PHYSICAL REVIEW C VOLUME 58, NUMBER 4 OCTOBER 1998

Nuclear orientation of $^{160}\mathrm{Ho}^{g+m}\mathrm{Gd}$: Sign change of $E2/M1$ mixing ratios in $\Delta K=2$ transitions of $^{160}\mathrm{Dy}$

T. I. Kracíková,[1,2,*] M. Finger,[1,2] M. Krmar,[3] K. Kumar,[4] A. Janata,[1,5] N. A. Lebedev,[1] V. N. Pavlov,[1] and E. Šimečková[6]
[1]*Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, 141 980 Dubna, Russia*
[2]*Faculty of Mathematics and Physics, Charles University, 180 00 Prague 8, Czech Republic*
[3]*Institute of Physics, Novi Sad, Yugoslavia*
[4]*Department of Physics, Tennessee Technological University, Cookeville, Tennessee 38505*
[5]*Nuclear Research Institute, 250 68 Řež, Czech Republic*
[6]*Institute of Nuclear Physics, Academy of Sciences, 250 68 Řež, Czech Republic*

(Received 26 March 1998)

Directional distribution of γ rays following the decay of $^{160}\mathrm{Ho}^{g+m}$ oriented in a gadolinium host at low temperature has been studied. The γ-ray anisotropies of transitions from the levels below 3 MeV in $^{160}\mathrm{Dy}$ were measured and multipole mixing ratios δ were determined. The variations in both sign and magnitude of δ for the $M1+E2$ transitions between the γ-vibrational ($K^{\pi}=2^{+}$) and ground-state ($K^{\pi}=0^{+}$) bands were observed. The $E2/M1$ mixing ratios for the $2\rightarrow2$ and $4\rightarrow4$ transitions from the β-vibrational ($K^{\pi}=0^{+}$) band to the ground-state band were determined, and the $E0/E2$ probability ratios obtained are consistent with the values for the β-vibrational bands. The $E2/M1$ mixing ratios of the $\gamma\rightarrow g$ transitions in $^{160}\mathrm{Dy}$ obtained previously by the nuclear orientation of $^{160}\mathrm{Tb}$ are compared with the present results. The dynamic deformation model is employed to calculate the collective bands, electromagnetic moments, transition probabilities, and mixing ratios in $^{160}\mathrm{Dy}$. A sign change of the $E2/M1$ mixing ratio is predicted for the $10_{\gamma}\rightarrow10_{g}$ transition. Our experimental results give such a sign change for the $4_{\gamma}\rightarrow4_{g}$ and $6_{\gamma}\rightarrow6_{g}$ transitions. Comparison with the presently determined experimental values of $X(E0/E2)$ for $\gamma\rightarrow g$ and $\beta\rightarrow g$ transitions is also given. [S0556-2813(98)04010-2]

PACS number(s): 23.20.En, 27.70.+q, 23.20.Gq, 21.60.Ev

I. INTRODUCTION

Studies of the directional distribution of γ rays following the decay of oriented nuclei give the experimental data providing a good possibility of testing the validity of various nuclear models. The experimental investigation of the magnitude and particularly of the sign of multipole mixing ratios, defined as the ratio of emission matrix elements, is one of those studies. The even-even $^{160}\mathrm{Dy}$ nucleus, in the family of the highly deformed nuclei with very rich level schemes and well-developed rotational and vibrational bands, is quite appropriate for such examinations.

A number of studies have been carried out in the past to determine the level structure of $^{160}\mathrm{Dy}$ through the β^{-} decay of $^{160}\mathrm{Tb}$ and the β^{+} and electron capture (EC) decay of $^{160}\mathrm{Ho}^{g+m}$. The γ rays, conversion electrons and γ-γ coincidences were measured from the decay of $^{160}\mathrm{Ho}^{g+m}$ [1], and the internal conversion coefficients (ICC) and multipolarities of many transitions in $^{160}\mathrm{Dy}$ were determined [2–5]. The levels of $^{160}\mathrm{Dy}$ have also been studied by several types of nuclear reactions and by Coulomb excitation. The results are summarized in the Nuclear Data Sheets [1]. Multipole mixing ratios of γ rays have been investigated extensively by the low-temperature nuclear orientation [6–9] and γ-γ angular correlation (see Ref. [1]) measurements of $^{160}\mathrm{Tb}$ and by the (α, 2n) reaction [5] and Coulomb excitation [10].

An excellent possibility for the nuclear orientation (NO) measurements is provided by the β^{+} and EC decay of two $^{160}\mathrm{Ho}$ isomers [1]: the ground state with $I^{\pi}=5^{+}$ [$T_{1/2}=25.6(3)$ min] and the first metastable state with $I^{\pi}=2^{-}$ [$T_{1/2}=5.02(5)$ h]. The decay scheme of $^{160}\mathrm{Ho}^{g+m}$ [1] is based largely on the work of Grigoriev *et al.* [2], extended and modified by the results of Refs. [3, 4]. The $^{160}\mathrm{Ho}^{g}$ decay scheme was proposed [1] from the data on the combined ground-state and isomeric-state decays of $^{160}\mathrm{Ho}$. Levels with energy up to 3 MeV in $^{160}\mathrm{Dy}$ are populated by the $^{160}\mathrm{Ho}^{g+m}$ decay, and high-spin members (up to $I=6$) of collective bands as well as low-spin states ($I=1$) are excited. An additional advantage is that both isomers are products of the much longer-lived $^{160}\mathrm{Er}$ ($T_{1/2}=28.58(9)$ h [1]), and only two soft γ rays, 7.1 and 60.0 keV, are emitted after the EC decay of $^{160}\mathrm{Er}$. Having in mind the very high hyperfine magnetic field of neighboring Dy and Er, dissolved in Gd [11], one could expect that the magnetic field at the Ho nuclei in a Gd host lattice could be about 700 T.

In the present work, the $^{160}\mathrm{Ho}^{g+m}$ nuclei were oriented in a Gd host at low temperatures. The directional distribution of γ rays was measured and multipole mixing ratios of the mixed transitions were determined. The dynamic deformation model (DDM) was used to interpret the experimental data. Preliminary results of our experiments are presented in Refs. [12,13].

Experimental details are given in Sec. II, while the data analysis is discussed in Sec. III. Our results are presented and discussed in Sec. IV. Section V gives a brief review of the DDM calculations, while Sec. VI gives the summary and conclusions.

---

*Electronic address: kracik@nusun.jinr.ru

© 1998 The American Physical Society

II. EXPERIMENTAL DETAILS

The radioactive source was produced in a spallation reaction on tantalum making use of the 600 MeV proton beam of the Dubna phasotron. Chemical separation of the erbium fraction from the irradiated target was carried out, and the ^{160}Er isotope was mass-separated and implanted into a gadolinium host. Thermal treatment of the sample was performed. Details of the sample preparation procedure are described in Ref. [14]. The ^{160}ErGd (^{160}HoGd) sample in the form of a disc of diameter 0.5 cm was soldered to the cold finger of a top-loading ^{3}He-^{4}He dilution refrigerator [15] which is able to maintain the sample temperature stable during long periods ($\geqslant 24$ h). An external magnetic field of 1.2 T was applied to polarize the sample.

The γ-ray spectra were taken simultaneously at the angles between the direction of the γ-ray emission and the orientation axis of $\Theta=\pi$ by a HPGe detector of the 20 cm^3 sensitive volume and of $\Theta=\pi/2$ by a 33 cm^3 coaxial Ge(Li) detector. The resolutions of these detectors are 1.9 and 4.5 keV at 1.33 MeV, respectively. The source-to-detector distances were 8–10 cm. The data were collected for periods of 2000 or 4000 s, and the relatively long half-life of ^{160}Er allowed us to take several ‘‘cold’’ [14(2) mK] and ‘‘warm’’ (~ 1.2 K) spectra when the source was oriented and random, respectively.

