Partial conservation of axial current constraints on pion production or absorption within nonrelativistic nuclear dynamics

V. Dmitrašinović[1] and T. Sato[2]
[1]Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208
[2]Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

(Received 8 May 1998)

We show the necessity of two-nucleon axial currents and associated pion emission or absorption operators for the partial conservation of the axial current (PCAC) nuclear matrix elements with arbitrary nuclear dynamics described by a nonrelativistic Schrödinger equation. As examples we construct such nonrelativistic axial two-body currents in the linear- and the heterotic ($g_A=1.26$) σ models, with an optional isoscalar vector (ω) meson exchange. The nuclear axial current matrix elements obey PCAC only if the nuclear wave functions used in the calculation are solutions to the Schrödinger equation with the static one-meson-exchange potential constructed in the respective (σ) model. The same holds true for the nuclear pion production amplitude, since it is proportional to the divergence of the axial current matrix element, by virtue of PCAC. Thus we found a consistency condition between the pion creation or absorption operator and the nuclear Hamiltonian. We present examples drawn from our models and discuss the implication of our results for one-pion-two-nucleon processes. [S0556-2813(98)00310-0]

PACS number(s): 25.30.−c, 11.40.Ha, 25.80.Hp, 25.10.+s

I. INTRODUCTION

In an earlier publication [1] a systematic study of axial current (partial) conservation was begun in the Bethe-Salpeter (BS) approach to nuclear bound states. There it was found that partial conservation of axial current (PCAC) puts constraints not only on the form of the axial current operator, but also on the nuclear wave function, by way of fixing the ''potential'' entering the nuclear BS equation. Since the pion production or absorption amplitude is an integral part of the conserved axial current nuclear matrix element, the same constraints are imposed on it as well. It stands to reason that the same kind of constraint will carry over into the nonrelativistic (NR) formalism.[1] In this paper we extend the work in Ref. [1] to the nonrelativistic description of the nucleus. We do not make a direct nonrelativistic reduction of the BS equations and amplitudes from Ref. [1], for they are well known not to have a good NR limit, but rather use this reference as a guide to developing the corresponding result within the Schrödinger equation approach. We shall show that even at this nonrelativistic level there are differences between the various versions of the σ model used in Ref. [1].

The aforementioned PCAC consistency condition between the (elementary) pion production mechanism and the NN potential determining the nuclear wave equation is a novel feature promising to introduce a higher level of logical coherence into present-day calculations of pion production on nuclear targets. As an example one may consider the $pp\to\pi^0 pp$ and $pp\to\pi^+ d$ reactions. There, in particular, the π-production mechanism (operator) has often been considered without any reference to the nuclear wave functions [2,3]. Several models involving scalar σ-meson-exchange pion production ''current,'' besides the π-exchange one, have been proposed [4,5]. We shall show here exactly which σ-meson-exchange π-production operator must be included and how when the two-nucleon potential contains a one-σ-exchange term, and *vice versa*. It turns out that this π-production operator is rather sensitive to the details of the σ model used. We also analyze the ω-meson-exchange π-production operator required by the presence of the ω-meson-exchange potential.

This paper falls into five sections. After the Introduction, in Sec. II, we define the context of our analysis and prove our most general result: the necessity of two-body axial currents for interacting two- (or more) particle systems obeying nonrelativistic quantum mechanics. In Sec. III we construct two-nucleon axial currents that respect PCAC at the level of *nuclear* matrix elements, starting from three underlying relativistic chirally symmetric meson-nucleon model Lagrangians. In Sec. IV we present the three corresponding sets of two-nucleon π-production operators as an example of the main proposition and we discuss the results. In Sec. V we summarize and draw the conclusions.

II. PARTIAL CONSERVATION OF NUCLEAR AXIAL CURRENT

The notion of PCAC is both historically and conceptually the foundation of chiral symmetry in hadronic interactions. Modern implementations of this symmetry, such as the nuclear chiral perturbation theory ($N\chi PT$), however, do not emphasize that point. In the following we shall try and present another viewpoint of PCAC that will bear significance to nuclear physics in general and pion-nuclear processes in particular. In short, PCAC states that the hadronic

[1]The idea that PCAC determines the pion-nuclear production operator has been around at least since the work of Blin-Stoyle and Tint [2]. But the notion that the nuclear wave functions entering the same pion production amplitude are also constrained by the PCAC appears to be new.

axial current $J^a_{\mu 5}$ must satisfy the following continuity equation:

$$\partial^\mu J^a_{\mu 5} = -f_\pi m_\pi^2 \Pi^a, \quad (1)$$

or equivalently

$$\nabla \cdot \mathbf{J}^a_5(\mathbf{R}) + \frac{\partial \rho^a_5(\mathbf{R})}{\partial t} = -f_\pi m_\pi^2 \Pi^a(\mathbf{R}), \quad (2)$$

where Π^a is the (canonical) pion field operator. In the quantum-mechanical framework this can be written as an equation relating the divergence of the three-current and the commutator of the Hamiltonian and the axial charge density:

$$\nabla \cdot \mathbf{J}^a_5(\mathbf{R}) + i[H, \rho^a_5(\mathbf{R})] = -f_\pi m_\pi^2 \Pi^a(\mathbf{R}). \quad (3)$$

This equation is a consequence of the (exact) Heisenberg equations of motion. Now we specialize to nonrelativistic nuclear physics by limiting ourselves to that subspace of the complete Hilbert space that contains at most two (real) nucleons interacting by exchanging one (virtual) meson at a time. There the total Hamiltonian of the nucleus H is the sum of the kinetic and potential energies $H=T+V$ of the nucleons, and the total axial current $\mathbf{J}^a_5(\mathbf{R})$ consists of one- and two-nucleon parts. We assume that the axial charge density ρ^a_5[2] is well approximated by its one-nucleon part[3] $\rho^a_{5,1-\mathrm{b}}$. This assumption agrees with—indeed it follows from—our fundamental assumption of nonrelativistic nuclear dynamics. This means that all (relativistic) operators are expanded in powers of $1/M$; this provides a convenient bookkeeping device in what follows. As shall be shown below, the (partial) continuity equation (3) strongly constrains (the longitudinal part of) the nuclear axial current, and in particular its two-nucleon, or meson-exchange part.

We may break up the axial current conservation equation into one- and two-body parts without loss of generality. The divergence of the complete one-body current equals $-i$ times the commutator of the kinetic energy T and the one-body axial charge density

$$\nabla \cdot \mathbf{J}^a_5(1-\mathrm{body}) = -i[T, \rho_5(1-\mathrm{body})] - f_\pi m_\pi^2 \Pi^a(1-\mathrm{body}), \quad (4)$$

is of $\mathcal{O}(M^{-2})$, i.e., zero to leading order in $1/M$, due to similar momentum dependences of the kinetic energy T and the axial charge density $\rho^a_5(1-\mathrm{body})$ operators, as well as to the absence of nondiagonal isospin operators from T. Therefore, the test of conservation of the complete nuclear axial current is whether or not the potential V commutes with the one-body axial charge density. It turns out that, due to the momentum operator inside of $\rho^a_5(1-\mathrm{body})$, only a completely trivial, viz. a spatially everywhere constant potential commutes with the axial charge. In nuclear physics, therefore, one *always* needs a two-body axial current axial current $\mathbf{J}^a_5(2-\mathrm{body}) = \Sigma^A_{j<k} \mathbf{J}_{5,(jk)}(2-\mathrm{body})$ to compensate for the temporal change of the axial charge density.

