rangle. \tag{10}$$

The necessary U(6)⊃SU(3) ISF's for this analysis are [17]

$$\langle [N-1,1](2N-2,1);[2,1^4](11)||[N](2N,0)\rangle=-\sqrt{\frac{2N-2}{N}}$$
$$\langle [N-1,1](2N-2,1);[2,1^4](11)||[N](2N-4,2)\rangle=\sqrt{\frac{2N+1}{N(2N-3)}}. \tag{11}$$

The U(6)-reduced matrix element of the one-body boson operator connecting FS and MS states, which is denoted with four bars in Eq. (10), has been obtained by using the F-spin formalism and is given as [16,17]

$$\langle [N_\pi]\otimes[N_\nu];[N]||||T^{[2,1^4]}_{\pi}||||[N_\pi]\otimes[N_\nu];[N-1,1]\rangle$$
$$=-\langle [N_\pi]\otimes[N_\nu];[N]||||T^{[2,1^4]}_{\nu}||||[N_\pi]\otimes[N_\nu];[N-1,1]\rangle=\sqrt{\frac{N_\pi N_\nu}{N-1}}. \tag{12}$$

As $E2$ and $M1$ transition operators, we take the standard expression in the SU(3) limit of the IBM-2:

$$T(E2)=e_\pi Q_\pi+e_\nu Q_\nu, \tag{13a}$$

$$T(M1)=\sqrt{\frac{3}{4\pi}}(g_\pi L_\pi+g_\nu L_\nu), \tag{13b}$$

where e_ρ and g_ρ ($\rho=\pi,\nu$) are the ρ-boson effective charge and the ρ-boson g factor, given in units of e b and μ_N, respectively. The matrix elements of the electromagnetic transition operators can be simply calculated by using Eqs. (9) and (10). The reduced $M1$ transition probabilities from the $K=1$ band the to the ground-state band are derived as follows:

$$B(M1;mL\to gL)=\frac{3}{4\pi}(g_\pi-g_\nu)^2\frac{(2N-L)(2N+L+1)}{N^2(2N-1)}N_\pi N_\nu, \tag{14a}$$

$$B(M1;mL+1\to gL)=\frac{3}{4\pi}(g_\pi-g_\nu)^2\frac{2(L+2)(2N-L)}{(2L+3)N(2N-1)}N_\pi N_\nu, \tag{14b}$$

$$B(M1;mL-1\to gL)=\frac{3}{4\pi}(g_\pi-g_\nu)^2\frac{2(L-1)(2N+L+1)}{(2L-1)N(2N-1)}N_\pi N_\nu. \tag{14c}$$

From Eq. (14b), the $B(M1)$ strength for the $1^+_m\to 0^+_g$ transition is given as the well-known result [9,11]

$$B(M1;1^+_m\to 0^+_g)=\frac{3}{4\pi}(g_\pi-g_\nu)^2\frac{8N_\pi N_\nu}{3(2N-1)}. \tag{15}$$

The $B(M1)$ strengths of the $m\to g$ transition for $N=12$ are shown in Fig. 1. From Eq. (14) and Fig. 1, an interesting result is obtained: for finite boson number N, the $B(M1)$ strength for the $mL-1\to gL$ transition increases, whereas $B(M1)$ strengths for $mL+1\to gL$ and $mL\to gL$ transitions decrease as an increase of the angular momentum L.

For a systematic analysis of the reduced $M1$ transition probabilities of the $K=1$ band to the ground-state band, it is useful to express $B(M1)$ strengths in Eq. (14) in the following form:

$$B(M1;mL'\to gL)=2\langle L'1,1-1|L0\rangle^2 M^2_{gm}(L,L'), \tag{16}$$

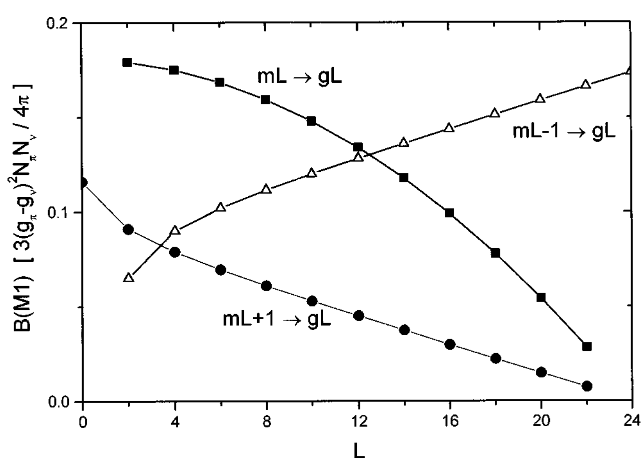

FIG. 1. $B(M1)$ strengths of the transition from the $K=1$ band to the ground-state band for the total boson number $N=12$ as a function of the angular momentum L.

