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One loop corrections to quantum hadrodynamics with vector mesons

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The renormalized elastic $\pi\pi$ scattering amplitude to one loop is calculated in the chiral limit in the σ model and in a quantum hadrodynamic model (QHD-III) with vector mesons. It is argued that QHD-III reduces to the linear σ model in the limit that the vector meson masses become large. The pion decay constant is also calculated to 1-loop in the σ model, and at tree level in QHD-III; it is shown that the coefficient of the tree level term in the scattering amplitude equals F_π^{-2}. The 1-loop correction of F_π in QHD-III violates strong isospin current conservation. Thus, it is concluded that QHD-III can, at best, only describe the strongly interacting nuclear sector. [S0556-2813(98)05309-6]

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QCD is very successful at describing hadronic interactions at high Q^2 where perturbation theory is applicable. At low Q^2 however, nonperturbative methods must be used and most quantitative predictions can not be extracted directly from QCD. Models that are designed to describe low Q^2 hadronic interactions must be guided by the symmetries of QCD and phenomenology. It is a fact that QCD possesses global $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ symmetry and that large scale processes must involve meson loops because of confinement. Phenomenology also reveals the existence of conserved vector and (partially) conserved axial vector currents. QHD-III [1] is a relativistic quantum field theory that incorporates these features: in this model, hadrons are the effective degrees of freedom and the vector mesons that couple to these currents, the ρ and the a_1, are introduced as the gauge bosons of a local $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ $\sigma-\omega$ model with pions (see below). The vector mesons are made massive via the Higgs mechanism (although the Higgs scalars do not contribute to the order considered in this paper) and the model is renormalizable.

A further motivation for QHD-III is the fact that simpler versions of QHD based on $\{N;\sigma,\omega\}$ and $\{N;\sigma,\omega,\pi\}$ have had significant phenomenological success [2]; hence, it is of interest to see how far this description can be extended. A similar model with vector mesons was developed in [3]. For models with vector mesons in nonlinear chiral Lagrangians, see [4(a),4(b),4(c),4(d),4(e)]. For early work on the subject, see [4(f)]. The consequences of the present model have not yet been explored. In the final analysis, the currents of QHD-III allow the theoretical exploration of strongly interacting systems and processes, while incorporating meson loop corrections.

This paper calculates to 1-loop the two simplest amplitudes in the meson sector of this model (the baryon sector is not included in this initial investigation): $\pi\pi$ scattering and pion decay. First, the invariant $\pi\pi$ scattering amplitude $\mathcal{M}_{\pi\pi\to\pi\pi}$ is calculated to 1-loop to $\mathcal{O}(g_\pi^4)$ and to $\mathcal{O}(g_\pi^2 g_\rho^2)$ in the limit $m_\sigma^2 \gg m_\rho^2 \gg s$ with $m_\pi=0$. To renormalize this scattering amplitude, the divergent parts of the counterterms are extracted from the σ, σ^2 and σ^3 vertex functions, and it is shown that these counterterms cancel all the divergences in $\mathcal{M}_{\pi\pi\to\pi\pi}$ to $\mathcal{O}(g_\pi^4)$ and to $\mathcal{O}(g_\pi^2 g_\rho^2)$, as well as the divergences in the σ 4-point function as expected in a renormalizable theory. To 1-loop, it is also argued that the gauge bosons decouple in the limit $m_\rho\to\infty$. Note that QHD-III reduces to the linear σ model when $g_\rho=0$. Second, pion decay is analyzed by looking at the axial current matrix element $\langle 0|A_\mu^i|\pi^j\rangle$ in the σ model to 1-loop; from it, the pion decay constant is identified. To renormalize the pion decay constant, the same counterterms evaluated to $\mathcal{O}(g_\pi^4)$ are used, and it is verified that the coefficient of the tree-level term in $\mathcal{M}_{\pi\pi\to\pi\pi}$ is the inverse square of the pion decay constant to this order. The matrix element $\langle 0|A_\mu^i|\pi^j\rangle$ is next considered in QHD-III so as to identify the pion decay constant in that model. At tree level, the pion decay constant of the σ model is replaced by $1/F_\pi^2=1/\sigma_0^2+g_\rho^2/m_\rho^2$. To next order, it is found that the 1-loop corrections violate local current conservation in this model; this matrix element to 1-loop is therefore neither gauge-invariant nor renormalizable: to that order, *it is not an S-matrix element of the theory*. Thus the model can at best provide a phenomenological description of the strongly interacting nuclear sector. In the process of performing these calculations, the QHD-III counterterm Lagrangian with coefficients $\{\delta_z,\delta_\mu,\delta_\lambda,\delta_{g_\rho},\dots\}$ is derived.