To show this formally we note that in general there are two possible sources of the noncommutativity of meson exchange potential V and the one-body axial charge density ρ^a_5: (i) noncommuting isospin factors; and (ii) noncommuting spin-spatial factors, as can be seen from the identity

$$\nabla \cdot \mathbf{J}^a_5(2-\mathrm{body}) = -i[V, \rho^a_5] - f_\pi m_\pi^2 \Pi^a(2-\mathrm{body}) = \frac{i}{2} \sum_{i=1}^{A} \sum_{j<k}^{A} \left[\frac{\tau^a_{(i)}}{2M}, I_{(jk)} \right] \{ \mathcal{V}_{(jk)}, \{ \boldsymbol{\sigma}_{(i)} \cdot \nabla_{(i)}, \delta(\mathbf{R} - \mathbf{r}_{(i)}) \} \}$$
$$- \frac{i}{2} \sum_{i=1}^{A} \sum_{j<k}^{A} \left\{ \frac{\tau^a_{(i)}}{2M}, I_{(jk)} \right\} [\mathcal{V}_{(jk)}, \{ \boldsymbol{\sigma}_{(i)} \cdot \nabla_{(i)}, \delta(\mathbf{R} - \mathbf{r}_{(i)}) \}] - f_\pi m_\pi^2 \Pi^a_\pi(2-\mathrm{body}), \quad (5)$$

where the two-body potential $V_{(jk)} = I_{(jk)} \mathcal{V}_{(jk)}$ is a product of its isospin $I_{(jk)}$ and spin-spatial $\mathcal{V}_{(jk)}$ parts. Since

$$[\mathcal{V}_{(jk)}, \{ \nabla_{(i)}, \delta(\mathbf{R} - \mathbf{r}_{(i)}) \}] \propto \delta(\mathbf{R} - \mathbf{r}_{(i)}) \delta_{ij} [\nabla_{(j)}, \mathcal{V}_{(jk)}] \neq 0, \quad (6)$$

and the isospin anticommutator $\{ \tau^a_{(i)}, I_{(jk)} \} \neq 0$ does not vanish for at least one value of $a=1,2,3$, we see that the next-to-the-last line in Eq. (5) does not vanish, and thence *the commutator of the potential and the axial charge density does not vanish for any potential* $\mathcal{V}_{jk}$, except the trivial one, i.e., a constant: $\mathcal{V}_{jk} = \mathrm{const}$.[4] In other words, *axial meson exchange currents (MEC's)* $\mathbf{J}^a_5(2-\mathrm{body})$ are always necessary,

[2]In the following we shall drop the ''index variable'' **R** in the current and charge operators, except when necessary to avoid confusion.

[3]The subleading (relativistic) correction to the axial charge-density operator contains two-body terms that modify the following analysis somewhat, but cannot change its main conclusion, due to an intrinsically different tensor structure of the one- and two-body operators.

[4]This argument does not furnish a formal proof so long as all possible spin and isospin operators have not been examined for accidental vanishing of their anticommutators with $\boldsymbol{\sigma}_{(i)}$ and $\tau^a_{(i)}$, respectively. Our proposition has been confirmed in all cases that we have studied so far, which cases constitute some of the most important parts of the nuclear NN potential.

so long as two nucleons interact and their dynamics can be described by quantum mechanics.[5] The same conclusions, of course, hold in nonrelativistic (NR) quark models, i.e., axial two-quark currents are always necessary in NR quark models, as well. It is the compelling nature of this argument that makes it distinct from previous arguments along similar lines [6,7].

This is perhaps a somewhat surprising result in view of the fact that the EM current conservation does *not* require MEC's for many parts of the nuclear potential, e.g., for the contributions from the exchange of neutral mesons. Hence it will be our task to construct axial MEC's associated with the exchange of the most important, i.e., the lightest, mesons. The π-exchange axial current has been known for some time [8,9], so we shall not repeat its derivation here. In this paper we shall concentrate on the isoscalar scalar (σ) and vector (ω) mesons. The isovector vector (ρ) meson cannot be introduced in a model-independent way, so we leave it for another occasion.

To reveal the necessity of consistency between the nuclear wave functions and the nuclear pion production/absorption amplitude we must remember that the operator equation (3) describing PCAC is just a shorthand for the same statement about *all* axial current nuclear matrix elements $\langle J^a_{\mu 5}\rangle_{fi}$. In momentum space the Heisenberg equations of motion lead to

$$\begin{aligned} q^\mu\langle J^a_{\mu 5}\rangle_{fi} &= \mathbf{q}\cdot\langle \mathbf{J}^a_5\rangle_{fi} - (E_f - E_i)\langle \rho^a_5\rangle_{fi} \\ &= \mathbf{q}\cdot\langle \mathbf{J}^a_5\rangle_{fi} - \langle [H,\rho^a_5]\rangle_{fi} \\ &= i\left(\frac{f_\pi m_\pi^2}{q^2 - m_\pi^2}\right)\langle \Gamma^a_\pi\rangle_{fi} \simeq -i\left(\frac{f_\pi m_\pi^2}{\mathbf{q}^2 + m_\pi^2}\right)\langle \Gamma^a_\pi\rangle_{fi}, \end{aligned} \tag{7}$$

only if the initial and final states $|\Psi_{f,i}\rangle$ are solutions to the nuclear Schrödinger equation $H|\Psi_n\rangle = E_n|\Psi_n\rangle$ (here $q_0 = E_f - E_i$). We used the abbreviation $\langle A\rangle_{fi} = \langle \Psi_f|A|\Psi_i\rangle$. Thus, *the nuclear pion absorption/emission amplitude* $\langle \Gamma^a_\pi\rangle_{fi}$ *must be evaluated using a pion absorption/emission operator* Γ^a_π *that matches the nuclear dynamics leading to the nuclear wave functions* $|\Psi_n\rangle$, *if the result is to agree with PCAC*. In the last line of Eq. (7) we used $q^2 = q_0^2 - \mathbf{q}^2$, which is true for elastic scattering in the Breit frame, i.e., when $q_0 = E_f - E_i = 0$.

The idea of consistency of nuclear wave functions and MEC's is not a new one: it has been a part of the electronuclear physics ''folklore'' for some time. The extension of this idea to the axial currents and pion production amplitudes seems to be less well known. In the following we shall apply this idea to nuclear axial currents in several variations of the σ model with ω mesons.

[5] This does not mean, however, that we know how to construct axial MEC's that obey PCAC for arbitrary *NN* potentials. Presently we know how to calculate only those axial MEC's that are related to potentials that are based on chiral Lagrangian models. Methods ordinarily used for electromagnetic (EM) MEC's do not seem to work here, cf. Ref. [1].

III. AXIAL CURRENTS IN σ MODELS

We shall use two of the σ models already developed in Ref. [1]. As is well known, the pseudoscalar and the pseudovector, or gradient, πNN couplings reduce to the same one-pion-exchange potential (OPEP) to leading order in nonrelativistic (NR) expansion. At first sight one might think that this implies identical axial MEC's in the linear and nonlinear-σ models. This is not so because the linear-σ model one-boson-exchange potential (OBEP) includes a σ-exchange potential as well, whereas the nonlinear model does not. Moreover, the difference between the ''heterotic σ model'' and the other two σ models persists, although in an unusual way: its (spatial) axial current is renormalized by a factor g_A, though the axial charge is not. In the following we look at each model separately. Subsequently we add the isoscalar-vector meson ω, which plays an important role by providing short-range repulsion and is chirally invariant by itself. That brings about new terms into the axial current and the pion production operator in a way that is consistent with PCAC.