TABLE I. Relative $M^2_{fm}(L,L')$ values between the $K=1$ band and the β,γ bands. The reference $M^2_{fm}(L,L)$ values are given in Eq. (19).

L'	$\left\|\frac{M_{\beta m}(L,L')}{M_{\beta m}(L,L)}\right\|^2$	$\left\|\frac{M_{\gamma m}(L,L')}{M_{\gamma m}(L,L)}\right\|^2$ (L=even)	$\left\|\frac{M_{\gamma m}(L,L')}{M_{\gamma m}(L,L)}\right\|^2$ (L=odd)
$L+1$	$\frac{2N+L+1}{2N}\left[\frac{(2N-2)^2-L}{(2N-2)^2-L(L+1)+1}\right]^2$	$\frac{2N+L+1}{2N}$	$\frac{2N}{2N-L-1}$
$L-1$	$\frac{2N-L}{2N}\left[\frac{(2N-2)^2+L+1}{(2N-2)^2-L(L+1)+1}\right]^2$	$\frac{2N-L}{2N}$	$\frac{2N}{2N+L}$

with

$$M^2_{gm}(L,L)=B(M1;mL\rightarrow gL), \tag{17a}$$

$$M^2_{gm}(L,L+1)=\frac{2N}{2N+L+1}\,M^2_{gm}(L,L), \tag{17b}$$

$$M^2_{gm}(L,L-1)=\frac{2N}{2N-L}\,M^2_{gm}(L,L). \tag{17c}$$

In the geometrical model, M^2_{gm} is interpreted as the square of the $M1$ intrinsic matrix element, which is independent of L [24]. However, in the present model, M^2_{gm} is dependent not only on N, but on L. The $B(M1)$ strengths from the $K=1$ band to the β and γ bands can be calculated from a similar method and expressed as

$$B(M1;mL'\rightarrow fL) =\langle L'1,1k_f-1|Lk_f\rangle^2 M^2_{fm}(L,L')\times\begin{cases}2 & \text{for } f=\beta,\\ 1 & \text{for } f=\gamma,\end{cases} \tag{18}$$

where $k_f=0$ for the β band and $k_f=2$ for the γ band, respectively. In Table I we list $M^2_{\beta m}$ and $M^2_{\gamma m}$ values relative to the following values:

$$M^2_{\beta m}(L,L)=B(M1;mL\rightarrow\beta L) =\frac{3}{4\pi}(g_\pi-g_\nu)^2 \times\frac{2[(2N-2)^2-L(L+1)+1]^2}{(N-1)^2(2N-1)(2N-3)\phi(N,L)}\,N_\pi N_\nu, \tag{19a}$$

$$M^2_{\gamma m}(L,L)=\frac{3}{4\pi}(g_\pi-g_\nu)^2 \times\frac{8(2N-L-2)(2N+L-1)}{(N-1)(2N-3)\phi(N,L)}\,N_\pi N_\nu \quad (L=\text{even}), \tag{19b}$$

$$=\frac{3}{4\pi}(g_\pi-g_\nu)^2 \times\frac{(2N-L-1)(2N+L)}{N(N-1)^2(2N-3)}\,N_\pi N_\nu \quad (L=\text{odd}), \tag{19c}$$

where $\phi(N,L)=2(2N-2)^2-L(L+1)$. In the present model the $B(M1)$ strengths for the transitions from the $K=1$ band to the ground-state, β, and γ bands have the common parameter $3(g_\pi-g_\nu)^2N_\pi N_\nu/4\pi$: therefore, the ratio of two $B(M1)$ strengths is independent of the adjustable parameters.