The model is constructed as follows: start with the $\sigma-\omega$ model with pions [2] (we use the conventions in [1]):

$$\begin{aligned}\mathcal{L}=&\bar\psi[i\gamma^\mu(\partial_\mu+ig_V V_\mu)-g_\pi(s+i\gamma_5\tau\cdot\pi)]\psi\\&+\frac{1}{2}(\partial_\mu s\partial^\mu s+\partial_\mu\pi\partial^\mu\pi)\\&-\frac{1}{4}\lambda(s^2+\pi^2-v^2)^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\&+\frac{1}{2}m_V^2 V_\mu V^\mu+\epsilon s+\mathcal{L}_{ct},\end{aligned}\tag{1}$$

where ϵs is the chiral symmetry violating term and $\mathcal{L}_{ct}$ is the counterterm Lagrangian. The QHD-III Lagrangian is constructed as follows (details are given in [1]): this Lagrangian is first made locally invariant under $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$; this results in the appearance of the l_μ and r_μ gauge bosons coupled to conserved currents. These bosons are given mass through the Higgs mechanism. The mass matrix is then diagonalized and the l_μ and r_μ fields are replaced by the new generalized coordinates, the ρ_μ and a_μ. The $O(4)$ symmetry is spontaneously broken by giving the scalar field a vacuum expectation value ($s=\sigma_0-\sigma$ with $\sigma_0\equiv M/g_\pi$). This in turn yields a bilinear term in the Lagrangian that must now be diagonalized by redefining the pion and a_1 field. The end result for the meson sector is

$$\mathcal{L}_{\sigma\pi}=\frac{1}{2}\left[\left(1-\frac{m_a^2}{m_\rho^2}\right)\partial_\mu\pi\partial^\mu\pi\right.$$
$$\left.+\left(\frac{m_a}{m_\rho}\partial_\mu\pi+g_\rho\sigma a_\mu+g_\rho\pi\times\rho_\mu\right)^2\right]$$
$$+\frac{1}{2}\left[\left(\partial_\mu\sigma-g_\rho\frac{m_a}{m_\rho}\pi\cdot a_\mu\right)^2-m_\sigma^2\sigma^2\right]$$
$$-g_\rho^2\sigma_0 a^\mu\left(\sigma a_\mu+\frac{m_a}{m_\rho}\pi\times\rho_\mu\right)$$
$$+g_\pi\frac{m_\sigma^2}{2M}\sigma\left(\sigma^2+\frac{m_a^2}{m_\rho^2}\pi^2\right)$$
$$-g_\pi^2\frac{m_\sigma^2}{8M^2}\left(\sigma^2+\frac{m_a^2}{m_\rho^2}\pi^2\right)^2+\mathcal{L}'_{\sigma\pi}. \tag{2}$$

In the above, the a_1 mass is given by $m_a^2=m_\rho^2+g_\rho^2\sigma_0^2$ with m_ρ the ρ mass and g_ρ the ρ-nucleon coupling constant. Equation (2) is referred to as the ''diagonalized Lagrangian'' and reduces to the linear σ model when $g_\rho=0$. $\mathcal{L}'_{\sigma\pi}$ contains the ''new'' interactions that appear when the pion and a_1 fields are redefined; to $\mathcal{O}(g_\rho^2)$ it is given by

$$\mathcal{L}'_{\sigma\pi}=\frac{g_\rho^2\sigma_0}{m_\rho^2}[\sigma\partial_\mu\pi\cdot\partial^\mu\pi-\pi\cdot\partial_\mu\pi\partial^\mu\sigma]. \tag{3}$$