III. EXPERIMENTAL DATA ANALYSIS

The directional distribution of γ rays from oriented nuclei is given by (see, for instance, Ref. [16])

$$W(\Theta)=\Sigma_{\lambda\ \mathrm{even}} B_\lambda U_\lambda A_\lambda Q_\lambda P_\lambda(\cos\Theta), \tag{3.1}$$

where B_λ are the orientation parameters of the initially oriented state I, U_λ account for the deorientation due to the unobserved β and γ transitions preceding the observed γ ray, A_λ describe the properties of the observed γ ray, Q_λ correct for the real geometry of the experiment, and $P_\lambda(\cos\Theta)$ are Legendre polynomials. Summation over the index λ is restricted as usually to the even values, 2 and 4.

The directional distribution coefficients A_λ are

$$A_\lambda=\frac{F_\lambda(LLI_fI_i)+2\delta F_\lambda(LL'I_fI_i)+\delta^2F_\lambda(L'L'I_fI_i)}{1+\delta^2}, \tag{3.2}$$

where F_λ are the angular-momentum-coupling factors determined by the spins I_i and I_f of the initial and final states, respectively, linking the observed γ ray, whose multipole orders are L and $L'=L+1$. The mixing ratio δ is defined as

$$\delta=\frac{\langle I_f\|L'\|I_i\rangle}{\langle I_f\|L\|I_i\rangle}. \tag{3.3}$$

In order to evaluate the directional distribution coefficients A_λ and consequently the multipole mixing ratios δ of γ rays, the orientation parameters B_λ and the deorientation coefficients U_λ must be determined. The experimental data obtained have been analyzed on the basis of the ^{160}Ho^{g+m} decay scheme of Ref. [1] which is illustrated in Fig. 1 (our results on multipolarities and spins are included). It may be seen from this figure that the produced ^{160}Ho source contains an equilibrium mixture of the ground-state, ^{160}Ho^g, and the first metastable-state, ^{160}Ho^m, activities. These activities have not been sufficiently well separated to allow a precise establishment of two separate decay schemes. However, by using the β-decay spin sequence and the intensity balance for each level, it is still possible to deduce the intensity of any particular $\beta^++\mathrm{EC}$ transition and to separate the ^{160}Ho^g decay scheme [1]. Although the decay scheme of ^{160}Ho^{g+m} is not complete because out of 343 γ rays from the decay of ^{160}Ho^{g+m}, 180 γ rays are not placed [1], the intensity balance is not influenced much as the intensity of the unplaced transitions represents only about 7% of the total. Thus, the major features of the ^{160}Ho^g and ^{160}Ho^m decays are known, and can be used to calculate the U_λ coefficients quite satisfactorily. Naturally, the population of the ^{160}Dy levels from ^{160}Ho^g, ^{160}Ho^m, and ^{160}Ho^{g+m} must be considered and the corresponding B_λ and U_λ values must be determined. We have evaluated [17] the appropriate expressions for all metastable states which have to be used in such cases.

The directional distribution of γ radiation emitted from a level populated by two different initially oriented states is

$$\begin{aligned}W(\Theta)=\Sigma_{\lambda\ \mathrm{even}}A_\lambda Q_\lambda[B_\lambda(g)U_\lambda(g)\\ +B_\lambda(m)U_\lambda(m)]P_\lambda(\cos\Theta).\end{aligned} \tag{3.4}$$

Here $B_\lambda(g)$ and $B_\lambda(m)$ are the orientation parameters of the ground and metastable states, and $U_\lambda(g)$ and $U_\lambda(m)$ are deorientation coefficients connected with these states, respectively. Thus, it is necessary to determine separately the values of $B_\lambda(g)$, $B_\lambda(m)$, and $U_\lambda(g)$, $U_\lambda(m)$. The orientation parameters are determined from the experimental anisotropies of pure multipole transitions. The deorientation coefficients are calculated on the basis of the decay scheme, branching intensities, and transition multipolarities. For the feeding from the ground state, the deorientation coefficients of the observed level are

$$U_\lambda(g)=\frac{I^\beta U_\lambda^\beta(g)+\Sigma_i^{N-1}I_i^{(\mathrm{to})}(g)U_{\lambda i}^{l}U_{\lambda i}^{\gamma}(g)}{\Sigma_j^M I_j^{(\mathrm{out})}}, \tag{3.5}$$

where $I_i^{(\mathrm{to})}$ and $I_j^{(\mathrm{out})}$ are the total (γ plus conversion electron) intensities of transitions populated (to) and depopulated (out of) the observed level, $I^\beta=\Sigma_j I_j^{(\mathrm{out})}-\Sigma_i I_i^{(\mathrm{to})}$, $U_\lambda^\beta(g)$ are the deorientation coefficients of the β radiation to the observed level, and $U_{\lambda i}^{l}$ and $U_{\lambda i}^{\gamma}$ are the deorientation coefficients of the ith intermediate level and of its depopulating transition. Both U_λ^β and U_λ^γ depend on the spins of the initial and final states, and on the multipole orders of β and γ radiations. The $U_\lambda(m)$ coefficients are calculated using Eq. (3.5), where the symbol g is replaced by m and the appropriate values of the intensities, mixing ratios and U_λ coefficients are taken.

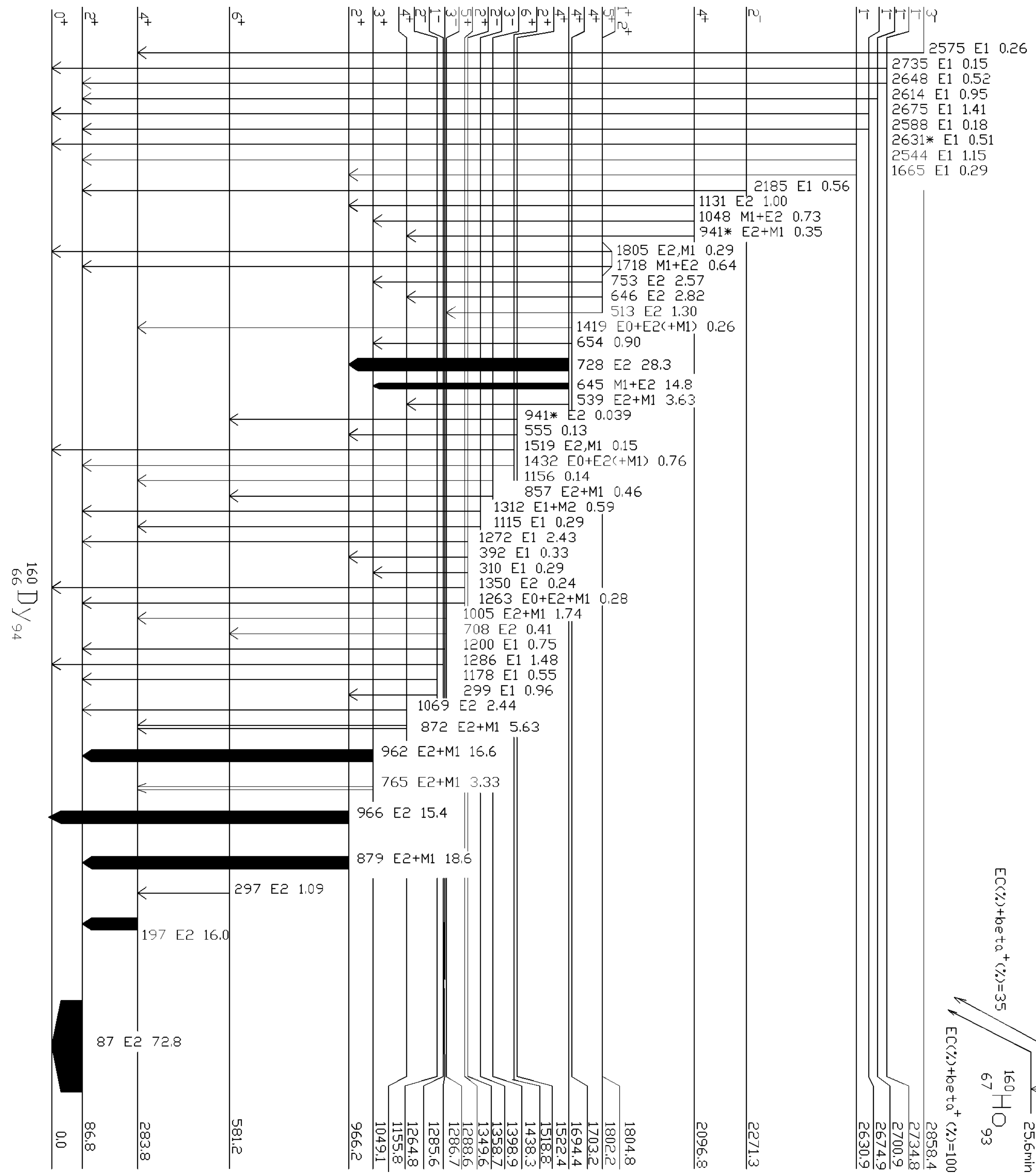

FIG. 1. Partial level scheme for the decay of $^{160}\mathrm{Ho}^{g+m}$ to $^{160}\mathrm{Dy}$. Transitions, with absolute intensities ($\gamma+\mathrm{ce}$) per 100 decays, observed in the present experiment are shown. Transitions located twice are denoted by a star.