A. The one-body current

The one-body part of the nuclear axial current is the sum over all nucleons of the *direct* axial current and the *pion pole term*, see Fig. 3(a) in Ref. [1]. In configuration space this takes its usual nonrelativistic form

$$\begin{aligned} \mathbf{J}^a_5(1-\text{body}) = g_A\sum_{i=1}^{A}\frac{\tau^a_{(i)}}{2}\left[\boldsymbol{\sigma}_{(i)} - \boldsymbol{\nabla}_{\mathbf{R}}\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\boldsymbol{\nabla}_{\mathbf{R}}}{(\nabla^2_{\mathbf{R}} - m_\pi^2)}\right)\right] \\ \times\,\delta(\mathbf{R} - \mathbf{r}_{(i)}). \end{aligned} \tag{8}$$

The axial charge density

$$\rho^a_5(1-\text{body}) = -ig_A\sum_{i=1}^{A}\frac{\tau^a_{(i)}}{2M}\{\boldsymbol{\sigma}_{(i)}\cdot\boldsymbol{\nabla}_{(i)},\delta(\mathbf{R}-\mathbf{r}_{(i)})\}, \tag{9}$$

on the other hand, is taken *without* the pion-pole contribution. This simplifying assumption is justified *ex post facto* by the fact that it does not prevent a successful construction of a consistent approximation. This does not mean that one cannot find, or should not search for approximations that avoid this assumption.

The axial coupling constant g_A is either unity, as in the linear-σ model, or its measured value 1.26 in the spatial part of the axial current Eq. (8), and unity in the axial charge Eq. (9) of the heterotic σ model of Ref. [1]. (For more on this, see the Appendix.) We also neglect all nucleon electroweak form factors. This should be adequate for the purpose of describing the static properties of the nucleus. The nuclear matrix element of the direct one-body axial current is depicted in Fig. 1(a), whereas the pion-pole part can be constructed by attaching the axial current ''wavy line'' to the external pion in Fig. 4(a).

(a) Linear σ model. The ith-nucleon current in the linear-σ model, i.e., with a unit (''normalized'') nucleon axial coupling, in momentum space reads

$$\mathbf{J}^a_{5,(i)}(\mathbf{p}'_i,\mathbf{p}_i) = \frac{\tau^a_{(i)}}{2}\left[\boldsymbol{\sigma}_{(i)} - \mathbf{q}\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\mathbf{q}}{\mathbf{q}^2 + m_\pi^2}\right)\right], \tag{10}$$

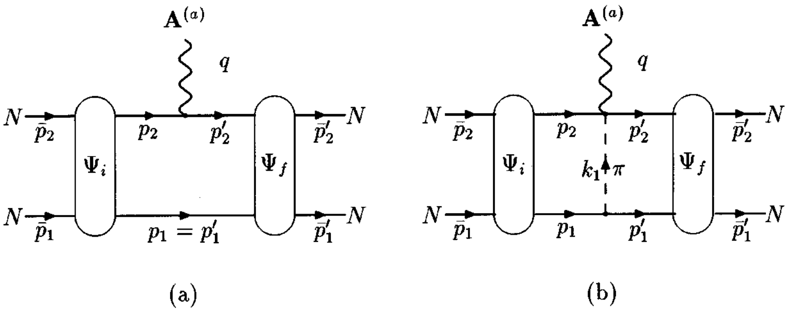

FIG. 1. Effective nonrelativistic Feynman diagrams contributing to the one- (a) and the two-body axial current nuclear matrix element (b). Each graph consists of a ''direct'' term and a ''pion pole'' term. We display only the direct terms here. In the nonlinear-σ model there are only graphs 1(a) and 1(b). The dashed line denotes a pion, the solid line denotes a nucleon, and the wavy line is the external axial current (source).

where $\mathbf{q}=\mathbf{p}_i'-\mathbf{p}_i$, and satisfies the nonrelativistic (static) version of the single-fermion axial Ward-Takahashi identity

$$\begin{aligned}\mathbf{q}\cdot\mathbf{J}_{5,(i)}a(\mathbf{p}_i',\mathbf{p}_i)&=\left(\frac{f_\pi m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)g_0\tau_{(i)}^a\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\mathbf{q}}{2M}\right)\\&\simeq -i\left(\frac{f_\pi m_\pi^2}{q^2-m_\pi^2}\right)\Gamma_\pi^a(\mathbf{p}_i',\mathbf{p}_i;1-\text{body})\\&=i\left(\frac{f_\pi m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)\Gamma_\pi^a(\mathbf{p}_i',\mathbf{p}_i;1-\text{body}),\quad(11)\end{aligned}$$

which follows from the Goldberger-Treiman (GT) relation $M=g_0 f_\pi$. We use the same symbol for operators in configuration and momentum space. The second line on the right-hand side of Eq. (11) is the single-nucleon pion absorption operator multiplied by the divergence of the axial current factor $f_\pi m_\pi^2$ and the static pion propagator $(\mathbf{q}^2+m_\pi^2)^{-1}$. We see that the pion absorption operator arises naturally from the divergence of the axial current. Both in this and in the heterotic σ model the one-body axial charge operator Eq. (9) does not contain the factor g_A, i.e., in momentum space it reads

$$\rho_{5,(i)}^a(\mathbf{p}_i',\mathbf{p}_i)=\frac{\tau_{(i)}^a}{2}\boldsymbol{\sigma}_{(i)}\cdot\left(\frac{\mathbf{p}_i'+\mathbf{p}_i}{2M}\right),\qquad(12)$$

for proof, see the Appendix.

(b) Heterotic σ model. In the heterotic σ model the GT relation is modified to $g_A M=g_{\pi NN}f_\pi$, where $g_A=1.26$ and the one-body current becomes

$$\mathbf{J}_{5,(i)}^a(\mathbf{p}_i',\mathbf{p}_i)=g_A\frac{\tau_{(i)}^a}{2}\left[\boldsymbol{\sigma}_{(i)}-\mathbf{q}\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\right],\qquad(13)$$

which satisfies the single-nucleon axial Ward-Takahashi identity

$$\begin{aligned}\mathbf{q}\cdot\mathbf{J}_{5,(i)}^a(\mathbf{p}_i',\mathbf{p}_i)&=f_\pi\left(\frac{m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)g_{\pi NN}\tau_{(i)}^a\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\mathbf{q}}{2M}\right)\\&\simeq -i\left(\frac{f_\pi m_\pi^2}{q^2-m_\pi^2}\right)\Gamma_\pi^a(\mathbf{p}_i',\mathbf{p}_i;1-\text{body})\\&=i\left(\frac{f_\pi m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)\Gamma_\pi^a(\mathbf{p}_i',\mathbf{p}_i;1-\text{body}),\quad(14)\end{aligned}$$

since $[T,\rho_5^a(1-\text{body})]=0+\mathcal{O}(M^{-2})$. The spatial parts of one-nucleon axial currents in the two models are almost identical in the NR limit, the only difference being the overall factor g_A. As stated above, the axial charges in the two models are identical (see the Appendix).

B. Two-nucleon axial current

Having shown that in order to have a partially conserved axial current in a nonrelativistic nuclear model, one must have two-nucleon axial currents, we turn towards constructing such MEC's. The two-body currents $\mathbf{J}_5(2-\text{b})$ appropriate to the meson exchange two-nucleon potentials will be constructed and we shall show that they lead to PCAC. We construct these ''meson exchange currents'' by nonrelativistic reduction of covariant Feynman amplitudes in specific chiral models.