The scattering amplitude is considered first to $\mathcal{O}(g_\pi^4)$ by putting $g_\rho=0$ and then to $\mathcal{O}(g_\pi^2 g_\rho^2)$. The loop integrals are done using dimensional regularization in the metric $(+,-,-,-)$, and it is assumed that $m_\pi=\epsilon=0$ as well as $m_\sigma^2\gg m_\rho^2\gg s,t,u$ where s, t and u are the Mandelstam variables (for discussions regarding the $m_\sigma\to\infty$ limit of the linear σ model, see [2,5,6,7]). To $\mathcal{O}(g_\pi^4)$ in the t-channel, the amplitude is $\mathcal{M}_{ac,bd}=\mathcal{M}\delta_{ac}\delta_{bd}$ with

$$\mathcal{M}=[\beta t+\alpha_1 t^2+\alpha_2(s^2+u^2)]$$
$$+\frac{1}{F^4}\frac{1}{16\pi^2}\left[-\frac{t^2}{2}\ln\frac{-t}{m_\sigma^2}-\frac{1}{12}(3s^2+u^2-t^2)\right.$$
$$\left.\times\ln\frac{-s}{m_\sigma^2}-\frac{1}{12}(3u^2+s^2-t^2)\ln\frac{-u}{m_\sigma^2}\right], \tag{4}$$

$$\beta\equiv\frac{1}{F^2}\left\{1-\delta_z-\frac{3}{16\pi^2}\frac{m_\sigma^2}{F^2}\left[\Gamma\left(\frac{\epsilon}{2}\right)+\ln 4\pi\right.\right.$$
$$\left.\left.-\ln\frac{m_\sigma^2}{\mu^2}+\frac{7}{6}\right]+2(\delta_\mu+\delta_\lambda)\right\}, \tag{5}$$

$$\alpha_1\equiv-\frac{1}{F^4}\frac{1}{16\pi^2}\frac{49}{18}+\frac{2}{m_\sigma^2F^2}[2\delta_\mu+\delta_\lambda],$$

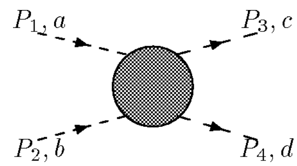

FIG. 1. $\pi\pi$ scattering.

$$\alpha_2\equiv-\frac{1}{F^4}\frac{1}{16\pi^2}\frac{2}{9}. \tag{6}$$

Here, $1/F^2\equiv 1/\sigma_0^2\equiv g_\pi^2/M^2$, $\{\delta_z,\delta_\mu,\delta_\lambda\}$ are of $\mathcal{O}(g_\pi^2)$, and μ parametrizes the renormalization conditions. The amplitude $\mathcal{M}$ does not depend on μ; the counterterms and the running coupling constant conspire to ensure that. The subscripts are defined in Fig. 1 and twenty diagrams contributed to the above result. Note the first term in β is just the tree-level amplitude. Note also that the unitary corrections in the second line of equation (4) has the log structure in Mandelstam variables first derived by Lehmann [8]. The divergent parts of the counterterms are obtained by considering the σ 1-point, 2-point and 3-point functions. The result to 1-loop is

$$\delta_\mu\doteq-\frac{3}{2}\frac{g_\pi^2}{M^2}\frac{m_\sigma^2}{16\pi^2}\Gamma\left(\frac{\epsilon}{2}\right);\quad \delta_\lambda\doteq-2\delta_\mu, \tag{7}$$

with δ_z finite. One can check by direct substitution that the above counterterms determined from the scalar sector cancel the divergences occurring in the $\pi\pi$ scattering amplitude. This verifies the counterterms calculated in [9].

For the scattering amplitude to $\mathcal{O}(g_\pi^2 g_\rho^2)$, the gauge bosons from QHD-III contribute another fifty diagrams. The corrections to the parameters β,α_1 and α_2 are

$$\delta\beta=\frac{g_\rho^2}{m_\rho^2}\left(1+\frac{1}{16\pi^2}\frac{m_\sigma^2}{F^2}\left\{-12+6\frac{m_\rho^2}{m_\sigma^2}\ln\frac{m_\sigma^2}{m_\rho^2}+9\frac{m_\rho^2}{m_\sigma^2}\right.\right.$$
$$\left.\left.\times\left[\Gamma\left(\frac{\epsilon}{2}\right)+\ln 4\pi-\ln\frac{m_\sigma^2}{\mu^2}+\frac{131}{162}\right]\right\}\right)$$
$$+2\frac{g_\rho^2}{m_\rho^2}\delta_{g_\rho}+\left(8\frac{g_\rho^2}{m_\rho^2}-\frac{1}{\sigma_0^2}\right)\delta_z+\frac{2}{\sigma_0^2}(\delta_\mu+\delta_\lambda), \tag{8}$$