IV. RESULTS

The directional distributions of 29 γ rays from the decay of oriented $^{160}\mathrm{Ho}^{g+m}$ were measured. Five ''cold'' and five ''warm'' spectra were chosen and after the peak area determination and the decay correction, the corresponding areas were averaged and the anisotropies $[W(\pi)-1]$ and $[1-W(\pi/2)]$ were determined. They are listed in Table I where the corresponding γ-ray intensities [1] are also presented.

Uncertainties, given in Table I, for $[W(\pi)-1]$ of the 728 and 966 keV γ rays are quite small (0.23, 0.67%), because their intensities are quite large (the first of these γ rays is the most intense one), and they have the largest anisotropies. When there are good detectors and high statistics, it is possible to obtain small uncertainties for intense γ rays with large anisotropies. For instance, Krane [8] obtained uncertainties of 0.93, 1.6% in $[W(\pi)-1]$ for the 299, 1178 keV γ rays in the nuclear orientation of $^{160}\mathrm{Tb}$ in Tb metal.

There are large differences in Table I in the uncertainties of $[W(\pi)-1]$ and $[1-W(\pi/2)]$ of the 728 and 966 keV γ rays, because different detectors were used for the two angles

TABLE I. Anisotropies of γ rays following the decay of oriented ^{160}Ho^{g+m}Gd. Relative γ-ray intensities are also given. Spins and parities of the initial and final levels are given in Table IV.

E_γ (keV)	$W(\pi)-1$ (%)	$1-W(\pi/2)$ (%)	$I_\gamma^{g+m\,a}$	$I_\gamma^{g\,a}$
197.0	−23.6(61)	−10.1(58)	1000(10)	780(40)
297.2	−39.9(61)	−17.8(99)	80(5)	80(5)
298.6	−12(11)	−9(16)	74(5)	
309.6	15.1(92)		22(3)	
538.6	23.9(23)	13.8(70)	280(20)	280(20)
728.2	−79.17(18)	−20.0(24)	2200(60)	2200(60)
753.1	−62.3(16)	−23.2(36)	200(20)	200(20)
765.3	18.1(28)	6.5(31)	260(20)	240(20)
856.9	23(90)	10.5(64)	36(3)	36(3)
872.0	10.9(51)	3.6(27)	440(40)	440(40)
879.4	17.6(15)	11.5(19)	1450(50)	1250(50)
941.0[b]	25.1(84)	10.6(50)	27(3)	27(3)
962.4	0.4(14)	1.6(22)	1300(50)	1200(50)
966.2	−74.45(50)	−20.8(27)	1200(50)	1030(50)
1004.7	23.4(35)	4.1(41)	136(10)	136(10)
1069.1	−37.0(19)	−17.6(33)	190(15)	190(15)
1271.9	−16(11)	−7.3(58)	190(18)	
1312.1	18(10)	11.0(89)	46(5)	
1419.0	−11(24)		20(3)	
1432.0	−18(10)		59(7)	
1717.7	−22.6(69)	−3(12)	50(6)	
2184.7	−18.0(73)		44(5)	
2544.1	6.7(58)		90(9)	
2574.6	24(17)		20(3)	
2588.4	11(19)		14(2)	
2614.5	3.0(72)		74(8)	
2648.0	15(12)		41(5)	
2674.8	48.2(72)		110(10)	
2735.1	50(18)		12(2)	

[a] Reference [1].

[b] The γ ray is probably an unresolved doublet.

and their resolutions differ by more than a factor of 2 (Sec. II). Also the two anisotropies differ by more than a factor of 3.

A. Deorientation coefficients

Several levels in ^{160}Dy are populated by β transitions and via β-γ cascades from both isomers of ^{160}Ho. The deorientation coefficients $U_\lambda(g)$ and $U_\lambda(m)$ were calculated on the basis of the ^{160}Ho^g and ^{160}Ho^{g+m} decay schemes [1] and all available data including our results. For all allowed β transitions from ^{160}Ho^g and ^{160}Ho^m to the positive- and negative-parity levels of ^{160}Dy, respectively, it is justifiable to assume that they are Gamow-Teller transitions with $\Delta j_\beta=1$. For β transitions with a change of parity, it is assumed that one unit of angular momentum is carried off. The values of U_λ are listed in Table II.

B. Orientation parameters

The anisotropies of three pure $E2$ transitions from the levels populated by the ^{160}Ho ground state, $I^\pi=5^+$, were chosen to determine the orientation parameters $B_\lambda(g)$. These transitions, the corresponding B_2 and B_4 values, and their weighted averages are presented in Table III. The B_4 values evaluated from the anisotropies of the 753 and 1069 keV transitions have large uncertainties. Particularly the latter, whose B_4 value has no significant contribution to the final result, is shown for the sake of completeness.

High-energy levels of ^{160}Dy are populated by the decay of the ^{160}Ho metastable state, $I^\pi=2^-$. There is a number of pure $E1$ transitions from the levels with spin 1^- to the 0^+ ground state. However, the majority of them are very weak. Three of the most intense transitions (2675 and 2735 keV, both $1^-\rightarrow 0^+$, and 2185 keV, $2^-\rightarrow 2^+$) were used to determine the $B_2(m)$ parameter. The fourth-order directional distribution coefficient $A_4(1\rightarrow 0)$ is zero as is the $U_4(2\rightarrow 1)$ deorientation coefficient. In agreement with the systematics of the $M2/E1$ mixing ratios and with the B_2 parameters obtained from the anisotropies of the 2675 and 2735 keV transitions, the value of $\delta=-0.05\pm 0.05$ was used to calculate the A_2 coefficient of the 2185 keV transition. The fourth-order term in Eq. (3.1) is very small in this case: $(B_4,U_4,A_4,Q_4,P_4)<0.0002$. Three values of $B_2(m)$ and their weighted mean value are listed in Table III. Since there is no suitable transition to determine the fourth-order orientation parameter, the $B_4(m)$ value was estimated from the theoretical orientation parameters [18]. The value of B_2 obtained is very close to the saturation value of $B_2(m)=1.195$ (as well as these values of ground-state parameters: $B_2(g)=1.698$ and $B_4(g)=1.177$), so we concluded that the theoretical value of $B_4(m)=0.24(3)$ corresponding to our $B_2^{\mathrm{exp}}(m)$, with similar uncertainty, is a good estimate.

C. Multipole mixing ratios

The directional distribution coefficients, A_λ, for the $M1+E2$, $E2$, and $E2(+M3)$ transitions were determined as

TABLE II. Deorientation coefficients.

E_{level} (keV)	U_2	U_4
283.8	g:0.318(19)	0.0327(26)
	m:0.0900(83)	0.0210(25)
581.2	g:0.587(63)	0.184(29)
	m:0.088(16)	0.0254(49)
966.2	g:0.606(23)	0.1955(73)
	m:0.0951(52)	−0.0324(20)
1049.1	g:0.527(28)	−0.0969(86)
	m:0.0499(34)	0.0141(17)
1155.8	g:0.643(54)	0.121(15)
1264.8	m:0.301(31)	−0.336(39)
1286.7	m:0.59(14)	0.230(61)
1288.6	g:0.714(66)	0.273(26)
	m:0.0606(70)	0.0160(39)
1349.6	m:0.60(10)	−0.030(30)
1358.7	m:0.405(49)	−0.509(64)
1398.9	m:0.779(49)	0.363(47)
1438.3	g:0.910(48)	0.67(12)
1694.4	g:0.935(29)	0.788(24)

TABLE III. Orientation parameters of the ground and metastable states of ^{160}HoGd.