In the classical limit, i.e., for a large boson number N, M_{fm} (where $f=g$, β, and γ) is independent of L, and so it is called the $M1$ intrinsic matrix element. For transitions of the $K=1$ band to the FS states, the magnitudes of the $M1$ intrinsic matrix elements are given as

$$|M_{gm}|=\sqrt{\frac{3N_\pi N_\nu}{2\pi N}}\,|g_\pi-g_\nu|, \tag{20a}$$

$$|M_{\beta m}|=\sqrt{\frac{3N_\pi N_\nu}{4\pi N^2}}\,|g_\pi-g_\nu|, \tag{20b}$$

$$|M_{\gamma m}|=\sqrt{\frac{3N_\pi N_\nu}{2\pi N^2}}\,|g_\pi-g_\nu|. \tag{20c}$$

The ratios of two $M1$ intrinsic matrix elements between the $K=1$ band and FS bands are free of the boson g factor: that is, those are dependent only on the boson number N, for example, $|M_{\beta m}/M_{gm}|=\sqrt{1/2N}$ and $|M_{\gamma m}/M_{gm}|=\sqrt{1/N}$. From Eqs. (16) and (18), the ratio of two reduced $M1$ transition probabilities from L' states of the $K=1$ band to L states of a band belongs to FS states depends on geometrical factors. For example, the ratio $B(M1{:}mL'\rightarrow gL)/B(M1{:}1^+_m\rightarrow 0^+_g)$ becomes

$$\frac{B(M1;mL'\rightarrow gL)}{B(M1;1^+_m\rightarrow 0^+_g)}=\left(\frac{\langle L'1,1-1|L0\rangle}{\langle 11,1-1|00\rangle}\right)^2\left|\frac{M_{gm}(L,L')}{M_{gm}(0,1)}\right|^2. \tag{21}$$

Since $|M_{gm}(L,L')/M_{gm}(0,1)|=1$ in the classical limit, the ratio $B(M1;mL'\rightarrow gL)/B(M1;1^+_m\rightarrow 0^+_g)$ is identical to the

TABLE II. Relative $M_{fm}'^2(L,L')$ values between the $K=1$ band and FS states. The reference $M_{fm}'^2(L,L)$ values are given in Eq. (23).

L'	$\left\lvert\frac{M'_{gm}(L,L')}{M'_{gm}(L,L)}\right\rvert^2$	$\left\lvert\frac{M'_{\beta m}(L,L')}{M'_{\beta m}(L,L)}\right\rvert^2$
$L+1$	$\frac{2N}{2N+L+1}$	$\frac{2N+L+1}{2N}\left[\frac{(2N-2)^2+L-2}{(2N-2)^2+L(L+1)-3}\right]^2$
$L+2$	$\frac{2N-L-2}{2N+L+1}$	$\frac{(2N+L+1)(2N-L-2)(2N+L-2)^2}{[(2N-2)^2+L(L+1)-3]^2}$
$L-1$	$\frac{2N}{2N-L}$	$\frac{2N-L}{2N}\left[\frac{(2N-2)^2-L-3}{(2N-2)^2+L(L+1)-3}\right]^2$
$L-2$	$\frac{2N+L-1}{2N-L}$	$\frac{(2N-L)(2N+L+1)(2N-L-3)^2}{[(2N-2)^2+L(L+1)-3]^2}$
L'	$\left\lvert\frac{M'_{\gamma m}(L,L')}{M'_{\gamma m}(L,L)}\right\rvert^2$ ($L=$even)	$\left\lvert\frac{M'_{\gamma m}(L,L')}{M'_{\gamma m}(L,L)}\right\rvert^2$ ($L=$odd)
$L+1$	$\frac{2N+L+1}{2N}$	$\frac{2N}{2N-L-1}$
$L+2$	$\frac{2N+L+1}{2N-L-2}$	$\frac{2N+L+2}{2N-L-1}$
$L-1$	$\frac{2N-L}{2N}$	$\frac{2N}{2N+L}$
$L-2$	$\frac{2N-L}{2N+L-1}$	$\frac{2N-L+1}{2N+L}$

result from the Alaga rule, which predicts it as the ratio of two Clebsch-Gordan coefficients [24]. Analogous results are obtained for $M1$ transitions of the $K=1$ band to the β and γ bands.