$$\delta\alpha_1=-\frac{1}{F^2}\frac{g_\rho^2}{m_\rho^2}\frac{6}{16\pi^2}\left[\Gamma\left(\frac{\epsilon}{2}\right)+\ln 4\pi-\ln\frac{m_\sigma^2}{\mu^2}-\frac{85}{108}\right]$$
$$-8\frac{g_\rho^2}{m_\rho^2}\frac{\delta_{g_\rho}}{m_\sigma^2}+2\frac{g_\rho^2}{m_\rho^2}\frac{\delta_z}{m_\sigma^2}-4\frac{g_\rho^2}{m_\rho^2}\frac{\delta_\mu}{m_\sigma^2}$$
$$+\frac{2}{m_\sigma^2\sigma_0^2}(2\delta_\mu+\delta_\lambda), \tag{9}$$

$$\delta\alpha_2=\frac{1}{F^2}\frac{g_\rho^2}{m_\rho^2}\frac{1}{16\pi^2}\left[\frac{26}{9}+\ln\frac{m_\sigma^2}{m_\rho^2}\right]+2\frac{g_\rho^2}{m_\rho^2}\frac{\delta_{g_\rho}}{m_\sigma^2}. \tag{10}$$

Now, in Eqs. (8)–(10) and in the coefficient of the log terms in Eq. (4), $1/F^2\equiv 1/\sigma_0^2+g_\rho^2/m_\rho^2$ to $\mathcal{O}(g_\pi^2 g_\rho^2)$ as required by unitarity. The counterterms become

$$\delta_\mu \doteq \frac{3}{2}\left(g_\rho^2 - \frac{m_\sigma^2}{\sigma_0^2}\right)\frac{\Gamma\left(\frac{\epsilon}{2}\right)}{16\pi^2}; \quad \delta_\lambda \doteq -2\,\delta_\mu; \quad \delta_z \doteq 6 g_\rho^2 \frac{\Gamma\left(\frac{\epsilon}{2}\right)}{16\pi^2}. \tag{11}$$

The calculation is carried out in the unitary gauge. It is shown that the divergent contributions to amplitudes in the scalar sector are gauge invariant; this is proved using the non-diagonalized Lagrangian which maintains explicit current conservation at each step. Here, δ_{g_ρ} can be determined from ρ decay, and it is finite. Note that δ_z has acquired a divergence. Upon substitution of Eq. (11) into Eqs. (8) and (9), it is found that the amplitude is now *finite* to $\mathcal{O}(g_\pi^2 g_\rho^2)$; note that the σ 4-point function is also made finite with these counterterms. Hence, the $\pi\pi$ scattering amplitude is now rid of all infinities (as is the entire scalar sector).

Consider pion decay in the σ model. From the axial current, F_π is calculated to 1-loop to be

$$F_\pi = \sigma_0\left\{1 + \frac{\delta_z}{2} + \frac{3}{32\pi^2}\frac{m_\sigma^2}{\sigma_0^2}\left[\Gamma\left(\frac{\epsilon}{2}\right) + \ln 4\pi - \ln\frac{m_\sigma^2}{\mu^2} + \frac{7}{6}\right] - (\delta_\mu + \delta_\lambda)\right\}. \tag{12}$$

First, note that the counterterms given in Eq. (7) cancel the divergences in F_π. Second, notice that to $\mathcal{O}(g_\pi^4)$, F_π^{-2} is *identical* to β given in Eq. (5). This verifies a well known property of the σ model in the chiral limit [10].

From the axial current of QHD-III, the tree level pion decay constant is found to be

$$F_\pi = \frac{m_\rho}{m_a}\sigma_0 = \left(1 + \frac{g_\rho^2}{m_\rho^2}\sigma_0^2\right)^{-1/2}\sigma_0, \tag{13}$$

or $1/F_\pi^2 = 1/\sigma_0^2 + g_\rho^2/m_\rho^2$. This result was first obtained by Gasiorowicz and Geffen [3] (see also [4(b)]). From the first term in Eq. (8), it is seen that the relationship between the pion decay constant and the $\pi\pi$ scattering amplitude in the chiral limit is also verified in QHD-III at tree level.