$E\gamma$(keV)	I_i^π	I_f^π	B_2	B_4[a]
		$^{160}\mathrm{Ho}^g$, $I^\pi=5^+$		
728.2	4^+	2^+	1.372(77)	0.97(12)
753.1	5^+	3^+	1.41(11)	0.62(29)
1069.1	4^+	2^+	1.30(17)	0.72(121)
Weighted average:			1.374(60)	0.92(11)
		$^{160}\mathrm{Ho}^m$, $I^\pi=2^-$		
2184.7	2^-	2^+	1.02(45)	
2674.8	1^-	0^+	1.17(17)	
2735.1	1^-	0^+	1.21(44)	
Weighted average:			1.16(15)	

[a] Theoretical value of $B_4=0.24(3)$ was used for the $^{160}\mathrm{Ho}^m$ 2^- state, see the text.

usual with the fourth-order term taken into account. In most cases the A_4 value has a large uncertainty which does not allow its use for the choice between two solutions of Eq. (3.2). Therefore these solutions were compared with the values of $|\delta(\alpha_K)|$ calculated using all experimental [3–5] and theoretical [19] ICC data. For the $E1+M2$ transitions, the fourth-order terms in $W(\Theta)$ were neglected as they are very small (considerably smaller than the uncertainties in the second-order terms). The values of A_2, A_4, and δ are presented in Table IV. The obtained results were verified by comparison of the experimental directional distribution coefficients for pure multipole transitions with the corresponding theoretical values. Moreover, when the same level is depopulated by the $M1+E2$ and pure multipole transitions, the directional distribution coefficients of the $M1+E2$ transition may be determined independently of $B_\lambda(I)$ and $U_\lambda(\beta\gamma)$ by using the deduced ratios of $(B_\lambda U_\lambda A_\lambda)$ for these two transitions and the theoretical A_λ coefficients for pure multipole transitions. In all three cases, the values of A_λ obtained in this way agree well (see Table IV) with those determined by using the B_λ^{exp} and $U_\lambda^{\mathrm{calc}}$: $A_2(539\ \mathrm{keV})=0.204(64)$, $A_2(872\ \mathrm{keV})=0.100(44)$, and $A_2(879\ \mathrm{keV})=0.221(26)$. This is additional evidence of the correct determination of B_λ and U_λ.

It was assumed in Ref. [1] that the 941 keV γ ray was an unresolved doublet since there were two possibilities to locate this transition in the $^{160}\mathrm{Ho}^{g+m}$ decay scheme: between the 1522 and 581 keV levels and between the 2097 and 1156 keV levels as the $4^+\rightarrow6^+$ and $4^+\rightarrow4^+$ transitions, respectively. However, the 1522 keV state is not excited in the decay of $^{160}\mathrm{Ho}^g$ and all intensity of the 941 keV γ ray, which is the same in the $^{160}\mathrm{Ho}^g$ and $^{160}\mathrm{Ho}^{g+m}$ decays [the ratios of the intensities of the 872 and 1069 keV transitions to the 941 keV transition intensity are 440(40):140(15):27(3), respectively, in both decays], is applied to the 2097 keV level. This means that the 1522 keV level may be depopulated as a maximum by 11% (the experimental error) of the 941 keV transition intensity only. Moreover, the calculated anisotropies of the $4\rightarrow6$ transition are $[W(\pi)-1]=-0.312(13)$ and $[1-W(\pi/2)]=-0.138(6)$, while the values of $[W(\pi)-1]=0.251(84)$ and $[1-W(\pi/2)]=0.106(50)$ were measured. Thus, the values of A_λ and δ, given in Table IV, were determined assuming that the 941 keV transition depopulates the 2097 keV level, and from two solutions of Eq. (3.2), $\delta=-0.83^{+0.10}_{-0.12}$ or $8.7^{+12.9}_{-3.1}$, the second value was preferred since the value of $|\delta(\alpha_K)|\geqslant2.3$ was evaluated from the α_K^{exp} [3].

An excellent discussion of the $M2/E1$ and $E2/M1$ mixing ratios of γ rays from the decay of ^{160}Tb, determined by the NO [6–9], $\gamma\gamma(\Theta)$, and $e_K\gamma(\Theta)$ measurements, was made by Marshak *et al.* [9]. (It is incomprehensible to us why the γ-ray anisotropies were not presented there.) Since no new information was obtained by the later $\gamma\gamma(\Theta)$ measurements (see Ref. [1]) and no new conclusions can be made on the $M2/E1$ mixing ratios, we do not discuss them here. It should be noted that the anisotropies of the $E1+M2$ 2185, 2544, 2575, 2588, 2614, and 2648 keV and pure $E1$ 2675 and 2735 keV γ rays were firstly measured from the decay of oriented $^{160}\mathrm{Ho}^{g+m}$.

The $M3/E2$ mixing ratios are also shown in Table IV, and all of them are smaller than their experimental errors. Systematic studies and theoretical estimates of $\delta(M3/E2)$ show that the $M3/E2$ mixing ratios are so small that we cannot measure yet with the accuracy $\delta>\Delta\delta$ such small quantities by the NO and correlation methods. Thus, if $\delta>\Delta\delta$ (except for special cases of forbidden transitions), such values are too large and do not look reliable. The theoretical estimates give very small values of $\delta(M3/E2)$. For instance, from the recommended upper limit for the $M3$ transition strength, the $M3/E2$ mixing ratios of the 197, 728, and 1069 keV transitions in ^{160}Dy were estimated [1] as $\delta<1.4\times10^{-5}$, $\delta<2.5\times10^{-4}$, and $\delta<1.3\times10^{-3}$, respectively. Therefore it is too early to make any conclusions on $\delta(M3/E2)$, and we consider (and call) the $E2(+M3)$ γ rays as pure multipole transitions when the experimental and theoretical values of A_λ are compared. The anisotropies of the 297 ($6^+\rightarrow4^+$), 728 ($4^+\rightarrow2^+$), 753 ($5^+\rightarrow3^+$), and 966 keV ($2^+\rightarrow0^+$) transitions were measured for the first time from the decay of the oriented $^{160}\mathrm{Ho}^{g+m}$.

The most interesting results of this work concern the ($4_\gamma\rightarrow4_g$) 872 and the ($6_\gamma\rightarrow6_g$) 857 keV transitions. There are two solutions of Eq. (3.2): $\delta=5.0^{+2.0}_{-1.1}$ or $-0.702^{+0.076}_{-0.086}$ and $\delta=5.1^{+5.8}_{-1.6}$ or $-1.06^{+0.17}_{-0.25}$, and the larger value was chosen in both cases since the ICC data [3,5] give $|\delta(\alpha_K)|=3.8^{+\infty}_{-1.4}$ and $2.5^{+1.9}_{-0.6}$, respectively. Thus, the sign change of the mixing ratios was observed for the $\Delta I=0$ transitions between the γ-vibrational and ground-state bands beginning

TABLE IV. Directional distribution coefficients, A_λ, and multipole mixing ratios, δ, of the γ transitions in ^{160}Dy.