Due to the presence of gradient operators it will be to our advantage to work in the momentum space. Definition of the Fourier transform of two-body currents into momentum space can be found in Ref. [8], among others. The current conservation relation in momentum space can now be written as

$$\begin{aligned}\mathbf{q}\cdot\mathbf{J}_5^a(2-\text{body})&=[V(2-\text{body}),\rho_5^a(1-\text{body})]\\&\quad-i\left(\frac{f_\pi m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)\Gamma_\pi^a(2-\text{body}).\qquad(15)\end{aligned}$$

We repeat that this equation follows from only two assumptions: (i) PCAC, and (ii) quantum mechanics. As stated above, the only ''sure-fire'' way of constructing an axial MEC that satisfies PCAC that we know of is to start from a relativistic chiral Lagrangian model. We shall use the two variations of the linear-σ model already utilized in Ref. [1] and the simplest ωNN interaction Lagrangian that preserves chiral symmetry.

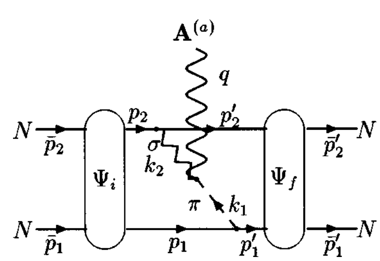

FIG. 2. The ''meson-in-flight'' graph contributing to the axial current in the linear and the heterotic σ models. The zig-zag line denotes a σ meson. One must keep all four graphs in Figs. 1(a), 1(b) and 2, 3 in both the linear and the heterotic σ model.

1. Linear-σ model

To construct the partially conserved nonrelativistic axial two-nucleon current in this model we start from the corresponding covariant amplitude. A nonrelativistic expansion in powers of $1/M$ leads to three terms of $O(1/M)$: (i) one due to the meson-in-flight diagrams, Fig. 2 plus their pion-pole counterparts, Fig. 5, [the covariant current that defines this amplitude can be found in Eqs. (33), (34) of Ref. [1]]:

$$\mathbf{A}_{\rm I}^a(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=\frac{g_0^2}{2M}\tau_{(1)}^a\left[\mathbf{k}_1-\mathbf{k}_2+\mathbf{q}\left(\frac{f_\pi g_{\sigma\pi\pi}}{\mathbf{q}^2+m_\pi^2}\right)\right]\times\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{(\mathbf{k}_2^2+m_\sigma^2)(\mathbf{k}_1^2+m_\pi^2)}+(1\leftrightarrow 2), \quad (16)$$

where $\mathbf{k}_i=\mathbf{p}_i-\mathbf{p}_i'$, $i=1,2$; and $f_\pi g_{\sigma\pi\pi}=m_\sigma^2-m_\pi^2$. Three-momentum conservation reads $\mathbf{k}_1+\mathbf{k}_2+\mathbf{q}=0$. This MEC alone is not sufficient for PCAC, as can be seen from the corresponding divergence, which reads

$$\begin{aligned}\mathbf{q}\cdot\mathbf{A}_{\rm I}^a(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})&=\frac{g_0^2}{2M}\tau_{(1)}^a\left[\mathbf{k}_2^2-\mathbf{k}_1^2+(m_\sigma^2-m_\pi^2)-f_\pi m_\pi^2\left(\frac{g_{\sigma\pi\pi}}{\mathbf{q}^2+m_\pi^2}\right)\right]\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{(\mathbf{k}_2^2+m_\sigma^2)(\mathbf{k}_1^2+m_\pi^2)}+(1\leftrightarrow 2)\\&=-\frac{g_0^2}{2M}\left[\tau_{(1)}^a\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_2^2+m_\sigma^2}+(1\leftrightarrow 2)\right]+\frac{g_0^2}{2M}\left[\tau_{(1)}^a\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right]+\mathcal{O}(f_\pi m_\pi^2)\\&=[V_\sigma,\rho_5^a]+\frac{g_0^2}{2M}\left[\tau_{(1)}^a\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right]+\frac{g_0^2}{2M}\left[\tau_{(1)}^a\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{\mathbf{k}_2^2+m_\sigma^2}+(1\leftrightarrow 2)\right]+\mathcal{O}(f_\pi m_\pi^2), \quad (17)\end{aligned}$$

where the one-meson-exchange NN potential in the linear-σ model reads

$$\begin{aligned}V_{2-\mathrm{b}}(\mathbf{p})&=V_\sigma(\mathbf{p})+V_\pi(\mathbf{p})\\&=-\frac{g_0^2}{\mathbf{p}^2+m_\sigma^2}+\vec{\tau}_{(1)}\cdot\vec{\tau}_{(2)}\left(\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{p}}{2M}\right)\left(\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{p}}{2M}\right)\\&\quad\times\left(\frac{g_0^2}{\mathbf{p}^2+m_\pi^2}\right)\\&=-\frac{g_0^2}{\mathbf{p}^2+m_\sigma^2}+\mathcal{O}(1/M^2). \quad (18)\end{aligned}$$

Note that the σ-exchange potential is of $O(1/M^0=1)$, whereas the π-exchange potential is of $O(1/M^2)$. Consequently, the commutator of the potential and the one-body axial charge also fall into two distinct orders in $1/M$. This fact allows an apparently clean and simple separation of the σ-exchange current effects from the π-exchange ones. In this paper we are primarily interested in the σ-exchange currents, so we leave the π-induced ones aside, as they have been extensively studied in the literature [8,9].

The commutator on the right-hand side of Eq. (15) is

$$\begin{aligned}[V_{2-\mathrm{b}},\rho_5^a]&=[V_\sigma,\rho_5^a]+\mathcal{O}(1/M^3)\\&=\frac{g_0^2}{2M}\left[\tau_{(1)}^a\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_2}{\mathbf{k}_2^2+m_\sigma^2}+(1\leftrightarrow 2)\right]+\mathcal{O}(1/M^3). \quad (19)\end{aligned}$$

We see that the σ exchange leads to terms of $O(1/M)$, whereas the π exchange leads to terms of $O(1/M^3)$ in the commutator. Comparing Eq. (19) with the divergence of the axial two-body current Eq. (17) we see that we are rather far from having the σ-exchange potential commutator on the right-hand side. There are four terms left over: two are proportional to the π propagator thus indicating perhaps a relationship to the π-exchange current, but also being of $\mathcal{O}(1/M)$, i.e., two orders in $1/M$ lower than the lowest expected π-exchange current contribution. The other two are proportional to the σ propagator. In other words, our expectations do not square well with these initial results. So, the question is: whence do these ''extra'' terms come from and is there something that might compensate for them? The answer is that among the apparently ''higher-order'' terms in the axial π-exchange currents there are two such ones.