With similar techniques, the reduced $E2$ transition probabilities for the $K=1$ band to the FS states can be calculated and written as

$$B(E2;mL'\rightarrow fL)=\langle L'1,2k_f-1|Lk_f\rangle^2 M_{fm}'^2(L,L')\times\begin{cases}2 & \text{for } f=g,\beta,\\ 1 & \text{for } f=\gamma,\end{cases}\tag{22}$$

where $k_f=0$ for g and β bands, and $k_f=2$ for the γ band. As for the case of the $M1$ transition, M'^2 is interpreted as the square of the $E2$ intrinsic matrix element and is independent of L in the geometrical model. In Table II, we list M'^2 values for the $E2$ transitions of the $K=1$ band to the FS states relative to the corresponding values for $mL\rightarrow fL$ transitions, which are given as

$$M_{gm}'^2(L,L)=(e_\pi-e_\nu)^2\,\frac{3(2N-L)(2N+L+1)}{8N^2(2N-1)}\,N_\pi N_\nu,\tag{23a}$$

$$M_{\beta m}'^2(L,L)=(e_\pi-e_\nu)^2\times\frac{3[(2N-2)^2+L(L+1)-3]^2}{4(N-1)^2(2N-1)(2N-3)\phi(N,L)}\,N_\pi N_\nu,\tag{23b}$$

$$M_{\gamma m}'^2(L,L)=(e_\pi-e_\nu)^2\times\frac{3(2N-L-2)(2N+L-1)}{(N-1)(2N-3)\phi(N,L)}\,N_\pi N_\nu\quad(L=\text{even}),\tag{23c}$$

$$=(e_\pi-e_\nu)^2\times\frac{3(2N-L-1)(2N+L)}{8N(N-1)^2(2N-3)}\,N_\pi N_\nu\quad(L=\text{odd}),\tag{23d}$$

where $\phi(N,L)=2(2N-2)^2-L(L+1)$. For the $M1$ and $E2$ transitions of 1_m^+ and 2_m^+ states to FS states, the $B(M1)$ and $B(E2)$ strengths are identical with the results obtained by Van Isacker *et al.* [11]. In this work, we have extended their work and calculated the electromagnetic transitions of all states of the $K=1$ band to all allowed states of the ground-state, the β, and the γ bands by using more general and simple processes.

In the classical limit, the magnitudes of the intrinsic matrix elements for the $E2$ transitions of the $K=1$ band are given as

$$|M'_{gm}|=\sqrt{\frac{3N_\pi N_\nu}{4N}}\,|e_\pi-e_\nu|,\tag{24a}$$

$$|M'_{\beta m}|=\sqrt{\frac{3N_\pi N_\nu}{8N^2}}\,|e_\pi-e_\nu|,\tag{24b}$$

$$|M'_{\gamma m}|=\sqrt{\frac{3N_\pi N_\nu}{4N^2}}\,|e_\pi-e_\nu|.\tag{24c}$$

From Eqs. (20) and (24), the magnitudes of the ratio of $E2$ and $M1$ intrinsic matrix elements for transitions from the $K=1$ band to the ground-state, the β, and the γ bands are all identical and independent of boson number N, i.e.,

$$\left|\frac{M'_{fm}}{M_{fm}}\right|=\sqrt{\frac{\pi}{2}}\left|\frac{e_\pi-e_\nu}{g_\pi-g_\nu}\right|,\tag{25}$$

where $f=g$, β, and γ. This interesting result is caused by the fact that $E2$ and $M1$ transition operators have the same tensor character under U(6) and SU(3).

Significant information for the electromagnetic transitions can also be obtained from the mixing ratio [25,26]

$$\delta(E2/M1)=0.835E_\gamma\Delta(L_i\rightarrow L_f),\tag{26}$$

with the reduced mixing ratio

TABLE III. Reduced mixing ratio $\delta(mL'\to fL)$ in units of $\sqrt{\pi/2}(e_\pi-e_\nu)/(g_\pi-g_\nu)$.

L'	$\Delta(mL'\to gL)$	$\Delta(mL'\to\beta L)$	$\Delta(mL'\to\gamma L)$
L	$-\sqrt{\frac{3}{(2L+1)(2L+3)}}$	$\sqrt{\frac{3}{(2L+1)(2L+3)}}\frac{(2N-2)^2+L(L+1)-3}{(2N-2)^2-L(L+1)+1}$	$-\frac{3\sqrt{3}}{(2L-1)(2L+3)}$
$L+1$	$\sqrt{\frac{L}{L+2}}$	$-\sqrt{\frac{L}{L+2}}\frac{(2N-2)^2+L-2}{(2N-2)^2-L}$	$-\frac{L+4}{\sqrt{L(L+2)}}$
$L-1$	$-\sqrt{\frac{L+1}{L-1}}$	$\sqrt{\frac{L+1}{L-1}}\frac{(2N-2)^2-L-3}{(2N-2)^2+L+1}$	$\frac{L+3}{\sqrt{(L-1)(L+1)}}$