$E_{\rm level}$ (keV)	I_i^π	E_γ (keV)	I_f^π	A_2[a]	A_4[a]	δ
283.8	4^+	197.0	2^+	$-0.40(14)$	$-0.6(28)$	$-0.05^{+0.13}_{-0.14}$
581.2	6^+	297.2	4^+	$-0.42(13)$	$-0.15(83)$	$0.02^{+0.12}_{-0.11}$
966.2	2^+	879.4[b]	2^+	0.222(26)	$-0.17(18)$	$-12.5^{+2.9}_{-5.0}$
		966.2	0^+	$-0.597(40)$	$-1.13(33)$	
1049.1	3^+	765.3	4^+	0.196(49)	$-0.36(58)$	$-12.8^{+3.9}_{-9.7}$
		962.4	2^+	0.026(33)	0.19(36)	$-12.8^{+2.3}_{-3.6}$
1155.8	4^+	872.0[b]	4^+	0.109(48)	0.20(60)[c]	$5.0^{+2.0}_{-1.1}$
		1069.1[d]	2^+	$-0.408(52)$	$-0.10(56)$	-0.038 ± 0.050
1264.8	2^-	298.6	2^+	$-0.37(31)$		$-0.04^{+0.30}_{-0.24}$
1288.6	5^+	1004.7	4^+	0.142(48)	0.35(26)	$-13.2^{+3.3}_{-6.7}$
1358.7	2^-	309.6	3^+	0.32(21)		$0.15^{+0.18}_{-0.16}$
		1271.9	2^+	$-0.33(18)$		$-0.07^{+0.15}_{-0.14}$
1398.9	3^-	1312.1	2^+	0.21(10)		0.071 ± 0.052
1438.3	6^+	856.9	6^+	0.178(67)	0.02(21)	$5.1^{+5.8}_{-1.6}$
1518.8	2^+	1432.0	2^+	$-0.31(18)$		$2.9^{+2.1}_{-1.0}$
1694.4	4^+	538.6[b]	4^+	0.205(64)	$-0.03(12)$	$12.1^{+141.3}_{-6.0}$
		728.2[d]	2^+	$-0.446(32)$	$-0.48(25)$	-0.002 ± 0.030
1703.2	4^+	1419.0	4^+	$-0.13(28)$		$2.1^{+4.6}_{-1.0}$
1802.2	5^+	753.1[d]	3^+	$-0.439(38)$	$-0.147(91)$	0.016 ± 0.034
1804.8	1^+	1717.7	2^+	$-0.31(11)$		$-1.4\leqslant\delta\leqslant-0.4$
	2^+	1717.7	2^+	$-0.23(24)$	$-0.6(10)$	$3.6^{+7.1}_{-1.6}$
2096.8	4^+	941.0[e]	4^+	0.179(53)	0.03(16)	$8.7^{+12.9}_{-3.1}$
2271.3	2^-	2184.7[d]	2^+	$-0.31(13)$		-0.09 ± 0.10
2630.9	1^-	2544.1	2^+	0.099(86)		$0.030^{+0.088}_{-0.092}$
2674.9	1^-	2588.4	2^+	0.16(28)		0.09 ± 0.32
		2674.8[c]	0^+	0.74(23)		
2700.9	1^-	2614.5	2^+	0.04(11)		$-0.03^{+0.12}_{-0.13}$
2734.8	1^-	2648.0	2^+	0.22(18)		$-0.15^{+0.20}_{-0.19}$
		2735.1	0^+	0.74(29)		
2858.4	3^-	2574.6	4^+	0.25(18)		0.07 ± 0.13

[a] Theoretical values of A_2 and A_4 of pure multipole transitions are:
$A_2(1\to0)=0.7071$, $A_4(1\to0)=0$;
$A_2(2\to0)=-0.5976$, $A_4(2\to0)=-1.069$;
$A_2(4\to2)=-0.4477$, $A_4(4\to2)=-0.3044$;
$A_2(5\to3)=-0.4206$, $A_4(5\to3)=-0.2428$;
$A_2(6\to4)=-0.4029$, $A_4(6\to4)=-0.2088$.

[b] A_λ are deduced independently of $B_\lambda(I)$ and $U_\lambda(\beta\gamma)$ by using the anisotropies of a pure $E2$ transition from the same level.

[c] The value of A_4 is determined using $B_4^{\rm exp}$ and $U_4^{\rm calc}$.

[d] A_λ are deduced using the weighted averages of B_λ except for the value determined from the anisotropies for this transition.

[e] See the text.

at the $4_\gamma\to4_g$ transition. In Table V, the $E2/M1$ mixing ratios of the $\gamma\to g$ transitions, determined by the previous NO measurements of ^{160}Tb [6–9], are compared with the present results. Note that the intensities of the $\gamma\to g$ transitions are considerably higher in the decay of ^{160}Ho^{g+m} than that of ^{160}Tb: $I_{765}:I_{872}:I_{879}:I_{962}=18:30:100:90$ and 7:0.73:100:33, respectively.

We should like to point out that in 1979, when our results of the NO study of ^{160}TbGd were reported [7], nothing was known about the mixing ratio sign change for the $\gamma\to g$ transitions. On the contrary, systematic studies of the $E2/M1$ mixing ratios of the $\gamma\to g$ and also $\beta\to g$ transitions (see, for instance, Refs. [20, 21]) have indicated that these mixing ratios do not change sign (in the same nucleus, for the same initial and final bands), and the mixing ratios of the $\gamma\to g$ transitions in deformed nuclei with $A>150$ were determined to be negative. Therefore for the $(4_\gamma\to4_g)$ 872 keV transition, the negative value of δ was chosen from two solutions of Eq. (3.2): $\delta=-0.70\pm0.10$ or $5.0^{+2.4}_{-1.3}$. The conversion electrons were not measured up to that time since the 872 keV γ ray is quite weak $[I_\gamma=0.723(12)]$ in the decay of ^{160}Tb and is affected by the close and intense $[I_\gamma$

TABLE V. Present $E2/M1$ mixing ratios of transitions from the γ-vibrational ($K^\pi=2^+$) band to the ground-state ($K^\pi=0^+$) band in ^{160}Dy obtained in this work are compared with previous results of the NO of ^{160}Tb.

$I_i^\pi \rightarrow I_f^\pi$	E_γ (keV)	HoGd This work	TbGd a	TbGd b	TbTb c	TbTb(sc) d
$2_\gamma^+ \rightarrow 2_g^+$	879.4	$-12.5_{-5.0}^{+2.9}$	-18_{-8}^{+4}	-12.8 ± 1.5	$-16.7_{-1.6}^{+1.3}$	-16.6 ± 0.5
$3_\gamma^+ \rightarrow 2_g^+$	962.4	$-12.8_{-3.6}^{+2.3}$		$-12.0_{-1.4}^{+1.1\text{e}}$	-11.0 ± 1.2	-13.8 ± 0.3
$3_\gamma^+ \rightarrow 4_g^+$	765.3	$-12.8_{-9.7}^{+3.9}$	$-7.7_{-0.7}^{+0.6}$	$-9.0_{-5.0}^{+2.4}$	$-8.3_{-0.9}^{+0.7}$	$-13.7_{-0.9}^{+0.8}$
$4_\gamma^+ \rightarrow 4_g^+$	872.0	$5.0_{-1.1}^{+2.0}$		$5.0_{-1.3}^{+2.4\text{f}}$		$21_{-10}^{+\infty\ \text{f}}$
$5_\gamma^+ \rightarrow 4_g^+$	1004.7	$-13.2_{-6.7}^{+3.3}$				
$6_\gamma^+ \rightarrow 6_g^+$	856.9	$5.1_{-1.6}^{+5.8}$				

[a] Reference [6].

[b] Our previous work [7].

[c] Reference [8].

[d] Reference [9].

[e] Anisotropies were corrected and reanalyzed.

[f] Mixing ratio δ which agrees with the value of $|\delta(\alpha_K)|=3.8_{-1.4}^{+\infty}$ obtained from α_K^{exp} [4,17], see the text.

$=100.0(2)]$ 879 keV γ ray. Ten years later, the same conclusion was made by Marshak *et al.* [9] who also preferred the negative value of $\delta=-0.953_{-0.105}^{+0.081}$ for the 872 keV γ ray from the decay of the oriented ^{160}TbTb (sc). In the decay of $^{160}\text{Ho}^{g+m}$, the 872 keV γ ray is much stronger $[I_{872}:I_{879}=30(3):100(3)]$, and its α_K value was determined [3,5].

Thus, the results of the NO measurements of ^{160}HoGd show that the multipole mixing ratios of the $2_\gamma \rightarrow 2_g$, $3_\gamma \rightarrow 2_g$, $3_\gamma \rightarrow 4_g$ and $5_\gamma \rightarrow 4_g$ transitions and the $4_\gamma \rightarrow 4_g$ and $6_\gamma \rightarrow 6_g$ transitions differ in both magnitude and sign, see Table V.