(i) One is the π-exchange axial current, Fig. 4(b),

$$\mathbf{A}_{\rm II}^a(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=-\frac{g_0^3}{2M^2}\left(\frac{f_\pi\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\left[\tau_{(1)}^a\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right], \quad (20)$$

that is actually one order in $1/M$ lower than naively ex-

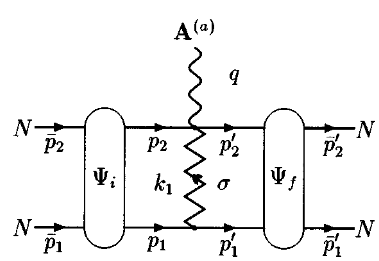

FIG. 3. σ-meson-exchange current contributing to the nuclear pion production matrix element. It consists of a ''pair,'' or Z-graph time-ordered contribution in the linear-σ model to which the elementary $\pi\sigma NN$ vertex is added in the heterotic model.

pected, due to the validity of the Goldberger-Treiman (GT) relation $f_\pi g_0 = M$ in the linear σ model. Thus

$$\mathbf{A}^a_{\rm II}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = -\frac{g_0^2}{2M}\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right]. \tag{21}$$

This current is due to an effective two-pion-nucleon vertex, which is really a time-ordered Z graph arising in the NR reduction of the nucleon Feynman propagator (the so-called pair current″), and is not an ‘‘elementary’’ interaction term in the Lagrangian. The divergence of this axial MEC is

$$\mathbf{q}\cdot\mathbf{A}^a_{\rm II}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = -\frac{g_0^2}{2M}\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right] + \mathcal{O}(f_\pi m_\pi^2). \tag{22}$$

(ii) Similarly, there is a σ-exchange Z graph, Fig. 6, as well

$$\mathbf{A}^a_{\rm III}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = -\frac{g_0^2}{2M}\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{\mathbf{k}_2^2+m_\sigma^2}+(1\leftrightarrow 2)\right]. \tag{23}$$

Its divergence is

$$\mathbf{q}\cdot\mathbf{A}^a_{\rm III}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) = -\frac{g_0^2}{2M}\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{(\mathbf{k}_2^2+m_\sigma^2)}+(1\leftrightarrow 2)\right] + \mathcal{O}(f_\pi m_\pi^2), \tag{24}$$

which is exactly what we need to bring the continuity equation (17) in the linear-σ model into the canonical form (15).

Thus, the complete nonrelativistic axial MEC to $\mathcal{O}(1/M)$ in the linear-σ model is

$$\begin{aligned}\mathbf{J}^a_{5,2-b}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) &= \mathbf{A}^a_{\rm I}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})+\mathbf{A}^a_{\rm II}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})+\mathbf{A}^a_{\rm III}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})\\ &= \frac{g_0^2}{2M}\tau^a_{(1)}\left\{\left[\mathbf{k}_1-\mathbf{k}_2+\mathbf{q}\left(\frac{f_\pi g_{\sigma\pi\pi}}{\mathbf{q}^2+m_\pi^2}\right)\right]\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{(\mathbf{k}_2^2+m_\sigma^2)(\mathbf{k}_1^2+m_\pi^2)}-\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\left[\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{\mathbf{k}_2^2+m_\sigma^2}\right]\right\}+(1\leftrightarrow 2).\end{aligned} \tag{25}$$

Thus, Eq. (25) uniquely fixes the pion production/absorption operator Γ^a_π in the one-boson-exchange potential approximation to the linear-σ model. This operator is to be used together with the $\mathcal{O}(1/M^3)\,\pi$-exchange operators and nuclear wave functions that are solutions to the Schrödinger equation with the above one-pion + σ-exchange NN potential.

The resulting MEC operator is perhaps somewhat unexpected: certainly the first (‘‘meson-in-flight’’) term is not a surprise, but the presence of the second (‘‘pion seagull’’) term might seem a little odd at first: One would have been hard pressed to correctly guess the second term in the MEC without the benefit of guidance by the linear-σ model. Thus we have expanded the nuclear interaction to include one-σ-exchange NN potential (beside the OPEP) in a manner that is consistent with PCAC, but still with $g_A=1$. Next, we shall relax that assumption. Once again, we resort to a chiral Lagrangian model for guidance.

2. Heterotic σ model

The heterotic σ model differs from the linear one by the presence of g_A in the GT relation, $g_{\pi NN} f_\pi = g_A M$, and in the spatial part of the axial current (8), but *not* in the axial charge (9). (For a proof of this statement, see the Appendix.) This variation induces some curious changes in the axial two-nucleon currents. To begin with, there are two types of relativistic Born approximation Feynman diagrams, depicted in Figs. 1(b), 4(b), and Fig. 6. [The complete relativistic current consists of the sum of Eqs. (41), (47), and (51), and its divergence in the sum of Eqs. (42), (48), and (52) in Ref. [1].] By evaluating its matrix element between on-shell single-nucleon states and making the nonrelativistic expansion in powers of $1/M$, one finds a host of π-exchange currents of $O(1/M^3)$, and two different σ-exchange currents of $O(1/M)$. One of them is the familiar meson-in-flight diagram (+ its exchange), Fig. 2, but rescaled by a factor of g_A:

$$\mathbf{A}^a_{\rm I}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) = g_A\frac{g_0^2}{2M}\tau^a_{(1)}\left[\mathbf{k}_1-\mathbf{k}_2+\mathbf{q}\left(\frac{f_\pi g_{\sigma\pi\pi}}{\mathbf{q}^2+m_\pi^2}\right)\right]\times\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{(\mathbf{k}_2^2+m_\sigma^2)(\mathbf{k}_1^2+m_\pi^2)}+(1\leftrightarrow 2), \tag{26}$$

for which the current divergence in momentum space reads

$$\begin{aligned}\mathbf{q}\cdot\mathbf{A}^a_{\rm I}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) &= g_A\frac{g_0^2}{2M}\tau^a_{(1)}\left[\mathbf{k}_2^2-\mathbf{k}_1^2+\mathbf{q}^2\left(\frac{f_\pi g_{\sigma\pi\pi}}{\mathbf{q}^2+m_\pi^2}\right)\right]\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{(\mathbf{k}_2^2+m_\sigma^2)(\mathbf{k}_1^2+m_\pi^2)}+(1\leftrightarrow 2)\\ &= g_A[V_\sigma,\rho^a_5]+g_A\frac{g_0^2}{2M}\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right]+g_A\frac{g_0^2}{2M}\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{\mathbf{k}_2^2+m_\sigma^2}+(1\leftrightarrow 2)\right]+\mathcal{O}(f_\pi m_\pi^2).\end{aligned} \tag{27}$$

The one-meson-exchange NN potential in the heterotic σ model reads

$$\begin{aligned} V_{2\text{-b}}(\mathbf{p}) &= V_\sigma(\mathbf{p}) + V_\pi(\mathbf{p}) \\ &= -\frac{g_0^2}{\mathbf{p}^2+m_\sigma^2} + \vec{\tau}_{(1)}\cdot\vec{\tau}_{(2)}\left(\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{p}}{2M}\right)\left(\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{p}}{2M}\right) \\ &\quad\times\left(\frac{g_{\pi NN}^2}{\mathbf{p}^2+m_\pi^2}\right) \\ &= -\frac{g_0^2}{\mathbf{p}^2+m_\sigma^2} + \mathcal{O}(1/M^2), \end{aligned} \tag{28}$$

and $g_{\pi NN}=g_A g_0$, due to the GT relation.