Consider the $\mathcal{O}(g_\pi^2 g_\rho^2)$ corrections to the $\pi\pi$ scattering amplitude, Eqs. (8)–(10). As $m_\rho \to \infty$, $\delta\alpha_1$, $\delta\alpha_2 \to 0$. This can be seen directly from Eqs. (9) and (10) by noticing that to this order, (i) δ_{g_ρ} is finite in this limit and (ii) the $\mathcal{O}(1/m_\sigma^2)$ terms are to be neglected so that the only surviving contributions from the counterterms are those of the σ model in Eq. (7). Note also that the tree-level correction of β [the first term of Eq. (8)] goes to zero in that limit. However, in this amplitude, the limit $m_\rho \to \infty$ must be taken without violating the constraint $m_\sigma^2/m_\rho^2 \gg 1$ but finite. This constraint implies the appearance of a new (quadratic) divergence in β proportional to m_σ^2. The $\ln m_\sigma^2/\mu^2$ divergences in β and $\delta\beta$ are already absorbed in the renormalization of the parameters of the Lagrangian and need not be considered further. The new quadratic divergence in β renormalizes the pion decay constant to 1-loop in exactly the same fashion; in [6,7], it is shown quite generally that the linear σ model reduces to the nonlinear σ model in the limit $m_\sigma \to \infty$ and that quadratic divergences have no observable effect to 1-loop. In $\delta\beta$, note that the 1-loop corrections are finite constants in the heavy mass limit subject to the above constraint. They are thus negligible with respect to the quadratic contributions in β. Hence, the gauge boson contribution to the scattering amplitude becomes negligible in the heavy mass limit: the QHD-III scattering amplitude reduces to the σ model amplitude in the limit $m_\rho \to \infty$ with $m_\sigma^2/m_\rho^2 \gg 1$ but finite.[1]

The decoupling of the a_1 and the ρ can be understood more generally as follows: from the Lagrangian given in Eq. (2) and the definition of m_a, it is seen that the gauge bosons decouple from the π and the σ when the gauge fields are rescaled according to $\rho_\mu = \rho'_\mu/m_\rho$ and $a_\mu = a'_\mu/m_\rho$ and the limit $m_\rho \to \infty$ is taken; this procedure results in the Lagrangian of the linear σ model (for a similar discussion in the linear σ model see [2]). This suppression in inverse powers of the mass is essentially the decoupling theorem [11].

Now consider pion decay in QHD-III. When the 1-loop correction of the matrix element $\langle 0|A_\mu^i|\pi^j\rangle$ is evaluated using the axial current derived from the QHD-III Lagrangian, it is found that the counterterms given in Eq. (11) *fail* to cancel all of the divergences; it is also explicitly found that this matrix element is not gauge invariant to 1-loop in QHD-III. This violation of current conservation occurs because the pion acquires a strong isospin charge when the vector mesons are introduced as gauge bosons as is the case in QHD-III; hence, in a process which destroys strong isospin charge such as pion decay, vector and axial-vector isospin current conservation is violated,[2] and that process is not an S-matrix element of QHD-III.

In *summary*, the renormalized $\pi\pi$ scattering amplitude has been calculated to 1-loop in the σ model and in QHD-III in the chiral limit for small external momenta with respect to the masses. The pion decay constant has been calculated to 1-loop in the σ model and it is shown explicitly that $F_\pi^{-2} = \beta$. It is also shown that this relation holds at tree level in QHD-III. It is argued that the gauge bosons decouple from the pion and the σ in the heavy mass limit. The 1-loop correction to F_π in QHD-III is seen to violate strong isospin current conservation; thus, the model can at best describe the strongly interacting nuclear sector.

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[1]Since the 1-loop corrections in β and $\delta\beta$ become larger than the tree-level terms in the limit $m_\rho, m_\sigma \to \infty$, taking these limits in our scattering amplitude is questionable since we used perturbation theory to obtain our result. However, 4-point functions can be used to construct effective Lagrangians in the heavy mass limit [6], and our scattering amplitude would have the same structure as that effective Lagrangian.

[2]Electromagnetic charge also disappears in pion decay. Of course, it is carried off in the lepton sector. The author knows of no simple way to fix up strong vector and axial vector isospin current conservation in this model for pion decay.

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