The mixing ratio sign change was also observed for the $\gamma \rightarrow g$ transitions in ^{166}Er by the NO study of $^{166}\text{Ho}^m$Ho [22], ^{166}TmGd [23], and of $^{166}\text{Ho}^m$Ho (sc) [24] and probably (the uncertainties are large) in ^{164}Er by the measurement of the directional distribution of γ rays in the $(\alpha,2n\gamma)$ nuclear reaction [25]. The possibility and significance of this phenomenon was presented and successfully interpreted by Hamilton *et al.* [24] and by Kumar [27] using the DDM [26]. Our results for ^{160}Dy suggest that the mixing ratio sign change is not a particular property of one nucleus, that it may be a more general and significant phenomenon.

Two $K^\pi=0^+$ bands are excited, but very weakly, in the decay of $^{160}\text{Ho}^{g+m}$ [1], and the first band is even weaker than the second one. The anisotropies of the $2 \rightarrow 2$ and $4 \rightarrow 4$ transitions at 1432 and 1419 keV between the levels of the β-vibrational ($K^\pi=0^+$) and ground-state ($K^\pi=0^+$) bands were measured. Since the anisotropies at $\Theta=\pi/2$ were not observed and the uncertainties in $[W(\pi)\text{-}1]$ are large, the fourth-order terms in Eq. (3.1) were not considered. Values of $\delta=2.9_{-1.0}^{+2.1}$ or $-0.09_{-0.14}^{+0.15}$ and $\delta=2.1_{-1.0}^{+4.6}$ or $-0.38_{-0.40}^{+0.34}$, respectively, were obtained, and the larger values were preferred since the $E0$ admixtures were admitted by the ICC data [4].

Using the $E2/M1$ mixing ratios, the corresponding experimental [4] and theoretical [19] ICC data and the relation $\alpha_K^{\text{exp}}=[\delta^2(1+q^2)\alpha_K(E2)+\alpha_K(M1)]/(1+\delta^2)$, the magnitudes of the $E0/E2$ mixing ratios $|q(E0/E2)|$ were calculated for the $\Delta I=0$ transitions from the $K^\pi=2^+$ and the second $K^\pi=0^+$ bands to the ground-state band. Then the values of $|q(E0/E2)|$ were used to calculate the dimensionless ratio [21],

$$X(E0/E2)=2.56\times10^9\,\frac{A^{4/3}q^2\alpha_K(E2)E_\gamma^5}{\Omega_K(Z,k)}. \tag{4.1}$$

Here E_γ is the transition energy in MeV and $\Omega_K(Z,k)$ is the electronic factor in s^{-1}. This factor is related to the $E0$ conversion coefficient, $A(E0)$, see Ref. [28], as

$$\Omega_K(Z,k)=8\pi\alpha kA(E0), \tag{4.2}$$

where α is the fine-structure constant, and k is the transition energy. The results are presented in Table VI. Though the uncertainties in $X(E0/E2)$ are large due to the large experimental errors in α_K^{exp}, the $X(E0/E2)$ values are generally consistent with the systematics: values for the $\gamma \rightarrow g$ transitions are an order of magnitude smaller than those for the $\beta \rightarrow g$ transitions.

Table VI shows that the second excited $K^\pi=0^+$ band has strong $E0$ transitions to the ground band. Furthermore, it is excited much more strongly than the first excited $K^\pi=0^+$ band (starting at 1280 keV). Hence, there is strong evidence

TABLE VI. Magnitudes of the $E0/E2$ mixing ratios and the relative $E0/E2$ probabilities of the $\Delta I=0$ transitions from the γ-vibrational ($K^\pi=2^+$) and the β-vibrational ($K^\pi=0^+$) bands to the ground-state ($K^\pi=0^+$) band.

$I_i \rightarrow I_f$	E_γ (keV)	E_i (keV)	$\lvert q(E0/E2)\rvert$	$X(E0/E2)$
		γ-vibrational ($K^\pi=2^+$) band		
$2 \rightarrow 2$	879.4	966.2	$0.24_{-0.16}^{+0.09}$	0.043 ± 0.038
$4 \rightarrow 4$	872.0	1155.8	$0.15_{-0.15}^{+0.18}$	$0.016_{-0.016}^{+0.064}$
$6 \rightarrow 6$	856.9	1438.3	$0.32_{-0.32}^{+0.17}$	$0.072_{-0.072}^{+0.079}$
		β-vibrational ($K^\pi=0^+$) band		
$2 \rightarrow 2$	1432.0	1518.8	$0.63_{-0.63}^{+0.36}$	$0.78_{-0.78}^{+1.13}$
$4 \rightarrow 4$	1419.0	1703.2	0.85 ± 0.22	$1.40_{-0.62}^{+0.79}$

TABLE VII. Calculated DDM properties of the ground-state, γ-vibrational, and β-vibrational bands in ^{160}Dy.

I	E_{level} (keV)	β	γ (deg)	Δ_p (keV)	Δ_n (keV)	Q^{a} ($e\cdot b$)	μ^{b} (n.m.)	K^{c}		
								0	2	4
				Ground-state ($K^\pi=0^+$) band						
0	0	0.30	13	711	547	0.00	0.00	100		
2	85	0.30	13	704	543	−1.83	0.71	100		
4	306	0.30	13	690	540	−2.31	1.42	99	1	
6	660	0.31	13	675	540	−2.51	2.12	98	2	
8	1149	0.32	13	660	539	−2.64	2.81	95	5	
10	1750	0.34	13	644	534	−2.70	3.51	91	8	1
				γ-vibrational ($K^\pi=2^+$) band						
2	850	0.30	17	740	564	1.78	0.60	2	98	
3	989	0.30	17	725	552	0.00	1.00	0	100	
4	1120	0.31	16	715	554	−0.72	1.38	15	83	2
5	1352	0.33	16	686	531	−1.34	1.75	0	98	2
6	1484	0.33	16	677	531	−1.23	2.12	20	77	3
7	1765	0.35	16	650	509	−1.96	2.44	0	95	5
8	1898	0.36	16	639	503	−1.72	2.82	26	72	3
9	2235	0.36	15	635	501	−2.24	3.14	0	91	9
10	2416	0.37	15	626	497	−2.17	3.51	22	73	5
				β-vibrational ($K^\pi=0^+$) band						
0	942	0.33	13	720	565	0.00	0.00	100		
2	1025	0.35	14	659	523	−2.08	0.70	99	1	
4	1231	0.37	15	634	496	−2.37	1.40	96	4	

[a] The experimental value is known only for the 87 keV (2_g) level and is $|Q|=1.8(4)e$b [10].

[b] Experimental values [1] are 0.723(19), 1.43(10), and 0.65(5) n.m., for $2_g, 4_g, 2_\gamma$, respectively.

[c] Components with $K=0$, 2, and 4 in %.

that the second band starting at 1519 keV is the β-vibrational band rather than the first one starting at 1280 keV.

D. Spin assignments

Experimental results obtained in this study allow for more precise, and in many cases unique, spin assignments of several levels of ^{160}Dy, especially as compared to the multiple spins allowed by the previously available data [1].

The 1804.8 keV level, $I^\pi=1^+$ *or* 2^+. The $E2$ and $E2$, $M1$ multipolarities of the 839, 1718, and 1805 keV transitions [3] to the 2^+ and 0^+ levels indicate positive parity and $I=1$ or 2. The 1805 keV level was interpreted as a band-head of the $K^\pi=1^+$ band and assigned as 1^+, see [1]. However, the measured anisotropies of the 1718 keV γ ray permit both spins: $-1.4\leqslant\delta(1^+\rightarrow 2^+)\leqslant -0.4$ and $\delta(2^+\rightarrow 2^+)=3.6^{+7.1}_{-1.6}$. The value of $|\delta(\alpha_K)|\geqslant 2.13$ was calculated from the ICC data [3,19] which prefers spin 2, but the accuracy of the results is poor and spin 1 is not ruled out.

The 2271.3 keV level, $I^\pi=2^-$. The $E1$ multipolarity of the 2185 keV transition [3] to the 2^+ state indicates negative parity and $I=1$, 2 or 3. For $I^\pi=1^-$ and 3^-, the M2 admixtures of (22^{+8}_{-15}) and of ($7.5^{+2.4}_{-2.1}$)%, respectively, obtained from the anisotropy of the 2185 keV γ ray, are too large for the $E1+M2$ transition. Thus, the 1^- and 3^- assignments are ruled out.