Similarly, the pion Z graph, Fig. 1(b), contribution is also renormalized by factor g_A:

$$\begin{aligned} \mathbf{A}^a_{\rm II}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) &= -g_A\frac{g_0^3}{2M^2}\left(\frac{\mathbf{q}f_\pi}{\mathbf{q}^2+m_\pi^2}\right) \\ &\quad\times\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right] \\ &= -g_A\frac{g_0^2}{2M}\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right) \\ &\quad\times\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right]. \end{aligned} \tag{29}$$

The divergence of this axial MEC is

$$\begin{aligned} \mathbf{q}\cdot\mathbf{A}^a_{\rm II}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) &= -g_A\frac{g_0^2}{2M}\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+(1\leftrightarrow 2)\right] \\ &\quad+\mathcal{O}(f_\pi m_\pi^2). \end{aligned} \tag{30}$$

Finally, in the heterotic σ model in addition to the σ-exchange Z graph, Fig. 6, contribution

$$\mathbf{A}^a_{\rm III}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) = -\frac{g_0^2}{2M}\tau^a_{(1)}\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{\mathbf{k}_2^2+m_\sigma^2}+(1\leftrightarrow 2), \tag{31}$$

there is also an elementary $A^a_\mu\sigma NN$ axial current vertex, Fig. 3, and a $\pi\sigma NN$ vertex-induced MEC, Fig. 6,

$$\begin{aligned} \mathbf{A}^a_{\rm IV}(\mathbf{k}_1,\mathbf{p}_2,\mathbf{q};h) &= -f_\pi\left(\frac{g_A-1}{f_\pi}\right)\frac{g_0^2}{2M}\frac{\tau^a_{(1)}}{(\mathbf{k}_2^2+m_\sigma^2)} \\ &\quad\times\left[2\boldsymbol{\sigma}_{(1)}+\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\boldsymbol{\sigma}_{(1)}\cdot(\mathbf{k}_2-\mathbf{q})\right] \\ &\quad+(1\leftrightarrow 2). \end{aligned} \tag{32}$$

The divergence of the sum of these last two axial MEC's is

$$\begin{aligned} \mathbf{q}\cdot\mathbf{A}^a_{\rm III+IV}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) &= -\frac{g_0^2}{2M}\frac{\tau^a_{(1)}}{\mathbf{k}_2^2+m_\sigma^2}[(g_A-1)\boldsymbol{\sigma}_{(1)}\cdot(\mathbf{k}_2 \\ &\quad+\mathbf{q})+\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}]+(1\leftrightarrow 2) \\ &\quad+\mathcal{O}(f_\pi m_\pi^2), \end{aligned} \tag{33}$$

which is exactly what we need to complete the continuity equation (15).

The sum of these three axial MEC's together with the one-body (impulse approximation) terms plus the $\mathcal{O}(1/M^2)$ pion-exchange currents which we consistently suppressed in this paper, constitute the complete, PCAC-obeying axial current:

$$\begin{aligned} \mathbf{J}^a_{5,2\text{-b}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h) &= \mathbf{A}^a_{\rm I}+\mathbf{A}^a_{\rm II}+\mathbf{A}^a_{\rm III}+\mathbf{A}^a_{\rm IV}=\frac{g_0^2}{2M}\tau^a_{(1)}\left\{g_A\left[\mathbf{k}_1-\mathbf{k}_2+\mathbf{q}\left(\frac{f_\pi g_{\sigma\pi\pi}}{\mathbf{q}^2+m_\pi^2}\right)\right]\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{(\mathbf{k}_2^2+m_\sigma^2)(\mathbf{k}_1^2+m_\pi^2)}\right. \\ &\quad\left.-\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\left[g_A\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_1}{\mathbf{k}_1^2+m_\pi^2}+\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{q}}{\mathbf{k}_2^2+m_\sigma^2}\right]-\left(\frac{g_A-1}{\mathbf{k}_2^2+m_\sigma^2}\right)\left[2\boldsymbol{\sigma}_{(1)}+\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\boldsymbol{\sigma}_{(1)}\cdot(\mathbf{k}_2-\mathbf{q})\right]\right\}+(1\leftrightarrow 2). \end{aligned} \tag{34}$$

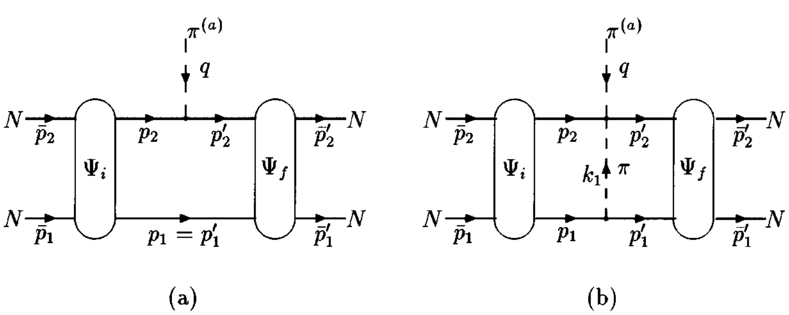

FIG. 4. Effective nonrelativistic Feynman diagrams contributing to the one- (a) and the two-body pion production nuclear matrix element (b). In the nonlinear-σ model there are only graphs 1(a) and 1(b). The pion-exchange current (b) consists of a ''pair,'' or Z-graph time-ordered contribution in the linear-σ model to which the elementary $\pi\pi NN$ vertex is added in the heterotic model. The dashed line denotes a pion, the solid one a nucleon.

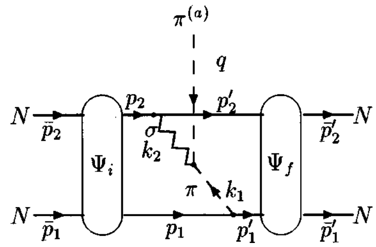

FIG. 5. The ''meson-in-flight'' graph contributing to the linear and the heterotic σ models. The zig-zag line denotes a σ meson. One must keep all four graphs in Figs. 1(a), 1(b) and 2, 3 in both the linear and the heterotic σ model.

Most of this current is new: specifically, terms (II, III, IV) have not been considered before, only the first term (I) having been derived in Refs. [10,11]. This current satisfies PCAC

$$\mathbf{q}\cdot\mathbf{J}^a_{5,2-\mathrm{b}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h)=[V_\sigma,\rho_5^a]+\mathcal{O}(f_\pi m_\pi^2). \tag{35}$$

This equation uniquely fixes the pion production/absorption operator in the OBEP approximation to the nuclear dynamics in the heterotic σ model as the ''$\mathcal{O}(f_\pi m_\pi^2)$'' term, to be evaluated in the next section. We see that, by a curious turn of events, the divergence of the complete axial current (35) equals the divergence found in the linear σ model (27), modulo different $\mathcal{O}(f_\pi m_\pi^2)$ terms, of course. In other words, we have exactly the same commutator of the one-σ-exchange potential (OΣEP) and the axial charge in these two σ models, despite manifest differences between their axial currents. This fact is a consequence of identical axial charges and OΣEP in the two models, the former being a subtle effect explained in the Appendix. Finally we turn to the ω exchange.

3. Omega meson exchange

One may include the ω-meson-exchange potential

$$V_\omega(\mathbf{p})=\frac{g_\omega^2}{\mathbf{p}^2+m_\omega^2} \tag{36}$$

into the nuclear Hamiltonian H at no peril to chiral symmetry because it is an isoscalar vector field which is chirally invariant by itself in both the linear and the nonlinear realization. As is well known, the main benefit of including this term into the nucleon-nucleon potential is that it provides short-range repulsion that is otherwise absent from the σ models. The nuclear potential with π,σ,ω exchange has sufficient attraction to bind the deuteron and enough repulsion to keep it weakly bound. The associated axial MEC is, Fig. 7,

$$\mathbf{J}^a_{5,\omega}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=-f_\pi g_{\pi NN}\frac{g_\omega^2}{2M^2}\left(\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\times\left[\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_2}{\mathbf{k}_2^2+m_\omega^2}+(1\leftrightarrow 2)\right]. \tag{37}$$

This completes the construction of the PCAC-constrained axial current in models with OBEP based on the π,σ, and ω mesons. The ρ meson was deliberately omitted since its contribution is (highly) model dependent.