$$\Delta(L_i\to L_f)=\frac{\langle L_f\|T(E2)\|L_i\rangle}{\langle L_f\|T(M1)\|L_i\rangle}, \tag{27}$$

where E_γ is the energy of the transition in MeV and $\Delta(L_i\to L_f)$ in e b/μ_N. The $E2/M1$ reduced mixing ratios for the transition of $K=1$ to the FS states are listed in Table III. From the results given in Table III, it is shown that the mixing ratio is dependent on the factor $(e_\pi-e_\nu)/(g_\pi-g_\nu)$ in the exact SU(3) limit, and thus the ratio of two $E2/M1$ mixing ratios is independent of the adjustable parameters given in Eq. (13). For $m\to g$ and $m\to\gamma$ transitions, the reduced mixing ratios are proportional to the ratio of two Clebsch-Gordan coefficients and have units of the ratio of $E2$ and $M1$ intrinsic matrix elements in Eq. (25), i.e.,

$$\Delta(mL'\to\kappa_f L)=-\frac{\langle L'1,2\kappa_f-1|L\kappa_f\rangle}{\langle L'1,1\kappa_f-1|L\kappa_f\rangle}\sqrt{\frac{\pi}{2}}\frac{e_\pi-e_\nu}{g_\pi-g_\nu}, \tag{28}$$

where $\kappa_f=0$ and 2 for the $m\to g$ and $m\to\gamma$ transitions, respectively. This interesting result is analogous to the result of the geometrical model, in which the reduced matrix element of the electromagnetic transition operator is expressed as the product of the geometrical factor (i.e., Clebsch-Gordan coefficient) and the intrinsic matrix element. However, for the $m\to\beta$ transition, the reduced mixing ratio contains an additional term which is dependent on N and L:

$$\Delta(mL'\to\beta L)=\frac{\langle L'1,2-1|L0\rangle}{\langle L'1,1-1|L0\rangle}f(N,L)\sqrt{\frac{\pi}{2}}\frac{e_\pi-e_\nu}{g_\pi-g_\nu}. \tag{29}$$

Since $f(N,L)$ approaches 1 in the limit $N\to\infty$, $\Delta(mL'\to\beta L)$ can also be expressed in the form of Eq. (28). Thus the classical limit of the present model corresponds to the geometrical model and the sign in Eq. (25) could be determined. The ratio of $E2$ and $M1$ intrinsic matrix elements is positive for the $m\to\beta$ transition and negative for $m\to g$ and $m\to\gamma$ transitions. If the effective charge and the g factor of the bosons were determined, there is a fixed-sign relationship between $E2$ and $M1$ intrinsic matrix elements; when e_π is larger than e_ν (the g factor of the proton boson is generally larger than that of the neutron boson), the signs of $E2$ and $M1$ intrinsic matrix elements are same for the $m\to\beta$ transition, whereas they are opposite for $m\to g$ and $m\to\gamma$ transitions, in the classical limit of the present model. It is of interest that the sign of the mixing ratios for the $mL'\to\beta L$ transition is always opposite to that for the $mL'\to gL$ transition.

III. SUMMARY

In this paper, $M1$ and $E2$ transition rates from the lowest mixed-symmetry $K=1$ band to fully symmetric states have been derived via the algebraic approach. To express the electromagnetic transition probabilities in closed forms, $M1$ and $E2$ transition operators have been defined in terms of the generators of $SU_\rho(3)$ $(\rho=\pi,\nu)$ and have obtained the tensor character in the SU(3) limit of the neutron-proton interacting boson model. $B(M1)$ and $B(E2)$ strengths contain a term which corresponds to the intrinsic matrix element in the geometrical model. To interpret this term as the intrinsic matrix element in the present model, we consider the classical limit $(N\to\infty)$, in which this term is independent of angular momentum. In the classical limit, the ratio of two reduced $M1$ (or $E2$) transition probabilities from the $K=1$ band to a band of the FS states is only dependent on the ratio of two Clebsch-Gordan coefficients. This result is analogous to the one from the Alaga rule. For $m\to g$, $m\to\beta$, and $m\to\gamma$ transitions, the ratios of $E2$ and $M1$ intrinsic matrix elements are independent of the boson number N and all are identical except the sign. It is shown that the $E2/M1$ mixing ratio of the $m\to\beta$ transition has a sign opposite to that of the $m\to g$ transition in the SU(3) limit of the IBM-2.

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