The 2630.9 keV level, $I^\pi=1^-$. The $E2$, $M1$, and $E1$ multipolarities of the 1345, 1665, and 2544 keV transitions [3] to the 1^-, 2^+ and 2^+ states, respectively, give 1^-, 2^- or 3^- as possible assignments. For $I^\pi=2^-$, the $M2$ admixture of ($16.2^{+6.4}_{-5.3}$)%, obtained from the anisotropy of the 2544 keV transition, rules out spin 2. In the case of $I^\pi=3^-$, the value of $M2\geqslant 1.24$%, for the $E1+M2$ 2544 keV transition is close to the upper limit of the admitted $M2$ admixtures. However, the doubly placed 2631 keV ground-state transition together with the more suitable $M2$ admixture of ($0.1^{+1.2}_{-0.2}$)% for $1^-\rightarrow 2^+$ transition practically rules out the 3^- assignment.

The 2858.4 keV level, $I^\pi=3^-$. The $E1$ multipolarity of the 2575 keV transition [3] to the 4^+ state permits $I^\pi=3^-$, 4^- or 5^-. The $M2$ admixture of (58^{+29}_{-25})% in the $4^-\rightarrow 4^+$ transition, obtained from the measured anisotropy of the 2575 keV γ ray, and the value of log ft$=7.16(9)$ for the $2^-\rightarrow 5^-$ β transition [1], rule out the 4^- and 5^- assignments.

V. MICROSCOPIC DDM CALCULATIONS FOR ^{160}DY

The dynamic deformation model [26,27], where a large configuration space is employed for the microscopic part of the calculation and a numerical integration method is used for the collective (band-mixing) part, has been employed to calculate the low-energy structure of ^{160}Dy. Two model parameters, the proton- and neutron-pairing strengths, were adjusted to fit the energy and magnetic moment of the first 2^+ state.

TABLE VIII. Theoretical and experimental reduced mixing ratios for the $\gamma\rightarrow g$ and $\beta\rightarrow g$ transitions in ^{160}Dy.

$I_i\rightarrow I_f$	Expt. E_γ (keV)	Expt. E_i (keV)	$\Delta(E2/M1)(\mathrm{MeV}^{-1})$		$X(E0/E2)$	
			Theor.	Expt.	Theor.	Expt.
$\gamma\rightarrow g,\ \Delta I=0$						
$2\rightarrow2$	879.4	966.2	-19.0	$-14.2^{+3.3}_{-5.7}$	0.00058	0.043 ± 0.038
$4\rightarrow4$	872.0	1155.8	-14.3	$5.7^{+2.3}_{-1.3}$	0.0064	$0.016^{+0.064}_{-0.016}$
$6\rightarrow6$	856.9	1438.3	-17.4	$6.0^{+6.8}_{-1.9}$	0.0017	$0.072^{+0.079}_{-0.072}$
$8\rightarrow8$	834.2	1801.2	-61.6			
$10\rightarrow10$	794.1	2222.8	55.2			
$\gamma\rightarrow g,\ \Delta I=+1$						
$3\rightarrow4$	765.3	1049.1	-11.2	$-16.7^{+5.1}_{-12.7}$		
$5\rightarrow6$	707.6	1288.6	-8.3			
$7\rightarrow8$	650.4	1617.4	-8.9			
$9\rightarrow10$		2022.0	-11.7			
$\gamma\rightarrow g,\ \Delta I=-1$						
$3\rightarrow2$	962.4	1049.1	-14.1	$-13.3^{+2.4}_{-3.7}$		
$5\rightarrow4$	1004.7	1288.6	-8.8	$-13.1^{+3.3}_{-6.7}$		
$7\rightarrow6$	1036.6	1617.4	-9.6			
$9\rightarrow8$	1055.4	2022.0	-23.1			
$\beta\rightarrow g,\ \Delta I=0$						
$2\rightarrow2$	1432.0	1518.8	160.2	$2.0^{+1.5}_{-0.7}$	0.70	$0.78^{+1.13}_{-0.78}$
$4\rightarrow4$	1419.0	1703.2	-7.0	$1.5^{+3.2}_{-0.7}$	3.22	$1.40^{+0.79}_{-0.62}$

The calculated structural properties of the ground-state, γ-vibrational and β-vibrational bands of ^{160}Dy are shown in Table VII. Values given in columns 3–6 are the rms values of the quadrupole and pairing deformations, that is they have been averaged over the β- and γ-dependent wave functions for each nuclear state. Note that the four deformations are not constant but vary by 10–30 %.

Values shown in columns 9–11 of Table VII give the percentages of the K components for each nuclear state. Mixing of all K values allowed for each I were taken into account. However, the components with $K=6$, 8, and 10 were negligible (much less than 1%), that is why they are not shown in the table. It is seen from Table VII that the K-mixing increases with I and with the excitation energy. For instance, the 2_γ state is 98% $K=2$ but the 8_γ state is only 72% $K=2$. Such variations play a crucial role in the values of the $E2/M1$ mixing ratios, since the $M1$ transitions are much weaker than the $E2$ transitions among collective bands. Hence, even minute variations in deformations and in the K mixing can have dramatic effects on the mixing ratios.

The calculated reduced mixing ratios, $\Delta(E2/M1) = \delta(E2/M1)/[E_\gamma\ (\mathrm{MeV})]$, and the $X(E0/E2)$ values are given and compared with the experimental values in Table VIII. Considering the sensitivity of the mixing ratios discussed above [note that $M1$ transitions are forbidden in the lowest-order collective models, rotational or vibrational, that is the $E2/M1$ mixing ratio is predicted to be infinity, whenever the $E2$ transition is allowed], the calculated $\Delta(E2/M1)$ values are remarkably close to the experimental mixing ratios, except that the sign change for $\gamma\rightarrow g$ $E2/M1$ mixing ratios (with $\Delta I=0$) is shifted from $I_i=4_\gamma$ to $I_i=10_\gamma$.

Sign change of the $E2/M1$ mixing ratio is caused by a sign change in the $M1$ matrix element, which comes from a sign change of the quantity $\partial g_-/\partial\gamma$, the γ-dependent derivative of the gyromagnetic ratio $g_-=(g_x-g_y)/2$, which is largely responsible for a $\Delta K=2$ $M1$ transition [21].

It might be pointed out here that at present, to the best of our knowledge, there is no other model which is able to predict the mixing ratio sign change for the $\gamma\rightarrow g$ transitions in the same nucleus.

As regards the $X(E0/E2)$ values, such ratios for $\Delta K=2$ transitions are predicted to vanish in the lowest order, since the $E0$ transitions are forbidden. The latter are allowed for the $\Delta K=0$ transitions, which is an important signature of a β-vibrational band, as indicated by the relatively large $X(E0/E2)$ values for the $\beta\rightarrow g$ transitions in Table VIII.

VI. SUMMARY AND CONCLUSIONS

Directional distributions of 29 γ rays (19 of them for the first time), following the decay of the oriented ^{160}Ho^{g+m}Gd, were measured and multipole mixing ratios of all mixed transitions were determined.

Sign change of the $E2/M1$ mixing ratios of transitions from the γ-vibrational ($K^\pi=2^+$) band to the ground-state band in ^{160}Dy was observed: Mixing ratios of the $\Delta I=0$ transitions, $4_\gamma\rightarrow4_g$ and $6_\gamma\rightarrow6_g$, differ in magnitude and sign from the negative values of δ for all other $\gamma\rightarrow g$ transitions. The mixing ratio sign change was also observed in ^{166}Er and probably in ^{164}Er, and in all three cases, ^{160}Dy, ^{164}Er, and ^{166}Er, the effect begins at $I_i=4_\gamma$, i.e., mixing ratio of the $4_\gamma\rightarrow4_g$ transition changes sign first. The available data also show that positive mixing ratios occur in all three nuclei and are smaller in magnitude (larger $M1$) than the negative δ values. This suggests that the mixing ratio sign change of the $\gamma\rightarrow g$ transitions is a more general and significant phenomenon. It seems that there is an ‘‘island’’ of nuclei with ‘‘anomalous’’ γ-vibrational bands and also with very weakly excited β-vibrational bands as compared with lighter (Sm, Gd) and heavier (Yb) even-even nuclei.