IV. PION PRODUCTION OPERATORS

The one-body pion production operator is well known, as well as the two-body one associated with pion exchange. We shall therefore concentrate on the MEC's that are associated with other meson (σ and ω) exchanges, as specified by the PCAC constraint Eq. (15).

A. Linear $\boldsymbol{\sigma}$ model

It follows from Eq. (15) and the linear-σ model axial current Eq. (25) that

$$\begin{aligned}\Gamma^a_{\pi 2-\mathrm{b}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})&=\Gamma^a_{\pi\mathrm{I}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})+\Gamma^a_{\pi\mathrm{II}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})\\&\quad+\Gamma^a_{\pi\mathrm{III}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})\\&=i\frac{g_0^3}{2M^2}\tau^a_{(2)}\left[\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{k}_2}{\mathbf{k}_2^2+m_\pi^2}+\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{q}}{\mathbf{k}_1^2+m_\sigma^2}\right]\\&\quad-ig_{\sigma\pi\pi}\frac{g_0^2}{2M}\tau^a_{(2)}\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{k}_2}{(\mathbf{k}_1^2+m_\sigma^2)(\mathbf{k}_2^2+m_\pi^2)}\\&\quad+(1\leftrightarrow 2),\end{aligned} \tag{38}$$

as the complete linear-σ model pion production operator to this order in $1/M$. This result corresponds to Figs. 4(b), 5, 6.

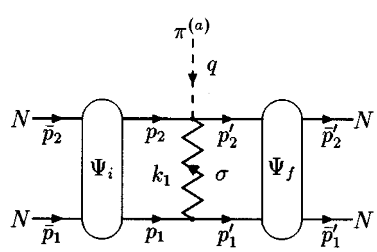

FIG. 6. σ-meson-exchange current contributing to the nuclear pion production matrix element. It consists of a ''pair,'' or Z-graph time-ordered contribution in the linear-σ model to which the elementary $\pi\sigma NN$ vertex is added in the heterotic model.

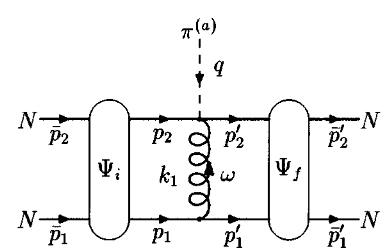

FIG. 7. The ω-exchange current contributing to the nuclear pion production matrix element. The curly line denotes an ω meson. This graph can be added to all σ models without disturbing their chiral symmetry.

Although it has long been known that the corresponding covariant amplitude is chirally symmetric, we are not aware of anyone having used the MEC (38) in nuclear physics.

B. Heterotic σ model

As stated before, the main difference between the linear and the heterotic σ models is the axial coupling constant g_A, which is induced by the new derivative-coupled interactions. The latter, in turn, renormalize one old graph, Fig. 5, and creates two new elementary diagrams: Figs. 6 and Fig. 4(b).[6] Thus one finds

$$\Gamma^a_{\pi 2\text{-b}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h)=\Gamma^a_{\pi\mathrm{I}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h)+\Gamma^a_{\pi\mathrm{II}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h)+\Gamma^a_{\pi\mathrm{III}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h)+\Gamma^a_{\pi\mathrm{IV}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q};h)=i\frac{g_0^3}{2M^2}\tau^a_{(2)}\left[g_A\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{k}_2}{\mathbf{k}_2^2+m_\pi^2}\right.$$
$$\left.+\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{q}}{\mathbf{k}_1^2+m_\sigma^2}+(g_A-1)\frac{\boldsymbol{\sigma}_{(2)}\cdot(\mathbf{k}_1-\mathbf{q})}{\mathbf{k}_1^2+m_\sigma^2}\right]-ig_{\sigma\pi\pi}g_A\frac{g_0^2}{2M}\tau^a_{(2)}\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{k}_2}{(\mathbf{k}_1^2+m_\sigma^2)(\mathbf{k}_2^2+m_\pi^2)}+(1\leftrightarrow 2). \quad (39)$$

This is the complete two-body pion-production operator in the heterotic σ model, though with all $\mathcal{O}(1/M^3)$ pion-exchange two-body operators excluded. This operator is only to be used with nuclear wave functions that are solutions to the Schrödinger equation with the aforementioned one-pion + σ-exchange NN potential, Eq. (28).

C. Omega meson exchange

The ω-exchange π-production operator, Fig. 7, to this order in $1/M$ is

$$\Gamma^a_{\pi\omega}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=-ig_\omega^2\frac{g_{\pi NN}}{2M}\left(\tau^a_{(1)}\frac{\boldsymbol{\sigma}_{(1)}\cdot\mathbf{k}_2}{\mathbf{k}_2^2+m_\omega^2}+(1\leftrightarrow 2)\right). \quad (40)$$

Note the utter functional dissimilarity between this and the two σ-MEC's Eqs. (38), and (39), despite the fact that both are due to terms in the NN potential that are indistinguishable to this order in nonrelativistic expansion, *viz.* the isoscalar one-vector- and the scalar-meson exchange potentials in Eqs. (36), and (28) respectively. Specifying the Lorentz structure of the NN potential distinguishes the vector- from the scalar-exchange induced terms, but even then the ambiguity between the two kinds of scalar MEC's is survives. This fact brings into focus the intrinsic and apparently intransigent ambiguities associated with attempts to construct consistent axial MEC's starting from the NN potential [12]. At this stage the only formalism that allows systematic construction of consistent is based on relativistic chiral Lagrangians.

We believe the preceding results to be important for two reasons: (i) They provide several explicit examples of how PCAC taken as an underlying principle leads to classification and construction of admissible (PCAC-consistent) approximations to nuclear axial current matrix elements in chiral models. (ii) They specify PCAC-consistent, apparently novel pion production operators in several OBEP nuclear models. Besides the $pp\rightarrow\pi^0 pp$ reaction, [13], these results ought to also be directly applicable to studies of the $pp\rightarrow\pi^+ d$ reaction [14].[7]

V. SUMMARY AND CONCLUSIONS

In summary, in this paper we have used PCAC to relate the nuclear axial current and pion production operators to each other, and to the two-nucleon potential, in nuclear theories based on the nonrelativistic Schrödinger equation. We focused in particular on the axial- and pion-production MEC's related to the exchange of the lightest isoscalar mesons with $J^P=0^+,1^-$, i.e., to σ,ω mesons. (Pions have been treated elsewhere [8,9], and the ρ meson contributions are highly model dependent.) We constructed axial currents and pion production operators in two variations of the linear-σ model, with $g_A=1$, or $g_A=1.26$, with or without ω exchange and showed explicitly that they satisfy the PCAC constraints.

In the process we also made several assumptions and approximations:

(i) We neglected the pion-pole term in the axial charge. This is justifiable since no need arose for it in our analysis.

(ii) We neglected all retardation effects, as well as the recoil MEC's.

(iii) We neglected all isovector-vector and/or axial vector meson exchanges.

(iv) We did not include nucleon or meson form factors, either electroweak or strong.

Another objection, perhaps of a more theoretical nature, can be raised against the present calculation: to talk about meson production in a NR potential theory (where such meson degrees of freedom have been ''integrated out'') seems self-contradictory. We have side-stepped this problem by defining the pion production operator as the $\mathcal{O}(f_\pi m_\pi^2)$ term in the divergence of the axial current Eq. (7), in analogy with the relativistic PCAC result. This issue can be addressed

[6]Scalar meson exchange currents have been introduced into the analysis of $pp\rightarrow\pi^0 pp$ on an *ad hoc* basis [4,5]. Those papers are different from ours in that they do not include the pion-σ-exchange graph, nor was there any concern shown for the consistency between the nuclear wave functions and the pion production operators.