The anisotropies of two transitions, $2 \rightarrow 2$ and $4 \rightarrow 4$, between the β-vibrational ($K^{\pi}=0^{+}$) and ground-state ($K^{\pi}=0^{+}$) bands were also measured, and the $E2/M1$ mixing ratios were determined. The magnitudes of the $E0/E2$ mixing ratios and consequently the $E0/E2$ probability ratios for all $\Delta I=0$ transitions were obtained. In general, the $|q(E0/E2)|$ and $X(E0/E2)$ values are consistent with the systematics: The $E0/E2$ probability ratios of the $\gamma \rightarrow g$ transitions are by an order of magnitude smaller than those for the $\beta \rightarrow g$ transitions. On this basis, we have identified the second-excited $K^{\pi}=0^{+}$ band, starting at 1518.8 keV, as the β-vibrational band.

The mixing ratio sign change (in the same nucleus and for the same initial and final bands) is described at present by the dynamic deformation model only. Although the calculated results for ^{160}Dy do not agree completely with the experiment, the mixing ratio sign change is predicted for the $10_{\gamma} \rightarrow 10_{g}$ transition while the effect was observed experimentally for the $4_{\gamma} \rightarrow 4_{g}$ and $6_{\gamma} \rightarrow 6_{g}$ transitions, excellent agreement was obtained for ^{166}Er [21].

ACKNOWLEDGMENTS

M. Krmar thanks Joint Institute for Nuclear Research, Dubna for granting him financial support and for the hospitality during his stay at this Institute, and Professor I. Bikit-from the Institute of Physics, Novi Sad (IPNS) for his support of this work. The assistance of J. Slivka (IPNS) in data analysis is gratefully acknowledged. This work was supported partially by the University Professor of Physics Grant, awarded (to K. Kumar) by the Tennessee Technological University, and partially by a travel grant awarded by the Joint Institute for Heavy-Ion Research, Oak Ridge National Laboratory. The Joint Institute has as member institutions the University of Tennessee, Vanderbilt University, and Oak Ridge National Laboratory. It is supported by the members and by the U. S. Department of Energy through Contract No. DE-FG05-87ER40361 with the University of Tennessee.

---

[1] C. W. Reich, Nucl. Data Sheets **68**, 405 (1993); **78**, 547 (1996).

[2] E. P. Grigoriev, K. Ya. Gromov, Z. T. Zhelev, T. A. Islamov, V. G. Kalinnikov, U. K. Nazarov, and S. S. Sabirov, Izv. Akad. Nauk SSSR, Ser. Fiz. **33**, 635 (1969) [Bull. Acad. Sci. USSR, Phys. Ser. **33**, 585 (1970)].

[3] A. A. Aleksandrov, V. S. Butsev, Ts. Vylov, E. P. Grigoriev, K. Ya. Gromov, V. G. Kalinnikov, and N. A. Lebedev, Izv. Akad. Nauk SSSR, Ser. Fiz. **38**, 2096 (1974) [Bull. Acad. Sci. USSR, Phys. Ser. **38**, 71 (1974)]; **38**, 2103 (1974) [**38**, 77 (1974)].

[4] E. P. Grigoriev and T. R. Dadamukhamedov, Izv. Akad. Nauk SSSR, Ser. Fiz. **51**, 1950 (1987) [Bull. Acad. Sci. USSR, Phys. Ser. **51**, 68 (1987)]; **54**, 575 (1990) [**54**, 196 (1990)].

[5] H. J. Riezebos, M. J. A. de Voigt, C. A. Fields, X. W. Cheng, R. J. Peterson, G. B. Hagemann, and A. Stolk, Nucl. Phys. **A465**, 1 (1987).

[6] R. A. Fox, W. D. Hamilton, and D. D. Warner, J. Phys. A **7**, 1716 (1974).

[7] I. I. Gromova, J. Dupák, J. Koníček, T. I. Kracíková, N. A. Lebedev, B. S. Neganov, V. N. Pavlov, I. Procházka, M. Finger, V. M. Tsupko-Sitnikov, A. F. Shchus, A. Machová, W. D. Hamilton, and R. A. Fox, Izv. Akad. Nauk SSSR, Ser. Fiz. **43**, 53 (1979) [Bull. Acad. Sci. USSR, Phys. Ser. **43**, 422 (1979)].

[8] K. S. Krane, Nucl. Phys. **A377**, 176 (1982).

[9] H. Marshak, W. D. Brewer, and P. Roman, Phys. Rev. C **40**, 1759 (1989).

[10] F. K. McGowan and W. T. Milner, Phys. Rev. C **23**, 1926 (1981).

[11] G. N. Rao, Hyperfine Interact. **24-26**, 1119 (1985).

[12] J. Slesinger, M. Finger, D. Nosek, and M. Vobecký, in *Proceedings of the 44th Annual Conference on Nuclear Spectroscopy and Structure of Atomic Nuclei*, Kharkov, 1994 (Nauka, Leningrad, 1994), p. 73.

[13] T. I. Kracíková, M. Krmar, M. Finger, A. Janata, N. A. Lebedev, V. N. Pavlov, D. E. Shabalin, and E. Šimečková, in *Proceedings of International Conference on Exotic Nuclei and Atomic Masses (ENAM 95)*, Arles, France, 1995, edited by M. de Saint Simon and O. Sorlin (Frontiers, France, 1995), p. 613.

[14] I. I. Gromova, J. Dupák, J. Koníček, T. I. Kracíková, N. A. Lebedev, A. Machová, B. S. Neganov, J. Ota, V. N. Pavlov, I. Procházka, J. Říkovská, M. Finger, M.I. Fominykh, V. I. Fominykh, Khan Khen Mo, V. M. Tsupko-Sitnikov, A. F. Shchus, and Z. Janout, in *Prikladnaya Yadernaya Spektroskopiya* (Atomizdat, Moskva, 1979), Vol. 9, p. 3.

[15] V. N. Pavlov, B. S. Neganov, J. Koníček, and J. Ota, Cryogenics **18**, 115 (1978).

[16] W. D. Hamilton, *The Electromagnetic Interaction in Nuclear Spectroscopy*, edited by W. D. Hamilton (North-Holland, Amsterdam, 1975), p. 645.

[17] M. Krmar, T. I. Kracíková, and M. Finger, Yad. Fiz. **61**, 1 (1998).

[18] K. S. Krane, Nucl. Data Tables **A11**, 407 (1973).

[19] F. Rössel, H. M. Fries, K. Alder, and H. C. Pauli, At. Data Nucl. Data Tables **21**, 92 (1978).

[20] K. S. Krane, At. Data Nucl. Data Tables **22**, 269 (1978).

[21] J. Lange, K. Kumar, and J. H. Hamilton, Rev. Mod. Phys. **54**, 119 (1983).

[22] K. S. Krane and J. D. Moses, Phys. Rev. C **24**, 654 (1981).

[23] S. Davaa, T. I. Kracíková, J. Ferencej, M. Finger, N. A. Lebedev, and V. N. Pavlov, in *Proceedings of the 35th Annual Conference on Nuclear Spectroscopy and Structure of Atomic Nuclei*, Leningrad, 1985 (Nauka, Leningrad, 1985), p. 123; T. I. Kracíková, S. Davaa, M. Finger, V. N. Pavlov, and V. M. Tsupko-Sitnikov, Hyperfine Interact. **34**, 127 (1987); T. I. Kracíková, M. Krmar, M. Finger, A. Janata, N. A. Lebedev, V. N. Pavlov, I. Procházka, and D. E. Shabalin, in *Proceedings of International Conference on Exotic Nuclei and Atomic Masses (ENAM 95)* (Ref. [13]), p. 615.

[24] W. D. Hamilton, H. Marshak, and K. Kumar, J. Phys. G **16**, L219 (1990).

[25] C. A. Fields, K. H. Hicks, R. A. Ristinen, F. W. N. de Boer, L. K. Peker, R. J. Peterson, and P. M. Walker, Nucl. Phys. **A422**, 215 (1984).

[26] K. Kumar, *Nuclear Models and the Search for Unity in Nuclear Physics* (Universitetsforlaget, Bergen, 1984).

[27] K. Kumar, Hyperfine Interact. **75**, 43 (1992).

[28] R. S. Hager and E. C. Seltzer, Nucl. Data Sheets **A6**, 1 (1969).