[7]This includes the nuclear chiral perturbation theory ($N\chi PT$), which in the present OBE approximation, is just the one-pion-exchange term that was previously treated in Refs. [8,9]

more deeply within the Fukuda-Sawada-Taketani-Okubo-Nishijima method for constructing effective nuclear theories [15], and we are currently working on it.

Partial conservation of the axial current ought to be an important criterion in the construction of both nuclear two-body, or meson-exchange axial current operators, and of nuclear two-body pion-production operators. The latter arise from two sources: (i) ''elementary'' meson-nucleon vertices present in the chiral Lagrangian; and (ii) ''effective'' meson-nucleon vertices due to the time-ordered Z graphs, also known as pair currents. They have not, to our knowledge, been examined from the present viewpoint, with the exception of Refs. [8,9] on the one-pion-exchange axial currents and the early work by Blin-Stoyle and Tint [2] on nuclear pion production. Our study is only the first step in joining these two ideas and extending them to include light isoscalar mesons (σ,ω).

This work is based on two fundamental assumptions: (i) quantum mechanics; and (i) PCAC. Although our examples were drawn from two specific sigma models, they exemplify a far more general relation between the nuclear Hamiltonian and the nuclear pion production operator. Indeed such a relation must hold in any calculation based on the two aforementioned principles, and in nuclear chiral perturbation theory in particular. This relationship is almost completely unexplored in the last mentioned setting, a situation that must be remedied.

Note added in proof. We learned of the related work by S. M. Ananyan [16] only after the present paper had been submitted.

ACKNOWLEDGMENTS

The authors would like to thank Kuniharu Kubodera and Fred Myhrer for valuable discussions and comments on the manuscript, as well as for providing an opportunity for this collaboration to take place. We also wish to thank T. Nogi for help in checking the calculations.

APPENDIX: AXIAL CHARGE DENSITY IN THE HETEROTIC σ MODEL

The heterotic σ model is a chirally symmetric field-theoretic model that leads to an axial current with arbitrary $g_A(\neq 1)$, and a mixture of pseudoscalar and pseudovector pion-nucleon couplings [1]. The Lagrangian density of this model is given by

$$\mathcal{L}=\bar{\psi}i\partial\!\!\!/\psi-g_0\bar{\psi}[\sigma+i\gamma_5\boldsymbol{\pi}\cdot\tau]\psi+\frac{1}{2}(\partial_\mu\boldsymbol{\phi})^2-V(\boldsymbol{\phi}^2)+\left(\frac{g_A-1}{f_\pi^2}\right)\left[\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\right)\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})+\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)\cdot(\sigma\partial^\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial^\mu\sigma)\right], \tag{A1}$$

where $\boldsymbol{\phi}=(\sigma,\boldsymbol{\pi})$ is a column vector and V is the same potential as in the linear σ model. The (partially conserved) axial-vector Noether current in this model reads

$$\mathbf{J}_{\mu 5}^a=\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a-(\boldsymbol{\pi}\partial_\mu\sigma-\sigma\partial_\mu\boldsymbol{\pi})^a+\left(\frac{g_A-1}{f_\pi^2}\right)\times\left[\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\cdot\boldsymbol{\pi}\right)\boldsymbol{\pi}^a+\sigma^2\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a+\sigma\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\times\boldsymbol{\pi}\right)^a\right]. \tag{A2}$$

After shifting the σ field this can be written as

$$\mathbf{J}_{\mu 5}^a=g_A\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)+f_\pi\partial_\mu\boldsymbol{\pi}+(s\partial_\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial_\mu s)^a+\left(\frac{g_A-1}{f_\pi^2}\right)\left[\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\cdot\boldsymbol{\pi}\right)\boldsymbol{\pi}^a+s(2f_\pi+s)\times\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a+(f_\pi+s)\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\times\boldsymbol{\pi}\right)^a\right]. \tag{A3}$$

Thus we see that the nucleon axial current has acquired a new coupling constant: $g_A\neq 1$, which was the purpose of this model.

The (new) derivative coupling terms in the Lagrangian (A1) modify the canonical momenta as follows:

$$\Pi_\sigma=\dot{\sigma}-\left(\frac{g_A-1}{f_\pi^2}\right)\left(\psi^\dagger\gamma_5\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2}\psi\right), \tag{A4}$$

$$\Pi_\pi^a=\dot{\boldsymbol{\pi}}^a+\left(\frac{g_A-1}{f_\pi^2}\right)\left[\left(\psi^\dagger\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{2}\psi\right)^a+\sigma\psi^\dagger\gamma_5\frac{\boldsymbol{\tau}^a}{2}\psi\right]. \tag{A5}$$

We see that the axial charge retains its linear-σ model form when written out in terms of canonical fields and their associated momenta:

$$\rho_5^a=\mathbf{J}_{05}^a=\psi^\dagger\gamma_5\frac{\boldsymbol{\tau}^a}{2}\psi-(\boldsymbol{\pi}^a\Pi_\sigma-\sigma\boldsymbol{\Pi}_\pi^a). \tag{A6}$$

Hence we see that the axial charge carried by the nucleon is unchanged as compared with the one in the linear-σ model, i.e., we have $g_A=1$ here, which was to be proven.

[1] V. Dmitrašinović, Phys. Rev. C **54**, 3247 (1996).
[2] R.J. Blin-Stoyle and M. Tint, Phys. Rev. **160**, 803 (1967).
[3] D. Koltun and A. Reitan, Phys. Rev. **141**, 1413 (1966).
[4] C.J. Horowitz, Phys. Rev. C **48**, 2920 (1993); C.J. Horowitz, H.O. Meyer, and D. K. Griegel, *ibid.* **49**, 1337 (1994); see, also, J.A. Niskanen, *ibid.* **53**, 526 (1996).
[5] T.-S.H. Lee and D.O. Riska, Phys. Rev. Lett. **70**, 2237 (1993).
[6] See e.g. R.J. Blin-Stoyle, in *Mesons in Nuclei*, edited by M. Rho and D. Wilkinson, Vol. I, pp. 3–66; J.-I. Fujita and M.

Ichimura, *ibid.* (North Holland, Amsterdam, 1979), Vol. II, pp. 625–650.

[7] Y. Futami, J.-I. Fujita, and H. Ohtsubo, Prog. Theor. Phys. Suppl. **60**, 83 (1976).

[8] K. Ohta, Nucl. Phys. **A314**, 333 (1979).

[9] J. Adam Jr., Ch. Hajduk, H. Henning, P.U. Sauer, and E. Truhlik, Nucl. Phys. **A531**, 623 (1991); I. S. Towner, *ibid.* **A542**, 631 (1992).

[10] K. Tsushima and D.O. Riska, Phys. Lett. B **333**, 17 (1994).

[11] S. Schramm and C.J. Horowitz, Phys. Lett. B **335**, 279 (1994).

[12] P. G. Blunden and D.O. Riska, Nucl. Phys. **A536**, 697 (1992).

[13] T. Sato, T.-S.H. Lee, F. Myhrer, and K. Kubodera, Phys. Rev. C **56**, 1246 (1997).

[14] L. Canton, A. Davini, and P.J. Dortmans, nucl-th/9801028.

[15] M. Kobayashi, T. Sato, and H. Ohstubo, Prog. Theor. Phys. **98**, 927 (1997).

[16] S.M. Ananyan, Phys. Rev. C **57**, 2669 (1998).